Computer algebra independent integration tests

8-Special-functions/8.2-Fresnel-integral-functions

Nasser M. Abbasi

July 28, 2021 Compiled on July 28, 2021 at 10:19am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [218]. This is test number [205].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (218)	% 0.00 (0)
Mathematica	% 87.16 (190)	% 12.84 (28)
Maple	% 70.64 (154)	% 29.36 (64)
Maxima	% 27.52 (60)	% 72.48 (158)
Fricas	% 27.52 (60)	% 72.48 (158)
Sympy	% 52.29 (114)	% 47.71 (104)
Giac	% 27.52 (60)	% 72.48 (158)
Mupad	% 27.52 (60)	% 72.48 (158)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

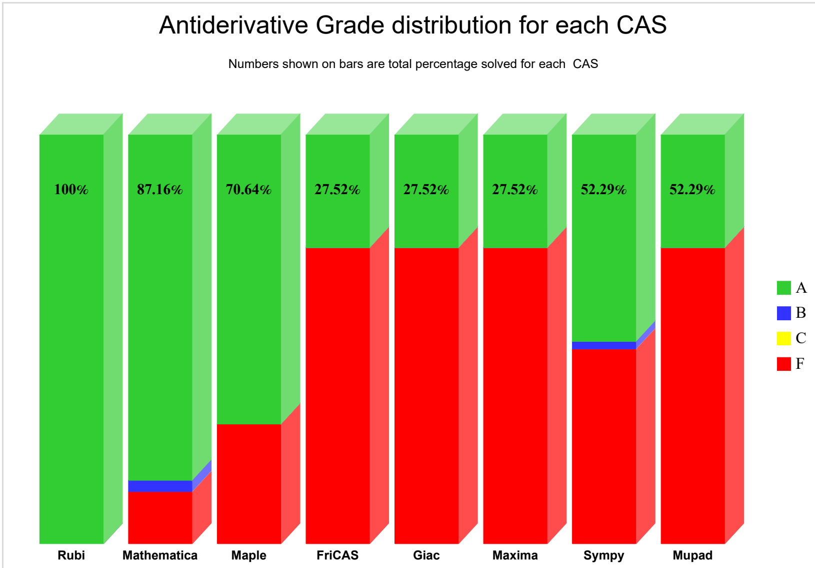
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

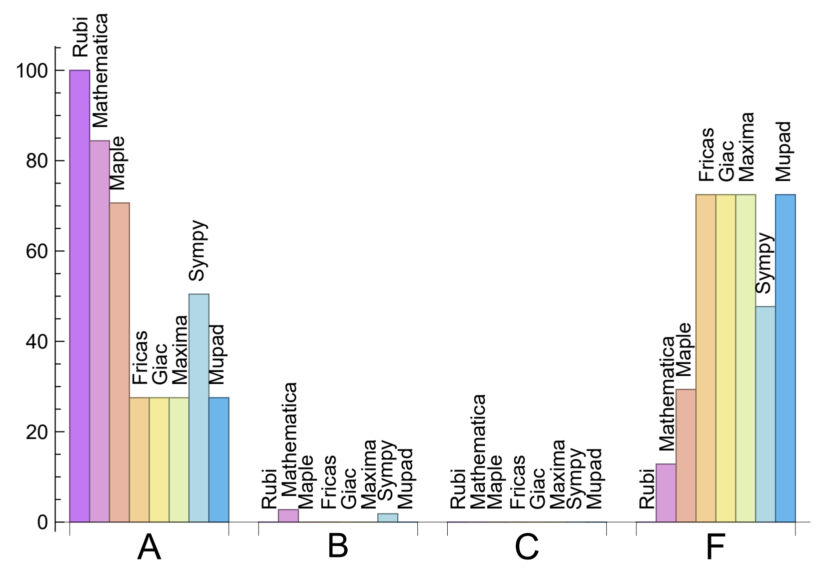
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	84.40	2.75	0.00	12.84
Maple	70.64	0.00	0.00	29.36
Maxima	27.52	0.00	0.00	72.48
Fricas	27.52	0.00	0.00	72.48
Sympy	50.46	1.83	0.00	47.71
Giac	27.52	0.00	0.00	72.48
Mupad	27.52	0.00	0.00	72.48

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	28	100.00 %	0.00 %	0.00 %
Maple	64	100.00 %	0.00 %	0.00 %
Maxima	158	100.00 %	0.00 %	0.00 %
Fricas	158	100.00 %	0.00 %	0.00 %
Sympy	104	96.15 %	1.92 %	1.92 %
Giac	158	100.00 %	0.00 %	0.00 %
Mupad	158	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

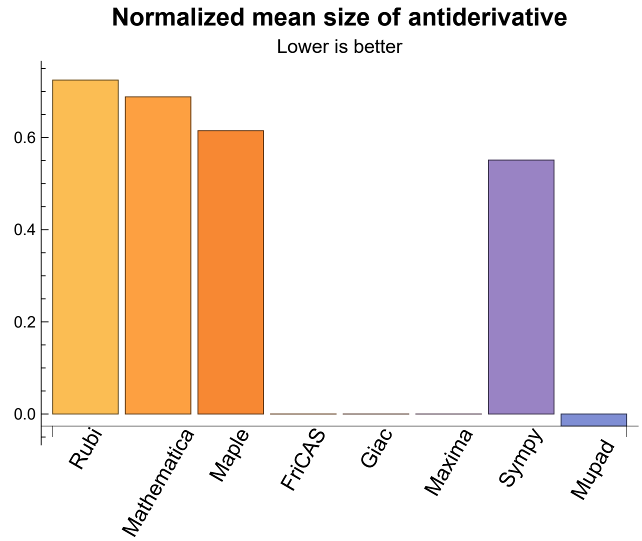
1.3 Performance

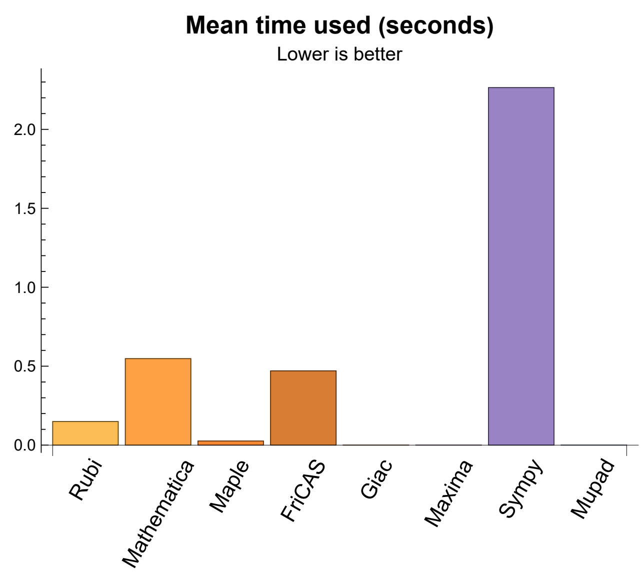
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.15	96.61	0.72	71.50	1.00
Mathematica	0.55	81.86	0.69	59.00	0.81
Maple	0.03	62.54	0.61	28.00	0.90
Maxima	0.00	0.00	0.00	0.00	0.00
Fricas	0.47	0.00	0.00	0.00	0.00
Sympy	2.26	36.75	0.55	0.00	0.00
Giac	0.00	0.00	0.00	0.00	0.00
Mupad	0.00	-1.00	-0.03	-1.00	-0.01

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {54, 55, 56, 163, 164, 165}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

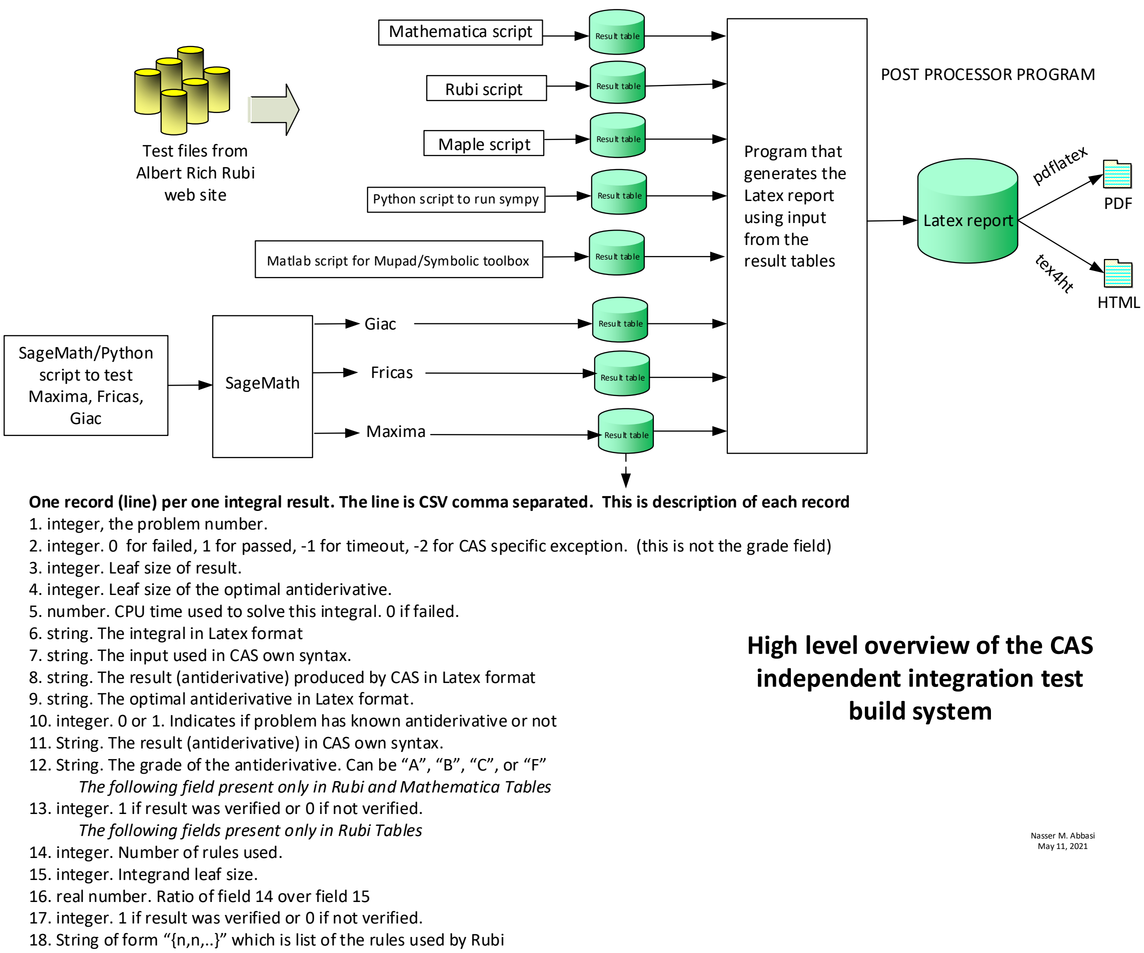
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 30, 31, 32, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 138, 139, 140, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 167, 168, 169, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 201, 202, 203, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218 }

B grade: { 22, 28, 57, 131, 137, 166 }

C grade: { }

F grade: { 9, 33, 37, 49, 50, 61, 62, 63, 64, 73, 77, 91, 95, 99, 118, 142, 146, 158, 159, 170, 171, 172, 173, 182, 186, 200, 204, 208 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 36, 38, 39, 40, 41, 42, 44, 45, 46, 48, 51, 52, 53, 57, 65, 66, 67, 68, 69, 70, 72, 74, 76, 78, 79, 80, 81, 82, 84, 85, 86, 88, 89, 90, 92, 94, 96, 98, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 143, 145, 147, 148, 149, 150, 151, 153, 154, 155, 157, 160, 161, 162, 166, 174, 175, 176, 177, 178, 179, 181, 183, 185, 187, 188, 189, 190, 191, 193, 194, 195, 197, 198, 199, 201, 203, 205, 207, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 31, 33, 35, 37, 43, 47, 49, 50, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 71, 73, 75, 77, 83, 87, 91, 93, 95, 97, 99, 101, 105, 109, 140, 142, 144, 146, 152, 156, 158, 159, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 180, 182, 184, 186, 192, 196, 200, 202, 204, 206, 208, 210, 214, 218 }

2.1.4 Maxima

A grade: { 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

2.1.5 FriCAS

A grade: { 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 14, 15, 16, 17, 18, 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 65, 66, 67, 68, 69, 70, 75, 79, 80, 81, 82, 84, 85, 86, 88, 89, 90, 93, 97, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 120, 121, 122, 123, 124, 125, 126, 127, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 174, 175, 176, 177, 178, 179, 184, 188, 189, 190, 191, 193, 194, 195, 197, 198, 199, 202, 206, 209, 211, 212, 213, 215, 216, 217 }

B grade: { 8, 10, 117, 119 }

C grade: { }

F grade: { 9, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 71, 72, 73, 74, 76, 77, 78, 83, 87, 91, 92, 94, 95, 96, 98, 99, 101, 105, 109, 118, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 180, 181, 182, 183, 185, 186, 187, 192, 196, 200, 201, 203, 204, 205, 207, 208, 210, 214, 218 }

2.1.7 Giac

A grade: { 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

2.1.8 Mupad

A grade: { 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	88	123	0	0	184	0	-1
normalized size	1	1.00	0.71	0.99	0.00	0.00	1.48	0.00	-0.01
time (sec)	N/A	0.088	0.080	0.030	0.000	0.537	1.977	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	83	107	0	0	156	0	-1
normalized size	1	1.00	0.76	0.98	0.00	0.00	1.43	0.00	-0.01
time (sec)	N/A	0.111	0.066	0.019	0.000	0.383	1.500	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	79	96	0	0	53	0	-1
normalized size	1	1.00	0.80	0.97	0.00	0.00	0.54	0.00	-0.01
time (sec)	N/A	0.064	0.095	0.020	0.000	0.419	0.853	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	80	0	0	121	0	-1
normalized size	1	1.00	0.85	0.95	0.00	0.00	1.44	0.00	-0.01
time (sec)	N/A	0.080	0.047	0.019	0.000	0.394	1.460	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	70	0	0	112	0	-1
normalized size	1	1.00	1.00	0.95	0.00	0.00	1.51	0.00	-0.01
time (sec)	N/A	0.047	0.024	0.023	0.000	0.393	0.852	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	0	0	80	0	-1
normalized size	1	1.00	1.00	0.92	0.00	0.00	1.36	0.00	-0.02
time (sec)	N/A	0.053	0.015	0.019	0.000	0.410	0.802	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	0	0	53	0	-1
normalized size	1	1.00	1.00	0.90	0.00	0.00	1.08	0.00	-0.02
time (sec)	N/A	0.024	0.013	0.022	0.000	0.405	0.517	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	0	0	48	0	-1
normalized size	1	1.00	1.00	1.04	0.00	0.00	1.85	0.00	-0.04
time (sec)	N/A	0.005	0.004	0.017	0.000	0.415	0.740	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	0	29	0	0	0	0	-1
normalized size	1	1.00	0.00	0.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.019	0.055	0.000	0.441	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	0	0	42	0	-1
normalized size	1	1.00	1.00	1.04	0.00	0.00	1.56	0.00	-0.04
time (sec)	N/A	0.022	0.015	0.021	0.000	0.527	0.635	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	43	0	0	51	0	-1
normalized size	1	1.00	1.00	0.98	0.00	0.00	1.16	0.00	-0.02
time (sec)	N/A	0.029	0.019	0.022	0.000	0.555	0.572	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	49	0	0	56	0	-1
normalized size	1	1.00	1.00	0.94	0.00	0.00	1.08	0.00	-0.02
time (sec)	N/A	0.064	0.020	0.019	0.000	0.480	1.014	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	65	0	0	110	0	-1
normalized size	1	1.00	1.00	0.94	0.00	0.00	1.59	0.00	-0.01
time (sec)	N/A	0.043	0.018	0.020	0.000	0.447	0.965	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	71	0	0	46	0	-1
normalized size	1	1.00	1.00	0.92	0.00	0.00	0.60	0.00	-0.01
time (sec)	N/A	0.089	0.036	0.022	0.000	0.401	0.910	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	76	86	0	0	56	0	-1
normalized size	1	1.00	0.81	0.91	0.00	0.00	0.60	0.00	-0.01
time (sec)	N/A	0.059	0.064	0.020	0.000	0.444	1.145	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	85	93	0	0	68	0	-1
normalized size	1	1.00	0.83	0.91	0.00	0.00	0.67	0.00	-0.01
time (sec)	N/A	0.122	0.080	0.025	0.000	0.440	2.053	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	84	109	0	0	185	0	-1
normalized size	1	1.00	0.71	0.92	0.00	0.00	1.55	0.00	-0.01
time (sec)	N/A	0.081	0.075	0.021	0.000	0.388	2.541	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	96	115	0	0	48	0	-1
normalized size	1	1.00	0.76	0.91	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.153	0.208	0.022	0.000	0.392	2.756	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	424	400	0	0	0	0	-1
normalized size	1	1.00	1.43	1.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.401	1.018	0.030	0.000	0.454	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	236	251	0	0	0	0	-1
normalized size	1	1.00	1.22	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.552	0.025	0.000	0.440	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	61	108	0	0	0	0	-1
normalized size	1	1.00	0.50	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.278	0.023	0.000	0.387	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	89	33	0	0	0	0	-1
normalized size	1	1.00	2.47	0.92	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.006	0.044	0.011	0.000	0.560	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.015	0.033	0.101	0.000	0.446	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.014	5.783	0.092	0.000	0.452	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	166	189	0	0	0	0	-1
normalized size	1	1.00	0.72	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.372	0.024	0.000	0.380	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	115	121	0	0	0	0	-1
normalized size	1	1.00	0.78	0.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.296	0.023	0.000	0.379	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	51	80	0	0	0	0	-1
normalized size	1	1.00	0.53	0.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.187	0.024	0.000	0.486	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	89	33	0	0	0	0	-1
normalized size	1	1.00	2.47	0.92	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.007	0.037	0.000	0.000	0.509	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.011	0.016	0.028	0.000	0.413	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.011	3.674	0.060	0.000	0.382	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	253	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.428	0.016	0.020	0.000	0.378	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	171	324	0	0	0	0	-1
normalized size	1	1.00	0.72	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.318	0.328	0.094	0.000	0.391	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.258	0.021	0.000	0.469	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	137	208	0	0	0	0	-1
normalized size	1	1.00	0.77	1.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.172	0.061	0.000	0.493	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	140	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.006	0.019	0.000	0.543	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	100	122	0	0	0	0	-1
normalized size	1	1.00	0.81	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.132	0.054	0.000	0.402	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.195	0.020	0.000	0.555	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	0	0	0	0	-1
normalized size	1	1.00	1.00	0.89	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.016	0.028	0.000	0.493	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.017	0.021	0.023	0.000	0.485	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.037	0.030	0.019	0.000	0.457	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.038	0.022	0.021	0.000	0.517	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.032	0.020	0.000	0.455	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.007	0.023	0.000	0.469	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.035	0.023	0.000	0.464	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.027	0.022	0.000	0.495	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	0.035	0.021	0.000	0.413	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	242	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.393	0.017	0.029	0.000	0.512	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	286	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.358	0.028	0.021	0.000	0.430	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	0.881	0.092	0.000	0.455	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.196	0.619	0.020	0.000	0.402	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	60	0	0	0	0	-1
normalized size	1	1.00	0.96	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.011	0.011	0.000	0.458	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	0.043	0.091	0.000	0.529	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	0.094	0.087	0.000	1.055	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	319	0	0	0	0	0	-1
normalized size	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.571	8.302	0.161	0.000	0.561	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	319	0	0	0	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	7.912	0.121	0.000	0.532	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	316	0	0	0	0	0	-1
normalized size	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.326	7.720	0.112	0.000	0.403	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	164	80	0	0	0	0	-1
normalized size	1	1.00	2.52	1.23	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.126	0.040	0.000	0.429	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	195	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.511	4.424	0.122	0.000	0.474	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	200	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.523	4.513	0.129	0.000	0.485	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	244	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.702	6.225	0.086	0.000	0.494	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.033	0.043	0.000	0.471	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	0.047	0.040	0.000	0.473	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.063	0.049	0.000	0.547	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.054	0.054	0.000	0.416	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0	-1
normalized size	1	1.00	1.00	0.92	0.00	0.00	0.77	0.00	-0.08
time (sec)	N/A	0.015	0.004	0.007	0.000	0.428	1.040	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0	-1
normalized size	1	1.00	1.00	0.92	0.00	0.00	0.77	0.00	-0.08
time (sec)	N/A	0.011	0.003	0.005	0.000	0.462	0.366	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	8	0	-1
normalized size	1	1.00	1.00	1.11	0.00	0.00	0.89	0.00	-0.11
time (sec)	N/A	0.015	0.010	0.082	0.000	0.427	0.217	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	0	10	0	-1
normalized size	1	1.00	1.00	1.09	0.00	0.00	0.91	0.00	-0.09
time (sec)	N/A	0.015	0.003	0.006	0.000	0.481	0.655	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	14	0	-1
normalized size	1	1.00	1.00	0.92	0.00	0.00	1.08	0.00	-0.08
time (sec)	N/A	0.015	0.003	0.007	0.000	0.558	1.531	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	0	31	0	-1
normalized size	1	1.00	1.00	1.06	0.00	0.00	1.82	0.00	-0.06
time (sec)	N/A	0.018	0.008	0.006	0.000	0.420	2.979	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	232	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.384	0.014	0.019	0.000	0.398	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	153	318	0	0	0	0	-1
normalized size	1	1.00	0.71	1.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.265	0.286	0.011	0.000	1.022	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.255	0.529	0.017	0.000	0.390	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	120	202	0	0	0	0	-1
normalized size	1	1.00	0.76	1.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.177	0.008	0.000	0.396	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	151	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	1.26	0.00	-0.01
time (sec)	N/A	0.118	0.007	0.016	0.000	0.390	17.565	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	83	115	0	0	0	0	-1
normalized size	1	1.00	0.79	1.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.112	0.006	0.000	0.408	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.241	0.014	0.000	0.405	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	46	0	0	0	0	-1
normalized size	1	1.00	0.90	0.94	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.033	0.007	0.000	0.583	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0	-1
normalized size	1	1.00	1.00	0.92	0.00	0.00	0.77	0.00	-0.08
time (sec)	N/A	0.011	0.002	0.000	0.000	0.482	0.340	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.018	0.035	0.016	0.000	0.395	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.018	0.042	0.015	0.000	0.544	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.044	0.014	0.000	0.471	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.013	0.016	0.000	0.446	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.033	0.016	0.000	0.406	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.036	0.016	0.000	0.389	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	241	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.207	0.031	0.019	0.000	0.425	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	224	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.353	0.017	0.016	0.000	0.552	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	0.036	0.015	0.000	0.483	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	263	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.477	0.042	0.020	0.000	0.414	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	0.088	0.040	0.000	0.432	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.442	0.058	0.039	0.000	0.574	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	163	321	0	0	0	0	-1
normalized size	1	1.00	0.75	1.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.271	0.212	0.069	0.000	0.479	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	184	0	0	0	264	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	1.43	0.00	-0.01
time (sec)	N/A	0.253	0.013	0.032	0.000	0.479	75.440	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	126	212	0	0	0	0	-1
normalized size	1	1.00	0.76	1.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.197	0.053	0.000	0.424	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.046	0.032	0.000	0.492	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	90	119	0	0	0	0	-1
normalized size	1	1.00	0.83	1.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.088	0.051	0.000	0.420	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	114	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	1.56	0.00	-0.01
time (sec)	N/A	0.055	0.007	0.040	0.000	0.436	2.891	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	52	0	0	0	0	-1
normalized size	1	1.00	0.81	0.88	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.039	0.030	0.000	1.454	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.016	0.025	0.000	0.441	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.018	0.038	0.033	0.000	0.403	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.008	0.034	0.000	0.399	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.044	0.033	0.000	0.532	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.046	0.037	0.000	0.543	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.038	0.033	0.000	0.444	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	163	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.009	0.030	0.000	0.422	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.039	0.030	0.000	0.515	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	0.036	0.034	0.000	0.467	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	0.036	0.033	0.000	0.416	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	278	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.516	0.022	0.032	0.000	0.398	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	89	123	0	0	184	0	-1
normalized size	1	1.00	0.72	0.99	0.00	0.00	1.48	0.00	-0.01
time (sec)	N/A	0.082	0.078	0.006	0.000	0.494	2.310	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	83	107	0	0	153	0	-1
normalized size	1	1.00	0.76	0.98	0.00	0.00	1.40	0.00	-0.01
time (sec)	N/A	0.114	0.082	0.005	0.000	0.539	2.228	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	80	97	0	0	49	0	-1
normalized size	1	1.00	0.81	0.98	0.00	0.00	0.49	0.00	-0.01
time (sec)	N/A	0.062	0.063	0.005	0.000	0.467	0.974	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	81	0	0	116	0	-1
normalized size	1	1.00	0.85	0.96	0.00	0.00	1.38	0.00	-0.01
time (sec)	N/A	0.077	0.058	0.005	0.000	0.495	1.204	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	70	0	0	112	0	-1
normalized size	1	1.00	1.00	0.95	0.00	0.00	1.51	0.00	-0.01
time (sec)	N/A	0.043	0.022	0.007	0.000	0.384	1.008	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	0	0	80	0	-1
normalized size	1	1.00	1.00	0.92	0.00	0.00	1.36	0.00	-0.02
time (sec)	N/A	0.054	0.015	0.006	0.000	0.461	1.110	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	0	0	49	0	-1
normalized size	1	1.00	1.00	0.90	0.00	0.00	1.00	0.00	-0.02
time (sec)	N/A	0.026	0.013	0.006	0.000	0.474	0.601	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	0	0	44	0	-1
normalized size	1	1.00	1.00	1.04	0.00	0.00	1.63	0.00	-0.04
time (sec)	N/A	0.005	0.003	0.006	0.000	0.445	0.704	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	0	23	0	0	0	0	-1
normalized size	1	1.00	0.00	0.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.018	0.042	0.000	0.478	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	0	0	53	0	-1
normalized size	1	1.00	1.00	1.04	0.00	0.00	1.96	0.00	-0.04
time (sec)	N/A	0.021	0.016	0.007	0.000	0.472	1.056	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	43	0	0	51	0	-1
normalized size	1	1.00	1.00	0.98	0.00	0.00	1.16	0.00	-0.02
time (sec)	N/A	0.029	0.024	0.006	0.000	0.399	0.697	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	49	0	0	42	0	-1
normalized size	1	1.00	1.00	0.94	0.00	0.00	0.81	0.00	-0.02
time (sec)	N/A	0.064	0.019	0.005	0.000	0.451	0.808	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	64	0	0	110	0	-1
normalized size	1	1.00	1.00	0.93	0.00	0.00	1.59	0.00	-0.01
time (sec)	N/A	0.041	0.021	0.005	0.000	0.498	1.348	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	71	0	0	65	0	-1
normalized size	1	1.00	1.00	0.92	0.00	0.00	0.84	0.00	-0.01
time (sec)	N/A	0.089	0.019	0.008	0.000	0.487	1.759	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	74	87	0	0	56	0	-1
normalized size	1	1.00	0.79	0.93	0.00	0.00	0.60	0.00	-0.01
time (sec)	N/A	0.059	0.123	0.005	0.000	0.427	1.422	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	84	93	0	0	44	0	-1
normalized size	1	1.00	0.82	0.91	0.00	0.00	0.43	0.00	-0.01
time (sec)	N/A	0.118	0.147	0.005	0.000	0.382	1.904	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	85	108	0	0	185	0	-1
normalized size	1	1.00	0.71	0.91	0.00	0.00	1.55	0.00	-0.01
time (sec)	N/A	0.078	0.072	0.004	0.000	0.464	2.908	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	96	115	0	0	76	0	-1
normalized size	1	1.00	0.76	0.91	0.00	0.00	0.60	0.00	-0.01
time (sec)	N/A	0.144	0.108	0.006	0.000	0.415	3.818	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	424	397	0	0	0	0	-1
normalized size	1	1.00	1.42	1.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	0.953	0.024	0.000	0.433	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	237	249	0	0	0	0	-1
normalized size	1	1.00	1.22	1.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.541	0.021	0.000	0.511	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	74	107	0	0	0	0	-1
normalized size	1	1.00	0.61	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.284	0.021	0.000	0.664	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	90	34	0	0	0	0	-1
normalized size	1	1.00	2.43	0.92	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.006	0.039	0.004	0.000	0.513	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.014	0.032	0.089	0.000	0.642	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.014	2.536	0.092	0.000	0.564	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	166	187	0	0	0	0	-1
normalized size	1	1.00	0.73	0.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.187	0.373	0.020	0.000	0.527	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	116	122	0	0	0	0	-1
normalized size	1	1.00	0.78	0.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.334	0.020	0.000	0.464	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	59	79	0	0	0	0	-1
normalized size	1	1.00	0.62	0.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.179	0.020	0.000	0.557	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	90	34	0	0	0	0	-1
normalized size	1	1.00	2.43	0.92	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.007	0.037	0.000	0.000	0.576	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.010	0.025	0.030	0.000	0.397	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.011	1.567	0.060	0.000	0.467	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	253	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	0.027	0.023	0.000	0.393	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	170	324	0	0	0	0	-1
normalized size	1	1.00	0.71	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	0.273	0.062	0.000	0.414	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.288	0.239	0.010	0.000	0.399	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	137	209	0	0	0	0	-1
normalized size	1	1.00	0.77	1.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.156	0.054	0.000	0.381	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	140	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.009	0.012	0.000	0.469	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	100	122	0	0	0	0	-1
normalized size	1	1.00	0.81	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.094	0.052	0.000	0.476	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.198	0.012	0.000	0.423	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	49	0	0	0	0	-1
normalized size	1	1.00	1.00	0.91	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.013	0.004	0.000	0.469	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.016	0.023	0.010	0.000	0.439	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.037	0.028	0.012	0.000	0.520	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.037	0.020	0.010	0.000	0.471	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.028	0.012	0.000	0.425	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.010	0.013	0.000	0.481	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.032	0.010	0.000	0.441	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.033	0.012	0.000	0.479	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.032	0.012	0.000	0.531	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	242	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.388	0.024	0.010	0.000	0.407	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	286	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	0.034	0.012	0.000	0.412	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.396	0.817	0.089	0.000	0.436	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.681	0.010	0.000	0.540	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	60	0	0	0	0	-1
normalized size	1	1.00	0.96	0.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.018	0.002	0.000	0.448	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	0.039	0.095	0.000	0.498	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	0.090	0.094	0.000	0.493	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	318	0	0	0	0	0	-1
normalized size	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.522	7.681	0.138	0.000	0.420	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	318	0	0	0	0	0	-1
normalized size	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.408	7.542	0.132	0.000	0.430	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	315	0	0	0	0	0	-1
normalized size	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.263	7.464	0.125	0.000	0.428	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	165	81	0	0	0	0	-1
normalized size	1	1.00	2.50	1.23	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.104	0.017	0.000	0.403	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	194	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	4.617	0.148	0.000	0.408	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	199	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	4.575	0.135	0.000	0.509	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	244	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.607	6.045	0.062	0.000	0.471	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.046	0.030	0.000	0.542	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.038	0.018	0.000	0.977	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.044	0.033	0.000	0.544	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.048	0.036	0.000	0.512	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0	-1
normalized size	1	1.00	1.00	0.92	0.00	0.00	0.77	0.00	-0.08
time (sec)	N/A	0.015	0.010	0.017	0.000	0.487	1.178	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0	-1
normalized size	1	1.00	1.00	0.92	0.00	0.00	0.77	0.00	-0.08
time (sec)	N/A	0.012	0.008	0.018	0.000	0.420	0.390	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	10	0	-1
normalized size	1	1.00	1.00	1.11	0.00	0.00	1.11	0.00	-0.11
time (sec)	N/A	0.016	0.013	0.085	0.000	0.395	0.241	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	0	12	0	-1
normalized size	1	1.00	1.00	1.09	0.00	0.00	1.09	0.00	-0.09
time (sec)	N/A	0.015	0.009	0.017	0.000	0.509	0.751	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	15	0	-1
normalized size	1	1.00	1.00	0.92	0.00	0.00	1.15	0.00	-0.08
time (sec)	N/A	0.016	0.009	0.018	0.000	0.418	1.660	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	0	34	0	-1
normalized size	1	1.00	1.00	1.06	0.00	0.00	2.00	0.00	-0.06
time (sec)	N/A	0.019	0.015	0.017	0.000	0.485	3.589	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	231	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	0.014	0.028	0.000	0.514	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	154	317	0	0	0	0	-1
normalized size	1	1.00	0.72	1.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	0.272	0.018	0.000	0.416	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.442	0.025	0.000	0.489	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	120	202	0	0	0	0	-1
normalized size	1	1.00	0.76	1.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.140	0.024	0.000	0.424	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	151	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	1.26	0.00	-0.01
time (sec)	N/A	0.118	0.012	0.026	0.000	0.525	18.870	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	83	114	0	0	0	0	-1
normalized size	1	1.00	0.80	1.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.095	0.018	0.000	0.507	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.239	0.027	0.000	0.446	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	44	45	0	0	0	0	-1
normalized size	1	1.00	0.92	0.94	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.031	0.017	0.000	0.508	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	0	10	0	-1
normalized size	1	1.00	1.00	0.92	0.00	0.00	0.77	0.00	-0.08
time (sec)	N/A	0.011	0.003	0.000	0.000	0.401	0.377	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.035	0.026	0.000	0.402	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	0.037	0.026	0.000	0.412	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.032	0.025	0.000	0.403	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.010	0.025	0.000	0.436	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.042	0.026	0.000	0.407	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.052	0.023	0.000	0.443	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	241	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.030	0.027	0.000	0.591	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	224	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.015	0.029	0.000	0.459	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	0.041	0.028	0.000	0.484	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	263	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.495	0.045	0.026	0.000	0.439	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	0.081	0.026	0.000	0.492	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.431	0.053	0.017	0.000	0.576	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	163	322	0	0	0	0	-1
normalized size	1	1.00	0.75	1.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.271	0.181	0.063	0.000	0.497	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	185	0	0	0	264	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	1.43	0.00	-0.01
time (sec)	N/A	0.254	0.012	0.015	0.000	0.480	77.907	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	126	212	0	0	0	0	-1
normalized size	1	1.00	0.75	1.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.195	0.054	0.000	0.464	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.046	0.020	0.000	0.596	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	90	120	0	0	0	0	-1
normalized size	1	1.00	0.83	1.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.096	0.049	0.000	0.428	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	0	114	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	1.54	0.00	-0.01
time (sec)	N/A	0.057	0.005	0.017	0.000	0.447	3.021	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	48	52	0	0	0	0	-1
normalized size	1	1.00	0.80	0.87	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.026	0.023	0.000	0.505	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.017	0.013	0.000	0.551	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.037	0.014	0.000	0.409	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.008	0.017	0.000	0.468	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.045	0.018	0.000	0.407	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.039	0.019	0.000	0.479	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.041	0.023	0.000	0.412	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	163	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.218	0.012	0.017	0.000	0.535	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.200	0.037	0.016	0.000	0.419	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.033	0.014	0.000	0.399	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	0.044	0.016	0.000	0.578	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	278	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.520	0.027	0.017	0.000	0.470	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [140] had the largest ratio of [1.100]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	8	0.500
2	A	6	4	1.00	8	0.500
3	A	5	4	1.00	8	0.500
4	A	5	4	1.00	8	0.500
5	A	4	4	1.00	8	0.500
6	A	4	4	1.00	8	0.500
7	A	3	3	1.00	6	0.500
8	A	1	1	1.00	4	0.250
9	A	3	3	1.00	8	0.375
10	A	2	2	1.00	8	0.250
11	A	3	3	1.00	8	0.375
12	A	4	4	1.00	8	0.500
13	A	4	4	1.00	8	0.500
14	A	5	4	1.00	8	0.500
15	A	5	4	1.00	8	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	6	4	1.00	8	0.500
17	A	6	4	1.00	8	0.500
18	A	7	4	1.00	8	0.500
19	A	14	10	1.00	14	0.714
20	A	11	9	1.00	14	0.643
21	A	8	7	1.00	12	0.583
22	A	1	1	1.00	6	0.167
23	A	0	0	0.00	0	0.000
24	A	0	0	0.00	0	0.000
25	A	14	10	1.00	10	1.000
26	A	11	9	1.00	10	0.900
27	A	8	7	1.00	8	0.875
28	A	1	1	1.00	6	0.167
29	A	0	0	0.00	0	0.000
30	A	0	0	0.00	0	0.000
31	A	23	10	1.00	10	1.000
32	A	19	10	1.00	10	1.000
33	A	16	9	1.00	10	0.900
34	A	12	9	1.00	10	0.900
35	A	10	9	1.00	10	0.900
36	A	8	6	1.00	10	0.600
37	A	5	5	1.00	8	0.625
38	A	4	4	1.00	6	0.667
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	0	0	0.00	0	0.000
43	A	9	9	1.00	10	0.900
44	A	0	0	0.00	0	0.000
45	A	0	0	0.00	0	0.000
46	A	0	0	0.00	0	0.000
47	A	20	10	1.00	10	1.000
48	A	0	0	0.00	0	0.000
49	A	18	13	1.00	16	0.812
50	A	10	9	1.00	14	0.643
51	A	4	3	1.00	8	0.375
52	A	0	0	0.00	0	0.000
53	A	0	0	0.00	0	0.000
54	A	14	9	1.00	17	0.529
55	A	14	9	1.00	15	0.600
56	A	14	9	1.00	13	0.692
57	A	3	1	1.00	17	0.059
58	A	14	9	1.00	17	0.529
59	A	14	9	1.00	17	0.529
60	A	16	10	1.00	19	0.526
61	A	4	4	1.00	22	0.182
62	A	4	4	1.00	22	0.182
63	A	4	4	1.00	19	0.210
64	A	4	4	1.00	19	0.210
65	A	2	2	1.00	19	0.105
66	A	2	2	1.00	17	0.118
67	A	2	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	2	2	1.00	19	0.105
69	A	2	2	1.00	19	0.105
70	A	2	2	1.00	19	0.105
71	A	22	9	1.00	20	0.450
72	A	18	9	1.00	20	0.450
73	A	15	8	1.00	20	0.400
74	A	11	8	1.00	20	0.400
75	A	9	8	1.00	20	0.400
76	A	7	5	1.00	20	0.250
77	A	4	4	1.00	20	0.200
78	A	2	2	1.00	18	0.111
79	A	2	2	1.00	17	0.118
80	A	0	0	0.00	0	0.000
81	A	0	0	0.00	0	0.000
82	A	0	0	0.00	0	0.000
83	A	8	8	1.00	20	0.400
84	A	0	0	0.00	0	0.000
85	A	0	0	0.00	0	0.000
86	A	0	0	0.00	0	0.000
87	A	19	9	1.00	20	0.450
88	A	0	0	0.00	0	0.000
89	A	0	0	0.00	0	0.000
90	A	0	0	0.00	0	0.000
91	A	23	8	1.00	20	0.400
92	A	18	8	1.00	20	0.400
93	A	16	9	1.00	20	0.450

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	13	9	1.00	20	0.450
95	A	10	8	1.00	20	0.400
96	A	7	6	1.00	20	0.300
97	A	5	5	1.00	20	0.250
98	A	4	3	1.00	18	0.167
99	A	1	1	1.00	17	0.059
100	A	0	0	0.00	0	0.000
101	A	4	4	1.00	20	0.200
102	A	0	0	0.00	0	0.000
103	A	0	0	0.00	0	0.000
104	A	0	0	0.00	0	0.000
105	A	13	9	1.00	20	0.450
106	A	0	0	0.00	0	0.000
107	A	0	0	0.00	0	0.000
108	A	0	0	0.00	0	0.000
109	A	26	9	1.00	20	0.450
110	A	6	4	1.00	8	0.500
111	A	6	4	1.00	8	0.500
112	A	5	4	1.00	8	0.500
113	A	5	4	1.00	8	0.500
114	A	4	4	1.00	8	0.500
115	A	4	4	1.00	8	0.500
116	A	3	3	1.00	6	0.500
117	A	1	1	1.00	4	0.250
118	A	3	3	1.00	8	0.375
119	A	2	2	1.00	8	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	3	3	1.00	8	0.375
121	A	4	4	1.00	8	0.500
122	A	4	4	1.00	8	0.500
123	A	5	4	1.00	8	0.500
124	A	5	4	1.00	8	0.500
125	A	6	4	1.00	8	0.500
126	A	6	4	1.00	8	0.500
127	A	7	4	1.00	8	0.500
128	A	14	10	1.00	14	0.714
129	A	11	9	1.00	14	0.643
130	A	8	7	1.00	12	0.583
131	A	1	1	1.00	6	0.167
132	A	0	0	0.00	0	0.000
133	A	0	0	0.00	0	0.000
134	A	14	10	1.00	10	1.000
135	A	11	9	1.00	10	0.900
136	A	8	7	1.00	8	0.875
137	A	1	1	1.00	6	0.167
138	A	0	0	0.00	0	0.000
139	A	0	0	0.00	0	0.000
140	A	23	11	1.00	10	1.100
141	A	19	10	1.00	10	1.000
142	A	16	10	1.00	10	1.000
143	A	12	9	1.00	10	0.900
144	A	10	10	1.00	10	1.000
145	A	8	6	1.00	10	0.600

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
146	A	5	5	1.00	8	0.625
147	A	4	4	1.00	6	0.667
148	A	0	0	0.00	0	0.000
149	A	0	0	0.00	0	0.000
150	A	0	0	0.00	0	0.000
151	A	0	0	0.00	0	0.000
152	A	9	9	1.00	10	0.900
153	A	0	0	0.00	0	0.000
154	A	0	0	0.00	0	0.000
155	A	0	0	0.00	0	0.000
156	A	20	10	1.00	10	1.000
157	A	0	0	0.00	0	0.000
158	A	18	13	1.00	16	0.812
159	A	10	9	1.00	14	0.643
160	A	4	3	1.00	8	0.375
161	A	0	0	0.00	0	0.000
162	A	0	0	0.00	0	0.000
163	A	14	9	1.00	17	0.529
164	A	14	9	1.00	15	0.600
165	A	14	9	1.00	13	0.692
166	A	3	1	1.00	17	0.059
167	A	14	9	1.00	17	0.529
168	A	14	9	1.00	17	0.529
169	A	16	10	1.00	19	0.526
170	A	4	4	1.00	22	0.182
171	A	4	4	1.00	22	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	A	4	4	1.00	19	0.210
173	A	4	4	1.00	19	0.210
174	A	2	2	1.00	19	0.105
175	A	2	2	1.00	17	0.118
176	A	2	2	1.00	19	0.105
177	A	2	2	1.00	19	0.105
178	A	2	2	1.00	19	0.105
179	A	2	2	1.00	19	0.105
180	A	22	10	1.00	20	0.500
181	A	18	9	1.00	20	0.450
182	A	15	9	1.00	20	0.450
183	A	11	8	1.00	20	0.400
184	A	9	9	1.00	20	0.450
185	A	7	5	1.00	20	0.250
186	A	4	4	1.00	20	0.200
187	A	2	2	1.00	18	0.111
188	A	2	2	1.00	17	0.118
189	A	0	0	0.00	0	0.000
190	A	0	0	0.00	0	0.000
191	A	0	0	0.00	0	0.000
192	A	8	8	1.00	20	0.400
193	A	0	0	0.00	0	0.000
194	A	0	0	0.00	0	0.000
195	A	0	0	0.00	0	0.000
196	A	19	9	1.00	20	0.450
197	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
198	A	0	0	0.00	0	0.000
199	A	0	0	0.00	0	0.000
200	A	23	9	1.00	20	0.450
201	A	18	8	1.00	20	0.400
202	A	16	10	1.00	20	0.500
203	A	13	9	1.00	20	0.450
204	A	10	9	1.00	20	0.450
205	A	7	6	1.00	20	0.300
206	A	5	5	1.00	20	0.250
207	A	4	3	1.00	18	0.167
208	A	1	1	1.00	17	0.059
209	A	0	0	0.00	0	0.000
210	A	4	4	1.00	20	0.200
211	A	0	0	0.00	0	0.000
212	A	0	0	0.00	0	0.000
213	A	0	0	0.00	0	0.000
214	A	13	9	1.00	20	0.450
215	A	0	0	0.00	0	0.000
216	A	0	0	0.00	0	0.000
217	A	0	0	0.00	0	0.000
218	A	26	9	1.00	20	0.450

Chapter 3

Listing of integrals

3.1 $\int x^7 S(bx)\,dx$

Optimal. Leaf size=124

$$-\frac{105S(bx)}{8\pi^4b^8}+\frac{x^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi b}+\frac{105x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^7}-\frac{35x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^3b^5}-\frac{7x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^2b^3}+\frac{1}{8}x^8S(bx)$$

```
[Out] -35/8*x^3*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/8*x^7*cos(1/2*b^2*Pi*x^2)/b/Pi-105
/8*FresnelS(b*x)/b^8/Pi^4+1/8*x^8*FresnelS(b*x)+105/8*x*sin(1/2*b^2*Pi*x^2)
/b^7/Pi^4-7/8*x^5*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3385, 3386, 3351}

$$-\frac{105S(bx)}{8\pi^4b^8}-\frac{7x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^2b^3}+\frac{105x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^7}+\frac{x^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi b}-\frac{35x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^3b^5}+\frac{1}{8}x^8S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelS[b*x],x]
```

```
[Out] (-35*x^3*Cos[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2])/(8*b
*Pi) - (105*FresnelS[b*x])/(8*b^8*Pi^4) + (x^8*FresnelS[b*x])/8 + (105*x*Si
n[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*Sin[(b^2*Pi*x^2)/2])/(8*b^3*Pi^2)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^7 S(bx)\,dx &= \frac{1}{8}x^8 S(bx) - \frac{1}{8}b\int x^8 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx \\
&= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8 S(bx) - \frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{8b\pi} \\
&= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8 S(bx) - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{35\int x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{8b^3\pi^2} \\
&= -\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8 S(bx) - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{105\int x^2\cos\left(\frac{1}{2}b^2\pi x\right.}{8b^5\pi^3} \\
&= -\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8 S(bx) + \frac{105x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} - \\
&= -\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} - \frac{105S(bx)}{8b^8\pi^4} + \frac{1}{8}x^8 S(bx) + \frac{105x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\sin}{8}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 88, normalized size = 0.71

$$\frac{(\pi^4b^8x^8-105)\,S(bx)-7bx\,(\pi^2b^4x^4-15)\sin\left(\frac{1}{2}\pi b^2x^2\right)+\pi b^3x^3\,(\pi^2b^4x^4-35)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelS[b*x],x]
```

```
[Out] (b^3*Pi*x^3*(-35 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8
)*FresnelS[b*x] - 7*b*x*(-15 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(8*b^8*Pi
^4)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(x^7\text{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^7*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\text{fresnels}\,(bx)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnels(b*x), x)
```

maple [A] time = 0.03, size = 123, normalized size = 0.99

$$\frac{\frac{\mathrm{S}(bx)b^8x^8}{8}+\frac{b^7x^7\cos\left(\frac{b^2\pi x^2}{2}\right)}{8\pi}-\frac{7\left(\frac{b^5x^5\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{5\left(-\frac{b^3x^3\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{3bx\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{3\,\mathrm{S}(bx)}{\pi}}{\pi}\right)}{\pi}\right)}{8\pi}}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x),x)
```

```
[Out] 1/b^8*(1/8*FresnelS(b*x)*b^8*x^8+1/8/Pi*b^7*x^7*cos(1/2*b^2*Pi*x^2)-7/8/Pi*
(1/Pi*b^5*x^5*sin(1/2*b^2*Pi*x^2)-5/Pi*(-1/Pi*b^3*x^3*cos(1/2*b^2*Pi*x^2)+3
/Pi*(1/Pi*b*x*sin(1/2*b^2*Pi*x^2)-1/Pi*FresnelS(b*x)))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \text{fresnels}\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^7\ \text{FresnelS}\,(b\,x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x),x)
```

```
[Out] int(x^7*FresnelS(b*x), x)
```

sympy [A] time = 1.98, size = 184, normalized size = 1.48

$$\frac{231x^8 S\,(bx)\,\Gamma\left(\frac{3}{4}\right)}{512\Gamma\left(\frac{15}{4}\right)}+\frac{231x^7\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi b\Gamma\left(\frac{15}{4}\right)}-\frac{1617x^5\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi^2b^3\Gamma\left(\frac{15}{4}\right)}-\frac{8085x^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi^3b^5\Gamma\left(\frac{15}{4}\right)}+\frac{24255x\sin\left(\frac{\pi b^2x^2}{2}\right.}{512\pi^4b^7\Gamma\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnels(b*x),x)
```

```
[Out] 231*x**8*fresnels(b*x)*gamma(3/4)/(512*gamma(15/4)) + 231*x**7*cos(pi*b**2*
x**2/2)*gamma(3/4)/(512*pi*b*gamma(15/4)) - 1617*x**5*sin(pi*b**2*x**2/2)*g
amma(3/4)/(512*pi**2*b**3*gamma(15/4)) - 8085*x**3*cos(pi*b**2*x**2/2)*gamm
a(3/4)/(512*pi**3*b**5*gamma(15/4)) + 24255*x*sin(pi*b**2*x**2/2)*gamma(3/4
)/(512*pi**4*b**7*gamma(15/4)) - 24255*fresnels(b*x)*gamma(3/4)/(512*pi**4*
b**8*gamma(15/4))
```

3.2 $\int x^6 S(bx)\,dx$

Optimal. Leaf size=109

$$\frac{x^6\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{48\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}-\frac{24x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}-\frac{6x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{1}{7}x^7S(bx)$$

```
[Out] -24/7*x^2*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/7*x^6*cos(1/2*b^2*Pi*x^2)/b/Pi+1/7
*x^7*FresnelS(b*x)+48/7*sin(1/2*b^2*Pi*x^2)/b^7/Pi^4-6/7*x^4*sin(1/2*b^2*Pi
*x^2)/b^3/Pi^2
```

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3379, 3296, 2637}

$$-\frac{6x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{48\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}+\frac{x^6\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}-\frac{24x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}+\frac{1}{7}x^7S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelS[b*x],x]
```

```
[Out] (-24*x^2*Cos[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) + (x^6*Cos[(b^2*Pi*x^2)/2])/(7*b
*Pi) + (x^7*FresnelS[b*x])/7 + (48*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x
^4*Sin[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^6 S(bx)\,dx &= \frac{1}{7}x^7 S(bx) - \frac{1}{7}b\int x^7 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{7}x^7 S(bx) - \frac{1}{14}b\,\text{Subst}\left(\int x^3 \sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right) \\
&= \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 S(bx) - \frac{3\,\text{Subst}\left(\int x^2\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{7b\pi} \\
&= \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 S(bx) - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{12\,\text{Subst}\left(\int x\sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{7b^3\pi^2} \\
&= -\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} + \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 S(bx) - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{24\,\text{Subst}\left(\int \cos\left(\frac{1}{2}b^2\right.\right.}{7b^5\pi^3} \\
&= -\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} + \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 S(bx) + \frac{48\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 83, normalized size = 0.76

$$
-\frac{6\left(\pi^2 b^4 x^4 - 8\right)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^4 b^7} + \frac{x^2\left(\pi^2 b^4 x^4 - 24\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{7\pi^3 b^5} + \frac{1}{7}x^7 S(bx)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelS[b*x],x]
```

```
[Out] (x^2*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) + (x^7*FresnelS
[b*x])/7 - (6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4)
```

fricas [F] time = 0.38, size = 0, normalized size = 0.00

$$
\text{integral}\left(x^6\,\text{fresnels}\,(bx)\,, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^6*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \text{fresnels}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnels(b*x), x)
```

maple [A] time = 0.02, size = 107, normalized size = 0.98

$$\frac{\frac{b^7x^7\mathrm{S}(bx)}{7}+\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}-\frac{6\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{7\pi}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x),x)
```

```
[Out] 1/b^7*(1/7*b^7*x^7*FresnelS(b*x)+1/7/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)-6/7/Pi*
(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2
/Pi^2*sin(1/2*b^2*Pi*x^2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \text{fresnels}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\,\mathrm{FresnelS}\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x),x)
```

```
[Out] int(x^6*FresnelS(b*x), x)
```

sympy [A] time = 1.50, size = 156, normalized size = 1.43

$$\frac{3x^7 S(bx)\Gamma\left(\frac{3}{4}\right)}{28\Gamma\left(\frac{7}{4}\right)}+\frac{3x^6\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{28\pi b\Gamma\left(\frac{7}{4}\right)}-\frac{9x^4\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{14\pi^2b^3\Gamma\left(\frac{7}{4}\right)}-\frac{18x^2\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{7\pi^3b^5\Gamma\left(\frac{7}{4}\right)}+\frac{36\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{7\pi^4b^7\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnels(b*x),x)
```

```
[Out] 3*x**7*fresnels(b*x)*gamma(3/4)/(28*gamma(7/4)) + 3*x**6*cos(pi*b**2*x**2/2
)*gamma(3/4)/(28*pi*b*gamma(7/4)) - 9*x**4*sin(pi*b**2*x**2/2)*gamma(3/4)/(
14*pi**2*b**3*gamma(7/4)) - 18*x**2*cos(pi*b**2*x**2/2)*gamma(3/4)/(7*pi**3
*b**5*gamma(7/4)) + 36*sin(pi*b**2*x**2/2)*gamma(3/4)/(7*pi**4*b**7*gamma(7
/4))
```

3.3 $\int x^5 S(bx)\, dx$

Optimal. Leaf size=99

$$\frac{5C(bx)}{2\pi^3 b^6}+\frac{x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi b}-\frac{5x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3 b^5}-\frac{5x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi^2 b^3}+\frac{1}{6}x^6S(bx)$$

```
[Out] -5/2*x*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/6*x^5*cos(1/2*b^2*Pi*x^2)/b/Pi+5/2*Fr
esnelC(b*x)/b^6/Pi^3+1/6*x^6*FresnelS(b*x)-5/6*x^3*sin(1/2*b^2*Pi*x^2)/b^3/
Pi^2
```

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3385, 3386, 3352}

$$\frac{5\text{FresnelC}(bx)}{2\pi^3 b^6}-\frac{5x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi^2 b^3}+\frac{x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi b}-\frac{5x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3 b^5}+\frac{1}{6}x^6S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelS[b*x],x]
```

```
[Out] (-5*x*Cos[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) + (x^5*Cos[(b^2*Pi*x^2)/2])/(6*b*Pi
) + (5*FresnelC[b*x])/(2*b^6*Pi^3) + (x^6*FresnelS[b*x])/6 - (5*x^3*Sin[(b^
2*Pi*x^2)/2])/(6*b^3*Pi^2)
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^5 S(bx)\,dx &= \frac{1}{6}x^6 S(bx) - \frac{1}{6}b\int x^6 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\
&= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{1}{6}x^6 S(bx) - \frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{6b\pi}\\
&= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{1}{6}x^6 S(bx) - \frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{5\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{2b^3\pi^2}\\
&= -\frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{1}{6}x^6 S(bx) - \frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{5\int\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{2b^5\pi^3}\\
&= -\frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5C(bx)}{2b^6\pi^3} + \frac{1}{6}x^6 S(bx) - \frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2}\end{aligned}$$

Mathematica [A] time = 0.10, size = 79, normalized size = 0.80

$$\frac{\pi^3 b^6 x^6 S(bx) + bx\left(\pi^2 b^4 x^4 - 15\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right) - 5\pi b^3 x^3\sin\left(\frac{1}{2}\pi b^2 x^2\right) + 15C(bx)}{6\pi^3 b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelS[b*x],x]
```

```
[Out] (b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + 15*FresnelC[b*x] + b^6*Pi^3
*x^6*FresnelS[b*x] - 5*b^3*Pi*x^3*Sin[(b^2*Pi*x^2)/2])/(6*b^6*Pi^3)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(x^5\text{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x),x, algorithm="fricas")
```

[Out] integral(x^5*fresnels(b*x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \text{fresnels}\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnels(b*x), x)
```

maple [A] time = 0.02, size = 96, normalized size = 0.97

$$\frac{\frac{b^6x^6\mathrm{S}(bx)}{6}+\frac{b^5x^5\cos\left(\frac{b^2\pi x^2}{2}\right)}{6\pi}-\frac{5\left(\frac{b^3x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{3\left(-\frac{bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\mathrm{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{6\pi}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x),x)
```

```
[Out] 1/b^6*(1/6*b^6*x^6*FresnelS(b*x)+1/6/Pi*b^5*x^5*cos(1/2*b^2*Pi*x^2)-5/6/Pi*
(1/Pi*b^3*x^3*sin(1/2*b^2*Pi*x^2)-3/Pi*(-1/Pi*b*x*cos(1/2*b^2*Pi*x^2)+1/Pi*
FresnelC(b*x))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \text{fresnels}\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,\mathrm{FresnelS}\,(b\,x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x),x)
```

```
[Out] int(x^5*FresnelS(b*x), x)
```

sympy [A] time = 0.85, size = 53, normalized size = 0.54

$$\frac{\pi b^3 x^9 \Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{9}{4}\right) {}_2F_3\left(\begin{matrix} \frac{3}{4}, \frac{9}{4} \\ \frac{3}{2}, \frac{7}{4}, \frac{13}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnels(b*x),x)
```

```
[Out] pi*b**3*x**9*gamma(3/4)*gamma(9/4)*hyper((3/4, 9/4), (3/2, 7/4, 13/4), -pi*
*2*b**4*x**4/16)/(32*gamma(7/4)*gamma(13/4))
```

3.4 $\int x^4 S(bx)\, dx$

Optimal. Leaf size=84

$$\frac{x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}-\frac{8\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}-\frac{4x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}+\frac{1}{5}x^5S(bx)$$

```
[Out] -8/5*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/5*x^4*cos(1/2*b^2*Pi*x^2)/b/Pi+1/5*x^5*
FresnelS(b*x)-4/5*x^2*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3379, 3296, 2638}

$$-\frac{4x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}+\frac{x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}-\frac{8\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}+\frac{1}{5}x^5S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelS[b*x],x]
```

```
[Out] (-8*Cos[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) + (x^4*Cos[(b^2*Pi*x^2)/2])/(5*b*Pi)
+ (x^5*FresnelS[b*x])/5 - (4*x^2*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^4 S(bx)\,dx &= \frac{1}{5}x^5 S(bx) - \frac{1}{5}b\int x^5 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\
&= \frac{1}{5}x^5 S(bx) - \frac{1}{10}b\,\mathrm{Subst}\left(\int x^2 \sin\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)\\
&= \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5 S(bx) - \frac{2\,\mathrm{Subst}\left(\int x\cos\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{5b\pi}\\
&= \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5 S(bx) - \frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{4\,\mathrm{Subst}\left(\int \sin\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{5b^3\pi^2}\\
&= -\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} + \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5 S(bx) - \frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.85

$$-\frac{4x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3} + \frac{\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5} + \frac{1}{5}x^5 S(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelS[b*x],x]
```

```
[Out] ((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) + (x^5*FresnelS[b*x]
)/5 - (4*x^2*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^4\mathrm{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^4*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \mathrm{fresnels}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnels(b*x), x)
```

maple [A] time = 0.02, size = 80, normalized size = 0.95

$$\frac{\frac{b^5x^5\mathrm{S}(bx)}{5}+\frac{b^4x^4\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{5\pi}-\frac{4\left(\frac{b^2x^2\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi^2}\right)}{5\pi}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x),x)
```

```
[Out] 1/b^5*(1/5*b^5*x^5*FresnelS(b*x)+1/5/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)-4/5/Pi*
(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \mathrm{fresnels}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{FresnelS}\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x),x)
```

```
[Out] int(x^4*FresnelS(b*x), x)
```

sympy [A] time = 1.46, size = 121, normalized size = 1.44

$$\frac{3x^5S\left(bx\right)\Gamma\left(\frac{3}{4}\right)}{20\Gamma\left(\frac{7}{4}\right)}+\frac{3x^4\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{20\pi b\Gamma\left(\frac{7}{4}\right)}-\frac{3x^2\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{5\pi^2b^3\Gamma\left(\frac{7}{4}\right)}-\frac{6\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{5\pi^3b^5\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnels(b*x),x)
```

```
[Out] 3*x**5*fresnels(b*x)*gamma(3/4)/(20*gamma(7/4)) + 3*x**4*cos(pi*b**2*x**2/2
)*gamma(3/4)/(20*pi*b*gamma(7/4)) - 3*x**2*sin(pi*b**2*x**2/2)*gamma(3/4)/(
5*pi**2*b**3*gamma(7/4)) - 6*cos(pi*b**2*x**2/2)*gamma(3/4)/(5*pi**3*b**5*g
amma(7/4))
```

3.5 $\int x^3 S(bx)\, dx$

Optimal. Leaf size=74

$$\frac{3S(bx)}{4\pi^2 b^4}+\frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b}-\frac{3x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2 b^3}+\frac{1}{4}x^4S(bx)$$

```
[Out] 1/4*x^3*cos(1/2*b^2*Pi*x^2)/b/Pi+3/4*FresnelS(b*x)/b^4/Pi^2+1/4*x^4*Fresnel
S(b*x)-3/4*x*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3385, 3386, 3351}

$$\frac{3S(bx)}{4\pi^2 b^4}-\frac{3x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2 b^3}+\frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b}+\frac{1}{4}x^4S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelS[b*x],x]
```

```
[Out] (x^3*Cos[(b^2*Pi*x^2)/2])/(4*b*Pi) + (3*FresnelS[b*x])/(4*b^4*Pi^2) + (x^4*
FresnelS[b*x])/4 - (3*x*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3 S(bx)\,dx &= \frac{1}{4}x^4S(bx) - \frac{1}{4}b\int x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\ &= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{1}{4}x^4S(bx) - \frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{4b\pi}\\ &= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{1}{4}x^4S(bx) - \frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3\int\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{4b^3\pi^2}\\ &= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3S(bx)}{4b^4\pi^2} + \frac{1}{4}x^4S(bx) - \frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 74, normalized size = 1.00

$$\frac{3S(bx)}{4\pi^2b^4} + \frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b} - \frac{3x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2b^3} + \frac{1}{4}x^4S(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelS[b*x],x]
[Out] (x^3*Cos[(b^2*Pi*x^2)/2])/(4*b*Pi) + (3*FresnelS[b*x])/(4*b^4*Pi^2) + (x^4*
FresnelS[b*x])/4 - (3*x*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2)
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\text{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x),x, algorithm="fricas")
[Out] integral(x^3*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{fresnels}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnels(b*x), x)
```

maple [A] time = 0.02, size = 70, normalized size = 0.95

$$\frac{\frac{b^4x^4\mathrm{S}(bx)}{4}+\frac{b^3x^3\cos\left(\frac{b^2\pi x^2}{2}\right)}{4\pi}-\frac{3\left(\frac{bx\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{\mathrm{S}(bx)}{\pi}\right)}{4\pi}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x),x)
```

```
[Out] 1/b^4*(1/4*b^4*x^4*FresnelS(b*x)+1/4/Pi*b^3*x^3*cos(1/2*b^2*Pi*x^2)-3/4/Pi*
(1/Pi*b*x*sin(1/2*b^2*Pi*x^2)-1/Pi*FresnelS(b*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\mathrm{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{FresnelS}\,(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x),x)
```

```
[Out] int(x^3*FresnelS(b*x), x)
```

sympy [A] time = 0.85, size = 112, normalized size = 1.51

$$\frac{21x^4S\left(bx\right)\Gamma\left(\frac{3}{4}\right)}{64\Gamma\left(\frac{11}{4}\right)}+\frac{21x^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{64\pi b\Gamma\left(\frac{11}{4}\right)}-\frac{63x\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{64\pi^2b^3\Gamma\left(\frac{11}{4}\right)}+\frac{63S\left(bx\right)\Gamma\left(\frac{3}{4}\right)}{64\pi^2b^4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnels(b*x),x)
```

```
[Out] 21*x**4*fresnels(b*x)*gamma(3/4)/(64*gamma(11/4)) + 21*x**3*cos(pi*b**2*x**
2/2)*gamma(3/4)/(64*pi*b*gamma(11/4)) - 63*x*sin(pi*b**2*x**2/2)*gamma(3/4)
/(64*pi**2*b**3*gamma(11/4)) + 63*fresnels(b*x)*gamma(3/4)/(64*pi**2*b**4*g
amma(11/4))
```

3.6 $\int x^2 S(bx)\,dx$

Optimal. Leaf size=59

$$\frac{x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}-\frac{2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{1}{3}x^3S(bx)$$

```
[Out] 1/3*x^2*cos(1/2*b^2*Pi*x^2)/b/Pi+1/3*x^3*FresnelS(b*x)-2/3*sin(1/2*b^2*Pi*x
^2)/b^3/Pi^2
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3379, 3296, 2637}

$$-\frac{2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}+\frac{1}{3}x^3S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[b*x],x]
```

```
[Out] (x^2*Cos[(b^2*Pi*x^2)/2])/(3*b*Pi) + (x^3*FresnelS[b*x])/3 - (2*Sin[(b^2*Pi
*x^2)/2])/(3*b^3*Pi^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 S(bx)\,dx &= \frac{1}{3}x^3 S(bx) - \frac{1}{3}b\int x^3 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx \\
&= \frac{1}{3}x^3 S(bx) - \frac{1}{6}b\,\text{Subst}\left(\int x\sin\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right) \\
&= \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{1}{3}x^3 S(bx) - \frac{\text{Subst}\left(\int \cos\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{3b\pi} \\
&= \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{1}{3}x^3 S(bx) - \frac{2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.00

$$\frac{x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b} - \frac{2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3} + \frac{1}{3}x^3 S(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelS[b*x],x]
[Out] (x^2*Cos[(b^2*Pi*x^2)/2])/(3*b*Pi) + (x^3*FresnelS[b*x])/3 - (2*Sin[(b^2*Pi
*x^2)/2])/(3*b^3*Pi^2)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\text{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x),x, algorithm="fricas")
[Out] integral(x^2*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{fresnels}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnels(b*x), x)
```

maple [A] time = 0.02, size = 54, normalized size = 0.92

$$\frac{\frac{b^3x^3\mathrm{S}(bx)}{3}+\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi}-\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x),x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*FresnelS(b*x)+1/3/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)-2/3/Pi^
2*sin(1/2*b^2*Pi*x^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\mathrm{FresnelS}\left(b\,x\right)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x),x)
```

```
[Out] int(x^2*FresnelS(b*x), x)
```

sympy [A] time = 0.80, size = 80, normalized size = 1.36

$$\frac{x^3S\left(bx\right)\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x^2\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{4\pi b\Gamma\left(\frac{7}{4}\right)}-\frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{2\pi^2b^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(b*x),x)
```

```
[Out] x**3*fresnels(b*x)*gamma(3/4)/(4*gamma(7/4)) + x**2*cos(pi*b**2*x**2/2)*gam
ma(3/4)/(4*pi*b*gamma(7/4)) - sin(pi*b**2*x**2/2)*gamma(3/4)/(2*pi**2*b**3*
gamma(7/4))
```

3.7 $\int xS(bx)\,dx$

Optimal. Leaf size=49

$$-\frac{C(bx)}{2\pi b^2}+\frac{x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{1}{2}x^2S(bx)$$

```
[Out] 1/2*x*cos(1/2*b^2*Pi*x^2)/b/Pi-1/2*FresnelC(b*x)/b^2/Pi+1/2*x^2*FresnelS(b*
x)
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3385, 3352}

$$-\frac{\text{FresnelC}(bx)}{2\pi b^2}+\frac{x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{1}{2}x^2S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[b*x],x]
```

```
[Out] (x*Cos[(b^2*Pi*x^2)/2])/(2*b*Pi) - FresnelC[b*x]/(2*b^2*Pi) + (x^2*FresnelS
[b*x])/2
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int xS(bx)\,dx &= \frac{1}{2}x^2S(bx) - \frac{1}{2}b\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\ &= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{1}{2}x^2S(bx) - \frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{2b\pi}\\ &= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} - \frac{C(bx)}{2b^2\pi} + \frac{1}{2}x^2S(bx)\end{aligned}$$

Mathematica [**A**] time = 0.01, size = 49, normalized size = 1.00

$$-\frac{C(bx)}{2\pi b^2} + \frac{x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b} + \frac{1}{2}x^2S(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelS[b*x],x]
```

```
[Out] (x*Cos[(b^2*Pi*x^2)/2])/(2*b*Pi) - FresnelC[b*x]/(2*b^2*Pi) + (x^2*FresnelS
[b*x])/2
```

fricas [**F**] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(x\text{fresnels}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x*fresnels(b*x), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*fresnels(b*x), x)
```

maple [A] time = 0.02, size = 44, normalized size = 0.90

$$\frac{\frac{b^2x^2\mathrm{S}(bx)}{2}+\frac{bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{2\pi}-\frac{\mathrm{FresnelC}(bx)}{2\pi}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x),x)
```

```
[Out] 1/b^2*(1/2*b^2*x^2*FresnelS(b*x)+1/2/Pi*b*x*cos(1/2*b^2*Pi*x^2)-1/2/Pi*Fres
nelC(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{FresnelS}\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x),x)
```

```
[Out] int(x*FresnelS(b*x), x)
```

sympy [A] time = 0.52, size = 53, normalized size = 1.08

$$\frac{\pi b^3x^5\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x),x)
```

```
[Out] pi*b**3*x**5*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -pi**
2*b**4*x**4/16)/(32*gamma(7/4)*gamma(9/4))
```

3.8 $\int S(bx)\,dx$

Optimal. Leaf size=26

$$\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(bx)$$

```
[Out] cos(1/2*b^2*Pi*x^2)/b/Pi+x*FresnelS(b*x)
```

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6418}

$$\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(bx)$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x],x]
```

```
[Out] Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]
```

Rule 6418

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelS[a +
 b*x])/b, x] + Simp[Cos[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int S(bx)\,dx=\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}+xS(bx)$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x],x]
```

```
[Out] Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnels}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x),x, algorithm="fricas")
[Out] integral(fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x),x, algorithm="giac")
[Out] integrate(fresnels(b*x), x)
```

maple [A] time = 0.02, size = 27, normalized size = 1.04

$$\frac{bx\,\mathrm{S}\left(bx\right)+\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x),x)
[Out] 1/b*(b*x*FresnelS(b*x)+1/Pi*cos(1/2*b^2*Pi*x^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x),x, algorithm="maxima")
[Out] integrate(fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \mathrm{FresnelS}\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x),x)
```

```
[Out] int(FresnelS(b*x), x)
```

sympy [B] time = 0.74, size = 48, normalized size = 1.85

$$\frac{3xS\left(bx\right)\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{4\pi b\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x),x)
```

```
[Out] 3*x*fresnels(b*x)*gamma(3/4)/(4*gamma(7/4)) + 3*cos(pi*b**2*x**2/2)*gamma(3
/4)/(4*pi*b*gamma(7/4))
```

3.9 $\int \frac{S(bx)}{x}\, dx$

Optimal. Leaf size=73

$$\frac{1}{2} i b x\, {}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2} i b^2 \pi x^2\right)-\frac{1}{2} i b x\, {}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2} i b^2 \pi x^2\right)$$

```
[Out] 1/2*I*b*x*HypergeometricPFQ([1/2, 1/2],[3/2, 3/2],-1/2*I*b^2*Pi*x^2)-1/2*I*
b*x*HypergeometricPFQ([1/2, 1/2],[3/2, 3/2],1/2*I*b^2*Pi*x^2)
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6424, 6358, 6360}

$$\frac{1}{2} i b x\, {}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2} i b^2 \pi x^2\right)-\frac{1}{2} i b x\, {}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2} i b^2 \pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x,x]
```

```
[Out] (I/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-I/2)*b^2*Pi*x^2] - (I
/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2]
```

Rule 6358

```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(2*b*x*HypergeometricPFQ[{1/2,
1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[b, x]
```

Rule 6360

```
Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(2*b*x*HypergeometricPFQ[{1/2,
 1/2}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[b, x]
```

Rule 6424

```
Int[FresnelS[(b_.)*(x_)]/(x_), x_Symbol] :> Dist[(1 + I)/4, Int[Erf[(Sqrt[P
i]*(1 + I)*b*x)/2]/x, x], x] + Dist[(1 - I)/4, Int[Erf[(Sqrt[Pi]*(1 - I)*b*
x)/2]/x, x], x] /; FreeQ[b, x]
```

Rubi steps

$$\int \frac{S(bx)}{x}\,dx = \left(-\frac{1}{4}-\frac{i}{4}\right)\int \frac{\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)}{x}\,dx + \left(\frac{1}{4}+\frac{i}{4}\right)\int \frac{\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)}{x}\,dx$$
$$= \frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]/x,x]
[Out] Integrate[FresnelS[b*x]/x, x]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x,x, algorithm="fricas")
[Out] integral(fresnels(b*x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{fresnels}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x,x, algorithm="giac")
[Out] integrate(fresnels(b*x)/x, x)
```

maple [A] time = 0.06, size = 29, normalized size = 0.40

$$\frac{\pi x^3 b^3 \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{3}{2},\frac{7}{4},\frac{7}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x,x)
[Out] 1/18*Pi*x^3*b^3*hypergeom([3/4,3/4],[3/2,7/4,7/4],-1/16*x^4*Pi^2*b^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}(bx)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{FresnelS}(b\,x)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x,x)
[Out] int(FresnelS(b*x)/x, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x,x)
[Out] Exception raised: AttributeError
```

3.10 $\int \frac{S(bx)}{x^2} \, dx$

Optimal. Leaf size=27

$$\frac{1}{2} b \operatorname{Si}\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{S(bx)}{x}$$

```
[Out] -FresnelS(b*x)/x+1/2*b*Si(1/2*b^2*Pi*x^2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6426, 3375}

$$\frac{1}{2} b \operatorname{Si}\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{S(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^2,x]
```

```
[Out] -(FresnelS[b*x]/x) + (b*SinIntegral[(b^2*Pi*x^2)/2])/2
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{S(bx)}{x^2} \, dx &= -\frac{S(bx)}{x} + b \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x} \, dx \\ &= -\frac{S(bx)}{x} + \frac{1}{2} b \operatorname{Si}\left(\frac{1}{2} b^2 \pi x^2\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.00

$$\frac{1}{2} b \operatorname{Si}\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{S(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^2,x]
```

```
[Out] -(FresnelS[b*x]/x) + (b*SinIntegral[(b^2*Pi*x^2)/2])/2
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^2, x)
```

maple [A] time = 0.02, size = 28, normalized size = 1.04

$$b\left(-\frac{\text{S}\,(bx)}{bx}+\frac{\text{Si}\left(\frac{b^2\pi\,x^2}{2}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^2,x)
```

```
[Out] b*(-FresnelS(b*x)/b/x+1/2*Si(1/2*b^2*Pi*x^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{FresnelS}(b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^2,x)
```

```
[Out] int(FresnelS(b*x)/x^2, x)
```

sympy [B] time = 0.64, size = 42, normalized size = 1.56

$$\frac{\pi b^3 x^2 \Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2}, \frac{3}{2}, \frac{7}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**2,x)
```

```
[Out] pi*b**3*x**2*gamma(3/4)*hyper((1/2, 3/4), (3/2, 3/2, 7/4), -pi**2*b**4*x**4
/16)/(16*gamma(7/4))
```

3.11 $\int \frac{S(bx)}{x^3} \, dx$

Optimal. Leaf size=44

$$\frac{1}{2}\pi b^2 C(bx) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{S(bx)}{2x^2}$$

```
[Out] 1/2*b^2*Pi*FresnelC(b*x)-1/2*FresnelS(b*x)/x^2-1/2*b*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6426, 3387, 3352}

$$\frac{1}{2}\pi b^2 \text{FresnelC}(bx) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{S(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[b*x])/2 - FresnelS[b*x]/(2*x^2) - (b*Sin[(b^2*Pi*x^2)/2])/
(2*x)
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{S(bx)}{x^3}\,dx = -\frac{S(bx)}{2x^2} + \frac{1}{2}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$
$$= -\frac{S(bx)}{2x^2} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x} + \frac{1}{2}\left(b^3\pi\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)dx$$
$$= \frac{1}{2}b^2\pi C(bx) - \frac{S(bx)}{2x^2} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$\frac{1}{2}\pi b^2 C(bx) - \frac{b\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{S(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[b*x])/2 - FresnelS[b*x]/(2*x^2) - (b*Sin[(b^2*Pi*x^2)/2])/
(2*x)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^3, x)
```

maple [A] time = 0.02, size = 43, normalized size = 0.98

$$b^{2}\left(-\frac{\mathrm{S}\left(bx\right)}{2b^{2}x^{2}}-\frac{\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{2bx}+\frac{\pi\,\mathrm{FresnelC}\left(bx\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^3,x)
```

```
[Out] b^2*(-1/2*FresnelS(b*x)/b^2/x^2-1/2*sin(1/2*b^2*Pi*x^2)/b/x+1/2*Pi*FresnelC
(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{fresnels}\left(bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\mathrm{FresnelS}\left(b\,x\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^3,x)
```

```
[Out] int(FresnelS(b*x)/x^3, x)
```

sympy [A] time = 0.57, size = 51, normalized size = 1.16

$$\frac{\pi b^{3}x\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_{2}F_{3}\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{5}{4},\frac{3}{2},\frac{7}{4}\end{matrix}\right|-\frac{\pi^{2}b^{4}x^{4}}{16}\right)}{32\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**3,x)
```

```
[Out] pi*b**3*x*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (5/4, 3/2, 7/4), -pi**2*b
**4*x**4/16)/(32*gamma(5/4)*gamma(7/4))
```

3.12 $\int \frac{S(bx)}{x^4}\,dx$

Optimal. Leaf size=52

$$-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^2}+\frac{1}{12}\pi b^3\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{S(bx)}{3x^3}$$

```
[Out] 1/12*b^3*Pi*Ci(1/2*b^2*Pi*x^2)-1/3*FresnelS(b*x)/x^3-1/6*b*sin(1/2*b^2*Pi*x
^2)/x^2
```

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3379, 3297, 3302}

$$\frac{1}{12}\pi b^3\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right)-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^2}-\frac{S(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[(b^2*Pi*x^2)/2])/12 - FresnelS[b*x]/(3*x^3) - (b*Sin[(b
^2*Pi*x^2)/2])/(6*x^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)}{x^4}\,dx &= -\frac{S(bx)}{3x^3}+\frac{1}{3}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx\\ &= -\frac{S(bx)}{3x^3}+\frac{1}{6}b\,\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right)\\ &= -\frac{S(bx)}{3x^3}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}+\frac{1}{12}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx,x,x^2\right)\\ &= \frac{1}{12}b^3\pi\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{S(bx)}{3x^3}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 1.00

$$-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^2}+\frac{1}{12}\pi b^3\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{S(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[(b^2*Pi*x^2)/2])/12 - FresnelS[b*x]/(3*x^3) - (b*Sin[(b
^2*Pi*x^2)/2])/(6*x^2)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\left(bx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^4, x)
```

maple [A] time = 0.02, size = 49, normalized size = 0.94

$$b^3\left(-\frac{\mathrm{S}\left(bx\right)}{3b^3x^3}-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{6b^2x^2}+\frac{\pi\,\mathrm{Ci}\left(\frac{b^2\pi\,x^2}{2}\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^4,x)
```

```
[Out] b^3*(-1/3*FresnelS(b*x)/b^3/x^3-1/6*sin(1/2*b^2*Pi*x^2)/b^2/x^2+1/12*Pi*Ci(
1/2*b^2*Pi*x^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\left(bx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^4,x)
```

```
[Out] int(FresnelS(b*x)/x^4, x)
```

<u>**sympy**</u> **[A]** time = 1.01, size = 56, normalized size = 1.08

$$-\frac{\pi^3 b^7 x^4 \Gamma\left(\frac{7}{4}\right) {}_3F_4\left(\begin{matrix} 1,1,\frac{7}{4} \\ 2,2,\frac{5}{2},\frac{11}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{768\Gamma\left(\frac{11}{4}\right)} + \frac{\pi b^3 \log\left(b^4 x^4\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**4,x)
```

```
[Out] -pi**3*b**7*x**4*gamma(7/4)*hyper((1, 1, 7/4), (2, 2, 5/2, 11/4), -pi**2*b*
*4*x**4/16)/(768*gamma(11/4)) + pi*b**3*log(b**4*x**4)/24
```

3.13 $\int \frac{S(bx)}{x^5} dx$

Optimal. Leaf size=69

$$-\frac{1}{12}\pi^2 b^4 S(bx) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{12x^3} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{12x} - \frac{S(bx)}{4x^4}$$

```
[Out] -1/12*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x-1/12*b^4*Pi^2*FresnelS(b*x)-1/4*FresnelS
(b*x)/x^4-1/12*b*sin(1/2*b^2*Pi*x^2)/x^3
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3387, 3388, 3351}

$$-\frac{1}{12}\pi^2 b^4 S(bx) - \frac{b \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{12x^3} - \frac{\pi b^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{12x} - \frac{S(bx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^5,x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(12*x) - (b^4*Pi^2*FresnelS[b*x])/12 - Fresne
lS[b*x]/(4*x^4) - (b*Sin[(b^2*Pi*x^2)/2])/(12*x^3)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)}{x^5}\,dx &= -\frac{S(bx)}{4x^4}+\frac{1}{4}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{S(bx)}{4x^4}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}+\frac{1}{12}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x}-\frac{S(bx)}{4x^4}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}-\frac{1}{12}\left(b^5\pi^2\right)\int \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x}-\frac{1}{12}b^4\pi^2S(bx)-\frac{S(bx)}{4x^4}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 1.00

$$-\frac{1}{12}\pi^2b^4S(bx)-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{12x^3}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{12x}-\frac{S(bx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^5,x]
```

```
[Out] -1/12*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x - (b^4*Pi^2*FresnelS[b*x])/12 - Fresne
lS[b*x]/(4*x^4) - (b*Sin[(b^2*Pi*x^2)/2])/(12*x^3)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\left(bx\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^5, x)
```

maple [A] time = 0.02, size = 65, normalized size = 0.94

$$b^4\left(-\frac{S(bx)}{4b^4x^4}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{12b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{bx}-\pi S(bx)\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^5,x)
```

```
[Out] b^4*(-1/4*FresnelS(b*x)/b^4/x^4-1/12*sin(1/2*b^2*Pi*x^2)/b^3/x^3+1/12*Pi*(-
1/b/x*cos(1/2*b^2*Pi*x^2)-Pi*FresnelS(b*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{FresnelS}(b\,x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^5,x)
```

```
[Out] int(FresnelS(b*x)/x^5, x)
```

sympy [A] time = 0.96, size = 110, normalized size = 1.59

$$\frac{\pi^2 b^4 S(bx)\,\Gamma\left(-\frac{1}{4}\right)}{64\Gamma\left(\frac{7}{4}\right)}+\frac{\pi b^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{1}{4}\right)}{64x\Gamma\left(\frac{7}{4}\right)}+\frac{b\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{1}{4}\right)}{64x^3\Gamma\left(\frac{7}{4}\right)}+\frac{3S(bx)\,\Gamma\left(-\frac{1}{4}\right)}{64x^4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**5,x)
```

```
[Out] pi**2*b**4*fresnels(b*x)*gamma(-1/4)/(64*gamma(7/4)) + pi*b**3*cos(pi*b**2*
x**2/2)*gamma(-1/4)/(64*x*gamma(7/4)) + b*sin(pi*b**2*x**2/2)*gamma(-1/4)/(
64*x**3*gamma(7/4)) + 3*fresnels(b*x)*gamma(-1/4)/(64*x**4*gamma(7/4))
```

3.14 $\int \frac{S(bx)}{x^6} \, dx$

Optimal. Leaf size=77

$$-\frac{b \sin \left(\frac{1}{2}\pi b^2 x^2\right)}{20x^4}-\frac{1}{80}\pi^2 b^5 \text{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\pi b^3 \cos \left(\frac{1}{2}\pi b^2 x^2\right)}{40x^2}-\frac{S(bx)}{5x^5}$$

```
[Out] -1/40*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^2-1/5*FresnelS(b*x)/x^5-1/80*b^5*Pi^2*Si
(1/2*b^2*Pi*x^2)-1/20*b*sin(1/2*b^2*Pi*x^2)/x^4
```

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3379, 3297, 3299}

$$-\frac{1}{80}\pi^2 b^5 \text{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{b \sin \left(\frac{1}{2}\pi b^2 x^2\right)}{20x^4}-\frac{\pi b^3 \cos \left(\frac{1}{2}\pi b^2 x^2\right)}{40x^2}-\frac{S(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^6,x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(40*x^2) - FresnelS[b*x]/(5*x^5) - (b*Sin[(b^
2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*SinIntegral[(b^2*Pi*x^2)/2])/80
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)}{x^6}\,dx &= -\frac{S(bx)}{5x^5}+\frac{1}{5}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx\\
&= -\frac{S(bx)}{5x^5}+\frac{1}{10}b\,\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx,x,x^2\right)\\
&= -\frac{S(bx)}{5x^5}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}+\frac{1}{40}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}-\frac{S(bx)}{5x^5}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{1}{80}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx,x,x^2\right)\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}-\frac{S(bx)}{5x^5}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{1}{80}b^5\pi^2\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 1.00

$$-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{20x^4}-\frac{1}{80}\pi^2b^5\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{40x^2}-\frac{S(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^6,x]
```

```
[Out] -1/40*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^2 - FresnelS[b*x]/(5*x^5) - (b*Sin[(b^
2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*SinIntegral[(b^2*Pi*x^2)/2])/80
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^6, x)
```

maple [A] time = 0.02, size = 71, normalized size = 0.92

$$b^5\left(-\frac{\mathrm{S}(bx)}{5b^5x^5}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{20b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^6,x)
```

```
[Out] b^5*(-1/5*FresnelS(b*x)/b^5/x^5-1/20*sin(1/2*b^2*Pi*x^2)/b^4/x^4+1/20*Pi*(-
1/2/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/4*Pi*Si(1/2*b^2*Pi*x^2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^6,x)
```

```
[Out] int(FresnelS(b*x)/x^6, x)
```

sympy [A] time = 0.91, size = 46, normalized size = 0.60

$$-\frac{\pi b^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{3}\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2},\frac{3}{2},\frac{7}{4}\end{matrix}\right|-\frac{\pi^{2}b^{4}x^{4}}{16}\right)}{16x^{2}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**6,x)
```

```
[Out] -pi*b**3*gamma(3/4)*hyper((-1/2, 3/4), (1/2, 3/2, 7/4), -pi**2*b**4*x**4/16
)/(16*x**2*gamma(7/4))
```

3.15 $\int \frac{S(bx)}{x^7}\,dx$

Optimal. Leaf size=94

$$-\frac{1}{90}\pi^3b^6C(bx)-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{30x^5}+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{90x}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{90x^3}-\frac{S(bx)}{6x^6}$$

```
[Out] -1/90*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^3-1/90*b^6*Pi^3*FresnelC(b*x)-1/6*Fresne
lS(b*x)/x^6-1/30*b*sin(1/2*b^2*Pi*x^2)/x^5+1/90*b^5*Pi^2*sin(1/2*b^2*Pi*x^2
)/x
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3387, 3388, 3352}

$$-\frac{1}{90}\pi^3b^6\text{FresnelC}(bx)+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{90x}-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{30x^5}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{90x^3}-\frac{S(bx)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^7,x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(90*x^3) - (b^6*Pi^3*FresnelC[b*x])/90 - Fres
nelS[b*x]/(6*x^6) - (b*Sin[(b^2*Pi*x^2)/2])/(30*x^5) + (b^5*Pi^2*Sin[(b^2*P
i*x^2)/2])/(90*x)
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{S(bx)}{x^7}\,dx &= -\frac{S(bx)}{6x^6}+\frac{1}{6}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx\\
&= -\frac{S(bx)}{6x^6}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{1}{30}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}-\frac{S(bx)}{6x^6}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}-\frac{1}{90}\left(b^5\pi^2\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}-\frac{S(bx)}{6x^6}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x}-\frac{1}{90}\left(b^7\pi^3\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}-\frac{1}{90}b^6\pi^3C(bx)-\frac{S(bx)}{6x^6}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.81

$$
\frac{1}{90}\left(-\pi^3b^6C(bx)+\frac{b\left(\pi^2b^4x^4-3\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^5}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^3}-\frac{15S(bx)}{x^6}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^7,x]
```

```
[Out] (-((b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^3) - b^6*Pi^3*FresnelC[b*x] - (15*Fresnel
S[b*x])/x^6 + (b*(-3 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x^5)/90
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^7},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^7,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^7, x)
```

maple [A] time = 0.02, size = 86, normalized size = 0.91

$$b^6\left(-\frac{S(bx)}{6b^6x^6}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{30b^5x^5}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\,\text{FresnelC}(bx)\right)}{3}\right)}{30}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^7,x)
```

```
[Out] b^6*(-1/6*FresnelS(b*x)/b^6/x^6-1/30*sin(1/2*b^2*Pi*x^2)/b^5/x^5+1/30*Pi*(-
1/3/b^3/x^3*cos(1/2*b^2*Pi*x^2)-1/3*Pi*(-sin(1/2*b^2*Pi*x^2)/b/x+Pi*Fresnel
C(b*x))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}\,(b\,x)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^7,x)
```

```
[Out] int(FresnelS(b*x)/x^7, x)
```

sympy [A] time = 1.15, size = 56, normalized size = 0.60

$$\frac{\pi b^3\Gamma\left(-\frac{3}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}-\frac{3}{4},\frac{3}{4}\\ \frac{1}{4},\frac{3}{2},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{32x^3\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**7,x)
```

```
[Out] pi*b**3*gamma(-3/4)*gamma(3/4)*hyper((-3/4, 3/4), (1/4, 3/2, 7/4), -pi**2*b
**4*x**4/16)/(32*x**3*gamma(1/4)*gamma(7/4))
```

3.16 $\int \frac{S(bx)}{x^8}\,dx$

Optimal. Leaf size=102

$$-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{42x^6}-\frac{1}{672}\pi^3b^7\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{336x^2}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{168x^4}-\frac{S(bx)}{7x^7}$$

```
[Out] -1/672*b^7*Pi^3*Ci(1/2*b^2*Pi*x^2)-1/168*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^4-1/7
*FresnelS(b*x)/x^7-1/42*b*sin(1/2*b^2*Pi*x^2)/x^6+1/336*b^5*Pi^2*sin(1/2*b^
2*Pi*x^2)/x^2
```

Rubi [A] time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3379, 3297, 3302}

$$-\frac{1}{672}\pi^3b^7\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right)+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{336x^2}-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{42x^6}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{168x^4}-\frac{S(bx)}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^8,x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(168*x^4) - (b^7*Pi^3*CosIntegral[(b^2*Pi*x^2
)/2])/672 - FresnelS[b*x]/(7*x^7) - (b*Sin[(b^2*Pi*x^2)/2])/(42*x^6) + (b^5
*Pi^2*Sin[(b^2*Pi*x^2)/2])/(336*x^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
```

```
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)}{x^8}\,dx &= -\frac{S(bx)}{7x^7}+\frac{1}{7}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx\\
&= -\frac{S(bx)}{7x^7}+\frac{1}{14}b\,\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx,x,x^2\right)\\
&= -\frac{S(bx)}{7x^7}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}+\frac{1}{84}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx,x,x^2\right)\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4}-\frac{S(bx)}{7x^7}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}-\frac{1}{336}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4}-\frac{S(bx)}{7x^7}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2}-\frac{1}{672}\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\cos}{}\right.\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4}-\frac{1}{672}b^7\pi^3\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{S(bx)}{7x^7}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 85, normalized size = 0.83

$$\frac{1}{672}\left(-\pi^3b^7\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)+\frac{2b\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^6}-\frac{4\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^4}-\frac{96S(bx)}{x^7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^8,x]
```

```
[Out] ((-4*b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^4 - b^7*Pi^3*CosIntegral[(b^2*Pi*x^2)/2]
 - (96*FresnelS[b*x])/x^7 + (2*b*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x
^6)/672
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^8, x)
```

maple [A] time = 0.02, size = 93, normalized size = 0.91

$$b^7\left(-\frac{\mathrm{S}\,(bx)}{7b^7x^7}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{42b^6x^6}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi\,\mathrm{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{42}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^8,x)
```

```
[Out] b^7*(-1/7*FresnelS(b*x)/b^7/x^7-1/42*sin(1/2*b^2*Pi*x^2)/b^6/x^6+1/42*Pi*(-
1/4/b^4/x^4*cos(1/2*b^2*Pi*x^2)-1/4*Pi*(-1/2*sin(1/2*b^2*Pi*x^2)/b^2/x^2+1/
4*Pi*Ci(1/2*b^2*Pi*x^2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{FresnelS}(b\,x)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^8,x)
```

```
[Out] int(FresnelS(b*x)/x^8, x)
```

sympy [A] time = 2.05, size = 68, normalized size = 0.67

$$\frac{\pi^5 b^{11} x^4 \Gamma\left(\frac{11}{4}\right) {}_3F_4\left(\begin{matrix} 1,1,\frac{11}{4} \\ 2,3,\frac{7}{2},\frac{15}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{61440\Gamma\left(\frac{15}{4}\right)} - \frac{\pi^3 b^7 \log\left(b^4 x^4\right)}{1344} - \frac{\pi b^3}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**8,x)
```

```
[Out] pi**5*b**11*x**4*gamma(11/4)*hyper((1, 1, 11/4), (2, 3, 7/2, 15/4), -pi**2*
b**4*x**4/16)/(61440*gamma(15/4)) - pi**3*b**7*log(b**4*x**4)/1344 - pi*b**
3/(24*x**4)
```

3.17 $\int \frac{S(bx)}{x^9} dx$

Optimal. Leaf size=119

$$\frac{1}{840}\pi^4 b^8 S(bx)-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{56x^7}+\frac{\pi^3b^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{840x}+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{840x^3}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{280x^5}-\frac{S(bx)}{8x^8}$$

```
[Out] -1/280*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^5+1/840*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)/x+
1/840*b^8*Pi^4*FresnelS(b*x)-1/8*FresnelS(b*x)/x^8-1/56*b*sin(1/2*b^2*Pi*x^
2)/x^7+1/840*b^5*Pi^2*sin(1/2*b^2*Pi*x^2)/x^3
```

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3387, 3388, 3351}

$$\frac{1}{840}\pi^4 b^8 S(bx)+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{840x^3}-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{56x^7}+\frac{\pi^3b^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{840x}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{280x^5}-\frac{S(bx)}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^9,x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(280*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2])/(8
40*x) + (b^8*Pi^4*FresnelS[b*x])/840 - FresnelS[b*x]/(8*x^8) - (b*Sin[(b^2*
Pi*x^2)/2])/(56*x^7) + (b^5*Pi^2*Sin[(b^2*Pi*x^2)/2])/(840*x^3)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{S(bx)}{x^9}\,dx = -\frac{S(bx)}{8x^8} + \frac{1}{8}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

$$= -\frac{S(bx)}{8x^8} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{1}{56}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{S(bx)}{8x^8} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} - \frac{1}{280}\left(b^5\pi^2\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{S(bx)}{8x^8} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} - \frac{1}{840}\left(b^7\pi^3\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x \cdots\right.}{x^2}$$

$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x} - \frac{S(bx)}{8x^8} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} + \frac{1}{840 \cdots}$$

$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x} + \frac{1}{840}b^8\pi^4 S(bx) - \frac{S(bx)}{8x^8} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\sin \cdots}{84 \cdots}$$

Mathematica [A] time = 0.08, size = 84, normalized size = 0.71

$$\frac{\left(\pi^4b^8x^8-105\right)S(bx)+bx\left(\pi^2b^4x^4-15\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)+\pi b^3x^3\left(\pi^2b^4x^4-3\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{840x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^9,x]
```

```
[Out] (b^3*Pi*x^3*(-3 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)
*FresnelS[b*x] + b*x*(-15 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(840*x^8)
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\left(bx\right)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^9,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^9, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^9, x)
```

maple [A] time = 0.02, size = 109, normalized size = 0.92

$$b^8\left(-\frac{\mathrm{S}\,(bx)}{8b^8x^8}-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{56b^7x^7}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{5b^5x^5}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{3b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{bx}-\pi\,\mathrm{S}(bx)\right)}{3}\right)}{5}\right)}{56}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^9,x)
```

```
[Out] b^8*(-1/8*FresnelS(b*x)/b^8/x^8-1/56*sin(1/2*b^2*Pi*x^2)/b^7/x^7+1/56*Pi*(-
1/5/b^5/x^5*cos(1/2*b^2*Pi*x^2)-1/5*Pi*(-1/3*sin(1/2*b^2*Pi*x^2)/b^3/x^3+1/
3*Pi*(-1/b/x*cos(1/2*b^2*Pi*x^2)-Pi*FresnelS(b*x)))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^9,x)
```

```
[Out] int(FresnelS(b*x)/x^9, x)
```

sympy [A] time = 2.54, size = 185, normalized size = 1.55

$$\frac{\pi^4 b^8 S\left(bx\right)\Gamma\left(-\frac{5}{4}\right)}{3584\Gamma\left(\frac{7}{4}\right)}+\frac{\pi^3 b^7\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x\Gamma\left(\frac{7}{4}\right)}+\frac{\pi^2 b^5\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^3\Gamma\left(\frac{7}{4}\right)}-\frac{3\pi b^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^5\Gamma\left(\frac{7}{4}\right)}-\frac{15b\sin\left(\frac{\pi b^2x^2}{2}\right)}{3584x^7\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**9,x)
```

```
[Out] pi**4*b**8*fresnels(b*x)*gamma(-5/4)/(3584*gamma(7/4)) + pi**3*b**7*cos(pi*
b**2*x**2/2)*gamma(-5/4)/(3584*x*gamma(7/4)) + pi**2*b**5*sin(pi*b**2*x**2/
2)*gamma(-5/4)/(3584*x**3*gamma(7/4)) - 3*pi*b**3*cos(pi*b**2*x**2/2)*gamma
(-5/4)/(3584*x**5*gamma(7/4)) - 15*b*sin(pi*b**2*x**2/2)*gamma(-5/4)/(3584*
x**7*gamma(7/4)) - 15*fresnels(b*x)*gamma(-5/4)/(512*x**8*gamma(7/4))
```

3.18 $\int \frac{S(bx)}{x^{10}}\,dx$

Optimal. Leaf size=127

$$-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{72x^8}+\frac{\pi^4b^9\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}+\frac{\pi^3b^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3456x^2}+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{1728x^4}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{432x^6}-\frac{S(bx)}{9x^9}$$

```
[Out] -1/432*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^6+1/3456*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)/x
^2-1/9*FresnelS(b*x)/x^9+1/6912*b^9*Pi^4*Si(1/2*b^2*Pi*x^2)-1/72*b*sin(1/2*
b^2*Pi*x^2)/x^8+1/1728*b^5*Pi^2*sin(1/2*b^2*Pi*x^2)/x^4
```

Rubi [A] time = 0.15, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6426, 3379, 3297, 3299}

$$\frac{\pi^4b^9\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{1728x^4}-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{72x^8}+\frac{\pi^3b^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3456x^2}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{432x^6}-\frac{S(bx)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^10,x]
```

```
[Out] -(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/(432*x^6) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2])/(3
456*x^2) - FresnelS[b*x]/(9*x^9) - (b*Sin[(b^2*Pi*x^2)/2])/(72*x^8) + (b^5*
Pi^2*Sin[(b^2*Pi*x^2)/2])/(1728*x^4) + (b^9*Pi^4*SinIntegral[(b^2*Pi*x^2)/2
])/6912
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
```

```
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6426

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelS[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{S(bx)}{x^{10}}\,dx = -\frac{S(bx)}{9x^9} + \frac{1}{9}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

$$= -\frac{S(bx)}{9x^9} + \frac{1}{18}b\,\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{S(bx)}{9x^9} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{1}{144}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{S(bx)}{9x^9} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} - \frac{1}{864}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{S(bx)}{9x^9} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} - \frac{\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right.}{x^2}\right.}{3456}$$

$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2} - \frac{S(bx)}{9x^9} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} + \frac{\left(b^9\right.}{}$$

$$= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2} - \frac{S(bx)}{9x^9} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} + \frac{b^9\pi}{}$$

Mathematica [A] time = 0.21, size = 96, normalized size = 0.76

$$\frac{\pi^4b^9\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) + \frac{4b\left(\pi^2b^4x^4-24\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^8} + \frac{2\pi b^3\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^6} - \frac{768S(bx)}{x^9}}{6912}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^10,x]
```

```
[Out] ((2*b^3*Pi*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^6 - (768*FresnelS[b*x
])/x^9 + (4*b*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x^8 + b^9*Pi^4*SinI
ntegral[(b^2*Pi*x^2)/2])/6912
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)}{x^{10}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)/x^10, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)/x^10, x)
```

maple [A] time = 0.02, size = 115, normalized size = 0.91

$$b^{9}\left(-\frac{\mathrm{S}\left(bx\right)}{9b^{9}x^{9}}-\frac{\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{72b^{8}x^{8}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{6b^{6}x^{6}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{4b^{4}x^{4}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{2b^{2}x^{2}}-\frac{\pi\,\mathrm{Si}\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^10,x)
```

```
[Out] b^9*(-1/9*FresnelS(b*x)/b^9/x^9-1/72*sin(1/2*b^2*Pi*x^2)/b^8/x^8+1/72*Pi*(-
1/6/b^6/x^6*cos(1/2*b^2*Pi*x^2)-1/6*Pi*(-1/4*sin(1/2*b^2*Pi*x^2)/b^4/x^4+1/
4*Pi*(-1/2/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/4*Pi*Si(1/2*b^2*Pi*x^2)))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnels}\left(bx\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)/x^10, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^10,x)
```

```
[Out] int(FresnelS(b*x)/x^10, x)
```

sympy [A] time = 2.76, size = 48, normalized size = 0.38

$$-\frac{\pi b^3\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\-\frac{1}{2},\frac{3}{2},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{48x^6\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**10,x)
```

```
[Out] -pi*b**3*gamma(3/4)*hyper((-3/2, 3/4), (-1/2, 3/2, 7/4), -pi**2*b**4*x**4/1
6)/(48*x**6*gamma(7/4))
```

3.19 $\int (c+dx)^3 S(a+bx)\,dx$

Optimal. Leaf size=296

$$-\frac{2d^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2b^4}+\frac{d^2(a+bx)^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{3d(bc-ad)^2C(a+bx)}{2\pi b^4}-\frac{(bc-ad)^4S(}{4b^4d}$$

```
[Out] (-a*d+b*c)^3*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+3/2*d*(-a*d+b*c)^2*(b*x+a)*cos(1/
2*Pi*(b*x+a)^2)/b^4/Pi+d^2*(-a*d+b*c)*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^4/P
i+1/4*d^3*(b*x+a)^3*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi-3/2*d*(-a*d+b*c)^2*Fresnel
C(b*x+a)/b^4/Pi-1/4*(-a*d+b*c)^4*FresnelS(b*x+a)/b^4/d+3/4*d^3*FresnelS(b*x
+a)/b^4/Pi^2+1/4*(d*x+c)^4*FresnelS(b*x+a)/d-2*d^2*(-a*d+b*c)*sin(1/2*Pi*(b
*x+a)^2)/b^4/Pi^2-3/4*d^3*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi^2
```

Rubi [A] time = 0.40, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352, 3296, 2637, 3386}

$$-\frac{2d^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2b^4}+\frac{d^2(a+bx)^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{3d(bc-ad)^2\text{FresnelC}(a+bx)}{2\pi b^4}-\frac{(bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*FresnelS[a + b*x],x]
```

```
[Out] ((b*c - a*d)^3*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi) + (3*d*(b*c - a*d)^2*(a +
b*x)*Cos[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) + (d^2*(b*c - a*d)*(a + b*x)^2*Cos
[(Pi*(a + b*x)^2)/2])/(b^4*Pi) + (d^3*(a + b*x)^3*Cos[(Pi*(a + b*x)^2)/2])/
(4*b^4*Pi) - (3*d*(b*c - a*d)^2*FresnelC[a + b*x])/(2*b^4*Pi) - ((b*c - a*d
)^4*FresnelS[a + b*x])/(4*b^4*d) + (3*d^3*FresnelS[a + b*x])/(4*b^4*Pi^2) +
 ((c + d*x)^4*FresnelS[a + b*x])/(4*d) - (2*d^2*(b*c - a*d)*Sin[(Pi*(a + b*
x)^2)/2])/(b^4*Pi^2) - (3*d^3*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^
2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
```

```
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 S(a+bx)\,dx = \frac{(c+dx)^4 S(a+bx)}{4d} - \frac{b\int (c+dx)^4 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{4d}$$

$$= \frac{(c+dx)^4 S(a+bx)}{4d} - \frac{\text{Subst}\left(\int \left(b^4c^4\left(1+\frac{ad\left(-4b^3c^3+6ab^2c^2d-4a^2bcd^2+a^3d^3\right)}{b^4c^4}\right)\sin\left(\frac{\pi x^2}{2}\right)+4\right.\right.}{}$$

$$= \frac{(c+dx)^4 S(a+bx)}{4d} - \frac{d^3\,\text{Subst}\left(\int x^4 \sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{4b^4} - \frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\right.}{}$$

$$= \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{d^3(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} - \frac{(bc-ad)}{4b}$$

$$= \frac{(bc-ad)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{d^2(bc-ad}{}$$

$$= \frac{(bc-ad)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{d^2(bc-ad}{}$$

Mathematica [A] time = 1.02, size = 424, normalized size = 1.43

$$\frac{-\pi a^3d^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+4\pi a^2bcd^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi a^2bd^3x\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+S(a+bx)\left(d^3\left(-\pi^2a^4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*FresnelS[a + b*x],x]
```

```
[Out] (4*b^3*c^3*Pi*Cos[(Pi*(a + b*x)^2)/2] - 6*a*b^2*c^2*d*Pi*Cos[(Pi*(a + b*x)^
2)/2] + 4*a^2*b*c*d^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - a^3*d^3*Pi*Cos[(Pi*(a +
b*x)^2)/2] + 6*b^3*c^2*d*Pi*x*Cos[(Pi*(a + b*x)^2)/2] - 4*a*b^2*c*d^2*Pi*x*
```

```
Cos[(Pi*(a + b*x)^2)/2] + a^2*b*d^3*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + 4*b^3*c*
d^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] - a*b^2*d^3*Pi*x^2*Cos[(Pi*(a + b*x)^2)/
2] + b^3*d^3*Pi*x^3*Cos[(Pi*(a + b*x)^2)/2] - 6*d*(b*c - a*d)^2*Pi*FresnelC
[a + b*x] + (4*b^3*c^3*Pi^2*(a + b*x) + 6*b^2*c^2*d*Pi^2*(-a^2 + b^2*x^2) +
 4*b*c*d^2*Pi^2*(a^3 + b^3*x^3) + d^3*(3 - a^4*Pi^2 + b^4*Pi^2*x^4))*Fresne
lS[a + b*x] - 8*b*c*d^2*Sin[(Pi*(a + b*x)^2)/2] + 5*a*d^3*Sin[(Pi*(a + b*x)
^2)/2] - 3*b*d^3*x*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^3x^3 + 3\,cd^2x^2 + 3\,c^2dx + c^3\right)\text{fresnels}\left(bx + a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*fresnels(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \text{fresnels}\left(bx + a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*fresnels(b*x + a), x)
```

maple [A] time = 0.03, size = 400, normalized size = 1.35

$$\frac{\mathrm{S}(bx+a)((bx+a)d-ad+bc)^4}{4b^3d} - \frac{-\frac{d^4(bx+a)^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{3d^4\left(\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{\mathrm{S}(bx+a)}{\pi}\right)}{\pi} - \frac{\left(-4a\,d^4+4bc\,d^3\right)(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{2\left(-4a\,d^4+4bc\,d^3\right)\sin\left(\frac{\pi(bx}{}\right.}{\pi^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*FresnelS(b*x+a),x)
```

```
[Out] 1/b*(1/4*FresnelS(b*x+a)*((b*x+a)*d-a*d+b*c)^4/b^3/d-1/4/b^3/d*(-d^4/Pi*(b*
x+a)^3*cos(1/2*Pi*(b*x+a)^2)+3*d^4/Pi*(1/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-1
/Pi*FresnelS(b*x+a))-(-4*a*d^4+4*b*c*d^3)/Pi*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2
)+2*(-4*a*d^4+4*b*c*d^3)/Pi^2*sin(1/2*Pi*(b*x+a)^2)-(6*a^2*d^4-12*a*b*c*d^3
+6*b^2*c^2*d^2)/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+(6*a^2*d^4-12*a*b*c*d^3+6*
b^2*c^2*d^2)/Pi*FresnelC(b*x+a)-(-4*a^3*d^4+12*a^2*b*c*d^3-12*a*b^2*c^2*d^2
```

```
+4*b^3*c^3*d)/Pi*cos(1/2*Pi*(b*x+a)^2)+a^4*d^4*FresnelS(b*x+a)-4*a^3*b*c*d^
3*FresnelS(b*x+a)+6*a^2*b^2*c^2*d^2*FresnelS(b*x+a)-4*a*b^3*c^3*d*FresnelS(
b*x+a)+b^4*c^4*FresnelS(b*x+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \operatorname{fresnels}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnels(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^3*fresnels(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{FresnelS}(a+b\,x)\,\left(c+d\,x\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)*(c + d*x)^3,x)
[Out] int(FresnelS(a + b*x)*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 S(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*fresnels(b*x+a),x)
[Out] Integral((c + d*x)**3*fresnels(a + b*x), x)
```

3.20 $\int (c+dx)^2 S(a+bx)\,dx$

Optimal. Leaf size=193

$$-\frac{d(bc-ad)C(a+bx)}{\pi b^3}-\frac{(bc-ad)^3S(a+bx)}{3b^3d}+\frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{d(a+bx)(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}$$

```
[Out] (-a*d+b*c)^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi+d*(-a*d+b*c)*(b*x+a)*cos(1/2*Pi*(
b*x+a)^2)/b^3/Pi+1/3*d^2*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi-d*(-a*d+b*c
)*FresnelC(b*x+a)/b^3/Pi-1/3*(-a*d+b*c)^3*FresnelS(b*x+a)/b^3/d+1/3*(d*x+c)
^3*FresnelS(b*x+a)/d-2/3*d^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi^2
```

Rubi [A] time = 0.23, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352, 3296, 2637}

$$-\frac{d(bc-ad)\text{FresnelC}(a+bx)}{\pi b^3}-\frac{(bc-ad)^3S(a+bx)}{3b^3d}+\frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{d(a+bx)(bc-ad)\cos\left(\frac{1}{2}\pi(a\right.}{\pi b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelS[a + b*x],x]
```

```
[Out] ((b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d*(b*c - a*d)*(a + b*x)
*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/
2])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x])/(b^3*Pi) - ((b*c - a*d)^
3*FresnelS[a + b*x])/(3*b^3*d) + ((c + d*x)^3*FresnelS[a + b*x])/(3*d) - (2
*d^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2 S(a+bx)\,dx = \frac{(c+dx)^3 S(a+bx)}{3d} - \frac{b\int (c+dx)^3 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{3d}$$

$$= \frac{(c+dx)^3 S(a+bx)}{3d} - \frac{\text{Subst}\left(\int \left(b^3c^3\left(1-\frac{ad(3b^2c^2-3abcd+a^2d^2)}{b^3c^3}\right)\sin\left(\frac{\pi x^2}{2}\right)+3b^2c^2d\left(1+\right.\right.\right.}{}$$

$$= \frac{(c+dx)^3 S(a+bx)}{3d} - \frac{d^2\,\text{Subst}\left(\int x^3 \sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{3b^3} - \frac{(d(bc-ad))\,\text{Subst}\left(\int x\right.}{}$$

$$= \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{(bc-ad)^3 S(a+bx)}{3b^3d} + \frac{(c+dx)^3 S(a+bx)}{3d} - \frac{d^2}{}$$

$$= \frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{d^2(a+bx)^2\cos}{3b}$$

$$= \frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{d^2(a+bx)^2\cos}{3b}$$

Mathematica [A] time = 0.55, size = 236, normalized size = 1.22

$$\frac{\pi a^2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi^2 S(a+bx)\left(a^3d^2-3a^2bcd+3ab^2c^2+b^3x\left(3c^2+3cdx+d^2x^2\right)\right)+3\pi b^2c^2\cos\left(\frac{1}{2}\pi(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*FresnelS[a + b*x],x]
```

```
[Out] (3*b^2*c^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - 3*a*b*c*d*Pi*Cos[(Pi*(a + b*x)^2)/2
] + a^2*d^2*Pi*Cos[(Pi*(a + b*x)^2)/2] + 3*b^2*c*d*Pi*x*Cos[(Pi*(a + b*x)^2
)/2] - a*b*d^2*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + b^2*d^2*Pi*x^2*Cos[(Pi*(a + b
*x)^2)/2] + 3*d*(-(b*c) + a*d)*Pi*FresnelC[a + b*x] + Pi^2*(3*a*b^2*c^2 - 3
*a^2*b*c*d + a^3*d^2 + b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*FresnelS[a + b*x]
 - 2*d^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2x^2+2\,cdx+c^2\right)\text{fresnels}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*fresnels(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \mathrm{fresnels}\left(bx+a\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*fresnels(b*x + a), x)
```

maple [A] time = 0.02, size = 251, normalized size = 1.30

$$\frac{\frac{S(bx+a)((bx+a)d-ad+bc)^3}{3b^2d} - \frac{-\frac{d^3(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{2d^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2} - \frac{\left(-3a\,d^3+3bc\,d^2\right)(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{\left(-3a\,d^3+3bc\,d^2\right)\mathrm{FresnelC}(bx+a)}{\pi} - \frac{\left(3a^2d^3-6a\cdots\right.}{\cdots}}{3b^2d}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*FresnelS(b*x+a),x)
```

```
[Out] 1/b*(1/3*FresnelS(b*x+a)*((b*x+a)*d-a*d+b*c)^3/b^2/d-1/3/b^2/d*(-d^3/Pi*(b*
x+a)^2*cos(1/2*Pi*(b*x+a)^2)+2*d^3/Pi^2*sin(1/2*Pi*(b*x+a)^2)-(-3*a*d^3+3*b
*c*d^2)/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+(-3*a*d^3+3*b*c*d^2)/Pi*FresnelC(b
*x+a)-(3*a^2*d^3-6*a*b*c*d^2+3*b^2*c^2*d)/Pi*cos(1/2*Pi*(b*x+a)^2)-a^3*d^3*
FresnelS(b*x+a)+3*a^2*b*c*d^2*FresnelS(b*x+a)-3*a*b^2*c^2*d*FresnelS(b*x+a)
+b^3*c^3*FresnelS(b*x+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \mathrm{fresnels}\left(bx+a\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*fresnels(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{FresnelS}\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)*(c + d*x)^2,x)
```

```
[Out] int(FresnelS(a + b*x)*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 S(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnels(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*fresnels(a + b*x), x)
```

3.21 $\int (c+dx)S(a+bx)\,dx$

Optimal. Leaf size=121

$$-\frac{(bc-ad)^2S(a+bx)}{2b^2d}+\frac{(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}-\frac{dC(a+bx)}{2\pi b^2}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{(c+dx)^2S(a+bx)}{2d}$$

```
[Out] (-a*d+b*c)*cos(1/2*Pi*(b*x+a)^2)/b^2/Pi+1/2*d*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)
/b^2/Pi-1/2*d*FresnelC(b*x+a)/b^2/Pi-1/2*(-a*d+b*c)^2*FresnelS(b*x+a)/b^2/d
+1/2*(d*x+c)^2*FresnelS(b*x+a)/d
```

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352}

$$-\frac{(bc-ad)^2S(a+bx)}{2b^2d}+\frac{(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}-\frac{d\text{FresnelC}(a+bx)}{2\pi b^2}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelS[a + b*x],x]
```

```
[Out] ((b*c - a*d)*Cos[(Pi*(a + b*x)^2)/2])/(b^2*Pi) + (d*(a + b*x)*Cos[(Pi*(a +
b*x)^2)/2])/(2*b^2*Pi) - (d*FresnelC[a + b*x])/(2*b^2*Pi) - ((b*c - a*d)^2*
FresnelS[a + b*x])/(2*b^2*d) + ((c + d*x)^2*FresnelS[a + b*x])/(2*d)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)S(a+bx)\,dx = \frac{(c+dx)^2S(a+bx)}{2d} - \frac{b\int (c+dx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{2d}$$

$$= \frac{(c+dx)^2S(a+bx)}{2d} - \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)\sin\left(\frac{\pi x^2}{2}\right)+2bcd\left(1-\frac{ad}{bc}\right)x\sin\left(\frac{\pi}{}\right.\right.\right.}{2b^2d}$$

$$= \frac{(c+dx)^2S(a+bx)}{2d} - \frac{d\,\text{Subst}\left(\int x^2\sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{2b^2} - \frac{(bc-ad)\,\text{Subst}\left(\int x\sin\left(\right.\right.}{b^2}$$

$$= \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{(bc-ad)^2S(a+bx)}{2b^2d} + \frac{(c+dx)^2S(a+bx)}{2d} - \frac{(bc-ad)\,\text{Su}}{}$$

$$= \frac{(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} + \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{dC(a+bx)}{2b^2\pi} - \frac{(bc-ad)^2S}{2b^2}$$

Mathematica [A] time = 0.28, size = 61, normalized size = 0.50

$$\frac{(-ad+2bc+bdx)\left(\pi(a+bx)S(a+bx)+\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\right)-dC(a+bx)}{2\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*FresnelS[a + b*x],x]
```

```
[Out] (-(d*FresnelC[a + b*x]) + (2*b*c - a*d + b*d*x)*(Cos[(Pi*(a + b*x)^2)/2] +
Pi*(a + b*x)*FresnelS[a + b*x]))/(2*b^2*Pi)
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)\text{fresnels}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*fresnels(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\text{fresnels}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*fresnels(b*x + a), x)
```

maple [A] time = 0.02, size = 108, normalized size = 0.89

$$\frac{\frac{\mathrm{S}(bx+a)\left(\frac{(bx+a)^2 d}{2}-ad(bx+a)+bc(bx+a)\right)}{b}-\frac{-\frac{d(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{d\,\mathrm{FresnelC}(bx+a)}{\pi}-\frac{(-2ad+2bc)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*FresnelS(b*x+a),x)
```

```
[Out] 1/b*(FresnelS(b*x+a)/b*(1/2*(b*x+a)^2*d-a*d*(b*x+a)+b*c*(b*x+a))-1/2/b*(-d/
Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+d/Pi*FresnelC(b*x+a)-(-2*a*d+2*b*c)/Pi*cos
(1/2*Pi*(b*x+a)^2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\text{fresnels}\,(bx + a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*fresnels(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \text{FresnelS}\,(a + b\,x)\ (c + d\,x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)*(c + d*x),x)
```

```
[Out] int(FresnelS(a + b*x)*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)\,S\,(a + bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a),x)
```

```
[Out] Integral((c + d*x)*fresnels(a + b*x), x)
```

3.22 $\int S(a+bx)\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx)S(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[Out] cos(1/2*Pi*(b*x+a)^2)/b/Pi+(b*x+a)*FresnelS(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6418}

$$\frac{(a+bx)S(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[a + b*x],x]
```

```
[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b
```

Rule 6418

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelS[a +
 b*x])/b, x] + Simp[Cos[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int S(a+bx)\,dx=\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)S(a+bx)}{b}$$

Mathematica [B] time = 0.04, size = 89, normalized size = 2.47

$$-\frac{\sin\left(\frac{\pi a^2}{2}\right)\sin\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}+\frac{\cos\left(\frac{\pi a^2}{2}\right)\cos\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(a+bx)+\frac{aS(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[a + b*x],x]
```

```
[Out] (Cos[(a^2*Pi)/2]*Cos[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi) + (a*FresnelS[a + b
*x])/b + x*FresnelS[a + b*x] - (Sin[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnels}\left(bx + a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\left(bx + a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a), x)
```

maple [A] time = 0.01, size = 33, normalized size = 0.92

$$\frac{(bx + a)\,\mathrm{S}\left(bx + a\right) + \frac{\cos\left(\frac{\pi (bx+a)^2}{2}\right)}{\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*FresnelS(b*x+a)+1/Pi*cos(1/2*Pi*(b*x+a)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\left(bx + a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \text{FresnelS}\,(a + b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x),x)
[Out] int(FresnelS(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int S\,(a + bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x)
[Out] Integral(fresnels(a + b*x), x)
```

3.23 $\int \frac{S(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\frac{S(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[a + b*x]/(c + d*x),x]
[Out] Defer[Int][FresnelS[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{S(a+bx)}{c+dx}\,dx = \int \frac{S(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]/(c + d*x),x]
[Out] Integrate[FresnelS[a + b*x]/(c + d*x), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\,(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(fresnels(b*x + a)/(d*x + c), x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)/(d*x+c),x)
[Out] int(FresnelS(b*x+a)/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate(fresnels(b*x + a)/(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{FresnelS}\,(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)/(c + d*x),x)
[Out] int(FresnelS(a + b*x)/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S\,(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(fresnels(a + b*x)/(c + d*x), x)
```

3.24 $\int \frac{S(a+bx)}{(c+dx)^2} \, dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{S(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{S(a+bx)}{(c+dx)^2} \, dx = \int \frac{S(a+bx)}{(c+dx)^2} \, dx$$

Mathematica [A] time = 5.78, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[FresnelS[a + b*x]/(c + d*x)^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(fresnels(b*x + a)/(d*x + c)^2, x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)/(d*x+c)^2,x)
[Out] int(FresnelS(b*x+a)/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(fresnels(b*x + a)/(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{FresnelS}(a+b\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)/(c + d*x)^2,x)
[Out] int(FresnelS(a + b*x)/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(fresnels(a + b*x)/(c + d*x)**2, x)
```

3.25 $\int x^3 S(a+bx)\,dx$

Optimal. Leaf size=229

$$-\frac{a^4S(a+bx)}{4b^4}-\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{3a^2C(a+bx)}{2\pi b^4}+\frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}+\frac{3S(a+bx)}{4\pi^2b^4}+\frac{2a\sin\left(\frac{1}{2}\pi(a+\right.}{\pi^2b^4}$$

```
[Out] -a^3*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+3/2*a^2*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^4
/Pi-a*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+1/4*(b*x+a)^3*cos(1/2*Pi*(b*x+
a)^2)/b^4/Pi-3/2*a^2*FresnelC(b*x+a)/b^4/Pi-1/4*a^4*FresnelS(b*x+a)/b^4+3/4
*FresnelS(b*x+a)/b^4/Pi^2+1/4*x^4*FresnelS(b*x+a)+2*a*sin(1/2*Pi*(b*x+a)^2)
/b^4/Pi^2-3/4*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi^2
```

Rubi [A] time = 0.18, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352, 3296, 2637, 3386}

$$-\frac{3a^2\text{FresnelC}(a+bx)}{2\pi b^4}-\frac{a^4S(a+bx)}{4b^4}-\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}+\frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}+\frac{3S(a+bx)}{4\pi^2b^4}+\frac{2a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelS[a + b*x],x]
```

```
[Out] -((a^3*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi)) + (3*a^2*(a + b*x)*Cos[(Pi*(a + b
*x)^2)/2])/(2*b^4*Pi) - (a*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi) +
((a + b*x)^3*Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi) - (3*a^2*FresnelC[a + b*x]
)/(2*b^4*Pi) - (a^4*FresnelS[a + b*x])/(4*b^4) + (3*FresnelS[a + b*x])/(4*b
^4*Pi^2) + (x^4*FresnelS[a + b*x])/4 + (2*a*Sin[(Pi*(a + b*x)^2)/2])/(b^4*P
i^2) - (3*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^3 S(a+bx)\,dx = \frac{1}{4}x^4 S(a+bx) - \frac{1}{4}b\int x^4 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx$$

$$= \frac{1}{4}x^4 S(a+bx) - \frac{\text{Subst}\left(\int\left(a^4\sin\left(\frac{\pi x^2}{2}\right) - 4a^3 x\sin\left(\frac{\pi x^2}{2}\right) + 6a^2x^2\sin\left(\frac{\pi x^2}{2}\right) - 4ax^3\sin\left(\frac{\pi x^2}{2}\right) +\right.}{4b^4}$$

$$= \frac{1}{4}x^4 S(a+bx) - \frac{\text{Subst}\left(\int x^4\sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{4b^4} + \frac{a\,\text{Subst}\left(\int x^3\sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{b^4}$$

$$= \frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} - \frac{a^4S(a+bx)}{4b^4} + \frac{1}{4}x^4S(a+bx)$$

$$= -\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{a(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{(a}{}$$

$$= -\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{a(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{(a}{}$$

Mathematica [A] time = 0.37, size = 166, normalized size = 0.72

$$\frac{\left(-\pi^2a^4+\pi^2b^4x^4+3\right)S(a+bx) - \pi a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - 6\pi a^2C(a+bx) + \pi a^2bx\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + \pi b^3x^3\,\text{c}}{4\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelS[a + b*x],x]
```

```
[Out] (-(a^3*Pi*Cos[(Pi*(a + b*x)^2)/2]) + a^2*b*Pi*x*Cos[(Pi*(a + b*x)^2)/2] - a
*b^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] + b^3*Pi*x^3*Cos[(Pi*(a + b*x)^2)/2] -
6*a^2*Pi*FresnelC[a + b*x] + (3 - a^4*Pi^2 + b^4*Pi^2*x^4)*FresnelS[a + b*x
] + 5*a*Sin[(Pi*(a + b*x)^2)/2] - 3*b*x*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^
2)
```

fricas [F] time = 0.38, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\text{fresnels}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnels(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{fresnels}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnels(b*x + a), x)
```

maple [A] time = 0.02, size = 189, normalized size = 0.83

$$\frac{\frac{\mathrm{S}(bx+a)b^4x^4}{4}+\frac{(bx+a)^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{4\pi}-\frac{3\left(\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{\mathrm{S}(bx+a)}{\pi}\right)}{4\pi}-\frac{a(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{2a\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2}+\frac{3a^2(bx+a)\cos\left(\frac{\pi(bx-}{2}\right.}{2\pi}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x+a),x)
```

```
[Out] 1/b^4*(1/4*FresnelS(b*x+a)*b^4*x^4+1/4/Pi*(b*x+a)^3*cos(1/2*Pi*(b*x+a)^2)-3
/4/Pi*(1/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-1/Pi*FresnelS(b*x+a))-a/Pi*(b*x+a
)^2*cos(1/2*Pi*(b*x+a)^2)+2*a/Pi^2*sin(1/2*Pi*(b*x+a)^2)+3/2*a^2/Pi*(b*x+a)
*cos(1/2*Pi*(b*x+a)^2)-3/2*a^2/Pi*FresnelC(b*x+a)-a^3/Pi*cos(1/2*Pi*(b*x+a)
^2)-1/4*a^4*FresnelS(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{fresnels}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnels(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\mathrm{FresnelS}\,(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(a + b*x),x)
```

```
[Out] int(x^3*FresnelS(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 S\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnels(b*x+a),x)
```

```
[Out] Integral(x**3*fresnels(a + b*x), x)
```

3.26 $\int x^2 S(a+bx)\,dx$

Optimal. Leaf size=147

$$\frac{a^3 S(a+bx)}{3b^3}+\frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{aC(a+bx)}{\pi b^3}-\frac{2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2 b^3}-\frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{(a+bx)^2 c}{}$$

```
[Out] a^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi-a*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi+1/3
*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi+a*FresnelC(b*x+a)/b^3/Pi+1/3*a^3*Fr
esnelS(b*x+a)/b^3+1/3*x^3*FresnelS(b*x+a)-2/3*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi^
2
```

Rubi [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352, 3296, 2637}

$$\frac{a^3 S(a+bx)}{3b^3}+\frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{a\text{FresnelC}(a+bx)}{\pi b^3}-\frac{2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2 b^3}-\frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[a + b*x],x]
```

```
[Out] (a^2*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (a*(a + b*x)*Cos[(Pi*(a + b*x)^2)/
2])/(b^3*Pi) + ((a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi) + (a*Fresne
lC[a + b*x])/(b^3*Pi) + (a^3*FresnelS[a + b*x])/(3*b^3) + (x^3*FresnelS[a +
 b*x])/3 - (2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2 S(a+bx)\,dx = \frac{1}{3}x^3 S(a+bx) - \frac{1}{3}b\int x^3 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx$$
$$= \frac{1}{3}x^3 S(a+bx) - \frac{\text{Subst}\left(\int\left(-a^3\sin\left(\frac{\pi x^2}{2}\right)+3a^2x\sin\left(\frac{\pi x^2}{2}\right)-3ax^2\sin\left(\frac{\pi x^2}{2}\right)+x^3\sin\left(\frac{\pi x^2}{2}\right)\right)\right.}{3b^3}$$
$$= \frac{1}{3}x^3 S(a+bx) - \frac{\text{Subst}\left(\int x^3\sin\left(\frac{\pi x^2}{2}\right)dx,x,a+bx\right)}{3b^3} + \frac{a\,\text{Subst}\left(\int x^2\sin\left(\frac{\pi x^2}{2}\right)dx,x,a+bx\right.}{b^3}$$
$$= -\frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{a^3S(a+bx)}{3b^3} + \frac{1}{3}x^3S(a+bx) - \frac{\text{Subst}\left(\int x\sin\left(\frac{\pi x}{2}\right)dx,x,(\right.}{6b^3}$$
$$= \frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} + \frac{aC(a}{b}$$
$$= \frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} + \frac{aC(a}{b}$$

Mathematica [A] time = 0.30, size = 115, normalized size = 0.78

$$\frac{\pi^2\left(a^3+b^3x^3\right)S(a+bx)+\pi a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi b^2x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+3\pi aC(a+bx)-2\sin\left(\frac{1}{2}\pi(a+b\right.}{3\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelS[a + b*x],x]
```

```
[Out] (a^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - a*b*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + b^2*Pi
*x^2*Cos[(Pi*(a + b*x)^2)/2] + 3*a*Pi*FresnelC[a + b*x] + Pi^2*(a^3 + b^3*x
^3)*FresnelS[a + b*x] - 2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

fricas [F] time = 0.38, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\text{fresnels}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnels(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{fresnels}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnels(b*x + a), x)
```

maple [A] time = 0.02, size = 121, normalized size = 0.82

$$\frac{\frac{b^3x^3\mathrm{S}(bx+a)}{3}+\frac{(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{3\pi}-\frac{2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{3\pi^2}-\frac{a(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{a\,\mathrm{FresnelC}(bx+a)}{\pi}+\frac{a^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{a^3\mathrm{S}(bx+a)}{3}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x+a),x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*FresnelS(b*x+a)+1/3/Pi*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)-2
/3/Pi^2*sin(1/2*Pi*(b*x+a)^2)-a/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+a/Pi*Fresn
elC(b*x+a)+a^2/Pi*cos(1/2*Pi*(b*x+a)^2)+1/3*a^3*FresnelS(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{fresnels}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnels(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{FresnelS}\,(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(a + b*x),x)
```

```
[Out] int(x^2*FresnelS(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 S\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(b*x+a),x)
```

```
[Out] Integral(x**2*fresnels(a + b*x), x)
```

3.27 $\int xS(a+bx)\,dx$

Optimal. Leaf size=96

$$-\frac{a^2S(a+bx)}{2b^2}-\frac{C(a+bx)}{2\pi b^2}-\frac{a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}+\frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{1}{2}x^2S(a+bx)$$

```
[Out] -a*cos(1/2*Pi*(b*x+a)^2)/b^2/Pi+1/2*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^2/Pi-1/
2*FresnelC(b*x+a)/b^2/Pi-1/2*a^2*FresnelS(b*x+a)/b^2+1/2*x^2*FresnelS(b*x+a
)
```

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6428, 3433, 3351, 3379, 2638, 3385, 3352}

$$-\frac{a^2S(a+bx)}{2b^2}-\frac{\text{FresnelC}(a+bx)}{2\pi b^2}-\frac{a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}+\frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{1}{2}x^2S(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[a + b*x],x]
```

```
[Out] -((a*Cos[(Pi*(a + b*x)^2)/2])/(b^2*Pi)) + ((a + b*x)*Cos[(Pi*(a + b*x)^2)/2
])/(2*b^2*Pi) - FresnelC[a + b*x]/(2*b^2*Pi) - (a^2*FresnelS[a + b*x])/(2*b
^2) + (x^2*FresnelS[a + b*x])/2
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6428

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelS[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int xS(a+bx)\,dx &= \frac{1}{2}x^2S(a+bx) - \frac{1}{2}b\int x^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx \\
&= \frac{1}{2}x^2S(a+bx) - \frac{\text{Subst}\left(\int\left(a^2\sin\left(\frac{\pi x^2}{2}\right) - 2ax\sin\left(\frac{\pi x^2}{2}\right) + x^2\sin\left(\frac{\pi x^2}{2}\right)\right)dx, x, a+bx\right)}{2b^2} \\
&= \frac{1}{2}x^2S(a+bx) - \frac{\text{Subst}\left(\int x^2\sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{2b^2} + \frac{a\,\text{Subst}\left(\int x\sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{b^2} - \\
&= \frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{a^2S(a+bx)}{2b^2} + \frac{1}{2}x^2S(a+bx) + \frac{a\,\text{Subst}\left(\int\sin\left(\frac{\pi x}{2}\right)dx, x, (a+b\right.}{2b^2} \\
&= -\frac{a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} + \frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{C(a+bx)}{2b^2\pi} - \frac{a^2S(a+bx)}{2b^2} + \frac{1}{2}x^2S(a+b
\end{aligned}$$

Mathematica [A] time = 0.19, size = 51, normalized size = 0.53

$$-\frac{C(a+bx)+(a-bx)\left(\pi(a+bx)S(a+bx)+\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\right)}{2\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelS[a + b*x],x]
```

```
[Out] -1/2*(FresnelC[a + b*x] + (a - b*x)*(Cos[(Pi*(a + b*x)^2)/2] + Pi*(a + b*x)
*FresnelS[a + b*x]))/(b^2*Pi)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(x\text{fresnels}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*fresnels(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnels}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*fresnels(b*x + a), x)
```

maple [A] time = 0.02, size = 80, normalized size = 0.83

$$\frac{\mathrm{S}\left(bx+a\right)\left(\frac{(bx+a)^2}{2}-a\left(bx+a\right)\right)+\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}-\frac{\mathrm{FresnelC}(bx+a)}{2\pi}-\frac{a\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x+a),x)
```

```
[Out] 1/b^2*(FresnelS(b*x+a)*(1/2*(b*x+a)^2-a*(b*x+a))+1/2/Pi*(b*x+a)*cos(1/2*Pi*
(b*x+a)^2)-1/2/Pi*FresnelC(b*x+a)-a/Pi*cos(1/2*Pi*(b*x+a)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnels}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnels(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\text{FresnelS}\,(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(a + b*x),x)
```

```
[Out] int(x*FresnelS(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xS\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x+a),x)
```

```
[Out] Integral(x*fresnels(a + b*x), x)
```

3.28 $\int S(a+bx)\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx)S(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[Out] cos(1/2*Pi*(b*x+a)^2)/b/Pi+(b*x+a)*FresnelS(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6418}

$$\frac{(a+bx)S(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[a + b*x],x]
```

```
[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b
```

Rule 6418

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelS[a +
 b*x])/b, x] + Simp[Cos[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int S(a+bx)\,dx=\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)S(a+bx)}{b}$$

Mathematica [B] time = 0.04, size = 89, normalized size = 2.47

$$-\frac{\sin\left(\frac{\pi a^2}{2}\right)\sin\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}+\frac{\cos\left(\frac{\pi a^2}{2}\right)\cos\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(a+bx)+\frac{aS(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[a + b*x],x]
```

```
[Out] (Cos[(a^2*Pi)/2]*Cos[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi) + (a*FresnelS[a + b
*x])/b + x*FresnelS[a + b*x] - (Sin[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnels}\left(bx + a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\left(bx + a\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a), x)
```

maple [A] time = 0.00, size = 33, normalized size = 0.92

$$\frac{(bx + a) \, \mathrm{S}\left(bx + a\right) + \frac{\cos\left(\frac{\pi (bx+a)^2}{2}\right)}{\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*FresnelS(b*x+a)+1/Pi*cos(1/2*Pi*(b*x+a)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\left(bx + a\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \text{FresnelS}\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x),x)
```

```
[Out] int(FresnelS(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int S\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x)
```

```
[Out] Integral(fresnels(a + b*x), x)
```

3.29 $\int \frac{S(a+bx)}{x}\,dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(\frac{S(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)/x,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[a + b*x]/x,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{S(a+bx)}{x}\,dx = \int \frac{S(a+bx)}{x}\,dx$$

Mathematica [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]/x,x]
```

```
[Out] Integrate[FresnelS[a + b*x]/x, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\left(bx+a\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)/x, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)/x,x)
```

```
[Out] int(FresnelS(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{FresnelS}\,(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)/x,x)
```

```
[Out] int(FresnelS(a + b*x)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S\,(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x,x)
```

```
[Out] Integral(fresnels(a + b*x)/x, x)
```

3.30 $\int \frac{S(a+bx)}{x^2} dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{S(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)/x^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/x^2, x]
```

Rubi steps

$$\int \frac{S(a+bx)}{x^2} dx = \int \frac{S(a+bx)}{x^2} dx$$

Mathematica [A] time = 3.67, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]/x^2,x]
```

```
[Out] Integrate[FresnelS[a + b*x]/x^2, x]
```

fricas [A] time = 0.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\left(bx+a\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(fresnels(b*x + a)/x^2, x)
```

maple [A] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)/x^2,x)
[Out] int(FresnelS(b*x+a)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x^2,x, algorithm="maxima")
[Out] integrate(fresnels(b*x + a)/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\text{FresnelS}\,(a + b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)/x^2,x)
[Out] int(FresnelS(a + b*x)/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S\,(a + bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x**2,x)
```

```
[Out] Integral(fresnels(a + b*x)/x**2, x)
```

3.31 $\int x^7 S(bx)^2\, dx$

Optimal. Leaf size=253

$$-\frac{105S(bx)^2}{8\pi^4b^8}-\frac{105x^2}{16\pi^4b^6}+\frac{x^7S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b}+\frac{7x^6}{48\pi^2b^2}+\frac{x^6\cos\left(\pi b^2x^2\right)}{16\pi^2b^2}+\frac{10\sin\left(\pi b^2x^2\right)}{\pi^5b^8}+\frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^4b^7}$$

```
[Out] -105/16*x^2/b^6/Pi^4+7/48*x^6/b^2/Pi^2-55/16*x^2*cos(b^2*Pi*x^2)/b^6/Pi^4+1
/16*x^6*cos(b^2*Pi*x^2)/b^2/Pi^2-35/4*x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)
/b^5/Pi^3+1/4*x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi-105/8*FresnelS(b*x
)^2/b^8/Pi^4+1/8*x^8*FresnelS(b*x)^2+105/4*x*FresnelS(b*x)*sin(1/2*b^2*Pi*x
^2)/b^7/Pi^4-7/4*x^5*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2+10*sin(b^2*
Pi*x^2)/b^8/Pi^5-5/8*x^4*sin(b^2*Pi*x^2)/b^4/Pi^3
```

Rubi [A] time = 0.43, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6430, 6454, 6462, 3379, 3309, 30, 3296, 2637, 2634, 6440}

$$-\frac{7x^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2b^3}+\frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^4b^7}+\frac{x^7S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b}-\frac{35x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^3b^5}-\frac{105S(bx)^2}{8\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelS[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) - (35*x^3*Cos[(b^2*Pi*
x^2)/2]*FresnelS[b*x])/(4*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x
])/(4*b*Pi) - (105*FresnelS[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelS[b*x]^2)/8
+ (105*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^7*Pi^4) - (7*x^5*FresnelS[
b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) -
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^7 S(bx)^2\,dx = \frac{1}{8}x^8 S(bx)^2 - \frac{1}{4}b\int x^8 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$
$$= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} + \frac{1}{8}x^8 S(bx)^2 - \frac{\int x^7\sin\left(b^2\pi x^2\right)dx}{8\pi} - \frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{4b\pi}$$
$$= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} + \frac{1}{8}x^8 S(bx)^2 - \frac{7x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35\int x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{4b^3\pi^2}$$
$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} + \frac{1}{8}x^8 S(bx)^2 - \frac{7x^5 S(bx)\,\mathrm{si}}{4b^3}$$
$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} + \frac{1}{8}x^8 S(bx)^2 + \frac{105xS(bx)}{4b}$$
$$= \frac{7x^6}{48b^2\pi^2} - \frac{41x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(}{4b\pi}$$
$$= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos}{}$$
$$= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos}{}$$

Mathematica [A] time = 0.02, size = 253, normalized size = 1.00

$$-\frac{105S(bx)^2}{8\pi^4 b^8} - \frac{105x^2}{16\pi^4 b^6} + \frac{x^7 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} + \frac{7x^6}{48\pi^2 b^2} + \frac{x^6\cos\left(\pi b^2 x^2\right)}{16\pi^2 b^2} + \frac{10\sin\left(\pi b^2 x^2\right)}{\pi^5 b^8} + \frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right.}{4\pi^4 b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelS[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) - (35*x^3*Cos[(b^2*Pi*
x^2)/2]*FresnelS[b*x])/(4*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x
])/(4*b*Pi) - (105*FresnelS[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelS[b*x]^2)/8
+ (105*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^7*Pi^4) - (7*x^5*FresnelS[
b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) -
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

fricas [F] time = 0.38, size = 0, normalized size = 0.00

$$\text{integral}\left(x^7 \text{fresnels}\,(bx)^2\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^7*fresnels(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \text{fresnels}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnels(b*x)^2, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^7 \mathrm{S}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x)^2,x)
```

```
[Out] int(x^7*FresnelS(b*x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \text{fresnels}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnels(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \operatorname{FresnelS}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x)^2,x)
[Out] int(x^7*FresnelS(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 S^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnels(b*x)**2,x)
[Out] Integral(x**7*fresnels(b*x)**2, x)
```

3.32 $\int x^6 S(bx)^2\,dx$

Optimal. Leaf size=239

$$\frac{531C\left(\sqrt{2}\,bx\right)}{56\sqrt{2}\,\pi^4b^7}-\frac{48x}{7\pi^4b^6}+\frac{2x^6S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{6x^5}{35\pi^2b^2}+\frac{x^5\cos\left(\pi b^2x^2\right)}{14\pi^2b^2}+\frac{96S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}-\frac{21x\cos\left(\pi b^2\right.}{8\pi^4b^6}$$

```
[Out] -48/7*x/b^6/Pi^4+6/35*x^5/b^2/Pi^2-21/8*x*cos(b^2*Pi*x^2)/b^6/Pi^4+1/14*x^5
*cos(b^2*Pi*x^2)/b^2/Pi^2-48/7*x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^5/Pi
^3+2/7*x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+1/7*x^7*FresnelS(b*x)^2+9
6/7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^7/Pi^4-12/7*x^4*FresnelS(b*x)*sin(1
/2*b^2*Pi*x^2)/b^3/Pi^2-17/28*x^3*sin(b^2*Pi*x^2)/b^4/Pi^3+531/112*FresnelC
(b*x*2^(1/2))/b^7/Pi^4*2^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6430, 6454, 6462, 3391, 30, 3386, 3385, 3352, 6460, 3357}

$$\frac{531\text{FresnelC}\left(\sqrt{2}\,bx\right)}{56\sqrt{2}\,\pi^4b^7}-\frac{12x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{96S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}+\frac{2x^6S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}-\frac{48x^2S(bx)\,\mathrm{co}}{7\pi^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelS[b*x]^2,x]
```

```
[Out] (-48*x)/(7*b^6*Pi^4) + (6*x^5)/(35*b^2*Pi^2) - (21*x*Cos[b^2*Pi*x^2])/(8*b^
6*Pi^4) + (x^5*Cos[b^2*Pi*x^2])/(14*b^2*Pi^2) + (531*FresnelC[Sqrt[2]*b*x])
/(56*Sqrt[2]*b^7*Pi^4) - (48*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*b^5*
Pi^3) + (2*x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*b*Pi) + (x^7*FresnelS[
b*x]^2)/7 + (96*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (12*x^4*F
resnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2) - (17*x^3*Sin[b^2*Pi*x^2])/(
28*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
```

```
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^6 S(bx)^2\,dx = \frac{1}{7}x^7 S(bx)^2 - \frac{1}{7}(2b)\int x^7 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$= \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b\pi} + \frac{1}{7}x^7 S(bx)^2 - \frac{\int x^6\sin\left(b^2\pi x^2\right)dx}{7\pi} - \frac{12\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{7b\pi}$$

$$= \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b\pi} + \frac{1}{7}x^7 S(bx)^2 - \frac{12x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{48\int x^3 S}{}$$

$$= \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b^5\pi^3} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b\pi} + \frac{1}{7}x^7 S(bx)^2 - \frac{12x^4 S(b}{}$$

$$= \frac{6x^5}{35b^2\pi^2} - \frac{111x\cos\left(b^2\pi x^2\right)}{56b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b^5\pi^3} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi}$$

$$= \frac{6x^5}{35b^2\pi^2} - \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{15C\left(\sqrt{2}\,bx\right)}{56\sqrt{2}\,b^7\pi^4} + \frac{6\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{7b^7\pi^4} - \frac{48x^2\cos\left(\frac{1}{2}\right.}{7b^5}$$

$$= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} - \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{51C\left(\sqrt{2}\,bx\right)}{56\sqrt{2}\,b^7\pi^4} + \frac{6\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{7b^7\pi^4} - \frac{4}{}$$

$$= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} - \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{51C\left(\sqrt{2}\,bx\right)}{56\sqrt{2}\,b^7\pi^4} + \frac{30\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{7b^7\pi^4} -$$

Mathematica [A] time = 0.33, size = 171, normalized size = 0.72

$$\frac{80\pi^4 b^7 x^7 S(bx)^2 + 160S(bx)\left(\pi b^2 x^2\left(\pi^2 b^4 x^4 - 24\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right) - 6\left(\pi^2 b^4 x^4 - 8\right)\sin\left(\frac{1}{2}\pi b^2 x^2\right)\right) + 2bx\left(5\left(4\pi^2 b\right.\right.}{560\pi^4 b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelS[b*x]^2,x]
```

```
[Out] (2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 80*b^7*Pi^4*x^7*FresnelS[b*x]^2 + 160
*FresnelS[b*x]*(b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8
 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(5*(-147 + 4*b^4*Pi^2*x^4)*Co
s[b^2*Pi*x^2] - 2*(960 - 24*b^4*Pi^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))
/(560*b^7*Pi^4)
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^6\mathrm{fresnels}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^6*fresnels(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6\mathrm{fresnels}\,(bx)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnels(b*x)^2, x)
```

maple [A] time = 0.09, size = 324, normalized size = 1.36

$$\frac{\frac{b^7x^7\mathrm{S}(bx)^2}{7}-2\,\mathrm{S}\,(bx)\left(-\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}+\frac{\frac{6b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}-\frac{24\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{7\pi}}{\pi}\right)+\frac{\frac{6}{35}\pi^2b^5x^5-\frac{48}{7}bx}{\pi^4}-\frac{6\left(\frac{\pi b^3x^3\sin\left(b^2\pi x^2\right)}{2}\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x)^2,x)
```

```
[Out] 1/b^7*(1/7*b^7*x^7*FresnelS(b*x)^2-2*FresnelS(b*x)*(-1/7/Pi*b^6*x^6*cos(1/2
*b^2*Pi*x^2)+6/7/Pi*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*c
os(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2))))+6/7/Pi^4*(1/5*Pi^2*b^5*x^5
-8*b*x)-6/7/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^
```

```
2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1
/2)))-1/7/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(
b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x
*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \text{fresnels}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnels(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\,\mathrm{FresnelS}\left(b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x)^2,x)
```

```
[Out] int(x^6*FresnelS(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 S^2\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**6*fresnels(b*x)**2, x)
```

3.33 $\int x^5 S(bx)^2\, dx$

Optimal. Leaf size=265

$$-\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}+\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}+\frac{5C(bx)S(bx)}{2\pi^3b^6}+\frac{x^5S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}+\frac{5x^4}{24\pi^2b^2}+\frac{x^4}{}$$

```
[Out] 5/24*x^4/b^2/Pi^2-11/6*cos(b^2*Pi*x^2)/b^6/Pi^4+1/12*x^4*cos(b^2*Pi*x^2)/b^
2/Pi^2-5*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^5/Pi^3+1/3*x^5*cos(1/2*b^2*P
i*x^2)*FresnelS(b*x)/b/Pi+5/2*FresnelC(b*x)*FresnelS(b*x)/b^6/Pi^3+1/6*x^6*
FresnelS(b*x)^2-5/8*I*x^2*HypergeometricPFQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x
^2)/b^4/Pi^3+5/8*I*x^2*HypergeometricPFQ([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/
b^4/Pi^3-5/3*x^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2-7/12*x^2*sin(b^
2*Pi*x^2)/b^4/Pi^3
```

Rubi [A] time = 0.30, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6430, 6454, 6462, 3379, 3309, 30, 3296, 2638, 6446}

$$-\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}+\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}+\frac{5\text{FresnelC}(bx)S(bx)}{2\pi^3b^6}-\frac{5x^3S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{x^5}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelS[b*x]^2,x]
```

```
[Out] (5*x^4)/(24*b^2*Pi^2) - (11*Cos[b^2*Pi*x^2])/(6*b^6*Pi^4) + (x^4*Cos[b^2*Pi
*x^2])/(12*b^2*Pi^2) - (5*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^5*Pi^3) +
 (x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*b*Pi) + (5*FresnelC[b*x]*Fresne
lS[b*x])/(2*b^6*Pi^3) + (x^6*FresnelS[b*x]^2)/6 - (((5*I)/8)*x^2*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^4*Pi^3) + (((5*I)/8)*x^2*H
ypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^4*Pi^3) - (5*x^3*F
resnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) - (7*x^2*Sin[b^2*Pi*x^2])/(1
2*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
```

```
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^5 S(bx)^2\,dx = \frac{1}{6}x^6 S(bx)^2 - \frac{1}{3}b\int x^6 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{6}x^6 S(bx)^2 - \frac{\int x^5\sin\left(b^2\pi x^2\right)dx}{6\pi} - \frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{3b\pi}$$

$$= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{6}x^6 S(bx)^2 - \frac{5x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{5\int x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^3\pi^2} +$$

$$= \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{6}x^6 S(bx)^2 - \frac{5x^3 S(bx)\sin}{3b^3\pi}$$

$$= \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{5C(bx)S(bx)}{2b^6\pi^3} + \frac{1}{6}x^6 S(bx)^2$$

$$= \frac{5x^4}{24b^2\pi^2} - \frac{17\cos\left(b^2\pi x^2\right)}{12b^6\pi^4} + \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi}$$

$$= \frac{5x^4}{24b^2\pi^2} - \frac{11\cos\left(b^2\pi x^2\right)}{6b^6\pi^4} + \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int x^5 S(bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^5*FresnelS[b*x]^2,x]
[Out] Integrate[x^5*FresnelS[b*x]^2, x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^5\mathrm{fresnels}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5*fresnels(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \text{fresnels}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnels(b*x)^2, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^5 \mathrm{S}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x)^2,x)
```

```
[Out] int(x^5*FresnelS(b*x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \text{fresnels}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnels(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \, \mathrm{FresnelS}(b\,x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x)^2,x)
```

```
[Out] int(x^5*FresnelS(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 S^2(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**5*fresnels(b*x)**2, x)
```

3.34 $\int x^4 S(bx)^2 \, dx$

Optimal. Leaf size=177

$$\frac{43S\left(\sqrt{2}\,bx\right)}{20\sqrt{2}\,\pi^3b^5}+\frac{2x^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}+\frac{4x^3}{15\pi^2b^2}+\frac{x^3\cos\left(\pi b^2x^2\right)}{10\pi^2b^2}-\frac{16S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}-\frac{11x\sin\left(\pi b^2x^2\right)}{20\pi^3b^4}-\frac{8x^2S}{}$$

```
[Out] 4/15*x^3/b^2/Pi^2+1/10*x^3*cos(b^2*Pi*x^2)/b^2/Pi^2-16/5*cos(1/2*b^2*Pi*x^2
)*FresnelS(b*x)/b^5/Pi^3+2/5*x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+1/5
*x^5*FresnelS(b*x)^2-8/5*x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2-11/
20*x*sin(b^2*Pi*x^2)/b^4/Pi^3+43/40*FresnelS(b*x*2^(1/2))/b^5/Pi^3*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6430, 6454, 6462, 3391, 30, 3386, 3351, 6452, 3385}

$$-\frac{8x^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}+\frac{2x^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}-\frac{16S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}+\frac{43S\left(\sqrt{2}\,bx\right)}{20\sqrt{2}\,\pi^3b^5}+\frac{4x^3}{15\pi^2b^2}-\frac{11x\sin\left(\pi b^2\right.}{20\pi^3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelS[b*x]^2,x]
```

```
[Out] (4*x^3)/(15*b^2*Pi^2) + (x^3*Cos[b^2*Pi*x^2])/(10*b^2*Pi^2) - (16*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(5*b^5*Pi^3) + (2*x^4*Cos[(b^2*Pi*x^2)/2]*Fresnel
S[b*x])/(5*b*Pi) + (x^5*FresnelS[b*x]^2)/5 + (43*FresnelS[Sqrt[2]*b*x])/(20
*Sqrt[2]*b^5*Pi^3) - (8*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
 - (11*x*Sin[b^2*Pi*x^2])/(20*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
```

```
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^4 S(bx)^2\,dx = \frac{1}{5}x^5 S(bx)^2 - \frac{1}{5}(2b)\int x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$
$$= \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 - \frac{\int x^4\sin\left(b^2\pi x^2\right)dx}{5\pi} - \frac{8\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{5b\pi}$$
$$= \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 - \frac{8x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{16\int xS(b\cdots}{\cdots}$$
$$= \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{16\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b^5\pi^3} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 - \frac{8x^2 S(bx)\,\mathrm{s}\cdots}{5b\cdots}$$
$$= \frac{4x^3}{15b^2\pi^2} + \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{16\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b^5\pi^3} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 + \cdots$$
$$= \frac{4x^3}{15b^2\pi^2} + \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{16\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b^5\pi^3} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 + \cdots$$

Mathematica [A] time = 0.17, size = 137, normalized size = 0.77

$$\frac{24\pi^3 b^5 x^5 S(bx)^2 + 32\pi b^3 x^3 - 66bx\sin\left(\pi b^2 x^2\right) + 48S(bx)\left(\left(\pi^2 b^4 x^4 - 8\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right) - 4\pi b^2 x^2\sin\left(\frac{1}{2}\pi b^2 x^2\right)\right)\cdots}{120\pi^3 b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelS[b*x]^2,x]
```

```
[Out] (32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 24*b^5*Pi^3*x^5*FresnelS[b
*x]^2 + 129*Sqrt[2]*FresnelS[Sqrt[2]*b*x] + 48*FresnelS[b*x]*((-8 + b^4*Pi^
2*x^4)*Cos[(b^2*Pi*x^2)/2] - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin
[b^2*Pi*x^2])/(120*b^5*Pi^3)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(x^4\text{fresnels}\,(bx)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*fresnels(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \text{fresnels}\,(bx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnels(b*x)^2, x)
```

maple [A] time = 0.06, size = 208, normalized size = 1.18

$$\frac{\frac{b^5x^5\mathrm{S}(bx)^2}{5}-2\,\mathrm{S}\,(bx)\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi^2}}{\pi}\right)+\frac{4b^3x^3}{15\pi^2}-\frac{4\left(\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}-\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{4\pi}\right)}{5\pi^2}-\frac{-\frac{\pi b^3x^3\cos\left(b^2\pi x^2\right)}{2}}{}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x)^2,x)
```

```
[Out] 1/b^5*(1/5*b^5*x^5*FresnelS(b*x)^2-2*FresnelS(b*x)*(-1/5/Pi*b^4*x^4*cos(1/2
*b^2*Pi*x^2)+4/5/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi
*x^2)))+4/15/Pi^2*b^3*x^3-4/5/Pi^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/
2)*FresnelS(b*x*2^(1/2)))-1/5/Pi^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*
(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)
*FresnelS(b*x*2^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \text{fresnels}\,(bx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4*fresnels(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{FresnelS}\left(b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x)^2,x)
```

```
[Out] int(x^4*FresnelS(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 S^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**4*fresnels(b*x)**2, x)
```

3.35 $\int x^3 S(bx)^2\,dx$

Optimal. Leaf size=140

$$\frac{3S(bx)^2}{4\pi^2b^4}+\frac{x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{3x^2}{8\pi^2b^2}+\frac{x^2\cos\left(\pi b^2x^2\right)}{8\pi^2b^2}-\frac{\sin\left(\pi b^2x^2\right)}{2\pi^3b^4}-\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3}+\frac{1}{4}x^4S(bx)^2$$

```
[Out] 3/8*x^2/b^2/Pi^2+1/8*x^2*cos(b^2*Pi*x^2)/b^2/Pi^2+1/2*x^3*cos(1/2*b^2*Pi*x^
2)*FresnelS(b*x)/b/Pi+3/4*FresnelS(b*x)^2/b^4/Pi^2+1/4*x^4*FresnelS(b*x)^2-
3/2*x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2-1/2*sin(b^2*Pi*x^2)/b^4/Pi
^3
```

Rubi [A] time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6430, 6454, 6462, 3379, 2634, 6440, 30, 3296, 2637}

$$-\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3}+\frac{x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{3S(bx)^2}{4\pi^2b^4}+\frac{3x^2}{8\pi^2b^2}-\frac{\sin\left(\pi b^2x^2\right)}{2\pi^3b^4}+\frac{x^2\cos\left(\pi b^2x^2\right)}{8\pi^2b^2}+\frac{1}{4}x^4S(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelS[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) + (x^3*Cos[(b^2*P
i*x^2)/2]*FresnelS[b*x])/(2*b*Pi) + (3*FresnelS[b*x]^2)/(4*b^4*Pi^2) + (x^4
*FresnelS[b*x]^2)/4 - (3*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b^3*Pi^2)
- Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^3 S(bx)^2\,dx = \frac{1}{4}x^4 S(bx)^2 - \frac{1}{2}b\int x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$
$$= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi} + \frac{1}{4}x^4 S(bx)^2 - \frac{\int x^3\sin\left(b^2\pi x^2\right)dx}{4\pi} - \frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{2b\pi}$$
$$= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi} + \frac{1}{4}x^4 S(bx)^2 - \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2} + \frac{3\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{2b^3\pi^2} + \frac{3}{}$$
$$= \frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} + \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi} + \frac{1}{4}x^4 S(bx)^2 - \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2} + \frac{3\,\text{Subst}(\int x\,d}{2b^4\pi}$$
$$= \frac{3x^2}{8b^2\pi^2} + \frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} + \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi} + \frac{3S(bx)^2}{4b^4\pi^2} + \frac{1}{4}x^4 S(bx)^2 - \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{2b^3\pi^2}$$

Mathematica [A] time = 0.01, size = 140, normalized size = 1.00

$$\frac{3S(bx)^2}{4\pi^2 b^4} + \frac{x^3 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi b} + \frac{3x^2}{8\pi^2 b^2} + \frac{x^2\cos\left(\pi b^2 x^2\right)}{8\pi^2 b^2} - \frac{\sin\left(\pi b^2 x^2\right)}{2\pi^3 b^4} - \frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi^2 b^3} + \frac{1}{4}x^4 S(bx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelS[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) + (x^3*Cos[(b^2*P
i*x^2)/2]*FresnelS[b*x])/(2*b*Pi) + (3*FresnelS[b*x]^2)/(4*b^4*Pi^2) + (x^4
*FresnelS[b*x]^2)/4 - (3*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b^3*Pi^2)
- Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\text{fresnels}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnels(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{fresnels}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnels(b*x)^2, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^3 \mathrm{S}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x)^2,x)
```

```
[Out] int(x^3*FresnelS(b*x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \mathrm{fresnels}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnels(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{FresnelS}\left(b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x)^2,x)
```

```
[Out] int(x^3*FresnelS(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} S^{2}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**3*fresnels(b*x)**2, x)
```

3.36 $\int x^2 S(bx)^2 \, dx$

Optimal. Leaf size=124

$$-\frac{5C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,\pi^2 b^3}+\frac{2x^2 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi b}+\frac{x\cos\left(\pi b^2 x^2\right)}{6\pi^2 b^2}+\frac{2x}{3\pi^2 b^2}-\frac{4S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi^2 b^3}+\frac{1}{3}x^3 S(bx)^2$$

```
[Out] 2/3*x/b^2/Pi^2+1/6*x*cos(b^2*Pi*x^2)/b^2/Pi^2+2/3*x^2*cos(1/2*b^2*Pi*x^2)*F
resnelS(b*x)/b/Pi+1/3*x^3*FresnelS(b*x)^2-4/3*FresnelS(b*x)*sin(1/2*b^2*Pi*
x^2)/b^3/Pi^2-5/12*FresnelC(b*x*2^(1/2))/b^3/Pi^2*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6430, 6454, 6460, 3357, 3352, 3385}

$$-\frac{5\text{FresnelC}\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,\pi^2 b^3}-\frac{4S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi^2 b^3}+\frac{2x^2 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3\pi b}+\frac{x\cos\left(\pi b^2 x^2\right)}{6\pi^2 b^2}+\frac{2x}{3\pi^2 b^2}+\frac{1}{3}x^3 S(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[b*x]^2,x]
```

```
[Out] (2*x)/(3*b^2*Pi^2) + (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (5*FresnelC[Sqrt[2]
*b*x])/(6*Sqrt[2]*b^3*Pi^2) + (2*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*
b*Pi) + (x^3*FresnelS[b*x]^2)/3 - (4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*
b^3*Pi^2)
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
```

```
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}
\int x^2 S(bx)^2\,dx &= \frac{1}{3}x^3 S(bx)^2 - \frac{1}{3}(2b)\int x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx \\
&= \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3 S(bx)^2 - \frac{\int x^2\sin\left(b^2\pi x^2\right)dx}{3\pi} - \frac{4\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{3b\pi} \\
&= \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} + \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3 S(bx)^2 - \frac{4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} - \frac{\int\cos\left(b^2\pi x^2\right.}{6b^2\pi^2} \\
&= \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2} + \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3 S(bx)^2 - \frac{4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \\
&= \frac{2x}{3b^2\pi^2} + \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2} + \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3 S(bx)^2 - \frac{4S(bx)\sin\left(\frac{1}{2}b\right.}{3b^3\pi^2} \\
&= \frac{2x}{3b^2\pi^2} + \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2} - \frac{\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{3b^3\pi^2} + \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3 S(bx)^2
\end{aligned}$$

Mathematica [A] time = 0.13, size = 100, normalized size = 0.81

$$\frac{4\pi^2 b^3 x^3 S(bx)^2 + 8S(bx)\left(\pi b^2 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) - 2\sin\left(\frac{1}{2}\pi b^2 x^2\right)\right) + 2bx\left(\cos\left(\pi b^2 x^2\right) + 4\right) - 5\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{12\pi^2 b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelS[b*x]^2,x]
```

```
[Out] (2*b*x*(4 + Cos[b^2*Pi*x^2]) - 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 4*b^3*Pi^2
*x^3*FresnelS[b*x]^2 + 8*FresnelS[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] - 2*
Sin[(b^2*Pi*x^2)/2]))/(12*b^3*Pi^2)
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2 \text{fresnels}\,(bx)^2\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnels(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \text{fresnels}\,(bx)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnels(b*x)^2, x)
```

maple [A] time = 0.05, size = 122, normalized size = 0.98

$$\frac{\frac{b^3 x^3 \mathrm{S}(bx)^2}{3} - 2\,\mathrm{S}\,(bx)\left(-\frac{b^2 x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi} + \frac{2\sin\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi^2}\right) + \frac{2bx}{3\pi^2} - \frac{\sqrt{2}\ \mathrm{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2} - \frac{-\frac{bx\cos\left(b^2 \pi x^2\right)}{2\pi} + \frac{\sqrt{2}\ \mathrm{FresnelC}\left(bx\sqrt{2}\right)}{4\pi}}{3\pi}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x)^2,x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*FresnelS(b*x)^2-2*FresnelS(b*x)*(-1/3/Pi*b^2*x^2*cos(1/2
*b^2*Pi*x^2)+2/3/Pi^2*sin(1/2*b^2*Pi*x^2))+2/3/Pi^2*b*x-1/3/Pi^2*2^(1/2)*Fr
```

```
esnelC(b*x*2^(1/2))-1/3/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fres
nelC(b*x*2^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \text{fresnels}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnels(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{FresnelS}\left(b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x)^2,x)
```

```
[Out] int(x^2*FresnelS(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 S^2\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**2*fresnels(b*x)**2, x)
```

3.37 $\int xS(bx)^2\,dx$

Optimal. Leaf size=143

$$\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{C(bx)S(bx)}{2\pi b^2}+\frac{xS(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^2}+\frac{1}{2}x^2$$

```
[Out] 1/4*cos(b^2*Pi*x^2)/b^2/Pi^2+x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi-1/2*F
resnelC(b*x)*FresnelS(b*x)/b^2/Pi+1/2*x^2*FresnelS(b*x)^2+1/8*I*x^2*Hyperge
ometricPFQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/Pi-1/8*I*x^2*HypergeometricPF
Q([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/Pi
```

Rubi [A] time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6430, 6454, 6446, 3379, 2638}

$$\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{\text{FresnelC}(bx)S(bx)}{2\pi b^2}+\frac{xS(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+\frac{\cos\left(\pi b^2x\right.}{4\pi^2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[b*x]^2,x]
```

```
[Out] Cos[b^2*Pi*x^2]/(4*b^2*Pi^2) + (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi)
 - (FresnelC[b*x]*FresnelS[b*x])/(2*b^2*Pi) + (x^2*FresnelS[b*x]^2)/2 + ((I
/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/Pi - ((I/8)
*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/Pi
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
```

```
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rubi steps

$$
\begin{aligned}
\int xS(bx)^2\,dx &= \frac{1}{2}x^2S(bx)^2 - b\int x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\
&= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} + \frac{1}{2}x^2S(bx)^2 - \frac{\int x\sin\left(b^2\pi x^2\right)dx}{2\pi} - \frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b\pi}\\
&= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} - \frac{C(bx)S(bx)}{2b^2\pi} + \frac{1}{2}x^2S(bx)^2 + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2\left(1,1\right.}{}\\
&= \frac{\cos\left(b^2\pi x^2\right)}{4b^2\pi^2} + \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} - \frac{C(bx)S(bx)}{2b^2\pi} + \frac{1}{2}x^2S(bx)^2 + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right.}{8\pi}
\end{aligned}
$$

Mathematica [F] time = 0.19, size = 0, normalized size = 0.00

$$\int xS(bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*FresnelS[b*x]^2,x]

[Out] Integrate[x*FresnelS[b*x]^2, x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(x\text{fresnels}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)^2,x, algorithm="fricas")
[Out] integral(x*fresnels(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnels}\,(bx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)^2,x, algorithm="giac")
[Out] integrate(x*fresnels(b*x)^2, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x\mathrm{S}\,(bx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x)^2,x)
[Out] int(x*FresnelS(b*x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnels}\,(bx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)^2,x, algorithm="maxima")
[Out] integrate(x*fresnels(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{FresnelS}\left(b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x)^2,x)
```

```
[Out] int(x*FresnelS(b*x)^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int xS^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)**2,x)
```

```
[Out] Integral(x*fresnels(b*x)**2, x)
```

3.38 $\int S(bx)^2\,dx$

Optimal. Leaf size=55

$$\frac{2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(bx)^2-\frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,\pi b}$$

```
[Out] 2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+x*FresnelS(b*x)^2-1/2*FresnelS(b*x
*2^(1/2))/b/Pi*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6420, 12, 6452, 3351}

$$\frac{2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(bx)^2-\frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^2,x]
```

```
[Out] (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) + x*FresnelS[b*x]^2 - FresnelS
[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6420

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelS[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Sin[(Pi*(a + b*x)^2)/2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
```

```
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int S(bx)^2\,dx &= xS(bx)^2 - 2\int bxS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\ &= xS(bx)^2 - (2b)\int xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\ &= \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} + xS(bx)^2 - \frac{\int\sin\left(b^2\pi x^2\right)dx}{\pi}\\ &= \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} + xS(bx)^2 - \frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b\pi}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 1.00

$$\frac{2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b} + xS(bx)^2 - \frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,\pi b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^2,x]
[Out] (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) + x*FresnelS[b*x]^2 - FresnelS
[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnels}\,(bx)^2\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2,x, algorithm="fricas")
[Out] integral(fresnels(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2, x)
```

maple [A] time = 0.03, size = 49, normalized size = 0.89

$$\frac{bx\mathrm{S}\left(bx\right)^{2}+\frac{2\,\mathrm{S}(bx)\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{\pi}-\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{2\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2,x)
```

```
[Out] 1/b*(b*x*FresnelS(b*x)^2+2*FresnelS(b*x)/Pi*cos(1/2*b^2*Pi*x^2)-1/2/Pi*2^(1
/2)*FresnelS(b*x*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{fresnels}\left(bx\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{FresnelS}\left(b\,x\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2,x)
```

```
[Out] int(FresnelS(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int S^{2}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2,x)
```

```
[Out] Integral(fresnels(b*x)**2, x)
```

3.39 $\int \frac{S(bx)^2}{x} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{S(bx)^2}{x},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x)^2/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)^2}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x,x]
[Out] Defer[Int][FresnelS[b*x]^2/x, x]
```

Rubi steps

$$\int \frac{S(bx)^2}{x} \, dx = \int \frac{S(bx)^2}{x} \, dx$$

Mathematica [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x,x]
[Out] Integrate[FresnelS[b*x]^2/x, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x,x)
```

```
[Out] int(FresnelS(b*x)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{FresnelS}\,(b\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x,x)
```

```
[Out] int(FresnelS(b*x)^2/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x,x)
```

```
[Out] Integral(fresnels(b*x)**2/x, x)
```

3.40 $\int \frac{S(bx)^2}{x^2}\,dx$

Optimal. Leaf size=38

$$2b\text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{S(bx)^2}{x}$$

```
[Out] -FresnelS(b*x)^2/x+2*b*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^2,x]
[Out] -(FresnelS[b*x]^2/x) + 2*b*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x
, x]
```

Rubi steps

$$\int \frac{S(bx)^2}{x^2}\,dx=-\frac{S(bx)^2}{x}+(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^2,x]
[Out] Integrate[FresnelS[b*x]^2/x^2, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^2,x, algorithm="fricas")
[Out] integral(fresnels(b*x)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^2,x, algorithm="giac")
[Out] integrate(fresnels(b*x)^2/x^2, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^2,x)
[Out] int(FresnelS(b*x)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^2,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)^2/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{FresnelS}\,(b\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^2,x)
```

```
[Out] int(FresnelS(b*x)^2/x^2, x)
```

sympy [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**2,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**2, x)
```

3.41 $\int \frac{S(bx)^2}{x^3} dx$

Optimal. Leaf size=39

$$b\text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)-\frac{S(bx)^2}{2x^2}$$

```
[Out] -1/2*FresnelS(b*x)^2/x^2+b*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x
^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)^2}{x^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^3,x]
```

```
[Out] -FresnelS[b*x]^2/(2*x^2) + b*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])
/x^2, x]
```

Rubi steps

$$\int \frac{S(bx)^2}{x^3} dx = -\frac{S(bx)^2}{2x^2} + b\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} dx$$

Mathematica [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^3,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^3, x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^3, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^3,x)
```

```
[Out] int(FresnelS(b*x)^2/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\text{FresnelS}\,(b\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^3,x)
```

```
[Out] int(FresnelS(b*x)^2/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**3,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**3, x)
```

3.42 $\int \frac{S(bx)^2}{x^4}\,dx$

Optimal. Leaf size=120

$$\frac{1}{3}\pi b^3 \text{Int}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)+\frac{\pi b^3 S\left(\sqrt{2}\,bx\right)}{3\sqrt{2}}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^2}+\frac{b^2\cos\left(\pi b^2 x^2\right)}{6x}-\frac{b^2}{6x}-\frac{S(bx)^2}{3x^3}$$

```
[Out] -1/6*b^2/x+1/6*b^2*cos(b^2*Pi*x^2)/x-1/3*FresnelS(b*x)^2/x^3-1/3*b*FresnelS
(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+1/6*b^3*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)+1/3*
b^3*Pi*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^4,x]
```

```
[Out] -b^2/(6*x) + (b^2*Cos[b^2*Pi*x^2])/(6*x) - FresnelS[b*x]^2/(3*x^3) + (b^3*P
i*FresnelS[Sqrt[2]*b*x])/(3*Sqrt[2]) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(3*x^2) + (b^3*Pi*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)^2}{x^4}\,dx &= -\frac{S(bx)^2}{3x^3}+\frac{1}{3}(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx\\
&= -\frac{b^2}{6x}-\frac{S(bx)^2}{3x^3}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2}-\frac{1}{6}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx+\frac{1}{3}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\\
&= -\frac{b^2}{6x}+\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{S(bx)^2}{3x^3}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2}+\frac{1}{3}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx+\frac{1}{3}\\
&= -\frac{b^2}{6x}+\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{S(bx)^2}{3x^3}+\frac{b^3\pi S\left(\sqrt{2}\,bx\right)}{3\sqrt{2}}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2}+\frac{1}{3}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi\right.}{x}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^4,x]
[Out] Integrate[FresnelS[b*x]^2/x^4, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^4,x, algorithm="fricas")
[Out] integral(fresnels(b*x)^2/x^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^4,x, algorithm="giac")
[Out] integrate(fresnels(b*x)^2/x^4, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^4,x)
[Out] int(FresnelS(b*x)^2/x^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}\,(b\,x)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^4,x)
[Out] int(FresnelS(b*x)^2/x^4, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**4,x)
[Out] Integral(fresnels(b*x)**2/x**4, x)
```

3.43 $\int \frac{S(bx)^2}{x^5}\,dx$

Optimal. Leaf size=127

$$-\frac{1}{12}\pi^2 b^4 S(bx)^2-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3}-\frac{b^2}{24x^2}+\frac{b^2\cos\left(\pi b^2x^2\right)}{24x^2}+\frac{1}{12}\pi b^4\text{Si}\left(b^2\pi x^2\right)-\frac{\pi b^3 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x}-\frac{S(bx}{4x}$$

```
[Out] -1/24*b^2/x^2+1/24*b^2*cos(b^2*Pi*x^2)/x^2-1/6*b^3*Pi*cos(1/2*b^2*Pi*x^2)*F
resnelS(b*x)/x-1/12*b^4*Pi^2*FresnelS(b*x)^2-1/4*FresnelS(b*x)^2/x^4+1/12*b
^4*Pi*Si(b^2*Pi*x^2)-1/6*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6430, 6456, 6464, 6440, 30, 3375, 3380, 3297, 3299}

$$-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3}-\frac{\pi b^3 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x}-\frac{1}{12}\pi^2 b^4 S(bx)^2+\frac{1}{12}\pi b^4\text{Si}\left(b^2\pi x^2\right)-\frac{b^2}{24x^2}+\frac{b^2\cos\left(\pi b^2x^2\right)}{24x^2}-\frac{S(bx}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^2/x^5,x]
```

```
[Out] -b^2/(24*x^2) + (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(6*x) - (b^4*Pi^2*FresnelS[b*x]^2)/12 - FresnelS[b*x]^2/(4
*x^4) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^3) + (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\int \frac{S(bx)^2}{x^5}\,dx = -\frac{S(bx)^2}{4x^4} + \frac{1}{2}b\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

$$= -\frac{b^2}{24x^2} - \frac{S(bx)^2}{4x^4} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} - \frac{1}{12}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx + \frac{1}{6}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S}{x^2}$$

$$= -\frac{b^2}{24x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x} - \frac{S(bx)^2}{4x^4} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} - \frac{1}{24}b^2\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi x\right.}{x^2}\right.$$

$$= -\frac{b^2}{24x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x} - \frac{S(bx)^2}{4x^4} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} + \frac{1}{24}b^4\pi\mathrm{Si}$$

$$= -\frac{b^2}{24x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x} - \frac{1}{12}b^4\pi^2S(bx)^2 - \frac{S(bx)^2}{4x^4} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\right.}{6x^3}$$

Mathematica [A] time = 0.01, size = 127, normalized size = 1.00

$$-\frac{1}{12}\pi^2b^4S(bx)^2 - \frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3} - \frac{b^2}{24x^2} + \frac{b^2\cos\left(\pi b^2x^2\right)}{24x^2} + \frac{1}{12}\pi b^4\mathrm{Si}\left(b^2\pi x^2\right) - \frac{\pi b^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x} - \frac{S(bx}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^2/x^5,x]
```

```
[Out] -1/24*b^2/x^2 + (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(6*x) - (b^4*Pi^2*FresnelS[b*x]^2)/12 - FresnelS[b*x]^2/(4
*x^4) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^3) + (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\,(bx)^2}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnels}\,(bx)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^5,x, algorithm="giac")
[Out] integrate(fresnels(b*x)^2/x^5, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\left(bx\right)^{2}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^5,x)
[Out] int(FresnelS(b*x)^2/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnels}\left(bx\right)^{2}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^5,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)^2/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)^{2}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^5,x)
[Out] int(FresnelS(b*x)^2/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^{2}\left(bx\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**5,x)
[Out] Integral(fresnels(b*x)**2/x**5, x)
```

3.44 $\int \frac{S(bx)^2}{x^6}\,dx$

Optimal. Leaf size=171

$$-\frac{1}{20}\pi^2 b^5 \text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)+\frac{7\pi^2 b^5 C\left(\sqrt{2}\,bx\right)}{60\sqrt{2}}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{10x^4}-\frac{b^2}{60x^3}+\frac{b^2\cos\left(\pi b^2 x^2\right)}{60x^3}-\frac{7\pi b^4 \sin}{12}$$

```
[Out] -1/60*b^2/x^3+1/60*b^2*cos(b^2*Pi*x^2)/x^3-1/20*b^3*Pi*cos(1/2*b^2*Pi*x^2)*
FresnelS(b*x)/x^2-1/5*FresnelS(b*x)^2/x^5-1/10*b*FresnelS(b*x)*sin(1/2*b^2*
Pi*x^2)/x^4-7/120*b^4*Pi*sin(b^2*Pi*x^2)/x+7/120*b^5*Pi^2*FresnelC(b*x*2^(1
/2))*2^(1/2)-1/20*b^5*Pi^2*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x
,x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^6,x]
```

```
[Out] -b^2/(60*x^3) + (b^2*Cos[b^2*Pi*x^2])/(60*x^3) + (7*b^5*Pi^2*FresnelC[Sqrt[
2]*b*x])/(60*Sqrt[2]) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(20*x^2)
 - FresnelS[b*x]^2/(5*x^5) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(10*x^4)
 - (7*b^4*Pi*Sin[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*Defer[Int][(FresnelS[b*x]
*Sin[(b^2*Pi*x^2)/2])/x, x])/20
```

Rubi steps

$$\int \frac{S(bx)^2}{x^6}\,dx = -\frac{S(bx)^2}{5x^5} + \frac{1}{5}(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

$$= -\frac{b^2}{60x^3} - \frac{S(bx)^2}{5x^5} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4} - \frac{1}{20}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx + \frac{1}{10}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S}{x^3}$$

$$= -\frac{b^2}{60x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{20x^2} - \frac{S(bx)^2}{5x^5} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4} + \frac{1}{40}\left(b^4\pi\right)$$

$$= -\frac{b^2}{60x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{20x^2} - \frac{S(bx)^2}{5x^5} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4} - \frac{7b^4\pi\sin(}{120}$$

$$= -\frac{b^2}{60x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3} + \frac{7b^5\pi^2 C\left(\sqrt{2}\,bx\right)}{60\sqrt{2}} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{20x^2} - \frac{S(bx)^2}{5x^5} - \frac{bS(bx)\sin\left(\frac{1}{2}b\right.}{10x^4}$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^6,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^6, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^6, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^6, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^6,x)
```

```
[Out] int(FresnelS(b*x)^2/x^6,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}(bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2/x^6, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{FresnelS}(b\,x)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^6,x)
```

```
[Out] int(FresnelS(b*x)^2/x^6, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**6,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**6, x)
```

3.45 $\int \frac{S(bx)^2}{x^7}\,dx$

Optimal. Leaf size=166

$$-\frac{1}{45}\pi^2 b^5 \text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2},x\right)-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^5}-\frac{b^2}{120x^4}+\frac{b^2\cos\left(\pi b^2 x^2\right)}{120x^4}+\frac{1}{72}\pi^2 b^6 \text{Ci}\left(b^2\pi x^2\right)-\frac{\pi b^4 \sin}{7}$$

```
[Out] -1/120*b^2/x^4+1/72*b^6*Pi^2*Ci(b^2*Pi*x^2)+1/120*b^2*cos(b^2*Pi*x^2)/x^4-1
/45*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3-1/6*FresnelS(b*x)^2/x^6-1/
15*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/72*b^4*Pi*sin(b^2*Pi*x^2)/x^2-
1/45*b^5*Pi^2*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Rubi [A] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)^2}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^7,x]
```

```
[Out] -b^2/(120*x^4) + (b^2*Cos[b^2*Pi*x^2])/(120*x^4) + (b^6*Pi^2*CosIntegral[b^
2*Pi*x^2])/72 - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(45*x^3) - Fresn
elS[b*x]^2/(6*x^6) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(15*x^5) - (b^4*
Pi*Sin[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/x^2, x])/45
```

Rubi steps

$$\int \frac{S(bx)^2}{x^7}\,dx = -\frac{S(bx)^2}{6x^6} + \frac{1}{3}b\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

$$= -\frac{b^2}{120x^4} - \frac{S(bx)^2}{6x^6} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} - \frac{1}{30}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx + \frac{1}{15}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}$$

$$= -\frac{b^2}{120x^4} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{45x^3} - \frac{S(bx)^2}{6x^6} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} - \frac{1}{60}b^2\,\text{Subst}\left(\int \frac{\cos\left(b^2\pi\right.}{x^3}\right.$$

$$= -\frac{b^2}{120x^4} + \frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{45x^3} - \frac{S(bx)^2}{6x^6} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} + \frac{1}{180}\left(b^4\right.$$

$$= -\frac{b^2}{120x^4} + \frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{45x^3} - \frac{S(bx)^2}{6x^6} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} - \frac{b^4\pi\sin}{72}$$

$$= -\frac{b^2}{120x^4} + \frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4} + \frac{1}{72}b^6\pi^2\text{Ci}\left(b^2\pi x^2\right) - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{45x^3} - \frac{S(bx)^2}{6x^6} - \frac{bS(bx)\sin}{1}$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^7,x]
[Out] Integrate[FresnelS[b*x]^2/x^7, x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\left(bx\right)^2}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^7,x, algorithm="fricas")
[Out] integral(fresnels(b*x)^2/x^7, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\left(bx\right)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^7,x, algorithm="giac")
[Out] integrate(fresnels(b*x)^2/x^7, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\left(bx\right)^{2}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^7,x)
[Out] int(FresnelS(b*x)^2/x^7,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnels}\left(bx\right)^{2}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^7,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)^2/x^7, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)^{2}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^7,x)
[Out] int(FresnelS(b*x)^2/x^7, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^{2}\left(bx\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**7,x)
[Out] Integral(fresnels(b*x)**2/x**7, x)
```

3.46 $\int \frac{S(bx)^2}{x^8}\,dx$

Optimal. Leaf size=259

$$-\frac{1}{168}\pi^3 b^7 \text{Int}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)-\frac{2}{315}\sqrt{2}\,\pi^3 b^7 S\left(\sqrt{2}\,bx\right)-\frac{\pi^3 b^7 S\left(\sqrt{2}\,bx\right)}{72\sqrt{2}}+\frac{\pi^2 b^6}{336x}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{21x^6}-\frac{2}{2}$$

```
[Out] -1/210*b^2/x^5+1/336*b^6*Pi^2/x+1/210*b^2*cos(b^2*Pi*x^2)/x^5-67/5040*b^6*P
i^2*cos(b^2*Pi*x^2)/x-1/84*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4-1/7
*FresnelS(b*x)^2/x^7-1/21*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6+1/168*b^5
*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-13/2520*b^4*Pi*sin(b^2*Pi*x^2)/
x^3-67/5040*b^7*Pi^3*FresnelS(b*x*2^(1/2))*2^(1/2)-1/168*b^7*Pi^3*Unintegra
ble(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [A] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)^2}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^8,x]
```

```
[Out] -b^2/(210*x^5) + (b^6*Pi^2)/(336*x) + (b^2*Cos[b^2*Pi*x^2])/(210*x^5) - (67
*b^6*Pi^2*Cos[b^2*Pi*x^2])/(5040*x) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/(84*x^4) - FresnelS[b*x]^2/(7*x^7) - (b^7*Pi^3*FresnelS[Sqrt[2]*b*x])
/(72*Sqrt[2]) - (2*Sqrt[2]*b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/315 - (b*Fresnel
S[b*x]*Sin[(b^2*Pi*x^2)/2])/(21*x^6) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*
x^2)/2])/(168*x^2) - (13*b^4*Pi*Sin[b^2*Pi*x^2])/(2520*x^3) - (b^7*Pi^3*Def
er[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/168
```

Rubi steps

$$\int \frac{S(bx)^2}{x^8}\,dx = -\frac{S(bx)^2}{7x^7} + \frac{1}{7}(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

$$= -\frac{b^2}{210x^5} - \frac{S(bx)^2}{7x^7} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} - \frac{1}{42}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx + \frac{1}{21}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}$$

$$= -\frac{b^2}{210x^5} + \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{84x^4} - \frac{S(bx)^2}{7x^7} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} + \frac{1}{168}\left(b^4\pi\right.$$

$$= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} + \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{84x^4} - \frac{S(bx)^2}{7x^7} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} +$$

$$= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} + \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{67b^6\pi^2\cos\left(b^2\pi x^2\right)}{5040x} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{84x^4} - \frac{S(bx)^2}{7x^7} -$$

$$= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} + \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{67b^6\pi^2\cos\left(b^2\pi x^2\right)}{5040x} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{84x^4} - \frac{S(bx)^2}{7x^7} -$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^8,x]
[Out] Integrate[FresnelS[b*x]^2/x^8, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\,(bx)^2}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^8,x, algorithm="fricas")
[Out] integral(fresnels(b*x)^2/x^8, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnels}\,(bx)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^8,x, algorithm="giac")
[Out] integrate(fresnels(b*x)^2/x^8, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\left(bx\right)^{2}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^8,x)
[Out] int(FresnelS(b*x)^2/x^8,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnels}\left(bx\right)^{2}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^8,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)^2/x^8, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)^{2}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^8,x)
[Out] int(FresnelS(b*x)^2/x^8, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^{2}\left(bx\right)}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**8,x)
[Out] Integral(fresnels(b*x)**2/x**8, x)
```

3.47 $\int \frac{S(bx)^2}{x^9} dx$

Optimal. Leaf size=242

$$\frac{1}{840}\pi^4 b^8 S(bx)^2+\frac{\pi^2 b^6}{1680x^2}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{28x^7}-\frac{b^2}{336x^6}+\frac{b^2\cos\left(\pi b^2x^2\right)}{336x^6}-\frac{1}{280}\pi^3 b^8 \mathrm{Si}\left(b^2\pi x^2\right)+\frac{\pi^3 b^7 S(bx)\cos\left(\frac{1}{2}\pi\right.}{420x}$$

```
[Out] -1/336*b^2/x^6+1/1680*b^6*Pi^2/x^2+1/336*b^2*cos(b^2*Pi*x^2)/x^6-1/336*b^6*
Pi^2*cos(b^2*Pi*x^2)/x^2-1/140*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5
+1/420*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x+1/840*b^8*Pi^4*FresnelS
(b*x)^2-1/8*FresnelS(b*x)^2/x^8-1/280*b^8*Pi^3*Si(b^2*Pi*x^2)-1/28*b*Fresne
lS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7+1/420*b^5*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi
*x^2)/x^3-1/420*b^4*Pi*sin(b^2*Pi*x^2)/x^4
```

Rubi [A] time = 0.39, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6430, 6456, 6464, 6440, 30, 3375, 3380, 3297, 3299, 3379}

$$\frac{\pi^2 b^5 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{420x^3}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{28x^7}+\frac{\pi^3 b^7 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{420x}-\frac{\pi b^3 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{140x^5}+\frac{1}{840}\pi^4 b^8 S(b$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^2/x^9,x]
```

```
[Out] -b^2/(336*x^6) + (b^6*Pi^2)/(1680*x^2) + (b^2*Cos[b^2*Pi*x^2])/(336*x^6) -
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS
[b*x])/(140*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(420*x) + (
b^8*Pi^4*FresnelS[b*x]^2)/840 - FresnelS[b*x]^2/(8*x^8) - (b*FresnelS[b*x]*
Sin[(b^2*Pi*x^2)/2])/(28*x^7) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(420*x^3) - (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) - (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
```

```
+ 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x])
/; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\int \frac{S(bx)^2}{x^9}\,dx = -\frac{S(bx)^2}{8x^8}+\frac{1}{4}b\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

$$= -\frac{b^2}{336x^6}-\frac{S(bx)^2}{8x^8}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7}-\frac{1}{56}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx+\frac{1}{28}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}$$

$$= -\frac{b^2}{336x^6}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5}-\frac{S(bx)^2}{8x^8}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7}-\frac{1}{112}b^2\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi x\right.}{x^4}\right.$$

$$= -\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}+\frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5}-\frac{S(bx)^2}{8x^8}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7}$$

$$= -\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}+\frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{420x}-\frac{S(b}{8x}$$

$$= -\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}+\frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6}-\frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5}+\frac{b^7\pi^3\cos\left(\right.}{4}$$

$$= -\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}+\frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6}-\frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5}+\frac{b^7\pi^3\cos\left(\right.}{4}$$

Mathematica [A] time = 0.02, size = 242, normalized size = 1.00

$$\frac{1}{840}\pi^4b^8S(bx)^2+\frac{\pi^2b^6}{1680x^2}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{28x^7}-\frac{b^2}{336x^6}+\frac{b^2\cos\left(\pi b^2x^2\right)}{336x^6}-\frac{1}{280}\pi^3b^8\mathrm{Si}\left(b^2\pi x^2\right)+\frac{\pi^3b^7S(bx)\cos\left(\frac{1}{2}\pi\right.}{420x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^2/x^9,x]
```

```
[Out] -1/336*b^2/x^6 + (b^6*Pi^2)/(1680*x^2) + (b^2*Cos[b^2*Pi*x^2])/(336*x^6) -
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS
[b*x])/(140*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(420*x) + (
b^8*Pi^4*FresnelS[b*x]^2)/840 - FresnelS[b*x]^2/(8*x^8) - (b*FresnelS[b*x]*
Sin[(b^2*Pi*x^2)/2])/(28*x^7) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(420*x^3) - (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) - (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)^2}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^9,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^9, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2/x^9, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^9,x)
```

```
[Out] int(FresnelS(b*x)^2/x^9,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^9,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)^2/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\text{FresnelS}\left(b\,x\right)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^9,x)
[Out] int(FresnelS(b*x)^2/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2\left(bx\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**9,x)
[Out] Integral(fresnels(b*x)**2/x**9, x)
```

3.48 $\int \frac{S(bx)^2}{x^{10}}\,dx$

Optimal. Leaf size=286

$$\frac{\pi^4 b^9 \text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)}{1728}-\frac{853\pi^4 b^9 C\left(\sqrt{2}\,bx\right)}{181440\sqrt{2}}+\frac{\pi^2 b^6}{5184x^3}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{36x^8}-\frac{b^2}{504x^7}+\frac{b^2\cos\left(\pi b^2x^2\right)}{504x^7}+\frac{853\pi$$

```
[Out] -1/504*b^2/x^7+1/5184*b^6*Pi^2/x^3+1/504*b^2*cos(b^2*Pi*x^2)/x^7-187/181440
*b^6*Pi^2*cos(b^2*Pi*x^2)/x^3-1/216*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x
)/x^6+1/1728*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2-1/9*FresnelS(b*
x)^2/x^9-1/36*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8+1/864*b^5*Pi^2*Fresne
lS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-19/15120*b^4*Pi*sin(b^2*Pi*x^2)/x^5+853/362
880*b^8*Pi^3*sin(b^2*Pi*x^2)/x-853/362880*b^9*Pi^4*FresnelC(b*x*2^(1/2))*2^
(1/2)+1/1728*b^9*Pi^4*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A] time = 0.36, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)^2}{x^{10}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^10,x]
```

```
[Out] -b^2/(504*x^7) + (b^6*Pi^2)/(5184*x^3) + (b^2*Cos[b^2*Pi*x^2])/(504*x^7) -
(187*b^6*Pi^2*Cos[b^2*Pi*x^2])/(181440*x^3) - (853*b^9*Pi^4*FresnelC[Sqrt[2
]*b*x])/(181440*Sqrt[2]) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(216*
x^6) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(1728*x^2) - FresnelS[b
*x]^2/(9*x^9) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(36*x^8) + (b^5*Pi^2*
FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(864*x^4) - (19*b^4*Pi*Sin[b^2*Pi*x^2])/
(15120*x^5) + (853*b^8*Pi^3*Sin[b^2*Pi*x^2])/(362880*x) + (b^9*Pi^4*Defer[I
nt][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/1728
```

Rubi steps

$$\int \frac{S(bx)^2}{x^{10}}\,dx = -\frac{S(bx)^2}{9x^9} + \frac{1}{9}(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

$$= -\frac{b^2}{504x^7} - \frac{S(bx)^2}{9x^9} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} - \frac{1}{72}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx + \frac{1}{36}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}$$

$$= -\frac{b^2}{504x^7} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{216x^6} - \frac{S(bx)^2}{9x^9} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} + \frac{1}{432}\left(b^4\pi\right.$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{216x^6} - \frac{S(bx)^2}{9x^9} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{216x^6} + \frac{b^7\pi^3\,\mathrm{co}}{}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{216x^6} + \frac{b^7\pi^3\,\mathrm{co}}{}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{67b^9\pi^4C\left(\sqrt{2}\,bx\right)}{25920\sqrt{2}} - \frac{1}{945}\sqrt{2}\,b^9\pi^4C$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^{10}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^10,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^10, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\,(bx)^2}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2/x^10, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^10,x, algorithm="giac")
[Out] integrate(fresnels(b*x)^2/x^10, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^10,x)
[Out] int(FresnelS(b*x)^2/x^10,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2/x^10,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)^2/x^10, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\text{FresnelS}\,(b\,x)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^10,x)
[Out] int(FresnelS(b*x)^2/x^10, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2\,(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**10,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**10, x)
```

3.49 $\int (c+dx)^2 S(a+bx)^2 \, dx$

Optimal. Leaf size=497

$$\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{d(bc-ad}{}$$

```
[Out] 2/3*d^2*x/b^2/Pi^2+1/2*d*(-a*d+b*c)*cos(Pi*(b*x+a)^2)/b^3/Pi^2+1/6*d^2*(b*x
+a)*cos(Pi*(b*x+a)^2)/b^3/Pi^2+2*(-a*d+b*c)^2*cos(1/2*Pi*(b*x+a)^2)*Fresnel
S(b*x+a)/b^3/Pi+2*d*(-a*d+b*c)*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a
)/b^3/Pi+2/3*d^2*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a)/b^3/Pi-d*(
-a*d+b*c)*FresnelC(b*x+a)*FresnelS(b*x+a)/b^3/Pi+(-a*d+b*c)^2*(b*x+a)*Fresn
elS(b*x+a)^2/b^3+d*(-a*d+b*c)*(b*x+a)^2*FresnelS(b*x+a)^2/b^3+1/3*d^2*(b*x+
a)^3*FresnelS(b*x+a)^2/b^3+1/4*I*d*(-a*d+b*c)*(b*x+a)^2*HypergeometricPFQ([
1, 1],[3/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^3/Pi-1/4*I*d*(-a*d+b*c)*(b*x+a)^2*Hyp
ergeometricPFQ([1, 1],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^3/Pi-4/3*d^2*FresnelS(
b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi^2-5/12*d^2*FresnelC((b*x+a)*2^(1/2))/b^
3/Pi^2*2^(1/2)-1/2*(-a*d+b*c)^2*FresnelS((b*x+a)*2^(1/2))/b^3/Pi*2^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {6432, 6420, 6452, 3351, 6430, 6454, 6446, 3379, 2638, 6460, 3357, 3352, 3385}

$$\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{d(bc-ad}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelS[a + b*x]^2,x]
```

```
[Out] (2*d^2*x)/(3*b^2*Pi^2) + (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) +
 (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (5*d^2*FresnelC[Sqrt[2]
*(a + b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (2*(b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2
]*FresnelS[a + b*x])/(b^3*Pi) + (2*d*(b*c - a*d)*(a + b*x)*Cos[(Pi*(a + b*x
)^2)/2]*FresnelS[a + b*x])/(b^3*Pi) + (2*d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)^
2)/2]*FresnelS[a + b*x])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x]*Fres
nelS[a + b*x])/(b^3*Pi) + ((b*c - a*d)^2*(a + b*x)*FresnelS[a + b*x]^2)/b^3
 + (d*(b*c - a*d)*(a + b*x)^2*FresnelS[a + b*x]^2)/b^3 + (d^2*(a + b*x)^3*F
resnelS[a + b*x]^2)/(3*b^3) - ((b*c - a*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(
Sqrt[2]*b^3*Pi) + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, (-I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - ((I/4)*d*(b*c - a*d)*(a + b*
x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^3*Pi) -
(4*d^2*FresnelS[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6420

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelS[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Sin[(Pi*(a + b*x)^2)/2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6432

```
Int[FresnelS[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelS[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int (c+dx)^2 S(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)S(x)^2+2bcd\left(1-\frac{ad}{bc}\right)xS(x)^2+d^2x^2S(x)^2\right)dx,x,a+\right.}{b^3}$$

$$= \frac{d^2\,\text{Subst}\left(\int x^2S(x)^2\,dx,x,a+bx\right)}{b^3}+\frac{(2d(bc-ad))\,\text{Subst}\left(\int xS(x)^2\,dx,x,a+bx\right)}{b^3}+$$

$$= \frac{(bc-ad)^2(a+bx)S(a+bx)^2}{b^3}+\frac{d(bc-ad)(a+bx)^2S(a+bx)^2}{b^3}+\frac{d^2(a+bx)^3S(a+bx)}{3b^3}$$

$$= \frac{2(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^3\pi}+\frac{2d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a}{b^3\pi}$$

$$= \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}+\frac{2(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^3\pi}+\frac{2d(bc-ad)}{}$$

$$= \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}+\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{d^2C\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}\,b^3\pi^2}+\frac{2(}{}$$

$$= \frac{2d^2x}{3b^2\pi^2}+\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}+\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{d^2C\left(\sqrt{2}(a+\right.}{6\sqrt{2}\,b^3\pi^2}$$

$$= \frac{2d^2x}{3b^2\pi^2}+\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}+\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{d^2C\left(\sqrt{2}(a+\right.}{6\sqrt{2}\,b^3\pi^2}$$

Mathematica [F] time = 0.88, size = 0, normalized size = 0.00

$$\int (c+dx)^2 S(a+bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^2*FresnelS[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^2*FresnelS[a + b*x]^2, x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2x^2+2\,cdx+c^2\right)\text{fresnels}\,(bx+a)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*fresnels(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \mathrm{fresnels}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*fresnels(b*x + a)^2, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (dx+c)^2\,\mathrm{S}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*FresnelS(b*x+a)^2,x)
[Out] int((d*x+c)^2*FresnelS(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \mathrm{fresnels}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnels(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^2*fresnels(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{FresnelS}\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)^2*(c + d*x)^2,x)
[Out] int(FresnelS(a + b*x)^2*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\,S^2\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnels(b*x+a)**2,x)
[Out] Integral((c + d*x)**2*fresnels(a + b*x)**2, x)
```

3.50 $\int (c+dx)S(a+bx)^2\,dx$

Optimal. Leaf size=279

$$\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}-\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}+\frac{(a+bx)(bc-ad)S(a+bx)^2}{b^2}$$

```
[Out] 1/4*d*cos(Pi*(b*x+a)^2)/b^2/Pi^2+2*(-a*d+b*c)*cos(1/2*Pi*(b*x+a)^2)*Fresnel
S(b*x+a)/b^2/Pi+d*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a)/b^2/Pi-1/2*
d*FresnelC(b*x+a)*FresnelS(b*x+a)/b^2/Pi+(-a*d+b*c)*(b*x+a)*FresnelS(b*x+a)
^2/b^2+1/2*d*(b*x+a)^2*FresnelS(b*x+a)^2/b^2+1/8*I*d*(b*x+a)^2*Hypergeometr
icPFQ([1, 1],[3/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^2/Pi-1/8*I*d*(b*x+a)^2*Hyperge
ometricPFQ([1, 1],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^2/Pi-1/2*(-a*d+b*c)*Fresne
lS((b*x+a)*2^(1/2))/b^2/Pi*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6432, 6420, 6452, 3351, 6430, 6454, 6446, 3379, 2638}

$$\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}-\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}+\frac{(a+bx)(bc-ad)S(a+bx)^2}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelS[a + b*x]^2,x]
```

```
[Out] (d*Cos[Pi*(a + b*x)^2])/(4*b^2*Pi^2) + (2*(b*c - a*d)*Cos[(Pi*(a + b*x)^2)/
2]*FresnelS[a + b*x])/(b^2*Pi) + (d*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*Fresn
elS[a + b*x])/(b^2*Pi) - (d*FresnelC[a + b*x]*FresnelS[a + b*x])/(2*b^2*Pi)
 + ((b*c - a*d)*(a + b*x)*FresnelS[a + b*x]^2)/b^2 + (d*(a + b*x)^2*Fresnel
S[a + b*x]^2)/(2*b^2) - ((b*c - a*d)*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*
b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*P
i*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {
3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6420

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelS[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Sin[(Pi*(a + b*x)^2)/2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6430

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
S[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Sin[(Pi*b^2*x^2)/
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6432

```
Int[FresnelS[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelS[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
```

```
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rubi steps

$$\int (c+dx)S(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(bc\left(1-\frac{ad}{bc}\right)S(x)^2+dxS(x)^2\right)dx,x,a+bx\right)}{b^2}$$
$$= \frac{d\,\text{Subst}\left(\int xS(x)^2\,dx,x,a+bx\right)}{b^2}+\frac{(bc-ad)\,\text{Subst}\left(\int S(x)^2\,dx,x,a+bx\right)}{b^2}$$
$$= \frac{(bc-ad)(a+bx)S(a+bx)^2}{b^2}+\frac{d(a+bx)^2S(a+bx)^2}{2b^2}-\frac{d\,\text{Subst}\left(\int x^2S(x)\sin\left(\frac{\pi x^2}{2}\right)d\right.}{b^2}$$
$$= \frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}+\frac{(b}{}$$
$$= \frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}-\frac{d(}{}$$
$$= \frac{d\cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2}+\frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a\right.}{b^2\pi}$$

Mathematica [F] time = 0.62, size = 0, normalized size = 0.00

$$\int (c+dx)S(a+bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)*FresnelS[a + b*x]^2,x]
[Out] Integrate[(c + d*x)*FresnelS[a + b*x]^2, x]
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)\text{fresnels}\,(bx+a)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)*fresnels(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\text{fresnels}\,(bx + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)*fresnels(b*x + a)^2, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int (dx + c)\,\mathrm{S}\,(bx + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*FresnelS(b*x+a)^2,x)
[Out] int((d*x+c)*FresnelS(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\text{fresnels}\,(bx + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)*fresnels(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \text{FresnelS}\,(a + b\,x)^2\,(c + d\,x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)^2*(c + d*x),x)
[Out] int(FresnelS(a + b*x)^2*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)\, S^2\,(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a)**2,x)
[Out] Integral((c + d*x)*fresnels(a + b*x)**2, x)
```

3.51 $\int S(a+bx)^2\,dx$

Optimal. Leaf size=70

$$\frac{(a+bx)S(a+bx)^2}{b}-\frac{S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,\pi b}+\frac{2S(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[Out] 2*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a)/b/Pi+(b*x+a)*FresnelS(b*x+a)^2/b-1/
2*FresnelS((b*x+a)*2^(1/2))/b/Pi*2^(1/2)
```

Rubi [**A**] time = 0.17, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6420, 6452, 3351}

$$\frac{(a+bx)S(a+bx)^2}{b}-\frac{S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,\pi b}+\frac{2S(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[a + b*x]^2,x]
```

```
[Out] (2*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x])/(b*Pi) + ((a + b*x)*FresnelS[
a + b*x]^2)/b - FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*Pi)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6420

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelS[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Sin[(Pi*(a + b*x)^2)/2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int S(a+bx)^2\,dx = \frac{(a+bx)S(a+bx)^2}{b} - 2\int (a+bx)S(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx$$
$$= \frac{(a+bx)S(a+bx)^2}{b} - \frac{2\,\mathrm{Subst}\left(\int xS(x)\sin\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{b}$$
$$= \frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b\pi} + \frac{(a+bx)S(a+bx)^2}{b} - \frac{\mathrm{Subst}\left(\int \sin\left(\pi x^2\right)dx, x, a+bx\right)}{b\pi}$$
$$= \frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b\pi} + \frac{(a+bx)S(a+bx)^2}{b} - \frac{S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,b\pi}$$

Mathematica [A] time = 0.01, size = 67, normalized size = 0.96

$$\frac{2\pi(a+bx)S(a+bx)^2 - \sqrt{2}\,S\left(\sqrt{2}\,(a+bx)\right) + 4S(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[a + b*x]^2,x]
```

```
[Out] (4*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x] + 2*Pi*(a + b*x)*FresnelS[a +
b*x]^2 - Sqrt[2]*FresnelS[Sqrt[2]*(a + b*x)])/(2*b*Pi)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\mathrm{fresnels}\,(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{fresnels}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)^2, x)
```

maple [A] time = 0.01, size = 60, normalized size = 0.86

$$\frac{(bx+a)\,\mathrm{S}\,(bx+a)^2+\frac{2\,\mathrm{S}(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{\sqrt{2}\,\mathrm{S}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)^2,x)
```

```
[Out] 1/b*((b*x+a)*FresnelS(b*x+a)^2+2*FresnelS(b*x+a)/Pi*cos(1/2*Pi*(b*x+a)^2)-1
/2/Pi*2^(1/2)*FresnelS((b*x+a)*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{fresnels}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{FresnelS}\,(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)^2,x)
```

```
[Out] int(FresnelS(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int S^2\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)**2,x)
```

```
[Out] Integral(fresnels(a + b*x)**2, x)
```

3.52 $\int \frac{S(a+bx)^2}{c+dx} \, dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{S(a+bx)^2}{c+dx},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)^2/(d*x+c),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(a+bx)^2}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[a + b*x]^2/(c + d*x),x]
[Out] Defer[Int][FresnelS[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{S(a+bx)^2}{c+dx} \, dx = \int \frac{S(a+bx)^2}{c+dx} \, dx$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)^2}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]^2/(c + d*x),x]
[Out] Integrate[FresnelS[a + b*x]^2/(c + d*x), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(fresnels(b*x + a)^2/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(fresnels(b*x + a)^2/(d*x + c), x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)^2/(d*x+c),x)
[Out] int(FresnelS(b*x+a)^2/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] integrate(fresnels(b*x + a)^2/(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\text{FresnelS}\,(a+b\,x)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)^2/(c + d*x),x)
[Out] int(FresnelS(a + b*x)^2/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(fresnels(a + b*x)**2/(c + d*x), x)
```

3.53 $\int \frac{S(a+bx)^2}{(c+dx)^2} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{S(a+bx)^2}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(a+bx)^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelS[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{S(a+bx)^2}{(c+dx)^2} dx = \int \frac{S(a+bx)^2}{(c+dx)^2} dx$$

Mathematica [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[FresnelS[a + b*x]^2/(c + d*x)^2, x]
```

fricas [A] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x + a)^2/(d*x + c)^2, x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(FresnelS(b*x+a)^2/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x + a)^2/(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{FresnelS}\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)^2/(c + d*x)^2,x)
```

```
[Out] int(FresnelS(a + b*x)^2/(c + d*x)^2, x)
```

<u>**sympy**</u> [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(fresnels(a + b*x)**2/(c + d*x)**2, x)
```

3.54 $\int x^2 S\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=231

$$\left(\frac{1}{12}-\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{3}{n}\right)}{\sqrt{\pi}\,bd}\right)+\left(\frac{1}{12}-\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}-\frac{}{2\pi}}$$

```
[Out] (1/12-1/12*I)*exp(-3*a/b/n+9/2*I/b^2/d^2/n^2/Pi)*x^3*erf((1/2+1/2*I)*(3/n+I
*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))+(1/12-1/1
2*I)*exp(-3*a/b/n-9/2*I/b^2/d^2/n^2/Pi)*x^3*erfi((1/2+1/2*I)*(3/n-I*a*b*d^2
*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))+1/3*x^3*FresnelS(
d*(a+b*ln(c*x^n)))
```

Rubi [A] time = 0.57, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6471, 4617, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{12}-\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{3}{n}\right)}{\sqrt{\pi}\,bd}\right)+\left(\frac{1}{12}-\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}-\frac{}{2\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/12 - I/12)*E^((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erf[((1/2
+ I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*
x^n)^(3/n) + ((1/12 - I/12)*E^((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))*x
^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(c*x^n)^(3/n) + (x^3*FresnelS[d*(a + b*Log[c*x^n])])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 4617

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6471

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelS[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Sin[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 S\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx=\frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{3}(bdn)\int x^2\sin\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)\,dx$$

$$=\frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}(ibdn)\int e^{-\frac{1}{2}id^2\pi\left(a+b\log\left(cx^n\right)\right)^2}x^2\,dx+\frac{1}{6}(ibdn)\int e^{\frac{1}{2}}$$

$$=\frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-iabd^2\pi\log\left(cx^n\right)-\frac{1}{2}i\right.$$

$$=\frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x$$

$$=\frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}\left(ibdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}\right.$$

$$=\frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}\left(ibdx^3\left(cx^n\right)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{1}{2}i\right.\right.$$

$$=\frac{1}{3}x^3 S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}\left(ibde^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3\left(cx^n\right)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\text{Subst}\left(\right.$$

$$=\left(\frac{1}{12}-\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3\left(cx^n\right)^{-3/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right.$$

Mathematica [**A**] time = 8.30, size = 319, normalized size = 1.38

$$\frac{1}{12}x^3\left(4S\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\sqrt[4]{-1}\sqrt{2}\left(cx^n\right)^{-3/n}\left(e^{\frac{9i}{\pi b^2d^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\pi abd^2n+\pi b^2d^2n\log\left(cx^n\right)-3i\right)}{\sqrt{\pi}\,bdn}\right)+i\text{er}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*(4*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^(((-6*a)/(b*
n) - (9*I)/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - L
og[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*(E^((9*I)/(b^2*d
^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n
]))/(b*d*n*Sqrt[Pi])] + I*Erfi[((-1)^(3/4)*(3*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(3/n)))/12
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\text{fresnels}\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(x^2*fresnels(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{fresnels}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x^2*fresnels((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int x^2\mathrm{S}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(d*(a+b*ln(c*x^n))),x)
[Out] int(x^2*FresnelS(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{fresnels}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(x^2*fresnels((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{FresnelS}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(d*(a + b*log(c*x^n))),x)
[Out] int(x^2*FresnelS(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 S\left(ad + bd\log\left(cx^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x**2*fresnels(a*d + b*d*log(c*x**n)), x)
```

3.55 $\int xS\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=227

$$\left(\frac{1}{8}-\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)+\left(\frac{1}{8}-\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{-\frac{2\left(\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\,\mathrm{e}$$

```
[Out] (1/8-1/8*I)*exp((2*I-2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x^2*erf((1/2+1/2*I)*(2
/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(2/n))+(1/8-
1/8*I)*x^2*erfi((1/2+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/P
i^(1/2))/exp(2*(I+a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/((c*x^n)^(2/n))+1/2*x^2*Fre
snelS(d*(a+b*ln(c*x^n)))
```

Rubi [A] time = 0.44, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6471, 4617, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{8}-\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)+\left(\frac{1}{8}-\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{-\frac{2\left(\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\,\mathrm{E}$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/8 - I/8)*E^((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*Erf[((1/2 + I/
2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)
^(2/n) + ((1/8 - I/8)*x^2*Erfi[((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*(c*x^n)^(2/n)) + (x^2*FresnelS[d*(a + b*Log[c*x^n])])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 4617

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6471

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelS[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Sin[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int xS\left(d\left(a+b\log\left(cx^n\right)\right)\right)dx = \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bdn)\int x\sin\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)dx$$

$$= \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}(ibdn)\int e^{-\frac{1}{2}id^2\pi\left(a+b\log\left(cx^n\right)\right)^2}x\,dx+\frac{1}{4}(ibdn)\int e^{\frac{1}{2}}$$

$$= \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-iabd^2\pi\log\left(cx^n\right)-\frac{1}{2}\right.$$

$$= \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x$$

$$= \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(ibdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-\right.$$

$$= \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(ibdx^2\left(cx^n\right)^{-iabd^2\pi-\frac{2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{1}{2}i\right.\right.$$

$$= \frac{1}{2}x^2S\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(ibde^{-\frac{2\left(i+abd^2n\pi\right)}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-iabd^2\pi-\frac{2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\right.$$

$$= \left(\frac{1}{8}-\frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-2/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)+$$

Mathematica [A] time = 7.91, size = 319, normalized size = 1.41

$$\frac{1}{8}x^2\left(4S\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\sqrt[4]{-1}\sqrt{2}\left(cx^n\right)^{-2/n}\left(e^{\frac{4i}{\pi b^2d^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\pi abd^2n+\pi b^2d^2n\log\left(cx^n\right)-2i\right)}{\sqrt{\pi}\,bdn}\right)+ie\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*(4*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^((-2*a)/(b*n
) - (2*I)/(b^2*d^2*n^2*Pi) - (I/2)*a^2*d^2*Pi + I*a*b*d^2*Pi*(n*Log[x] - Lo
g[c*x^n]) - (I/2)*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)*(E^((4*I)/(b^2*d
^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n
]))/(b*d*n*Sqrt[Pi])] + I*Erfi[((-1)^(3/4)*(2*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(2/n)))/8
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(x\text{fresnels}\left(bd\log\left(cx^{n}\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(x*fresnels(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnels}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x*fresnels((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int x\,\mathrm{S}\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(d*(a+b*ln(c*x^n))),x)
[Out] int(x*FresnelS(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnels}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(x*fresnels((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{FresnelS}\left(d\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(d*(a + b*log(c*x^n))),x)
[Out] int(x*FresnelS(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xS\left(ad+bd\log\left(cx^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x*fresnels(a*d + b*d*log(c*x**n)), x)
```

3.56 $\int S\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=214

$$\left(\frac{1}{4}-\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)+\left(\frac{1}{4}-\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{erfi}\left(\left(\right.\right.$$

```
[Out] (1/4-1/4*I)*x*erf((1/2+1/2*I)*(1/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d
/Pi^(1/2))/exp(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))+(1/4-1/4*I)*
x*erfi((1/2+1/2*I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/
exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))+x*FresnelS(d*(a+b*ln(c*
x^n)))
```

Rubi [A] time = 0.33, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {6468, 4615, 2277, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{4}-\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)+\left(\frac{1}{4}-\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{Erfi}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/4 - I/4)*x*Erf[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c
*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^
(-1)) + ((1/4 - I/4)*x*Erfi[((1/2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(
c*x^n)^n^(-1)) + x*FresnelS[d*(a + b*Log[c*x^n])]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2277

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)), x_Symbol] :> Int[
F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b,
c, d, n}, x]
```

Rule 4615

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)], x_Symbol] :> Dist[I
/2, Int[E^(-(I*d*(a + b*Log[c*x^n])^2)), x], x] - Dist[I/2, Int[E^(I*d*(a +
 b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 6468

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Sim
p[x*FresnelS[d*(a + b*Log[c*x^n])], x] - Dist[b*d*n, Int[Sin[(Pi*(d*(a + b*
Log[c*x^n]))^2)/2], x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rubi steps

$$\int S\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx = xS\left(d\left(a+b\log\left(cx^n\right)\right)\right)-(bdn)\int \sin\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)\,dx$$
$$= xS\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(ibdn)\int e^{-\frac{1}{2}id^2\pi\left(a+b\log\left(cx^n\right)\right)^2}\,dx+\frac{1}{2}(ibdn)\int e^{\frac{1}{2}id^2\pi\left(a+b\right.}$$
$$= xS\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-iabd^2\pi\log\left(cx^n\right)-\frac{1}{2}ib^2d^2\right.$$
$$= xS\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)\left(cx^n\right)^{-}$$
$$= xS\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}\left(ibdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\right.$$
$$= xS\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}\left(ibdx\left(cx^n\right)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi\right.\right.$$
$$= xS\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}\left(ibde^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x\left(cx^n\right)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(\right.\right.$$
$$= \left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x\left(cx^n\right)^{-1/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)+\left(\frac{1}{4}\right.$$

Mathematica [A] time = 7.72, size = 316, normalized size = 1.48

$$xS\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\frac{\sqrt[4]{-1}\,x\left(cx^n\right)^{-1/n}\left(e^{\frac{i}{\pi b^2d^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\pi abd^2n+\pi b^2d^2n\log\left(cx^n\right)-i\right)}{\sqrt{\pi}\,bdn}\right)+i\,\text{erfi}\left(\frac{(-1)^{3/4}\left(\pi abd^2n+\pi b^2d^2n\log\right.}{\sqrt{2\pi}\,bdn}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*E^(((-2*a)/(b*n) - I/(b^2*d^
2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b^2
*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*x*(E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1
/2 + I/2)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]
+ I*Erfi[((-1)^(3/4)*(I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*S
qrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2] + I*Sin[(d^
2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(2*Sqrt[2]*(c*x^n)^n^(-1))
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\mathrm{fresnels}\left(bd\log\left(cx^{n}\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(fresnels(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{fresnels}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(fresnels((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \mathrm{S}\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a+b*ln(c*x^n))),x)
[Out] int(FresnelS(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{fresnels}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(fresnels((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{FresnelS}\left(d\ \left(a+b\ \ln\left(c\,x^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a + b*log(c*x^n))),x)
[Out] int(FresnelS(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int S\left(d\left(a+b\log\left(cx^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n))),x)
[Out] Integral(fresnels(d*(a + b*log(c*x**n))), x)
```

3.57 $\int \frac{S\left(d\left(a+b\log(cx^n)\right)\right)}{x}\,dx$

Optimal. Leaf size=65

$$\frac{\cos\left(\frac{1}{2}\pi d^2\left(a+b\log\left(cx^n\right)\right)^2\right)}{\pi bdn}+\frac{\left(a+b\log\left(cx^n\right)\right)S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

```
[Out] cos(1/2*d^2*Pi*(a+b*ln(c*x^n))^2)/b/d/n/Pi+FresnelS(d*(a+b*ln(c*x^n)))*(a+b
*ln(c*x^n))/b/n
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6418}

$$\frac{\cos\left(\frac{1}{2}\pi d^2\left(a+b\log\left(cx^n\right)\right)^2\right)}{\pi bdn}+\frac{\left(a+b\log\left(cx^n\right)\right)S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] Cos[(d^2*Pi*(a + b*Log[c*x^n])^2)/2]/(b*d*n*Pi) + (FresnelS[d*(a + b*Log[c*
x^n])]*(a + b*Log[c*x^n]))/(b*n)
```

Rule 6418

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelS[a +
 b*x])/b, x] + Simp[Cos[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int S(d(a+bx))\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{\text{Subst}\left(\int S(x)\,dx,x,ad+bd\log\left(cx^n\right)\right)}{bdn}\\ &= \frac{\cos\left(\frac{1}{2}\pi\left(ad+bd\log\left(cx^n\right)\right)^2\right)}{bdn\pi}+\frac{S\left(ad+bd\log\left(cx^n\right)\right)\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [B] time = 0.13, size = 164, normalized size = 2.52

$$-\frac{\sin\left(\frac{1}{2}\pi a^2d^2\right)\sin\left(\pi abd^2\log\left(cx^n\right)+\frac{1}{2}\pi b^2d^2\log^2\left(cx^n\right)\right)}{\pi bdn}+\frac{\cos\left(\frac{1}{2}\pi a^2d^2\right)\cos\left(\pi abd^2\log\left(cx^n\right)+\frac{1}{2}\pi b^2d^2\log^2\left(c\right.\right.}{\pi bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (Cos[(a^2*d^2*Pi)/2]*Cos[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/
2])/(b*d*n*Pi) + (a*FresnelS[d*(a + b*Log[c*x^n])])/(b*n) + (FresnelS[d*(a
+ b*Log[c*x^n])]*Log[c*x^n])/n - (Sin[(a^2*d^2*Pi)/2]*Sin[a*b*d^2*Pi*Log[c*
x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2])/(b*d*n*Pi)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\left(bd\log\left(cx^{n}\right)+ad\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*d*log(c*x^n) + a*d)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x, x)
```

maple [A] time = 0.04, size = 80, normalized size = 1.23

$$\frac{\ln\left(c\,x^{n}\right)\mathrm{S}\left(ad+bd\ln\left(c\,x^{n}\right)\right)}{n}+\frac{\mathrm{S}\left(ad+bd\ln\left(c\,x^{n}\right)\right)a}{nb}+\frac{\cos\left(\frac{\pi\left(ad+bd\ln\left(c\,x^{n}\right)\right)^{2}}{2}\right)}{nbd\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x,x)
```

```
[Out] 1/n*ln(c*x^n)*FresnelS(a*d+b*d*ln(c*x^n))+1/n/b*FresnelS(a*d+b*d*ln(c*x^n))
*a+1/n/b/d/Pi*cos(1/2*Pi*(a*d+b*d*ln(c*x^n))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{FresnelS}\left(d\ \left(a+b\ \ln\left(c\,x^n\right)\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int(FresnelS(d*(a + b*log(c*x^n)))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S\left(ad+bd\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x, x)
```

3.58 $\int \frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx$

Optimal. Leaf size=217

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\left(cx^n\right)^{\frac{1}{n}} e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}} \operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\left(cx^n\right)^{\frac{1}{n}} e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}} \operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)\right.}{\sqrt{\pi}\,bd}\right.}{x}$$

```
[Out] (1/4-1/4*I)*exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erf((1/2+1/2*
I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x+(1/4-1/4*I)*ex
p(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erfi((1/2+1/2*I)*(1/n+I*a*b
*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x-FresnelS(d*(a+b*ln(c*x^n)))
/x
```

Rubi [A] time = 0.51, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6471, 4617, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\left(cx^n\right)^{\frac{1}{n}} e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}} \operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\left(cx^n\right)^{\frac{1}{n}} e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)\right.}{\sqrt{\pi}\,bd}\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] ((1/4 - I/4)*E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Erf[((1/
2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])]
)/x + ((1/4 - I/4)*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Er
fi[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqr
t[Pi])])/x - FresnelS[d*(a + b*Log[c*x^n])]/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 4617

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6471

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelS[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Sin[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx = -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + (bdn)\int \frac{\sin\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)}{x^2}\,dx$$

$$= -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + \frac{1}{2}(ibdn)\int \frac{e^{-\frac{1}{2}id^2\pi\left(a+b\log(cx^n)\right)^2}}{x^2}\,dx - \frac{1}{2}(ibdn)\int \frac{e^{\frac{1}{2}id^2\pi(a\cdots}}{\cdots}$$

$$= -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + \frac{1}{2}(ibdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log\left(cx^n\right) - \frac{1}{2}ib^2d^2\cdots\right.}{x^2}$$

$$= -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + \frac{1}{2}(ibdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)\left(cx^n\right)^{\cdots}}{x^2}$$

$$= -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + \frac{1}{2}\left(ibdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2\cdots\right.$$

$$= -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + \frac{\left(ibd\left(cx^n\right)^{-iabd^2\pi-\frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi + \cdots\right.\right.}{2x}$$

$$= -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + \frac{\left(ibde^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}\left(cx^n\right)^{-iabd^2\pi-\frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(-1-\cdots\right.}{\cdots}\right.\right.}{2x}$$

$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}\left(cx^n\right)^{\frac{1}{n}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} + \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}\left(cx^n\cdots\right.}{\cdots}$$

Mathematica [A] time = 4.42, size = 195, normalized size = 0.90

$$-\frac{4S\left(d\left(a+b\log\left(cx^n\right)\right)\right) + \sqrt[4]{-1}\sqrt{2}\left(cx^n\right)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\left(e^{\frac{i}{\pi b^2d^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\pi abd^2n+\pi b^2d^2n\log(cx^n)+i\right)}{\sqrt{\pi}bdn}\right) + i\,\text{erfi}\left(\frac{(-1)^{3/4}\left(\pi ab\cdots\right.}{\cdots}\right.\right.}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[2]*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(
-1)*(I*Erfi[((-1)^(3/4)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d
*n*Sqrt[2*Pi])] + E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(I + a*b*d^2*n*P
i + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]) + 4*FresnelS[d*(a + b*Log[
c*x^n])])/x
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\left(bd\log\left(cx^{n}\right)+ad\right)}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
[Out] integral(fresnels(b*d*log(c*x^n) + a*d)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x^2, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x^2,x)
[Out] int(FresnelS(d*(a+b*ln(c*x^n)))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}\left(d\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int(FresnelS(d*(a + b*log(c*x^n)))/x^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S\left(ad + bd\log\left(cx^{n}\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x**2, x)
```

3.59 $\int \frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$

Optimal. Leaf size=228

$$\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\left(cx^n\right)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log\left(cx^n\right)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x^2}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\left(cx^n\right)^{2/n}e^{-\frac{2\left(-\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d\right.}{\sqrt{\pi}\,bd}\right.}{x^2}$$

```
[Out] (1/8-1/8*I)*exp((2*I+2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*(c*x^n)^(2/n)*erf((1/2
+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x^2+(1/8-1/
8*I)*(c*x^n)^(2/n)*erfi((1/2+1/2*I)*(2/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n
))/b/d/Pi^(1/2))/exp(2*(I-a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/x^2-1/2*FresnelS(d*
(a+b*ln(c*x^n)))/x^2
```

Rubi [A] time = 0.52, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6471, 4617, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\left(cx^n\right)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log\left(cx^n\right)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x^2}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\left(cx^n\right)^{2/n}e^{-\frac{2\left(-\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2\right.}{\sqrt{\pi}\,b}\right.}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] ((1/8 - I/8)*E^((2*I + 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*(c*x^n)^(2/n)*Erf[
((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])
])/x^2 + ((1/8 - I/8)*(c*x^n)^(2/n)*Erfi[((1/2 + I/2)*(2/n + I*a*b*d^2*Pi +
 I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I - a*b*d^2*n*Pi))/(b^2
*d^2*n^2*Pi))*x^2) - FresnelS[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 4617

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6471

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelS[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Sin[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx = -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{2}(bdn)\int \frac{\sin\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)}{x^3}\,dx$$

$$= -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{4}(ibdn)\int \frac{e^{-\frac{1}{2}id^2\pi\left(a+b\log\left(cx^n\right)\right)^2}}{x^3}\,dx-\frac{1}{4}(ibdn)\int \frac{e^{\frac{1}{2}id^2\pi(}}{}$$

$$= -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{4}(ibdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi-iabd^2\pi\log\left(cx^n\right)-\frac{1}{2}ib^2d\right.}{x^3}$$

$$= -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{4}(ibdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)\left(cx^n\right.}{x^3}$$

$$= -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{1}{4}\left(ibdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib\right.$$

$$= -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{\left(ibd\left(cx^n\right)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\pi\right.\right.}{4x^2}$$

$$= -\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}+\frac{\left(ibde^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}\left(cx^n\right)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\operatorname{Subst}\left(\int \exp\left(\right.\right.}{4x^2}$$

$$= \frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}\left(cx^n\right)^{2/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)}{x^2}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}}{}$$

Mathematica [A] time = 4.51, size = 200, normalized size = 0.88

$$-\frac{S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2}-\frac{\sqrt[4]{-1}\left(e^{\frac{4i}{\pi b^2d^2n^2}}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}\left(\pi abd^2n+\pi b^2d^2n\log\left(cx^n\right)+2i\right)}{\sqrt{2\pi}\,bdn}\right)+i\operatorname{erfi}\left(\frac{(-1)^{3/4}\left(\pi abd^2n+\pi b^2d^2n\log\left(cx^n\right)-2i\right)}{\sqrt{2\pi}\,bdn}\right)\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((2*((a*n)/b - I/(b^2*d^2*Pi) + n*(-(n*Log[x]) + Log[c*x
^n])))/n^2)*(I*Erfi[((-1)^(3/4)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[2*Pi])] + E^((4*I)/(b^2*d^2*n^2*Pi))*Erfi[((-1)^(1/4)*(2*
I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])]))/Sqrt[2]
- FresnelS[d*(a + b*Log[c*x^n])]/(2*x^2)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\left(bd\log\left(cx^{n}\right)+ad\right)}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
[Out] integral(fresnels(b*d*log(c*x^n) + a*d)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnels}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x^3, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x^3,x)
[Out] int(FresnelS(d*(a+b*ln(c*x^n)))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnels}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
[Out] integrate(fresnels((b*log(c*x^n) + a)*d)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}\left(d\ \left(a+b\ \ln\left(c\,x^{n}\right)\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int(FresnelS(d*(a + b*log(c*x^n)))/x^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S\left(ad + bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x**3, x)
```

3.60 $\int (ex)^m S\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=280

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-\frac{m+1}{n}}\exp\left(\frac{i(m+1)\left(2i\pi abd^2n+m+1\right)}{2\pi b^2d^2n^2}\right)\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2n+i\pi b^2d^2n\log(cx^n)+m+1\right)}{\sqrt{\pi}\,bdn}\right)}{m+1}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-}}{}$$

```
[Out] (1/4-1/4*I)*exp(1/2*I*(1+m)*(1+m+2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x*(e*x)^
m*erf((1/2+1/2*I)*(1+m+I*a*b*d^2*n*Pi+I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1
/2))/(1+m)/((c*x^n)^((1+m)/n))+(1/4-1/4*I)*x*(e*x)^m*erfi((1/2+1/2*I)*(1+m-
I*a*b*d^2*n*Pi-I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1/2))/exp(1/2*I*(1+m)*(1
+m-2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/(1+m)/((c*x^n)^((1+m)/n))+(e*x)^(1+m)*
FresnelS(d*(a+b*ln(c*x^n)))/e/(1+m)
```

Rubi [A] time = 0.70, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {6471, 4617, 2278, 2274, 15, 20, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-\frac{m+1}{n}}\exp\left(\frac{i(m+1)\left(2i\pi abd^2n+m+1\right)}{2\pi b^2d^2n^2}\right)\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2n+i\pi b^2d^2n\log(cx^n)+m+1\right)}{\sqrt{\pi}\,bdn}\right)}{m+1}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/4 - I/4)*E^(((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*x*(e*x)^m*Erf[((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log
[c*x^n]))/(b*d*n*Sqrt[Pi])])/((1 + m)*(c*x^n)^((1 + m)/n)) + ((1/4 - I/4)*x
*(e*x)^m*Erfi[((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])])/(E^(((I/2)*(1 + m)*(1 + m - (2*I)*a*b*d^2*n*Pi))/(
b^2*d^2*n^2*Pi))*(1 + m)*(c*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*FresnelS[d*(
a + b*Log[c*x^n])])/(e*(1 + m))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
```

```
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 4617

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6471

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelS[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Sin[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (ex)^m S\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx = \frac{(ex)^{1+m}S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{(bdn)\int (ex)^m \sin\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)\,d}{1+m}$$

$$= \frac{(ex)^{1+m}S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{(ibdn)\int e^{-\frac{1}{2}id^2\pi\left(a+b\log\left(cx^n\right)\right)^2}(ex)^m\,dx}{2(1+m)} + \frac{(ibdn}{}$$

$$= \frac{(ex)^{1+m}S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log\left(cx^n\right)\right.}{2(1+m)}$$

$$= \frac{(ex)^{1+m}S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right.\right.}{2(1+m)}$$

$$= \frac{(ex)^{1+m}S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{\left(ibdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi\right.}{2(1+}$$

$$= \frac{(ex)^{1+m}S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{\left(ibdnx^{-m+iabd^2n\pi}(ex)^m\left(cx^n\right)^{-iabd^2\pi}\right)\int \exp\left(\right.}{2(1}$$

$$= \frac{(ex)^{1+m}S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{\left(ibdx(ex)^m\left(cx^n\right)^{-iabd^2\pi-\frac{1+m-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{(ex)^{1+m}S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{\left(ibd\exp\left(-\frac{i(1+m)\left(1+m-2iabd^2n\pi\right)}{2b^2d^2n^2\pi}\right)x(ex)^m\left(cx^n\right)\right.}{}$$

$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\exp\left(\frac{i(1+m)\left(1+m+2iabd^2n\pi\right)}{2b^2d^2n^2\pi}\right)x(ex)^m\left(cx^n\right)^{-\frac{1+m}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+m+iabd^2n\pi+ib^2\right.}{bdn\sqrt{\pi}}\right.}{1+m}$$

Mathematica [A] time = 6.22, size = 244, normalized size = 0.87

$$\frac{(ex)^m \Bigl(4xS\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\sqrt[4]{-1}\,\sqrt{2}\,x^{-m}\exp\left(-\frac{(m+1)\left(2\pi abd^2n+2\pi b^2d^2n\left(\log(cx^n)-n\log(x)\right)+im+i\right)}{2\pi b^2d^2n^2}\right)\Bigl(\text{erf}\Bigl(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(\pi ab \cdots}{\cdots}}{4(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^m*(-(((-1)^(1/4)*Sqrt[2]*(Erf[((1/2 + I/2)*(I + I*m + a*b*d^2*n*Pi +
 b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])] + E^((I*(1 + m)^2)/(b^2*d^2*n^
2*Pi))*Erfi[((-1)^(3/4)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x^n]
))/(b*d*n*Sqrt[2*Pi])]))/(E^(((1 + m)*(I + I*m + 2*a*b*d^2*n*Pi + 2*b^2*d^2
*n*Pi*(-(n*Log[x]) + Log[c*x^n])))/(2*b^2*d^2*n^2*Pi))*x^m)) + 4*x*FresnelS
[d*(a + b*Log[c*x^n])]))/(4*(1 + m))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m\,\text{fresnels}\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnels(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*fresnels(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m\,\text{fresnels}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnels(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*fresnels((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (ex)^m\,\mathrm{S}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*FresnelS(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*FresnelS(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \,\mathrm{fresnels}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnels(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*fresnels((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{FresnelS}\left(d\ \left(a+b\ \ln\left(c\,x^n\right)\right)\right)\,\left(e\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

```
[Out] int(FresnelS(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m\, S\left(ad+bd\log\left(cx^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*fresnels(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*fresnels(a*d + b*d*log(c*x**n)), x)
```

3.61 $\int e^{c+\frac{1}{2}ib^2\pi x^2} S(bx)\,dx$

Optimal. Leaf size=64

$$\frac{1}{4}ibe^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{e^c\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}\,bx\right)^2}{8b}$$

```
[Out] 1/8*exp(c)*erf((1/2-1/2*I)*b*x*Pi^(1/2))^2/b+1/4*I*b*exp(c)*x^2*Hypergeomet
ricPFQ([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
```

Rubi [A] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6436, 6376, 6375, 30}

$$\frac{1}{4}ibe^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{e^c\text{Erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}\,bx\right)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

```
[Out] -(E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/(8*b) + (I/4)*b*E^c*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c
*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6436

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelS[(b_.)*(x_)], x_Symbol] :> Dist[(1 + I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 + I)*b*x)/2], x], x] + Dist[(1 - I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 - I)*b*x)/2], x], x] /; FreeQ[{b, c,
 d}, x] && EqQ[d^2, -((Pi^2*b^4)/4)]
```

Rubi steps

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}S(bx)\,dx = \left(-\frac{1}{4}-\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\mathrm{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)dx + \left(\frac{1}{4}+\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\mathrm{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}$$

$$= \frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) - \frac{e^c\,\mathrm{Subst}\left(\int x\,dx, x, \mathrm{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)\right)}{4b}$$

$$= -\frac{e^c\mathrm{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)^2}{8b} + \frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.03, size = 0, normalized size = 0.00

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}S(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

```
[Out] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(e^{\left(\frac{1}{2}i\pi b^2x^2+c\right)}\mathrm{fresnels}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(e^(1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(\frac{1}{2}i\pi b^2x^2+c\right)}\mathrm{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{c+\frac{i b^2 \pi x^2}{2}} \mathrm{S}\left(b x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(\frac{1}{2} i \pi b^2 x^2 + c\right)} \mathrm{fresnels}\left(b x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{\frac{1\mathrm{i}\,\Pi\,b^2\,x^2}{2}+c}\,\mathrm{FresnelS}\left(b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelS(b*x),x)
[Out] int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelS(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{c} \int e^{\frac{i \pi b^{2} x^{2}}{2}} S\left(b x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b**2*pi*x**2)*fresnels(b*x),x)
[Out] exp(c)*Integral(exp(I*pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.62 $\int e^{c-\frac{1}{2}ib^2\pi x^2} S(bx)\,dx$

Optimal. Leaf size=64

$$\frac{e^c \operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}\,bx\right)^2}{8b}-\frac{1}{4}ibe^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/8*exp(c)*erf((1/2+1/2*I)*b*x*Pi^(1/2))^2/b-1/4*I*b*exp(c)*x^2*Hypergeomet
ricPFQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)
```

Rubi [A] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6436, 6373, 30, 6378}

$$\frac{e^c \operatorname{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}\,bx\right)^2}{8b}-\frac{1}{4}ibe^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

```
[Out] (E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/(8*b) - (I/4)*b*E^c*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 6436

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelS[(b_.)*(x_)], x_Symbol] :> Dist[(1 + I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 + I)*b*x)/2], x], x] + Dist[(1 - I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 - I)*b*x)/2], x], x] /; FreeQ[{b, c,
 d}, x] && EqQ[d^2, -((Pi^2*b^4)/4)]
```

Rubi steps

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}S(bx)\,dx = \left(-\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)dx + \left(\frac{1}{4}+\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\right.$$

$$= -\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{e^c\,\text{Subst}\left(\int x\,dx, x, \text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)\right)}{4b}$$

$$= \frac{e^c\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)^2}{8b} - \frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}S(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
[Out] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)}\text{fresnels}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
[Out] integral(e^(-1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)}\text{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{c-\frac{i b^2 \pi x^2}{2}} \mathrm{S}\left(b x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(-\frac{1}{2} i \pi b^2 x^2 + c\right)} \mathrm{fresnels}\left(b x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{c-\frac{\Pi\, b^2\, x^2\, 1\mathrm{i}}{2}}\, \mathrm{FresnelS}\left(b\, x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelS(b*x),x)
[Out] int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelS(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{c} \int e^{- \frac{i \pi b^{2} x^{2}}{2}} S\left(b x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b**2*pi*x**2)*fresnels(b*x),x)
[Out] exp(c)*Integral(exp(-I*pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.63 $\int S(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=101

$$-\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\sin(c)C(bx)S(bx)}{2b}+\frac{\cos(c)S(bx)^2}{2b}$$

```
[Out] 1/2*cos(c)*FresnelS(b*x)^2/b+1/2*FresnelC(b*x)*FresnelS(b*x)*sin(c)/b-1/8*I
*b*x^2*HypergeometricPFQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)*sin(c)+1/8*I*b*
x^2*HypergeometricPFQ([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)*sin(c)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6442, 6446, 6440, 30}

$$-\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\sin(c)\text{FresnelC}(bx)S(bx)}{2b}+\cos($$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```

```
[Out] (Cos[c]*FresnelS[b*x]^2)/(2*b) + (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
 - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]*Sin[c
] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin[c
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6442

```
Int[FresnelS[(b_.)*(x_)]*Sin[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[Sin[c]
, Int[Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[Cos[c], Int[Sin[d*x^2]*Fresne
lS[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rubi steps

$$\int S(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)dx=\cos(c)\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx+\sin(c)\int\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx$$
$$=\frac{C(bx)S(bx)\sin(c)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2\right.$$
$$=\frac{\cos(c)S(bx)^2}{2b}+\frac{C(bx)S(bx)\sin(c)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c)+\frac{1}{8}ib$$

Mathematica [F] time = 0.06, size = 0, normalized size = 0.00

$$\int S(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```

```
[Out] Integrate[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \mathrm{S}(bx)\sin\left(c+\frac{b^{2}\pi\,x^{2}}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
[Out] int(FresnelS(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{\Pi\,b^{2}\,x^{2}}{2}+c\right)\mathrm{FresnelS}\,(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + (Pi*b^2*x^2)/2)*FresnelS(b*x),x)
[Out] int(sin(c + (Pi*b^2*x^2)/2)*FresnelS(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{\pi b^{2}x^{2}}{2}+c\right)S\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(c+1/2*b**2*pi*x**2),x)
[Out] Integral(sin(pi*b**2*x**2/2 + c)*fresnels(b*x), x)
```

3.64 $\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=101

$$-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\cos(c)C(bx)S(bx)}{2b}-\frac{\sin(c)S(bx)^2}{2b}$$

```
[Out] 1/2*cos(c)*FresnelC(b*x)*FresnelS(b*x)/b-1/8*I*b*x^2*cos(c)*HypergeometricP
FQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)+1/8*I*b*x^2*cos(c)*HypergeometricPFQ(
[1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)-1/2*FresnelS(b*x)^2*sin(c)/b
```

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6448, 6446, 6440, 30}

$$-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\cos(c)\text{FresnelC}(bx)S(bx)}{2b}-\frac{\sin(c)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*Cos[c]*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] + (I/8)*b*x^2*Cos[c]*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] - (FresnelS[b*x]^2*Sin[c])/(2*b
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 6448

```
Int[Cos[(c_) + (d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Dist[Cos[c]
, Int[Cos[d*x^2]*FresnelS[b*x], x], x] - Dist[Sin[c], Int[Sin[d*x^2]*Fresne
lS[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx = \cos(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx - \sin(c)\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$
$$= \frac{\cos(c)C(bx)S(bx)}{2b} - \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\Big($$
$$= \frac{\cos(c)C(bx)S(bx)}{2b} - \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\Big($$

Mathematica [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x],x]
[Out] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\cos\left(\frac{1}{2}\,\pi b^2x^2+c\right)\text{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}\,\pi b^2x^2+c\right)\text{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \cos\left(c+\frac{b^2\pi\,x^2}{2}\right)\mathrm{S}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c+1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(cos(c+1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}\,\pi b^2x^2+c\right)\mathrm{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(\frac{\Pi\,b^2\,x^2}{2}+c\right)\,\mathrm{FresnelS}\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + (Pi*b^2*x^2)/2)*FresnelS(b*x),x)
[Out] int(cos(c + (Pi*b^2*x^2)/2)*FresnelS(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{\pi b^2x^2}{2}+c\right)S\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b**2*pi*x**2)*fresnels(b*x),x)
[Out] Integral(cos(pi*b**2*x**2/2 + c)*fresnels(b*x), x)
```

3.65 $\int S(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=13

$$\frac{S(bx)^3}{3b}$$

```
[Out] 1/3*FresnelS(b*x)^3/b
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6440, 30}

$$\frac{S(bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^2*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int S(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, S(bx)\right)}{b}\\ &= \frac{S(bx)^3}{3b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{S(bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^2*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^3/(3*b)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnels}\,(bx)^2\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^2*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\,(bx)^2\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^2*sin(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.01, size = 12, normalized size = 0.92

$$\frac{\mathrm{S}\,(bx)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] 1/3*FresnelS(b*x)^3/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\,(bx)^2\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^2*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \mathrm{FresnelS}(b\,x)^2\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2*sin((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)^2*sin((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 1.04, size = 10, normalized size = 0.77

$$\begin{cases} \frac{S^{3}(bx)}{3b} & \text{for}\: b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2*sin(1/2*b**2*pi*x**2),x)
[Out] Piecewise((fresnels(b*x)**3/(3*b), Ne(b, 0)), (0, True))
```

3.66 $\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=13

$$\frac{S(bx)^2}{2b}$$

```
[Out] 1/2*FresnelS(b*x)^2/b
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6440, 30}

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= \frac{\text{Subst}(\int x\,dx, x, S(bx))}{b}\\ &= \frac{S(bx)^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] FresnelS[b*x]^2/(2*b)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{\mathrm{S}\,(bx)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] 1/2*FresnelS(b*x)^2/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \mathrm{FresnelS}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 0.37, size = 10, normalized size = 0.77

$$\begin{cases} \frac{S^{2}(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Piecewise((fresnels(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

3.67 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)}\,dx$

Optimal. Leaf size=9

$$\frac{\log(S(bx))}{b}$$

[Out] ln(FresnelS(b*x))/b

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6440, 29}

$$\frac{\log(S(bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x],x]

[Out] Log[FresnelS[b*x]]/b

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, S(bx)\right)}{b}\\ &= \frac{\log(S(bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{\log(S(bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x],x]
```

```
[Out] Log[FresnelS[b*x]]/b
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{fresnels}\,(bx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(sin(1/2*pi*b^2*x^2)/fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{fresnels}\,(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x), x)
```

maple [A] time = 0.08, size = 10, normalized size = 1.11

$$\frac{\ln\left(\mathrm{S}\,(bx)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x),x)
```

```
[Out] ln(FresnelS(b*x))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{fresnels}\,(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{\sin\left(\frac{\Pi b^2 x^2}{2}\right)}{\mathrm{FresnelS}\,(b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x),x)
```

```
[Out] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x), x)
```

sympy [A] time = 0.22, size = 8, normalized size = 0.89

$$\begin{cases} \frac{\log\left(S(bx)\right)}{b} & \text{for}\ b \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x),x)
```

```
[Out] Piecewise((log(fresnels(b*x))/b, Ne(b, 0)), (nan, True))
```

3.68 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^2}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{bS(bx)}$$

```
[Out] -1/b/FresnelS(b*x)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6440, 30}

$$-\frac{1}{bS(bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^2,x]
```

```
[Out] -(1/(b*FresnelS[b*x]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, S(bx)\right)}{b}\\ &= -\frac{1}{bS(bx)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$-\frac{1}{bS(bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^2,x]
[Out] -(1/(b*FresnelS[b*x]))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{\mathrm{fresnels}\,(bx)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^2,x, algorithm="fricas")
[Out] integral(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{\mathrm{fresnels}\,(bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^2,x, algorithm="giac")
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^2, x)
```

maple [A] time = 0.01, size = 12, normalized size = 1.09

$$-\frac{1}{b\,\mathrm{S}\,(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x)^2,x)
[Out] -1/b/FresnelS(b*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{\mathrm{fresnels}\,(bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{\sin\left(\frac{\Pi\, b^2\, x^2}{2}\right)}{\mathrm{FresnelS}\,(b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^2,x)
```

```
[Out] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^2, x)
```

sympy [A] time = 0.66, size = 10, normalized size = 0.91

$$\begin{cases} -\frac{1}{bS(bx)} & \text{for}\ b \neq 0 \\ \mathrm{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x)**2,x)
```

```
[Out] Piecewise((-1/(b*fresnels(b*x)), Ne(b, 0)), (nan, True))
```

3.69 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2bS(bx)^2}$$

```
[Out] -1/2/b/FresnelS(b*x)^2
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6440, 30}

$$-\frac{1}{2bS(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^3,x]
```

```
[Out] -1/(2*b*FresnelS[b*x]^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, S(bx)\right)}{b}\\ &= -\frac{1}{2bS(bx)^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{2bS(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^3,x]
```

```
[Out] -1/2*1/(b*FresnelS[b*x]^2)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}\,(bx)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}\,(bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^3, x)
```

maple [A] time = 0.01, size = 12, normalized size = 0.92

$$-\frac{1}{2b\mathrm{S}\,(bx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x)^3,x)
```

```
[Out] -1/2/b/FresnelS(b*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{fresnels}\,(bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnels(b*x)^3,x, algorithm="maxima")
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnels(b*x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\sin\left(\frac{\Pi b^2 x^2}{2}\right)}{\mathrm{FresnelS}\left(b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^3,x)
[Out] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^3, x)
```

sympy [A] time = 1.53, size = 14, normalized size = 1.08

$$\begin{cases} -\frac{1}{2bS^2(bx)} & \text{for } b \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x)**3,x)
[Out] Piecewise((-1/(2*b*fresnels(b*x)**2), Ne(b, 0)), (nan, True))
```

3.70 $\int S(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=17

$$\frac{S(bx)^{n+1}}{b(n+1)}$$

```
[Out] FresnelS(b*x)^(1+n)/b/(1+n)
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6440, 30}

$$\frac{S(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^(1 + n)/(b*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int S(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= \frac{\text{Subst}\left(\int x^n \, dx, x, S(bx)\right)}{b}\\ &= \frac{S(bx)^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{S(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^(1 + n)/(b*(1 + n))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnels}\,(bx)^n\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\,(bx)^n\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.01, size = 18, normalized size = 1.06

$$\frac{\mathrm{S}\,(bx)^{1+n}}{b\,(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] FresnelS(b*x)^(1+n)/b/(1+n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\,(bx)^n\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \text{FresnelS}\,(b\,x)^n\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^n*sin((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)^n*sin((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 2.98, size = 31, normalized size = 1.82

$$\begin{cases} 0 & \text{for}\ b = 0 \wedge (b = 0 \vee n = -1) \\ \frac{\log{(S(bx))}}{b} & \text{for}\ n = -1 \\ \frac{S(bx)S^{n}(bx)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**n*sin(1/2*b**2*pi*x**2),x)
[Out] Piecewise((0, Eq(b, 0) & (Eq(b, 0) | Eq(n, -1))), (log(fresnels(b*x))/b, Eq
(n, -1)), (fresnels(b*x)*fresnels(b*x)**n/(b*n + b), True))
```

3.71 $\int x^8 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) \, dx$

Optimal. Leaf size=232

$$\frac{105S(bx)^2}{2\pi^4b^9}+\frac{105x^2}{4\pi^4b^7}-\frac{7x^6}{12\pi^2b^3}-\frac{x^7S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{40\sin\left(\pi b^2x^2\right)}{\pi^5b^9}-\frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}+\frac{55x^2\cos\left(\pi b^2x\right.}{4\pi^4b^7}$$

```
[Out] 105/4*x^2/b^7/Pi^4-7/12*x^6/b^3/Pi^2+55/4*x^2*cos(b^2*Pi*x^2)/b^7/Pi^4-1/4*
x^6*cos(b^2*Pi*x^2)/b^3/Pi^2+35*x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^6/P
i^3-x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+105/2*FresnelS(b*x)^2/b^9/
Pi^4-105*x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^8/Pi^4+7*x^5*FresnelS(b*x)*s
in(1/2*b^2*Pi*x^2)/b^4/Pi^2-40*sin(b^2*Pi*x^2)/b^9/Pi^5+5/2*x^4*sin(b^2*Pi*
x^2)/b^5/Pi^3
```

Rubi [A] time = 0.38, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6454, 6462, 3379, 3309, 30, 3296, 2637, 2634, 6440}

$$\frac{7x^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{x^7S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{35x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{105S(bx)^2}{2\pi^4b^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (35*x^3*Cos[(b^2*Pi*x^2)
/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^
2*Pi) + (105*FresnelS[b*x]^2)/(2*b^9*Pi^4) - (105*x*FresnelS[b*x]*Sin[(b^2*
Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^
2) - (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$
\begin{aligned}
\int x^8 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx &= -\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi}+\frac{\int x^7\sin\left(b^2\pi x^2\right)\,dx}{2b\pi} \\
&= -\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{7x^5S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{35\int x^4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\cdots}{b^4\pi^2} \\
&= -\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{7x^5S(b\cdots}{\cdots} \\
&= -\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}-\frac{105xS(b\cdots}{\cdots} \\
&= -\frac{7x^6}{12b^3\pi^2}+\frac{41x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4}-\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^7\cdots}{\cdots} \\
&= \frac{105x^2}{4b^7\pi^4}-\frac{7x^6}{12b^3\pi^2}+\frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4}-\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b\cdots}{b^6\pi^3} \\
&= \frac{105x^2}{4b^7\pi^4}-\frac{7x^6}{12b^3\pi^2}+\frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4}-\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b\cdots}{b^6\pi^3}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 232, normalized size = 1.00

$$
\frac{105S(bx)^2}{2\pi^4b^9}+\frac{105x^2}{4\pi^4b^7}-\frac{7x^6}{12\pi^2b^3}-\frac{x^7S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{40\sin\left(\pi b^2x^2\right)}{\pi^5b^9}-\frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}+\frac{55x^2\cos\left(\pi b^2x\cdots\right.}{4\pi^4b^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (35*x^3*Cos[(b^2*Pi*x^2)
/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^
2*Pi) + (105*FresnelS[b*x]^2)/(2*b^9*Pi^4) - (105*x*FresnelS[b*x]*Sin[(b^2*
Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^
2) - (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$
\text{integral}\left(x^8\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(x^8*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \text{fresnels}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^8*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^8 \mathrm{S}\,(bx) \sin\left(\frac{b^2 \pi\, x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \text{fresnels}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^8*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^8\,\mathrm{FresnelS}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^8*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Timed out
```

3.72 $\int x^7 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=216

$$-\frac{531C\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,\pi^4b^8}+\frac{24x}{\pi^4b^7}-\frac{3x^5}{5\pi^2b^3}-\frac{x^6S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{48S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}+\frac{147x\cos\left(\pi b^2x^2\right)}{16\pi^4b^7}+\frac{24x^2S(bx)\cos}{\pi^3b}$$

```
[Out] 24*x/b^7/Pi^4-3/5*x^5/b^3/Pi^2+147/16*x*cos(b^2*Pi*x^2)/b^7/Pi^4-1/4*x^5*co
s(b^2*Pi*x^2)/b^3/Pi^2+24*x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^6/Pi^3-x^
6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi-48*FresnelS(b*x)*sin(1/2*b^2*Pi*
x^2)/b^8/Pi^4+6*x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+17/8*x^3*sin
(b^2*Pi*x^2)/b^5/Pi^3-531/32*FresnelC(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6454, 6462, 3391, 30, 3386, 3385, 3352, 6460, 3357}

$$-\frac{531\text{FresnelC}\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,\pi^4b^8}+\frac{6x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{48S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{x^6S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{24x^2S(bx)\cos\left(\right.}{\pi^3b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (24*x)/(b^7*Pi^4) - (3*x^5)/(5*b^3*Pi^2) + (147*x*Cos[b^2*Pi*x^2])/(16*b^7*
Pi^4) - (x^5*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (531*FresnelC[Sqrt[2]*b*x])/(1
6*Sqrt[2]*b^8*Pi^4) + (24*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3)
 - (x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (48*FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (6*x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b
^4*Pi^2) + (17*x^3*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
```

1]

Rubi steps

$$\int x^7 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi} + \frac{\int x^6\sin\left(b^2\pi x^2\right)dx}{2b\pi}$$

$$= -\frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{6x^4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{24\int x^3S(b}{}$$

$$= -\frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{6x^4S(bx)}{}$$

$$= -\frac{3x^5}{5b^3\pi^2} + \frac{111x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^6\text{co}}{}$$

$$= -\frac{3x^5}{5b^3\pi^2} + \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{15C\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,b^8\pi^4} - \frac{3\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{b^8\pi^4} +$$

$$= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} + \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{51C\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,b^8\pi^4} - \frac{3\sqrt{2}\,C\left(\sqrt{2}\right.}{b^8\pi^4}$$

$$= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} + \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{51C\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,b^8\pi^4} - \frac{15\sqrt{2}\,C\left(\sqrt{}\right.}{b^8\pi^4}$$

Mathematica [A] time = 0.29, size = 153, normalized size = 0.71

$$\frac{-160S(bx)\left(\pi b^2x^2\left(\pi^2b^4x^4-24\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)-6\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\right)+2bx\left(2\left(-24\pi^2b^4x^4+85\pi b^2x^2\,\text{s}\right.\right.}{160\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 160*FresnelS[b*x]*(b^2*Pi*x^2*(-24 +
 b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)
/2]) + 2*b*x*((735 - 20*b^4*Pi^2*x^4)*Cos[b^2*Pi*x^2] + 2*(960 - 24*b^4*Pi^
2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/(160*b^8*Pi^4)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(x^7\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^7*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \text{fresnels}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.01, size = 318, normalized size = 1.47

$$\frac{\frac{\mathrm{S}(bx)\left(-\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{6b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{24\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}}{\pi}\right)}{b^7}-\frac{\frac{\frac{3}{5}\pi^2b^5x^5-24bx}{\pi^4}-\frac{3\left(\frac{\pi b^3x^3\sin\left(b^2\pi x^2\right)}{2}-\frac{3\pi\left(-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi}+\frac{\sqrt{2}\ \mathrm{Fresn}}{}\right.}{2}\right.}{\pi^4}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (FresnelS(b*x)/b^7*(-1/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)+6/Pi*(1/Pi*b^4*x^4*si
n(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^
2*Pi*x^2))))-1/b^7*(3/Pi^4*(1/5*Pi^2*b^5*x^5-8*b*x)-3/Pi^4*(1/2*Pi*b^3*x^3*
sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC
(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))-1/2/Pi^3*(-1/2*Pi*b^5*x^5*c
os(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*c
os(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*
x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \text{fresnels}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^7*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \operatorname{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^7*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**7*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.73 $\int x^6 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=248

$$\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}-\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}-\frac{15C(bx)S(bx)}{2\pi^3b^7}-\frac{5x^4}{8\pi^2b^3}-\frac{x^5S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+$$

```
[Out] -5/8*x^4/b^3/Pi^2+11/2*cos(b^2*Pi*x^2)/b^7/Pi^4-1/4*x^4*cos(b^2*Pi*x^2)/b^3
/Pi^2+15*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^6/Pi^3-x^5*cos(1/2*b^2*Pi*x^
2)*FresnelS(b*x)/b^2/Pi-15/2*FresnelC(b*x)*FresnelS(b*x)/b^7/Pi^3+15/8*I*x^
2*HypergeometricPFQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^5/Pi^3-15/8*I*x^2*
HypergeometricPFQ([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^5/Pi^3+5*x^3*FresnelS
(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+7/4*x^2*sin(b^2*Pi*x^2)/b^5/Pi^3
```

Rubi [A] time = 0.25, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6454, 6462, 3379, 3309, 30, 3296, 2638, 6446}

$$\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}-\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}-\frac{15\text{FresnelC}(bx)S(bx)}{2\pi^3b^7}+\frac{5x^3S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-5*x^4)/(8*b^3*Pi^2) + (11*Cos[b^2*Pi*x^2])/(2*b^7*Pi^4) - (x^4*Cos[b^2*Pi
*x^2])/(4*b^3*Pi^2) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) -
 (x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (15*FresnelC[b*x]*Fresn
elS[b*x])/(2*b^7*Pi^3) + (((15*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}
, (-I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (((15*I)/8)*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^5*Pi^3) + (5*x^3*FresnelS[b*x]*Sin[(b^2*P
i*x^2)/2])/(b^4*Pi^2) + (7*x^2*Sin[b^2*Pi*x^2])/(4*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}
\int x^6 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx &= -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi}+\frac{\int x^5\sin\left(b^2\pi x^2\right)dx}{2b\pi} \\
&= -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{5x^3S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{15\int x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\
&= -\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{5x^3S(bx)}{} \\
&= -\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}-\frac{15C(bx)S}{2b^7\pi} \\
&= -\frac{5x^4}{8b^3\pi^2}+\frac{17\cos\left(b^2\pi x^2\right)}{4b^7\pi^4}-\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^5\cos}{} \\
&= -\frac{5x^4}{8b^3\pi^2}+\frac{11\cos\left(b^2\pi x^2\right)}{2b^7\pi^4}-\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^5\cos}{}
\end{aligned}$$

Mathematica [F] time = 0.53, size = 0, normalized size = 0.00

$$\int x^6 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(x^6\text{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(x^6*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6\text{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^6*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^6 \mathrm{S}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^6*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \mathrm{fresnels}(bx) \sin\left(\frac{1}{2} \pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^6*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6 \, \mathrm{FresnelS}(b\,x) \, \sin\left(\frac{\Pi\, b^2\, x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^6*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**6*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.74 $\int x^5 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=158

$$-\frac{43S\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,\pi^3b^6}-\frac{2x^3}{3\pi^2b^3}-\frac{x^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{8S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{11x\sin\left(\pi b^2x^2\right)}{8\pi^3b^5}+\frac{4x^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}$$

```
[Out] -2/3*x^3/b^3/Pi^2-1/4*x^3*cos(b^2*Pi*x^2)/b^3/Pi^2+8*cos(1/2*b^2*Pi*x^2)*Fr
esnelS(b*x)/b^6/Pi^3-x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+4*x^2*Fre
snelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+11/8*x*sin(b^2*Pi*x^2)/b^5/Pi^3-43/
16*FresnelS(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6454, 6462, 3391, 30, 3386, 3351, 6452, 3385}

$$\frac{4x^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{8S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}-\frac{43S\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,\pi^3b^6}-\frac{2x^3}{3\pi^2b^3}+\frac{11x\sin\left(\pi b^2x^2\right)}{8\pi^3b^5}-$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-2*x^3)/(3*b^3*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (8*Cos[(b^2*Pi
*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]
)/(b^2*Pi) - (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) + (4*x^2*Fresn
elS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi
^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
```

```
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi} + \frac{\int x^4\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= -\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{4x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{8\int xS(bx}{}$$

$$= -\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{4x^2S(bx)\,\mathrm{si}}{b^4}$$

$$= -\frac{2x^3}{3b^3\pi^2} - \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} - \frac{3S}{8}$$

$$= -\frac{2x^3}{3b^3\pi^2} - \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} - \frac{11}{8}$$

Mathematica [A] time = 0.18, size = 120, normalized size = 0.76

$$-\frac{32\pi b^3x^3 - 66bx\sin\left(\pi b^2x^2\right) + 48S(bx)\left(\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right) - 4\pi b^2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)\right) + 12\pi b^3x^3\cos\left(\pi\right.}{48\pi^3b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -1/48*(32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 129*Sqrt[2]*FresnelS
[Sqrt[2]*b*x] + 48*FresnelS[b*x]*((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] -
 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin[b^2*Pi*x^2])/(b^6*Pi^3)
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^5\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^5*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.01, size = 202, normalized size = 1.28

$$\frac{\frac{S(bx)\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{b^5}-\frac{\frac{2b^3x^3}{3\pi^2}-\frac{2\left(\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}-\frac{\sqrt{2}\,S\left(bx\sqrt{2}\right)}{4\pi}\right)}{\pi^2}-\frac{-\frac{\pi b^3x^3\cos\left(b^2\pi x^2\right)}{2}+\frac{3\pi\left(\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}-\frac{\sqrt{2}\,S\left(bx\sqrt{2}\right)}{4\pi}\right)}{2}}{2\pi^3}}{b^5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (FresnelS(b*x)/b^5*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*si
n(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))-1/b^5*(2/3/Pi^2*b^3*x^3-2/Pi
^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi
^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/
Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,\mathrm{FresnelS}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^5*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \sin\left(\frac{\pi b^2 x^2}{2}\right) S\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**5*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.75 $\int x^4 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=120

$$-\frac{3S(bx)^2}{2\pi^2 b^5}-\frac{3x^2}{4\pi^2 b^3}-\frac{x^3 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}+\frac{\sin\left(\pi b^2 x^2\right)}{\pi^3 b^5}+\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4}-\frac{x^2\cos\left(\pi b^2 x^2\right)}{4\pi^2 b^3}$$

```
[Out] -3/4*x^2/b^3/Pi^2-1/4*x^2*cos(b^2*Pi*x^2)/b^3/Pi^2-x^3*cos(1/2*b^2*Pi*x^2)*
FresnelS(b*x)/b^2/Pi-3/2*FresnelS(b*x)^2/b^5/Pi^2+3*x*FresnelS(b*x)*sin(1/2
*b^2*Pi*x^2)/b^4/Pi^2+sin(b^2*Pi*x^2)/b^5/Pi^3
```

Rubi [A] time = 0.12, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6454, 6462, 3379, 2634, 6440, 30, 3296, 2637}

$$\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4}-\frac{x^3 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}-\frac{3S(bx)^2}{2\pi^2 b^5}-\frac{3x^2}{4\pi^2 b^3}+\frac{\sin\left(\pi b^2 x^2\right)}{\pi^3 b^5}-\frac{x^2\cos\left(\pi b^2 x^2\right)}{4\pi^2 b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (x^3*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (3*FresnelS[b*x]^2)/(2*b^5*Pi^2) + (3*
x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi} + \frac{\int x^3\sin\left(b^2\pi x^2\right)dx}{2b\pi}$$

$$= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{3\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^4\pi^2} - \cdots$$

$$= -\frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{3\,\text{Subst}(\int x \cdots}{b^5 \cdots}$$

$$= -\frac{3x^2}{4b^3\pi^2} - \frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} - \frac{3S(bx)^2}{2b^5\pi^2} + \frac{3xS(bx)\sin\left(\frac{1}{2}b^2 \cdots\right.}{b^4\pi^2}$$

Mathematica [A] time = 0.01, size = 120, normalized size = 1.00

$$-\frac{3S(bx)^2}{2\pi^2b^5} - \frac{3x^2}{4\pi^2b^3} - \frac{x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\sin\left(\pi b^2x^2\right)}{\pi^3b^5} + \frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{x^2\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (x^3*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (3*FresnelS[b*x]^2)/(2*b^5*Pi^2) + (3*
x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(x^4\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^4*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^4*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^4 \mathrm{S}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \mathrm{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^4*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^4*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 17.56, size = 151, normalized size = 1.26

$$\begin{cases} -\frac{x^3\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi b^2} - \frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} - \frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{\pi^2b^3} + \frac{3x\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi^2b^4} + \frac{2\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{\pi^3b^5} - \frac{3S^2(bx)}{2\pi^2b^5} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((-x**3*cos(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) - x**2*sin(pi*
b**2*x**2/2)**2/(2*pi**2*b**3) - x**2*cos(pi*b**2*x**2/2)**2/(pi**2*b**3) +
 3*x*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi**2*b**4) + 2*sin(pi*b**2*x**2/2)
*cos(pi*b**2*x**2/2)/(pi**3*b**5) - 3*fresnels(b*x)**2/(2*pi**2*b**5), Ne(b
, 0)), (0, True))
```

3.76 $\int x^3 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=105

$$\frac{5C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,\pi^2b^4}-\frac{x}{\pi^2b^3}-\frac{x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

```
[Out] -x/b^3/Pi^2-1/4*x*cos(b^2*Pi*x^2)/b^3/Pi^2-x^2*cos(1/2*b^2*Pi*x^2)*FresnelS
(b*x)/b^2/Pi+2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+5/8*FresnelC(b*x*
2^(1/2))/b^4/Pi^2*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6454, 6460, 3357, 3352, 3385}

$$\frac{5\text{FresnelC}\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,\pi^2b^4}+\frac{2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x}{\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -(x/(b^3*Pi^2)) - (x*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (5*FresnelC[Sqrt[2]*b*
x])/(4*Sqrt[2]*b^4*Pi^2) - (x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)
 + (2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2)
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}
\int x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= -\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi}+\frac{\int x^2\sin\left(b^2\pi x^2\right)\,dx}{2b\pi} \\
&= -\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{\int\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} \\
&= -\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\ldots \\
&= -\frac{x}{b^3\pi^2}-\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{2S(bx)\sin\left(\frac{1}{2}b^2\pi\ldots\right.}{b^4\pi^2} \\
&= -\frac{x}{b^3\pi^2}-\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{5C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{2S(bx)\sin\left(\frac{1}{2}b^2\ldots\right.}{b^4\pi^2}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 83, normalized size = 0.79

$$\frac{-8S(bx)\left(\pi b^2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)-2\sin\left(\frac{1}{2}\pi b^2x^2\right)\right)-2bx\left(\cos\left(\pi b^2x^2\right)+4\right)+5\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{8\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-2*b*x*(4 + Cos[b^2*Pi*x^2]) + 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 8*Fresnel
S[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] - 2*Sin[(b^2*Pi*x^2)/2]))/(8*b^4*Pi^
2)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^3\mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.01, size = 115, normalized size = 1.10

$$\frac{\frac{\mathrm{S}(bx)\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{b^3}-\frac{\frac{bx}{\pi^2}-\frac{\sqrt{2}\;\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{2\pi^2}-\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi}+\frac{\sqrt{2}\;\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (FresnelS(b*x)/b^3*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi
*x^2))-1/b^3*(1/Pi^2*b*x-1/2/Pi^2*2^(1/2)*FresnelC(b*x*2^(1/2))-1/2/Pi*(-1/
2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^3*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{FresnelS}\,(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^3*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sin\left(\frac{\pi b^2x^2}{2}\right)S\,(bx)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**3*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.77 $\int x^2 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=137

$$-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}+\frac{C(bx)S(bx)}{2\pi b^3}-\frac{xS(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

```
[Out] -1/4*cos(b^2*Pi*x^2)/b^3/Pi^2-x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+1/
2*FresnelC(b*x)*FresnelS(b*x)/b^3/Pi-1/8*I*x^2*HypergeometricPFQ([1, 1],[3/
2, 2],-1/2*I*b^2*Pi*x^2)/b/Pi+1/8*I*x^2*HypergeometricPFQ([1, 1],[3/2, 2],1
/2*I*b^2*Pi*x^2)/b/Pi
```

Rubi [A] time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6454, 6446, 3379, 2638}

$$-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}+\frac{\text{FresnelC}(bx)S(bx)}{2\pi b^3}-\frac{xS(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -Cos[b^2*Pi*x^2]/(4*b^3*Pi^2) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*
Pi) + (FresnelC[b*x]*FresnelS[b*x])/(2*b^3*Pi) - ((I/8)*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b*Pi) + ((I/8)*x^2*Hypergeometric
PFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b*Pi)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
```

```
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rubi steps

$$\int x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi}+\frac{\int x\sin\left(b^2\pi x^2\right)dx}{2b\pi}$$
$$= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{C(bx)S(bx)}{2b^3\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}+\frac{ix^2\,{}_2F_2\left(1,\right.}{}$$
$$= -\frac{\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{C(bx)S(bx)}{2b^3\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right.}{8b\pi}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \text{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^2*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{S}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \text{fresnels}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^2*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \, \mathrm{FresnelS}(b\,x) \, \sin\left(\frac{\Pi\, b^2\, x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^2*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**2*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.78 $\int xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=49

$$\frac{S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,\pi b^2}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

```
[Out] -cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+1/4*FresnelS(b*x*2^(1/2))/b^2/Pi*
2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6452, 3351}

$$\frac{S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,\pi b^2}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + FresnelS[Sqrt[2]*b*x]/(2*
Sqrt[2]*b^2*Pi)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx&=-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{\int\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\&=-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,b^2\pi}\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.90

$$\frac{\sqrt{2}\,S\left(\sqrt{2}\,bx\right)-4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x] + Sqrt[2]*FresnelS[Sqrt[2]*b*x])/(4*b
^2*Pi)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(x\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$\frac{-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{S}(bx)}{\pi b}+\frac{\mathrm{S}\left(bx\sqrt{2}\right)\sqrt{2}}{4b\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (-cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/Pi/b+1/4*FresnelS(b*x*2^(1/2))/b/Pi*2^(
1/2))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x*fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\text{FresnelS}\,(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin\left(\frac{\pi b^2x^2}{2}\right)S\,(bx)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.79 $\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=13

$$\frac{S(bx)^2}{2b}$$

```
[Out] 1/2*FresnelS(b*x)^2/b
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6440, 30}

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= \frac{\text{Subst}(\int x\,dx, x, S(bx))}{b}\\ &= \frac{S(bx)^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] FresnelS[b*x]^2/(2*b)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{\mathrm{S}\left(bx\right)^{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] 1/2*FresnelS(b*x)^2/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \mathrm{FresnelS}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 0.34, size = 10, normalized size = 0.77

$$\begin{cases} \frac{S^2(bx)}{2b} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((fresnels(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

3.80 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

```
[Out] Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="fricas")
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="giac")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx)\sin\left(\frac{b^2 \pi\, x^2}{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{FresnelS}\,(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x,x)
```

```
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S\left(bx\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x, x)
```

3.81 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

```
[Out] Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{fresnels}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="fricas")
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="giac")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx)\sin\left(\frac{b^2 \pi\, x^2}{2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{FresnelS}\,(b\,x)\;\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^2,x)
```

```
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^2, x)
```

sympy **[A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S\left(bx\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**2,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**2, x)
```

3.82 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$

Optimal. Leaf size=102

$$\frac{1}{2}\pi b^2\text{Int}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{\pi b^2S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}+\frac{b\cos\left(\pi b^2x^2\right)}{4x}-\frac{b}{4x}$$

```
[Out] -1/4*b/x+1/4*b*cos(b^2*Pi*x^2)/x-1/2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+
1/4*b^2*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)+1/2*b^2*Pi*Unintegrable(cos(1/2*b^
2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

```
[Out] -b/(4*x) + (b*Cos[b^2*Pi*x^2])/(4*x) + (b^2*Pi*FresnelS[Sqrt[2]*b*x])/(2*Sq
rt[2]) - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*x^2) + (b^2*Pi*Defer[Int][(
Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/2
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx &= -\frac{b}{4x}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}-\frac{1}{4}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b\\ &= -\frac{b}{4x}+\frac{b\cos\left(b^2\pi x^2\right)}{4x}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx+\\ &= -\frac{b}{4x}+\frac{b\cos\left(b^2\pi x^2\right)}{4x}+\frac{b^2\pi S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\end{aligned}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="fricas")
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="giac")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^3,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**3,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**3, x)
```

3.83 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$

Optimal. Leaf size=109

$$-\frac{1}{6}\pi^2b^3S(bx)^2-\frac{\pi b^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}+\frac{b\cos\left(\pi b^2x^2\right)}{12x^2}+\frac{1}{6}\pi b^3\text{Si}\left(b^2\pi x^2\right)-\frac{b}{12x^2}$$

```
[Out] -1/12*b/x^2+1/12*b*cos(b^2*Pi*x^2)/x^2-1/3*b^2*Pi*cos(1/2*b^2*Pi*x^2)*Fresn
elS(b*x)/x-1/6*b^3*Pi^2*FresnelS(b*x)^2+1/6*b^3*Pi*Si(b^2*Pi*x^2)-1/3*Fresn
elS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3
```

Rubi [A] time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6456, 6464, 6440, 30, 3375, 3380, 3297, 3299}

$$-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{\pi b^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x}-\frac{1}{6}\pi^2b^3S(bx)^2+\frac{1}{6}\pi b^3\text{Si}\left(b^2\pi x^2\right)+\frac{b\cos\left(\pi b^2x^2\right)}{12x^2}-\frac{b}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] -b/(12*x^2) + (b*Cos[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*Fr
esnelS[b*x])/(3*x) - (b^3*Pi^2*FresnelS[b*x]^2)/6 - (FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/(3*x^3) + (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = -\frac{b}{12x^2} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} - \frac{1}{6}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx + \frac{1}{3}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{x^2}$$

$$= -\frac{b}{12x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} - \frac{1}{12}b\,\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^2}\right.$$

$$= -\frac{b}{12x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} + \frac{1}{12}b^3\pi\text{Si}$$

$$= -\frac{b}{12x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x} - \frac{1}{6}b^3\pi^2S(bx)^2 - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{3x^3}$$

Mathematica [A] time = 0.01, size = 109, normalized size = 1.00

$$-\frac{1}{6}\pi^2b^3S(bx)^2 - \frac{\pi b^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x} - \frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3} + \frac{b\cos\left(\pi b^2x^2\right)}{12x^2} + \frac{1}{6}\pi b^3\text{Si}\left(b^2\pi x^2\right) - \frac{b}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] -1/12*b/x^2 + (b*Cos[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*Fr
esnelS[b*x])/(3*x) - (b^3*Pi^2*FresnelS[b*x]^2)/6 - (FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/(3*x^3) + (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="giac")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^4,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^{2} x^{2}}{2}\right) S\left(bx\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**4,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**4, x)
```

3.84 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$

Optimal. Leaf size=153

$$-\frac{1}{8}\pi^2 b^4 \mathrm{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)+\frac{7\pi^2 b^4 C\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}-\frac{\pi b^2 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^2}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4x^4}+\frac{b\cos\left(\pi b^2\right.}{24x^3}$$

```
[Out] -1/24*b/x^3+1/24*b*cos(b^2*Pi*x^2)/x^3-1/8*b^2*Pi*cos(1/2*b^2*Pi*x^2)*Fresn
elS(b*x)/x^2-1/4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-7/48*b^3*Pi*sin(b^2*
Pi*x^2)/x+7/48*b^4*Pi^2*FresnelC(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Unintegr
able(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

```
[Out] -b/(24*x^3) + (b*Cos[b^2*Pi*x^2])/(24*x^3) + (7*b^4*Pi^2*FresnelC[Sqrt[2]*b
*x])/(24*Sqrt[2]) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(8*x^2) - (F
resnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*x^4) - (7*b^3*Pi*Sin[b^2*Pi*x^2])/(48*
x) - (b^4*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/8
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx &= -\frac{b}{24x^3}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{1}{8}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx+\frac{1}{4}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{x^3}\\
&= -\frac{b}{24x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^2}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}+\frac{1}{16}\left(b^3\pi\right)\\
&= -\frac{b}{24x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^2}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{7b^3\pi\sin}{48}\\
&= -\frac{b}{24x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}+\frac{7b^4\pi^2 C\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^2}-\frac{S(bx)\sin\left(\frac{1}{2}\right.}{4x^4}
\end{aligned}$$

Mathematica [A]   time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5, x]
```

fricas [A]   time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="fricas")
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

giac [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="giac")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

maple [A]   time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{fresnels}(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^5,x)
```

```
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^5, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right)S(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**5,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**5, x)
```

3.85 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

Optimal. Leaf size=148

$$-\frac{1}{15}\pi^2 b^4 \text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2},x\right)-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5}-\frac{\pi b^2 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^3}+\frac{b\cos\left(\pi b^2 x^2\right)}{40x^4}+\frac{1}{24}\pi^2 b^5 \text{Ci}$$

```
[Out] -1/40*b/x^4+1/24*b^5*Pi^2*Ci(b^2*Pi*x^2)+1/40*b*cos(b^2*Pi*x^2)/x^4-1/15*b^
2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3-1/5*FresnelS(b*x)*sin(1/2*b^2*Pi
*x^2)/x^5-1/24*b^3*Pi*sin(b^2*Pi*x^2)/x^2-1/15*b^4*Pi^2*Unintegrable(Fresne
lS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] -b/(40*x^4) + (b*Cos[b^2*Pi*x^2])/(40*x^4) + (b^5*Pi^2*CosIntegral[b^2*Pi*x
^2])/24 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15*x^3) - (FresnelS[b
*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) - (b^3*Pi*Sin[b^2*Pi*x^2])/(24*x^2) - (b^4
*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/15
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = -\frac{b}{40x^4} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} - \frac{1}{10}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx + \frac{1}{5}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(}{x^4}$$

$$= -\frac{b}{40x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x^3} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} - \frac{1}{20}b\,\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^3}\right.$$

$$= -\frac{b}{40x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{40x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x^3} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{1}{60}\left(b^3\pi\right)$$

$$= -\frac{b}{40x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{40x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x^3} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} - \frac{b^3\pi\sin\left(\right.}{24x}$$

$$= -\frac{b}{40x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{40x^4} + \frac{1}{24}b^5\pi^2\text{Ci}\left(b^2\pi x^2\right) - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x^3} - \frac{S(bx)\sin}{5x}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6, x]
```

fricas [A] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="fricas")
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\left(bx\right) \sin\left(\frac{b^{2} \pi\, x^{2}}{2}\right)}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnels}\left(bx\right) \sin\left(\frac{1}{2}\, \pi b^{2} x^{2}\right)}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^6,x)
```

```
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^6, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^{2} x^{2}}{2}\right) S\left(bx\right)}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**6,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**6, x)
```

3.86 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$

Optimal. Leaf size=241

$$-\frac{1}{48}\pi^3 b^6 \text{Int}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{1}{45}\sqrt{2}\,\pi^3 b^6 S\left(\sqrt{2}\,bx\right)-\frac{7\pi^3 b^6 S\left(\sqrt{2}\,bx\right)}{144\sqrt{2}}+\frac{\pi^2 b^5}{96x}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{\pi b^2 S}{}$$

```
[Out] -1/60*b/x^5+1/96*b^5*Pi^2/x+1/60*b*cos(b^2*Pi*x^2)/x^5-67/1440*b^5*Pi^2*cos
(b^2*Pi*x^2)/x-1/24*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4-1/6*Fresne
lS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6+1/48*b^4*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*
x^2)/x^2-13/720*b^3*Pi*sin(b^2*Pi*x^2)/x^3-67/1440*b^6*Pi^3*FresnelS(b*x*2^
(1/2))*2^(1/2)-1/48*b^6*Pi^3*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)
/x,x)
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

```
[Out] -b/(60*x^5) + (b^5*Pi^2)/(96*x) + (b*Cos[b^2*Pi*x^2])/(60*x^5) - (67*b^5*Pi
^2*Cos[b^2*Pi*x^2])/(1440*x) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
24*x^4) - (7*b^6*Pi^3*FresnelS[Sqrt[2]*b*x])/(144*Sqrt[2]) - (Sqrt[2]*b^6*P
i^3*FresnelS[Sqrt[2]*b*x])/45 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^6)
 + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(48*x^2) - (13*b^3*Pi*Sin[b
^2*Pi*x^2])/(720*x^3) - (b^6*Pi^3*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/x, x])/48
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = -\frac{b}{60x^5} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} - \frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx + \frac{1}{6}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}$$

$$= -\frac{b}{60x^5} + \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{24x^4} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{1}{48}\left(b^3\right.$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} + \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{24x^4} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} +$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} + \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{24x^4} -$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} + \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{24x^4} -$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="fricas")
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="giac")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^7,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^7, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)S\left(bx\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**7,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**7, x)
```

3.87 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

Optimal. Leaf size=224

$$\frac{1}{210}\pi^4 b^7 S(bx)^2+\frac{\pi^2 b^5}{420x^2}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{\pi b^2 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}+\frac{b\cos\left(\pi b^2x^2\right)}{84x^6}-\frac{1}{70}\pi^3 b^7\mathrm{Si}\left(b^2\pi x^2\right)+\frac{\pi^3 b^{}}{}$$

```
[Out] -1/84*b/x^6+1/420*b^5*Pi^2/x^2+1/84*b*cos(b^2*Pi*x^2)/x^6-1/84*b^5*Pi^2*cos
(b^2*Pi*x^2)/x^2-1/35*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5+1/105*b^
6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x+1/210*b^7*Pi^4*FresnelS(b*x)^2-1
/70*b^7*Pi^3*Si(b^2*Pi*x^2)-1/7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7+1/105
*b^4*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-1/105*b^3*Pi*sin(b^2*Pi*x^2
)/x^4
```

Rubi [A] time = 0.35, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6456, 6464, 6440, 30, 3375, 3380, 3297, 3299, 3379}

$$\frac{\pi^2 b^4 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{105x^3}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{\pi^3 b^6 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{105x}-\frac{\pi b^2 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}+\frac{1}{210}\pi^4 b^7 S(b$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

```
[Out] -b/(84*x^6) + (b^5*Pi^2)/(420*x^2) + (b*Cos[b^2*Pi*x^2])/(84*x^6) - (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
35*x^5) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x) + (b^7*Pi^4*
FresnelS[b*x]^2)/210 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*P
i^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x^3) - (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) - (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
```

```
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = -\frac{b}{84x^6} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} - \frac{1}{14}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx + \frac{1}{7}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}$$

$$= -\frac{b}{84x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} - \frac{1}{28}b\,\text{Subst}\left(\int \frac{\cos\left(b^2\pi x\right)}{x^4}\right.$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S}{105x}$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5} +$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} + \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5} +$$

Mathematica [A] time = 0.02, size = 224, normalized size = 1.00

$$\frac{1}{210}\pi^4b^7S(bx)^2+\frac{\pi^2b^5}{420x^2}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{\pi b^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}+\frac{b\cos\left(\pi b^2x^2\right)}{84x^6}-\frac{1}{70}\pi^3b^7\text{Si}\left(b^2\pi x^2\right)+\frac{\pi^3b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

```
[Out] -1/84*b/x^6 + (b^5*Pi^2)/(420*x^2) + (b*Cos[b^2*Pi*x^2])/(84*x^6) - (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
35*x^5) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x) + (b^7*Pi^4*
FresnelS[b*x]^2)/210 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*P
i^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x^3) - (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) - (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="fricas")
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="giac")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}\,(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^8,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**8,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**8, x)
```

3.88 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

Optimal. Leaf size=268

$$\frac{1}{384}\pi^4 b^8 \text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) - \frac{853\pi^4 b^8 C\left(\sqrt{2}\,bx\right)}{40320\sqrt{2}} + \frac{\pi^2 b^5}{1152x^3} - \frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8} - \frac{\pi b^2 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{48x^6}$$

```
[Out] -1/112*b/x^7+1/1152*b^5*Pi^2/x^3+1/112*b*cos(b^2*Pi*x^2)/x^7-187/40320*b^5*
Pi^2*cos(b^2*Pi*x^2)/x^3-1/48*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6+
1/384*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2-1/8*FresnelS(b*x)*sin(
1/2*b^2*Pi*x^2)/x^8+1/192*b^4*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-19
/3360*b^3*Pi*sin(b^2*Pi*x^2)/x^5+853/80640*b^7*Pi^3*sin(b^2*Pi*x^2)/x-853/8
0640*b^8*Pi^4*FresnelC(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrable(Fre
snelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
```

```
[Out] -b/(112*x^7) + (b^5*Pi^2)/(1152*x^3) + (b*Cos[b^2*Pi*x^2])/(112*x^7) - (187
*b^5*Pi^2*Cos[b^2*Pi*x^2])/(40320*x^3) - (853*b^8*Pi^4*FresnelC[Sqrt[2]*b*x
])/(40320*Sqrt[2]) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(48*x^6) +
(b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(384*x^2) - (FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(8*x^8) + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(1
92*x^4) - (19*b^3*Pi*Sin[b^2*Pi*x^2])/(3360*x^5) + (853*b^7*Pi^3*Sin[b^2*Pi
*x^2])/(80640*x) + (b^8*Pi^4*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])
/x, x])/384
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = -\frac{b}{112x^7} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} - \frac{1}{16}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx + \frac{1}{8}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right.}{x^7}$$

$$= -\frac{b}{112x^7} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^6} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{1}{96}\left(b^3\right.$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^6} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right.}{8x^8}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{48x^6}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{48x^6}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{853b^8\pi^4C\left(\sqrt{2}\,bx\right)}{40320\sqrt{2}} -$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="fricas")
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}\,(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^9,x)
```

```
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^9, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**9,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**9, x)
```

3.89 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

Optimal. Leaf size=263

$$\frac{1}{945}\pi^4 b^8 \text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)+\frac{\pi^2b^5}{2520x^4}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{\pi b^2 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{63x^7}+\frac{b\cos\left(\pi b^2x^2\right)}{144x^8}-\frac{5\pi}{}$$

```
[Out] -1/144*b/x^8+1/2520*b^5*Pi^2/x^4-5/2016*b^9*Pi^4*Ci(b^2*Pi*x^2)+1/144*b*cos
(b^2*Pi*x^2)/x^8-67/30240*b^5*Pi^2*cos(b^2*Pi*x^2)/x^4-1/63*b^2*Pi*cos(1/2*
b^2*Pi*x^2)*FresnelS(b*x)/x^7+1/945*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b
*x)/x^3-1/9*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9+1/315*b^4*Pi^2*FresnelS(b
*x)*sin(1/2*b^2*Pi*x^2)/x^5-11/3024*b^3*Pi*sin(b^2*Pi*x^2)/x^6+5/2016*b^7*P
i^3*sin(b^2*Pi*x^2)/x^2+1/945*b^8*Pi^4*Unintegrable(FresnelS(b*x)*sin(1/2*b
^2*Pi*x^2)/x^2,x)
```

Rubi [A] time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

```
[Out] -b/(144*x^8) + (b^5*Pi^2)/(2520*x^4) + (b*Cos[b^2*Pi*x^2])/(144*x^8) - (67*
b^5*Pi^2*Cos[b^2*Pi*x^2])/(30240*x^4) - (5*b^9*Pi^4*CosIntegral[b^2*Pi*x^2]
)/2016 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(63*x^7) + (b^6*Pi^3*Co
s[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x^3) - (FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(9*x^9) + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5) - (1
1*b^3*Pi*Sin[b^2*Pi*x^2])/(3024*x^6) + (5*b^7*Pi^3*Sin[b^2*Pi*x^2])/(2016*x
^2) + (b^8*Pi^4*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/945
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = -\frac{b}{144x^8} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} - \frac{1}{18}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^9}\,dx + \frac{1}{9}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}$$

$$= -\frac{b}{144x^8} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{63x^7} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} - \frac{1}{36}b\,\mathrm{Subst}\left(\int\frac{\cos\left(b^2\pi\right.}{x^5}\right.$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{63x^7} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right.}{9x^9}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{63x^7} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right.}{945x^3}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{63x^7}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{63x^7}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{5b^9\pi^4\mathrm{Ci}\left(b^2\pi x^2\right)}{2016} - \frac{b^2}{}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnels}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnels}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnels(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^10,x)
```

```
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^10, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S\left(bx\right)}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**10,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**10, x)
```

3.90 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\cos\left(\frac{1}{2}\pi b^2x^2\right) S(bx)^n, x\right)$$

[Out] Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n\,dx$$

Verification is Not applicable to the result.

[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n,x]

[Out] Defer[Int][Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]

Rubi steps

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n\,dx = \int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n\,dx$$

Mathematica [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n,x]

[Out] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnels}\,(bx)^n \cos\left(\frac{1}{2}\,\pi b^2x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)^n,x, algorithm="fricas")
[Out] integral(fresnels(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}(bx)^n \cos\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)^n,x, algorithm="giac")
[Out] integrate(fresnels(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{b^2\pi x^2}{2}\right) \mathrm{S}(bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnels}(bx)^n \cos\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)^n,x, algorithm="maxima")
[Out] integrate(fresnels(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \text{FresnelS}(b\,x)^n \cos\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^n*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)^n*cos((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{\pi b^2 x^2}{2}\right) S^n(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)**n,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)**n, x)
```

3.91 $\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=307

$$-\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}+\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}+\frac{105C(bx)S(bx)}{2\pi^4b^9}+\frac{35x^4}{8\pi^3b^5}+\frac{x^7S(bx)\sin\left(\frac{1}{2}\pi b^2\right.}{\pi b^2}$$

```
[Out] 35/8*x^4/b^5/Pi^3-1/16*x^8/b/Pi-40*cos(b^2*Pi*x^2)/b^9/Pi^5+5/2*x^4*cos(b^2
*Pi*x^2)/b^5/Pi^3-105*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^8/Pi^4+7*x^5*co
s(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2+105/2*FresnelC(b*x)*FresnelS(b*x)/
b^9/Pi^4-105/8*I*x^2*HypergeometricPFQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b
^7/Pi^4+105/8*I*x^2*HypergeometricPFQ([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^7
/Pi^4-35*x^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^7*FresnelS(b*x)*s
in(1/2*b^2*Pi*x^2)/b^2/Pi-55/4*x^2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^6*sin(b^2
*Pi*x^2)/b^3/Pi^2
```

Rubi [A] time = 0.44, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6462, 3379, 3309, 30, 3296, 2638, 6454, 6446}

$$-\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}+\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}+\frac{105\text{FresnelC}(bx)S(bx)}{2\pi^4b^9}+\frac{x^7S(bx)\sin\left(\frac{1}{2}\pi b^2\right.}{\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (35*x^4)/(8*b^5*Pi^3) - x^8/(16*b*Pi) - (40*Cos[b^2*Pi*x^2])/(b^9*Pi^5) + (
5*x^4*Cos[b^2*Pi*x^2])/(2*b^5*Pi^3) - (105*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b
*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (1
05*FresnelC[b*x]*FresnelS[b*x])/(2*b^9*Pi^4) - (((105*I)/8)*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^7*Pi^4) + (((105*I)/8)*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (35*x^3
*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelS[b*x]*Sin[(b^
2*Pi*x^2)/2])/(b^2*Pi) - (55*x^2*Sin[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*Sin[b
^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
```

```
1]
```

Rubi steps

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx = \frac{x^7 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{7\int x^6 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^7\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$

$$= \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{x^7 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,d}{b^4\pi^2}$$

$$= \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{35x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{1(}{}$$

$$= -\frac{x^8}{16b\pi} + \frac{7x^4\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx}{b^4\pi^2}$$

$$= -\frac{x^8}{16b\pi} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx}{b^4\pi^2}$$

$$= \frac{35x^4}{8b^5\pi^3} - \frac{x^8}{16b\pi} - \frac{119\cos\left(b^2\pi x^2\right)}{4b^9\pi^5} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx}{b^8\pi^4}$$

$$= \frac{35x^4}{8b^5\pi^3} - \frac{x^8}{16b\pi} - \frac{40\cos\left(b^2\pi x^2\right)}{b^9\pi^5} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4}$$

Mathematica [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
[Out] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^8\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
[Out] integral(x^8*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnels}\,(bx)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^8 \cos\left(\frac{b^2 \pi\, x^2}{2}\right) \mathrm{S}\,(bx)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnels}\,(bx)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^8\,\mathrm{FresnelS}\,(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^8*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x**8*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.92 $\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\, dx$

Optimal. Leaf size=217

$$\frac{531S\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,\pi^4b^8}+\frac{4x^3}{\pi^3b^5}+\frac{x^6S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{48S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{147x\sin\left(\pi b^2x^2\right)}{16\pi^4b^7}-\frac{24x^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}$$

```
[Out] 4*x^3/b^5/Pi^3-1/14*x^7/b/Pi+17/8*x^3*cos(b^2*Pi*x^2)/b^5/Pi^3-48*cos(1/2*b
^2*Pi*x^2)*FresnelS(b*x)/b^8/Pi^4+6*x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b
^4/Pi^2-24*x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^6*FresnelS(b*x)
*sin(1/2*b^2*Pi*x^2)/b^2/Pi-147/16*x*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^5*sin(b
^2*Pi*x^2)/b^3/Pi^2+531/32*FresnelS(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6462, 3391, 30, 3386, 3385, 3351, 6454, 6452}

$$\frac{x^6S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{24x^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{6x^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{48S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}+\frac{531S\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (4*x^3)/(b^5*Pi^3) - x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3)
- (48*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^8*Pi^4) + (6*x^4*Cos[(b^2*Pi*x^
2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*b
^8*Pi^4) - (24*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^6*Fre
snelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7*
Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx = \frac{x^6S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{6\int x^5S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^6\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$

$$= \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{x^6S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{24\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^4\pi^2}$$

$$= -\frac{x^7}{14b\pi} + \frac{3x^3\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{24x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} +$$

$$= -\frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4} + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} -$$

$$= \frac{4x^3}{b^5\pi^3} - \frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4} + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S}{b^4\pi^2}$$

$$= \frac{4x^3}{b^5\pi^3} - \frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4} + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S}{b^4\pi^2}$$

Mathematica [A] time = 0.21, size = 163, normalized size = 0.75

$$\frac{-16\pi^3b^7x^7 + 896\pi b^3x^3 - 2058bx\sin\left(\pi b^2x^2\right) + 56\pi^2b^5x^5\sin\left(\pi b^2x^2\right) + 224S(bx)\left(\pi b^2x^2\left(\pi^2b^4x^4 - 24\right)\sin\left(\frac{1}{2}\pi\right.\right.}{224\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (896*b^3*Pi*x^3 - 16*b^7*Pi^3*x^7 + 476*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 3717*S
qrt[2]*FresnelS[Sqrt[2]*b*x] + 224*FresnelS[b*x]*(6*(-8 + b^4*Pi^2*x^4)*Cos
[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) - 2
058*b*x*Sin[b^2*Pi*x^2] + 56*b^5*Pi^2*x^5*Sin[b^2*Pi*x^2])/(224*b^8*Pi^4)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(x^7\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnels}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^7*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}\,(bx)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

maple [A] time = 0.07, size = 321, normalized size = 1.48

$$\frac{\frac{\mathrm{S}(bx)\left(\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{6\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{\pi}\right)}{b^7}-\frac{\frac{\frac{1}{7}\pi^2b^7x^7-8b^3x^3}{2\pi^3}+\frac{-\frac{3\pi b^3x^3\cos\left(b^2\pi x^2\right)}{2}+\frac{9\pi\left(\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}-\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right.}{4\pi}\right.}{2}}{\pi^4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] (FresnelS(b*x)/b^7*(1/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)-6/Pi*(-1/Pi*b^4*x^4*co
s(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2
*Pi*x^2))))-1/b^7*(1/2/Pi^3*(1/7*Pi^2*b^7*x^7-8*b^3*x^3)+3/Pi^4*(-1/2*Pi*b^
3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*Fre
snelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^5*x
^5*sin(b^2*Pi*x^2)-5/2*Pi*(-1/2/Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2/Pi*(1/2/Pi*b
*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2))))-12/Pi*b*x*sin(b^2
*Pi*x^2)+6/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}\,(bx)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \operatorname{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^7*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x**7*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.93 $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=184

$$\frac{15S(bx)^2}{2\pi^3 b^7}+\frac{15x^2}{4\pi^3 b^5}+\frac{x^5 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{11\sin\left(\pi b^2x^2\right)}{2\pi^4 b^7}-\frac{15xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3 b^6}+\frac{7x^2\cos\left(\pi b^2x^2\right)}{4\pi^3 b^5}+\frac{5x^3S(bx)}{}$$

```
[Out] 15/4*x^2/b^5/Pi^3-1/12*x^6/b/Pi+7/4*x^2*cos(b^2*Pi*x^2)/b^5/Pi^3+5*x^3*cos(
1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2+15/2*FresnelS(b*x)^2/b^7/Pi^3-15*x*F
resnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^5*FresnelS(b*x)*sin(1/2*b^2*Pi*
x^2)/b^2/Pi-11/2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] time = 0.25, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6462, 3379, 3309, 30, 3296, 2637, 6454, 2634, 6440}

$$\frac{x^5 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{15xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3 b^6}+\frac{5x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2 b^4}+\frac{15S(bx)^2}{2\pi^3 b^7}+\frac{15x^2}{4\pi^3 b^5}+\frac{x^4\sin\left(\pi b^2x^2\right)}{4\pi^2 b^3}-\frac{11}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (15*x^2)/(4*b^5*Pi^3) - x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^3
) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (15*FresnelS[b*x
]^2)/(2*b^7*Pi^3) - (15*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (
x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*Sin[b^2*Pi*x^2])/(2*b
^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx = \frac{x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{5\int x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^5 \sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$

$$= \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{15\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,d}{b^4\pi^2}$$

$$= \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{15xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + 15$$

$$= -\frac{x^6}{12b\pi} + \frac{5x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{15xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3}$$

$$= \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{15S(bx)^2}{2b^7\pi^3} - \frac{15xS}{}$$

$$= \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{15S(bx)^2}{2b^7\pi^3} - \frac{15xS}{}$$

Mathematica [A] time = 0.01, size = 184, normalized size = 1.00

$$\frac{15S(bx)^2}{2\pi^3 b^7} + \frac{15x^2}{4\pi^3 b^5} + \frac{x^5 S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{11\sin\left(\pi b^2 x^2\right)}{2\pi^4 b^7} - \frac{15xS(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^3 b^6} + \frac{7x^2\cos\left(\pi b^2 x^2\right)}{4\pi^3 b^5} + \frac{5x^3 S(bx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (15*x^2)/(4*b^5*Pi^3) - x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^3
) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (15*FresnelS[b*x
]^2)/(2*b^7*Pi^3) - (15*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (
x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*Sin[b^2*Pi*x^2])/(2*b
^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(x^6\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{fresnels}(bx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^6*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}\,(bx)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^6 \cos\left(\frac{b^2\pi\, x^2}{2}\right) \mathrm{S}\,(bx)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{fresnels}\,(bx)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\, \mathrm{FresnelS}\,(b\, x)\; \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^6*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 75.44, size = 264, normalized size = 1.43

$$\begin{cases} -\frac{x^6\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{12\pi b} - \frac{x^6\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{12\pi b} + \frac{x^5\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi b^2} + \frac{x^4\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} + \frac{5x^3\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi^2b^4} + \frac{2x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{\pi^3b^5} + \frac{11}{} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Piecewise((-x**6*sin(pi*b**2*x**2/2)**2/(12*pi*b) - x**6*cos(pi*b**2*x**2/2
)**2/(12*pi*b) + x**5*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) + x**4*si
n(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + 5*x**3*cos(pi*b**2*x
**2/2)*fresnels(b*x)/(pi**2*b**4) + 2*x**2*sin(pi*b**2*x**2/2)**2/(pi**3*b*
*5) + 11*x**2*cos(pi*b**2*x**2/2)**2/(2*pi**3*b**5) - 15*x*sin(pi*b**2*x**2
/2)*fresnels(b*x)/(pi**3*b**6) - 11*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)
/(pi**4*b**7) + 15*fresnels(b*x)**2/(2*pi**3*b**7), Ne(b, 0)), (0, True))
```

3.94 $\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=166

$$-\frac{43C\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,\pi^3b^6}+\frac{4x}{\pi^3b^5}+\frac{x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{8S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{11x\cos\left(\pi b^2x^2\right)}{8\pi^3b^5}+\frac{4x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+$$

```
[Out] 4*x/b^5/Pi^3-1/10*x^5/b/Pi+11/8*x*cos(b^2*Pi*x^2)/b^5/Pi^3+4*x^2*cos(1/2*b^
2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2-8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi
^3+x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1/4*x^3*sin(b^2*Pi*x^2)/b^3
/Pi^2-43/16*FresnelC(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6462, 3391, 30, 3386, 3385, 3352, 6454, 6460, 3357}

$$-\frac{43\text{FresnelC}\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,\pi^3b^6}+\frac{x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{8S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{4x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^3\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (4*x)/(b^5*Pi^3) - x^5/(10*b*Pi) + (11*x*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3) - (4
3*FresnelC[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) + (4*x^2*Cos[(b^2*Pi*x^2)/2]*
FresnelS[b*x])/(b^4*Pi^2) - (8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3
) + (x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) + (x^3*Sin[b^2*Pi*x^2]
)/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx = \frac{x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{4\int x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^4\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$

$$= \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{8\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^4\pi^2} - \cdots$$

$$= -\frac{x^5}{10b\pi} + \frac{x\cos\left(b^2\pi x^2\right)}{b^5\pi^3} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{8S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^4S(\cdots}{\cdots}$$

$$= -\frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{C\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b^6\pi^3} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{8S(bx)\sin\left(\cdots\right.}{b^6\pi^3}$$

$$= \frac{4x}{b^5\pi^3} - \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{11C\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,b^6\pi^3} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{8S(\cdots}{\cdots}$$

$$= \frac{4x}{b^5\pi^3} - \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{11C\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,b^6\pi^3} - \frac{2\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{b^6\pi^3} + \frac{4x^2\cos\left(\frac{1}{2}b^2\cdots\right.}{b^4\pi\cdots}$$

Mathematica [A] time = 0.20, size = 126, normalized size = 0.76

$$\frac{80S(bx)\left(4\pi b^2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right) + \left(\pi^2b^4x^4 - 8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\right) + 2bx\left(-4\pi^2b^4x^4 + 10\pi b^2x^2\sin\left(\pi b^2x^2\right) + 55\cos\left(\cdots\right.\right.}{80\pi^3b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (-215*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 80*FresnelS[b*x]*(4*b^2*Pi*x^2*Cos[(b
^2*Pi*x^2)/2] + (-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(160 - 4*b
^4*Pi^2*x^4 + 55*Cos[b^2*Pi*x^2] + 10*b^2*Pi*x^2*Sin[b^2*Pi*x^2]))/(80*b^6*
Pi^3)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(x^5\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

[Out] integral(x^5*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)

maple [A] time = 0.05, size = 212, normalized size = 1.28

$$\frac{\frac{\mathrm{S}(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{b^5}-\frac{\frac{\frac{1}{5}\pi^2b^5x^5-8bx}{2\pi^3}+\frac{-\frac{bx\cos(b^2\pi x^2)}{\pi}+\frac{\sqrt{2}\ \mathrm{FresnelC}(bx\sqrt{2})}{2\pi}}{\pi^2}-\frac{\frac{\pi b^3x^3\sin(b^2\pi x^2)}{2}-\frac{3\pi\left(-\frac{bx\cos(b^2\pi x^2)}{2\pi}\cdots\right.}{\cdots}}{\cdots}}{b^5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

[Out] (FresnelS(b*x)/b^5*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))-1/b^5*(1/2/Pi^3*(1/5*Pi^2*b^5*x^5-8*b*x)+2/Pi^2*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,\mathrm{FresnelS}\,(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^5*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x**5*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.95 $\int x^4 \cos \left(\frac{1}{2}b^2\pi x^2\right) S(bx)\, dx$

Optimal. Leaf size=195

$$\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}-\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}-\frac{3C(bx)S(bx)}{2\pi^2b^5}+\frac{x^3S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^3b^5}$$

```
[Out] -1/8*x^4/b/Pi+cos(b^2*Pi*x^2)/b^5/Pi^3+3*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x
)/b^4/Pi^2-3/2*FresnelC(b*x)*FresnelS(b*x)/b^5/Pi^2+3/8*I*x^2*Hypergeometri
cPFQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^3/Pi^2-3/8*I*x^2*HypergeometricPF
Q([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^3/Pi^2+x^3*FresnelS(b*x)*sin(1/2*b^2*
Pi*x^2)/b^2/Pi+1/4*x^2*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] time = 0.16, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6462, 3379, 3309, 30, 3296, 2638, 6454, 6446}

$$\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}-\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}-\frac{3\text{FresnelC}(bx)S(bx)}{2\pi^2b^5}+\frac{x^3S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{3xS}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -x^4/(8*b*Pi) + Cos[b^2*Pi*x^2]/(b^5*Pi^3) + (3*x*Cos[(b^2*Pi*x^2)/2]*Fresn
elS[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]*FresnelS[b*x])/(2*b^5*Pi^2) + (((3*
I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^3*Pi^2
) - (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(
b^3*Pi^2) + (x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) + (x^2*Sin[b^2
*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rule 6454

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelS[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*Si
n[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m
, 1]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx = \frac{x^3S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3\int x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^3\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$
$$= \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{x^3S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3\int\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^4\pi^2} - $$
$$= \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{3C(bx)S(bx)}{2b^5\pi^2} + \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} - \frac{3ix^2\,{}_2}{}$$
$$= -\frac{x^4}{8b\pi} + \frac{3\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{3C(bx)S(bx)}{2b^5\pi^2} + \frac{3ix^2\,{}_2F_2\left(1,1\right.}{8}$$
$$= -\frac{x^4}{8b\pi} + \frac{\cos\left(b^2\pi x^2\right)}{b^5\pi^3} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{3C(bx)S(bx)}{2b^5\pi^2} + \frac{3ix^2\,{}_2F_2\left(1,1;\right.}{8b^3}$$

Mathematica [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
[Out] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(x^4\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
[Out] integral(x^4*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^4 \cos\left(\frac{b^2 \pi x^2}{2}\right) \mathrm{S}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \mathrm{fresnels}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \,\mathrm{FresnelS}(b\,x)\, \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^4*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
[Out] Integral(x**4*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.96 $\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=108

$$-\frac{5S\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,\pi^2 b^4}+\frac{x^2 S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}+\frac{2S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4}+\frac{x\sin\left(\pi b^2 x^2\right)}{4\pi^2 b^3}-\frac{x^3}{6\pi b}$$

```
[Out] -1/6*x^3/b/Pi+2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2+x^2*FresnelS(b*x
)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1/4*x*sin(b^2*Pi*x^2)/b^3/Pi^2-5/8*FresnelS(b*
x*2^(1/2))/b^4/Pi^2*2^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6462, 3391, 30, 3386, 3351, 6452}

$$\frac{x^2 S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}+\frac{2S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4}-\frac{5S\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,\pi^2 b^4}+\frac{x\sin\left(\pi b^2 x^2\right)}{4\pi^2 b^3}-\frac{x^3}{6\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -x^3/(6*b*Pi) + (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) - (5*Fresn
elS[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2) + (x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(b^2*Pi) + (x*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3391

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6452

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelS[b*x])/(2*d), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx &= \frac{x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{2\int xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\
&= \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \sin\left(b^2\pi x^2\right)\,dx}{b^3\pi^2} - \frac{\int x^2\,dx}{2b\pi} + \int \\
&= -\frac{x^3}{6b\pi} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b^4\pi^2} + \frac{x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x\sin\left(b^2\pi\right.}{4b^3\pi^2} \\
&= -\frac{x^3}{6b\pi} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{5S\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2} + \frac{x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x\sin\left(b^2\right.}{4b^3\pi^2}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 90, normalized size = 0.83

$$\frac{-4\pi b^3x^3 + 24S(bx)\left(\pi b^2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right) + 2\cos\left(\frac{1}{2}\pi b^2x^2\right)\right) + 6bx\sin\left(\pi b^2x^2\right) - 15\sqrt{2}\,S\left(\sqrt{2}\,bx\right)}{24\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (-4*b^3*Pi*x^3 - 15*Sqrt[2]*FresnelS[Sqrt[2]*b*x] + 24*FresnelS[b*x]*(2*Cos
[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) + 6*b*x*Sin[b^2*Pi*x^2])
/(24*b^4*Pi^2)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

maple [A] time = 0.05, size = 119, normalized size = 1.10

$$\frac{\frac{\mathrm{S}(bx)\left(\frac{b^2x^2\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi^2}\right)}{b^3}-\frac{\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{2\pi^2}+\frac{b^3x^3}{6\pi}-\frac{\frac{bx\sin\left(b^2\pi\,x^2\right)}{2\pi}-\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] (FresnelS(b*x)/b^3*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*
x^2))-1/b^3*(1/2/Pi^2*2^(1/2)*FresnelS(b*x*2^(1/2))+1/6/Pi*b^3*x^3-1/2/Pi*(
1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^3*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x**3*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.97 $\int x^2 \cos\left(\frac{1}{2} b^2 \pi x^2\right) S(bx)\, dx$

Optimal. Leaf size=73

$$-\frac{S(bx)^2}{2\pi b^3}+\frac{xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x^2}{4\pi b}$$

```
[Out] -1/4*x^2/b/Pi-1/2*FresnelS(b*x)^2/b^3/Pi+x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2
)/b^2/Pi+1/4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6462, 3379, 2634, 6440, 30}

$$\frac{xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{S(bx)^2}{2\pi b^3}+\frac{\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x^2}{4\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -x^2/(4*b*Pi) - FresnelS[b*x]^2/(2*b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*Pi*x
^2)/2])/(b^2*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6462

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelS[b*x])/(2*d), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx &= \frac{xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi} \\
&= \frac{xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\text{Subst}(\int x\,dx, x, S(bx))}{b^3\pi} - \frac{\text{Subst}\left(\int \sin^2\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right.}{2b\pi} \\
&= -\frac{x^2}{4b\pi} - \frac{S(bx)^2}{2b^3\pi} + \frac{xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 73, normalized size = 1.00

$$-\frac{S(bx)^2}{2\pi b^3} + \frac{xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\sin\left(\pi b^2x^2\right)}{4\pi^2b^3} - \frac{x^2}{4\pi b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -1/4*x^2/(b*Pi) - FresnelS[b*x]^2/(2*b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*Pi
*x^2)/2])/(b^2*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
[Out] integral(x^2*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnels}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^2 \cos\left(\frac{b^2 \pi\, x^2}{2}\right) \mathrm{S}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnels}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{FresnelS}\,(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^2*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 2.89, size = 114, normalized size = 1.56

$$\begin{cases} -\frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} - \frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} + \frac{x\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi b^2} + \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} - \frac{S^2(bx)}{2\pi b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Piecewise((-x**2*sin(pi*b**2*x**2/2)**2/(4*pi*b) - x**2*cos(pi*b**2*x**2/2)
**2/(4*pi*b) + x*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) + sin(pi*b**2*
x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) - fresnels(b*x)**2/(2*pi*b**3),
Ne(b, 0)), (0, True))
```

3.98 $\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\, dx$

Optimal. Leaf size=59

$$\frac{C\left(\sqrt{2}\, bx\right)}{2\sqrt{2}\,\pi b^2}+\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{x}{2\pi b}$$

```
[Out] -1/2*x/b/Pi+FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1/4*FresnelC(b*x*2^(1/
2))/b^2/Pi*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6460, 3357, 3352}

$$\frac{\text{FresnelC}\left(\sqrt{2}\, bx\right)}{2\sqrt{2}\,\pi b^2}+\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{x}{2\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -x/(2*b*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*Pi) + (FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 6460

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelS[b*x])/(2*d), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx &= \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi} \\ &= \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \left(\frac{1}{2}-\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)dx}{b\pi} \\ &= -\frac{x}{2b\pi} + \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{\int \cos\left(b^2\pi x^2\right)dx}{2b\pi} \\ &= -\frac{x}{2b\pi} + \frac{C\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,b^2\pi} + \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.81

$$\frac{4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)+\sqrt{2}\,C\left(\sqrt{2}\,bx\right)-2bx}{4\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (-2*b*x + Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/
2])/(4*b^2*Pi)
```

fricas [F] time = 1.45, size = 0, normalized size = 0.00

$$\text{integral}\left(x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(x*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

maple [A] time = 0.03, size = 52, normalized size = 0.88

$$\frac{\frac{\mathrm{S}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{b\pi}-\frac{\frac{bx}{2}-\frac{\sqrt{2}\ \mathrm{FresnelC}\left(bx\sqrt{2}\right)}{4}}{b\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] (FresnelS(b*x)/b*sin(1/2*b^2*Pi*x^2)/Pi-1/b/Pi*(1/2*b*x-1/4*2^(1/2)*Fresnel
C(b*x*2^(1/2))))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{FresnelS}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(\frac{\pi b^2x^2}{2}\right)S\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.99 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=80

$$-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{C(bx)S(bx)}{2b}$$

```
[Out] 1/2*FresnelC(b*x)*FresnelS(b*x)/b-1/8*I*b*x^2*HypergeometricPFQ([1, 1],[3/2
, 2],-1/2*I*b^2*Pi*x^2)+1/8*I*b*x^2*HypergeometricPFQ([1, 1],[3/2, 2],1/2*I
*b^2*Pi*x^2)
```

Rubi [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6446}

$$-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)S(bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-I/2)*b^2*Pi*x^2] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2,
 2}, (I/2)*b^2*Pi*x^2]
```

Rule 6446

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(FresnelC[b*x
]*FresnelS[b*x])/(2*b), x] + (-Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}, {3
/2, 2}, -((I*b^2*Pi*x^2)/2)])/8, x] + Simp[(1*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (1*I*b^2*Pi*x^2)/2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2,
(Pi^2*b^4)/4]
```

Rubi steps

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx=\frac{C(bx)S(bx)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnels}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{b^2 \pi\, x^2}{2}\right)\mathrm{S}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnels}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \text{FresnelS}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{\pi b^2 x^2}{2}\right) S\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.100 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)$$

```
[Out] Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x,x]
```

```
[Out] Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnels}\left(bx\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\operatorname{fresnels}\left(bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\mathrm{S}\left(bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\operatorname{fresnels}\left(bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x,x)
```

```
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x, x)
```

3.101 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)-\frac{1}{2}\pi bS(bx)^2$$

```
[Out] -cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x-1/2*b*Pi*FresnelS(b*x)^2+1/4*b*Si(b^2*
Pi*x^2)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6464, 6440, 30, 3375}

$$-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)-\frac{1}{2}\pi bS(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^2,x]
```

```
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*Si
nIntegral[b^2*Pi*x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
```

```
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^2}\,dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}+\frac{1}{2}b\int \frac{\sin\left(b^2\pi x^2\right)}{x}\,dx-\left(b^2\pi\right)\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}+\frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)-(b\pi)\,\mathrm{Subst}\left(\int x\,dx,x,S(bx)\right)\\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}-\frac{1}{2}b\pi S(bx)^2+\frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)-\frac{1}{2}\pi bS(bx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^2,x]
```

```
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*Si
nIntegral[b^2*Pi*x^2])/4
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnels}\left(bx\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnels}\left(bx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)\mathrm{S}\left(bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^{2} x^{2}\right)\mathrm{fresnels}\left(bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^2,x)
```

```
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(\frac{\pi b^{2} x^{2}}{2} \right)} S\left(bx\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**2,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**2, x)
```

3.102 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,dx$

Optimal. Leaf size=94

$$-\frac{1}{2}\pi b^2\text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{\pi b^2C\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{b\sin\left(\pi b^2x^2\right)}{4x}$$

```
[Out] -1/2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2-1/4*b*sin(b^2*Pi*x^2)/x+1/4*b^2*
Pi*FresnelC(b*x*2^(1/2))*2^(1/2)-1/2*b^2*Pi*Unintegrable(FresnelS(b*x)*sin(
1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) - (b^2*Pi*Defer[Int][(FresnelS[b*
x]*Sin[(b^2*Pi*x^2)/2])/x, x])/2
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2x^2}+\frac{1}{4}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{2}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx\\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}-\frac{1}{2}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx+\frac{1}{2}\left(b^3\right.\\ &= \frac{b^2\pi C\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}-\frac{1}{2}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{x}\end{aligned}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnels}\,(bx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^3,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnels}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^3,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^3, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{S}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnels}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^3,x)
```

```
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S\left(bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**3,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**3, x)
```

3.103 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{3}\pi b^2 \text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b\sin\left(\pi b^2x^2\right)}{12x^2}+\frac{1}{12}\pi b^3\text{Ci}\left(b^2\pi x^2\right)$$

```
[Out] 1/12*b^3*Pi*Ci(b^2*Pi*x^2)-1/3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3-1/12*b
*sin(b^2*Pi*x^2)/x^2-1/3*b^2*Pi*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x
^2)/x^2,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Defer[Int][(FresnelS[b*x]*S
in[(b^2*Pi*x^2)/2])/x^2, x])/3
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4}\,dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x^3}+\frac{1}{6}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^3}\,dx-\frac{1}{3}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x^3}+\frac{1}{12}b\,\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^2}\,dx,x,x^2\right)-\frac{1}{3}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\right.}{x^2}\\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}-\frac{1}{3}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx+\frac{1}{12}\left(b^3\right.\\ &= \frac{1}{12}b^3\pi\text{Ci}\left(b^2\pi x^2\right)-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}-\frac{1}{3}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\right.}{x^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnels}\,(bx)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^4,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnels}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^4,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^4, x)
```

maple [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\mathrm{S}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \operatorname{fresnels}(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^4,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^4,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^4, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**4,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**4, x)
```

3.104 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5}\,dx$

Optimal. Leaf size=156

$$-\frac{1}{8}\pi^2b^4\text{Int}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{7\pi^2b^4S\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}+\frac{\pi b^3}{16x}+\frac{\pi b^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^2}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b\sin\ldots}{\ldots}$$

```
[Out] 1/16*b^3*Pi/x-7/48*b^3*Pi*cos(b^2*Pi*x^2)/x-1/4*cos(1/2*b^2*Pi*x^2)*Fresnel
S(b*x)/x^4+1/8*b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-1/24*b*sin(b^2*
Pi*x^2)/x^3-7/48*b^4*Pi^2*FresnelS(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Uninte
grable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5,x]
```

```
[Out] (b^3*Pi)/(16*x) - (7*b^3*Pi*Cos[b^2*Pi*x^2])/(48*x) - (Cos[(b^2*Pi*x^2)/2]*
FresnelS[b*x])/(4*x^4) - (7*b^4*Pi^2*FresnelS[Sqrt[2]*b*x])/(24*Sqrt[2]) +
(b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^2) - (b*Sin[b^2*Pi*x^2])/(2
4*x^3) - (b^4*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/8
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5}\,dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4x^4}+\frac{1}{8}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx-\frac{1}{4}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx \\
&= \frac{b^3\pi}{16x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4x^4}+\frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}+\frac{1}{16}\left(b^3\pi\right)\ldots \\
&= \frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4x^4}+\frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}-\frac{b\sin\ldots}{24\ldots} \\
&= \frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4x^4}-\frac{7b^4\pi^2S\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}+\frac{b^2\pi S(bx)\sin\ldots}{8x^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnels}\left(bx\right)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^5,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^5, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnels}\left(bx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^5,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^5, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\mathrm{S}\left(bx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\operatorname{fresnels}(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^5, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^5,x)
```

```
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^5, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)S(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**5,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**5, x)
```

3.105 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^6}\,dx$

Optimal. Leaf size=163

$$\frac{1}{30}\pi^3 b^5 S(bx)^2+\frac{\pi b^3}{60x^2}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{\pi b^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3}-\frac{b\sin\left(\pi b^2x^2\right)}{40x^4}-\frac{7}{120}\pi^2 b^5\,\mathrm{Si}\left(b^2\pi x^2\right)+\frac{\pi^2 b^4 S(\cdots}{\cdots}$$

```
[Out] 1/60*b^3*Pi/x^2-1/24*b^3*Pi*cos(b^2*Pi*x^2)/x^2-1/5*cos(1/2*b^2*Pi*x^2)*Fre
snelS(b*x)/x^5+1/15*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x+1/30*b^5*P
i^3*FresnelS(b*x)^2-7/120*b^5*Pi^2*Si(b^2*Pi*x^2)+1/15*b^2*Pi*FresnelS(b*x)
*sin(1/2*b^2*Pi*x^2)/x^3-1/40*b*sin(b^2*Pi*x^2)/x^4
```

Rubi [A] time = 0.21, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6464, 6456, 6440, 30, 3375, 3380, 3297, 3299, 3379}

$$\frac{\pi b^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3}+\frac{\pi^2 b^4 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{15x}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{30}\pi^3 b^5 S(bx)^2-\frac{7}{120}\pi^2 b^5\,\mathrm{Si}\left(b^2\pi x^2\right)+\frac{\pi b\cdots}{60\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^6,x]
```

```
[Out] (b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(5*x^5) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15
*x) + (b^5*Pi^3*FresnelS[b*x]^2)/30 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(15*x^3) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2
*Pi*x^2])/120
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
```

```
ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^6}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{1}{10}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx - \frac{1}{5}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

$$= \frac{b^3\pi}{60x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} + \frac{1}{20}b\,\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^3}\,d\right.$$

$$= \frac{b^3\pi}{60x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} -$$

$$= \frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x} + \frac{b^2\pi S(}{}$$

$$= \frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x} + \frac{1}{30}b^5\pi$$

Mathematica [A] time = 0.01, size = 163, normalized size = 1.00

$$\frac{1}{30}\pi^3b^5S(bx)^2+\frac{\pi b^3}{60x^2}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{\pi b^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3}-\frac{b\sin\left(\pi b^2x^2\right)}{40x^4}-\frac{7}{120}\pi^2b^5\text{Si}\left(b^2\pi x^2\right)+\frac{\pi^2b^4S(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^6,x]
```

```
[Out] (b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(5*x^5) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15
*x) + (b^5*Pi^3*FresnelS[b*x]^2)/30 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(15*x^3) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2
*Pi*x^2])/120
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnels}\,(bx)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^6,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnels}\,(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^6,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^6, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2 \pi\, x^2}{2}\right) \mathrm{S}\,(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnels}\,(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^6,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}\,(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^6,x)
```

```
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S\left(bx\right)}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**6,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**6, x)
```

3.106 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}\,dx$

Optimal. Leaf size=231

$$\frac{1}{48}\pi^3 b^6 \text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x}, x\right) - \frac{1}{45}\sqrt{2}\,\pi^3 b^6 C\left(\sqrt{2}\,bx\right) - \frac{7\pi^3 b^6 C\left(\sqrt{2}\,bx\right)}{144\sqrt{2}} + \frac{\pi b^3}{144x^3} - \frac{S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^6} +$$

```
[Out] 1/144*b^3*Pi/x^3-13/720*b^3*Pi*cos(b^2*Pi*x^2)/x^3-1/6*cos(1/2*b^2*Pi*x^2)*
FresnelS(b*x)/x^6+1/48*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2+1/24*
b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-1/60*b*sin(b^2*Pi*x^2)/x^5+67/
1440*b^5*Pi^2*sin(b^2*Pi*x^2)/x-67/1440*b^6*Pi^3*FresnelC(b*x*2^(1/2))*2^(1
/2)+1/48*b^6*Pi^3*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7,x]
```

```
[Out] (b^3*Pi)/(144*x^3) - (13*b^3*Pi*Cos[b^2*Pi*x^2])/(720*x^3) - (7*b^6*Pi^3*Fr
esnelC[Sqrt[2]*b*x])/(144*Sqrt[2]) - (Sqrt[2]*b^6*Pi^3*FresnelC[Sqrt[2]*b*x
])/45 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(6*x^6) + (b^4*Pi^2*Cos[(b^2*Pi
*x^2)/2]*FresnelS[b*x])/(48*x^2) + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2
])/(24*x^4) - (b*Sin[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/
(1440*x) + (b^6*Pi^3*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/
48
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x^6} + \frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx - \frac{1}{6}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

$$= \frac{b^3\pi}{144x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x^6} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} - \frac{b\sin\left(b^2\pi x^2\right)}{60x^5} + \frac{1}{48}\left(b^3\pi\right)$$

$$= \frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x^6} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^2} + \frac{b^2}{}$$

$$= \frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x^6} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^2} + \frac{b^2}{}$$

$$= \frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3} - \frac{7b^6\pi^3C\left(\sqrt{2}\,bx\right)}{144\sqrt{2}} - \frac{1}{45}\sqrt{2}\,b^6\pi^3C\left(\sqrt{2}\,bx\right) - \frac{\cos\left(\frac{1}{2}b^2\pi\right.}{6}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7, x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnels}\left(bx\right)}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^7,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^7, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnels}\left(bx\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^7,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^7, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)\mathrm{S}\left(bx\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{fresnels}\left(bx\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^7,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^7, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^{2}\,x^{2}}{2}\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^7,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^7, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^{2}x^{2}}{2}\right)S\left(bx\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**7,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**7, x)
```

3.107 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8}\,dx$

Optimal. Leaf size=202

$$\frac{1}{105}\pi^3 b^6 \text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)+\frac{\pi b^3}{280x^4}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{\pi b^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}-\frac{b\sin\left(\pi b^2x^2\right)}{84x^6}-\frac{1}{84}\pi^3$$

```
[Out] 1/280*b^3*Pi/x^4-1/84*b^7*Pi^3*Ci(b^2*Pi*x^2)-1/105*b^3*Pi*cos(b^2*Pi*x^2)/
x^4-1/7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7+1/105*b^4*Pi^2*cos(1/2*b^2*Pi
*x^2)*FresnelS(b*x)/x^3+1/35*b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1
/84*b*sin(b^2*Pi*x^2)/x^6+1/84*b^5*Pi^2*sin(b^2*Pi*x^2)/x^2+1/105*b^6*Pi^3*
Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Rubi [A] time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8,x]
```

```
[Out] (b^3*Pi)/(280*x^4) - (b^3*Pi*Cos[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*CosInte
gral[b^2*Pi*x^2])/84 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*x^7) + (b^4*P
i^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x^3) + (b^2*Pi*FresnelS[b*x]*Si
n[(b^2*Pi*x^2)/2])/(35*x^5) - (b*Sin[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*Sin[
b^2*Pi*x^2])/(84*x^2) + (b^6*Pi^3*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/x^2, x])/105
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7} + \frac{1}{14}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^7}\,dx - \frac{1}{7}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

$$= \frac{b^3\pi}{280x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} + \frac{1}{28}b\,\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right.}{x^4}\right.$$

$$= \frac{b^3\pi}{280x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{105x^3} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5}$$

$$= \frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{105x^3} + \frac{b^2\pi}{}$$

$$= \frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{105x^3} + \frac{b^2\pi}{}$$

$$= \frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{1}{84}b^7\pi^3\text{Ci}\left(b^2\pi x^2\right) - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\right.}{1}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnels}\,(bx)}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^8, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\operatorname{fresnels}(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^8,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^8, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\mathrm{S}(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\operatorname{fresnels}(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^8, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^8,x)
```

```
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^8, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S\left(bx\right)}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**8,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**8, x)
```

3.108 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9}\,dx$

Optimal. Leaf size=271

$$\frac{1}{384}\pi^4 b^8 \text{Int}\left(\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)+\frac{853\pi^4 b^8 S\left(\sqrt{2}\,bx\right)}{40320\sqrt{2}}-\frac{\pi^3 b^7}{768x}+\frac{\pi b^3}{480x^5}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8}+\frac{\pi b^2 S(bx)\sin\left(\frac{1}{2}\pi\right.}{48x^6}$$

```
[Out] 1/480*b^3*Pi/x^5-1/768*b^7*Pi^3/x-19/3360*b^3*Pi*cos(b^2*Pi*x^2)/x^5+853/80
640*b^7*Pi^3*cos(b^2*Pi*x^2)/x-1/8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8+1/
192*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4+1/48*b^2*Pi*FresnelS(b*x
)*sin(1/2*b^2*Pi*x^2)/x^6-1/384*b^6*Pi^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/
x^2-1/112*b*sin(b^2*Pi*x^2)/x^7+187/40320*b^5*Pi^2*sin(b^2*Pi*x^2)/x^3+853/
80640*b^8*Pi^4*FresnelS(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrable(co
s(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [A] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9,x]
```

```
[Out] (b^3*Pi)/(480*x^5) - (b^7*Pi^3)/(768*x) - (19*b^3*Pi*Cos[b^2*Pi*x^2])/(3360
*x^5) + (853*b^7*Pi^3*Cos[b^2*Pi*x^2])/(80640*x) - (Cos[(b^2*Pi*x^2)/2]*Fre
snelS[b*x])/(8*x^8) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(192*x^4
) + (853*b^8*Pi^4*FresnelS[Sqrt[2]*b*x])/(40320*Sqrt[2]) + (b^2*Pi*FresnelS
[b*x]*Sin[(b^2*Pi*x^2)/2])/(48*x^6) - (b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x
^2)/2])/(384*x^2) - (b*Sin[b^2*Pi*x^2])/(112*x^7) + (187*b^5*Pi^2*Sin[b^2*P
i*x^2])/(40320*x^3) + (b^8*Pi^4*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*
x])/x, x])/384
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{1}{16}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}\,dx - \frac{1}{8}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

$$= \frac{b^3\pi}{480x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7} + \frac{1}{96}\left(b^3\pi\right.$$

$$= \frac{b^3\pi}{480x^5} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{192x^4} + \cdots$$

$$= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S}{192x^4}$$

$$= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8}$$

$$= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnels}\,(bx)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^9,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^9, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnels}\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^9,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^9, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2 \pi x^2}{2}\right) \mathrm{S}\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnels}\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^9,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^9, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}\,(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^9,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^9, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S\left(bx\right)}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**9,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**9, x)
```

3.109 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^{10}}\,dx$

Optimal. Leaf size=278

$$-\frac{\pi^5 b^9 S(bx)^2}{1890}-\frac{\pi^3 b^7}{3780x^2}+\frac{\pi b^3}{756x^6}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+\frac{\pi b^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{63x^7}-\frac{b\sin\left(\pi b^2x^2\right)}{144x^8}+\frac{83\pi^4 b^9\text{Si}\left(b^2\pi x^2\right)}{30240}$$

```
[Out] 1/756*b^3*Pi/x^6-1/3780*b^7*Pi^3/x^2-11/3024*b^3*Pi*cos(b^2*Pi*x^2)/x^6+5/2
016*b^7*Pi^3*cos(b^2*Pi*x^2)/x^2-1/9*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9+
1/315*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5-1/945*b^8*Pi^4*cos(1/2
*b^2*Pi*x^2)*FresnelS(b*x)/x-1/1890*b^9*Pi^5*FresnelS(b*x)^2+83/30240*b^9*P
i^4*Si(b^2*Pi*x^2)+1/63*b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7-1/945*
b^6*Pi^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-1/144*b*sin(b^2*Pi*x^2)/x^8+
67/30240*b^5*Pi^2*sin(b^2*Pi*x^2)/x^4
```

Rubi [A] time = 0.52, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6464, 6456, 6440, 30, 3375, 3380, 3297, 3299, 3379}

$$-\frac{\pi^3 b^6 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{945x^3}+\frac{\pi b^2 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{63x^7}-\frac{\pi^4 b^8 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{945x}+\frac{\pi^2 b^4 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{315x^5}-\frac{S(bx)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^10,x]
```

```
[Out] (b^3*Pi)/(756*x^6) - (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(3
024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (Cos[(b^2*Pi*x^2)/2]*F
resnelS[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(315*x
^5) - (b^8*Pi^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x) - (b^9*Pi^5*Fres
nelS[b*x]^2)/1890 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(63*x^7) - (
b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) - (b*Sin[b^2*Pi*x^2])
/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinIn
tegral[b^2*Pi*x^2])/30240
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6440

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6456

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelS[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[(d*x^(m + 2))/(Pi*b*(m + 1)*(m + 2)), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6464

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelS[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^{10}}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{1}{18}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^9}\,dx - \frac{1}{9}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

$$= \frac{b^3\pi}{756x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} + \frac{1}{36}b\,\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^5}\right.$$

$$= \frac{b^3\pi}{756x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{315x^5} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7}$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S}{315x^5}$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S}{315x^5}$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} + \frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} +$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} + \frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} +$$

Mathematica [A] time = 0.02, size = 278, normalized size = 1.00

$$-\frac{\pi^5b^9S(bx)^2}{1890} - \frac{\pi^3b^7}{3780x^2} + \frac{\pi b^3}{756x^6} - \frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9} + \frac{\pi b^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{63x^7} - \frac{b\sin\left(\pi b^2x^2\right)}{144x^8} + \frac{83\pi^4b^9\text{Si}\left(b^2\pi x^2\right)}{30240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^10,x]
```

```
[Out] (b^3*Pi)/(756*x^6) - (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(3
024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (Cos[(b^2*Pi*x^2)/2]*F
resnelS[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(315*x
^5) - (b^8*Pi^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x) - (b^9*Pi^5*Fres
nelS[b*x]^2)/1890 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(63*x^7) - (
b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) - (b*Sin[b^2*Pi*x^2])
/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinIn
tegral[b^2*Pi*x^2])/30240
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnels}\left(bx\right)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^10,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^10, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnels}\left(bx\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^10,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^10, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2 \pi x^2}{2}\right)\mathrm{S}\left(bx\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^10,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^10,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnels}\left(bx\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnels(b*x)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnels(b*x)/x^10, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^10,x)
```

```
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^{2} x^{2}}{2}\right) S\left(b x\right)}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**10,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**10, x)
```

3.110 $\int x^7 C(bx)\,dx$

Optimal. Leaf size=124

$$-\frac{105C(bx)}{8\pi^4b^8}-\frac{x^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi b}+\frac{105x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^7}+\frac{35x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^3b^5}-\frac{7x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^2b^3}+\frac{1}{8}x^8C(bx)$$

```
[Out] 105/8*x*cos(1/2*b^2*Pi*x^2)/b^7/Pi^4-7/8*x^5*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2-1
05/8*FresnelC(b*x)/b^8/Pi^4+1/8*x^8*FresnelC(b*x)+35/8*x^3*sin(1/2*b^2*Pi*x
^2)/b^5/Pi^3-1/8*x^7*sin(1/2*b^2*Pi*x^2)/b/Pi
```

Rubi [A] time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3386, 3385, 3352}

$$-\frac{105\text{FresnelC}(bx)}{8\pi^4b^8}-\frac{x^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi b}+\frac{35x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^3b^5}-\frac{7x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^2b^3}+\frac{105x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^7}+\frac{1}{8}x^8\text{Fresne}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelC[b*x],x]
```

```
[Out] (105*x*Cos[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*Cos[(b^2*Pi*x^2)/2])/(8*b
^3*Pi^2) - (105*FresnelC[b*x])/(8*b^8*Pi^4) + (x^8*FresnelC[b*x])/8 + (35*x
^3*Sin[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) - (x^7*Sin[(b^2*Pi*x^2)/2])/(8*b*Pi)
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^7 C(bx)\,dx = \frac{1}{8}x^8 C(bx) - \frac{1}{8}b\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)dx$$
$$= \frac{1}{8}x^8 C(bx) - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{7\int x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{8b\pi}$$
$$= -\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{1}{8}x^8 C(bx) - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{8b^3\pi^2}$$
$$= -\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{1}{8}x^8 C(bx) + \frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} - \frac{105\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right.}{8b^5\pi^3}$$
$$= \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{1}{8}x^8 C(bx) + \frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} -$$
$$= \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} - \frac{105C(bx)}{8b^8\pi^4} + \frac{1}{8}x^8 C(bx) + \frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} - \frac{x^7\sin}{8}$$

Mathematica [A] time = 0.08, size = 89, normalized size = 0.72

$$\frac{\left(\pi^4 b^8 x^8 - 105\right)C(bx) - 7bx\left(\pi^2 b^4 x^4 - 15\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right) + \pi b^3 x^3\left(35 - \pi^2 b^4 x^4\right)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8\pi^4 b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelC[b*x],x]
```

```
[Out] (-7*b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*Fr
esnelC[b*x] + b^3*Pi*x^3*(35 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(8*b^8*Pi
^4)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(x^7\,\text{fresnelc}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^7*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \text{fresnelc}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnelc(b*x), x)
```

maple [A] time = 0.01, size = 123, normalized size = 0.99

$$\frac{\frac{\text{FresnelC}(bx)b^8x^8}{8} - \frac{b^7x^7\sin\left(\frac{b^2\pi x^2}{2}\right)}{8\pi} + \frac{-\frac{7b^5x^5\cos\left(\frac{b^2\pi x^2}{2}\right)}{8\pi} + \frac{7\left(\frac{5b^3x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi} - \frac{15\left(-\frac{bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{\text{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{8\pi}}{\pi}}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x),x)
```

```
[Out] 1/b^8*(1/8*FresnelC(b*x)*b^8*x^8-1/8/Pi*b^7*x^7*sin(1/2*b^2*Pi*x^2)+7/8/Pi*
(-1/Pi*b^5*x^5*cos(1/2*b^2*Pi*x^2)+5/Pi*(1/Pi*b^3*x^3*sin(1/2*b^2*Pi*x^2)-3
/Pi*(-1/Pi*b*x*cos(1/2*b^2*Pi*x^2)+1/Pi*FresnelC(b*x)))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \text{fresnelc}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^7\, \text{FresnelC}(b\, x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*FresnelC(b*x),x)

[Out] int(x^7*FresnelC(b*x), x)

sympy [A] time = 2.31, size = 184, normalized size = 1.48

$$\frac{45x^8C(bx)\Gamma\left(\frac{1}{4}\right)}{512\Gamma\left(\frac{13}{4}\right)}-\frac{45x^7\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi b\Gamma\left(\frac{13}{4}\right)}-\frac{315x^5\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi^2b^3\Gamma\left(\frac{13}{4}\right)}+\frac{1575x^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi^3b^5\Gamma\left(\frac{13}{4}\right)}+\frac{4725x\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma}{512\pi^4b^7\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*fresnelc(b*x),x)

[Out] 45*x**8*fresnelc(b*x)*gamma(1/4)/(512*gamma(13/4)) - 45*x**7*sin(pi*b**2*x**2/2)*gamma(1/4)/(512*pi*b*gamma(13/4)) - 315*x**5*cos(pi*b**2*x**2/2)*gamma(1/4)/(512*pi**2*b**3*gamma(13/4)) + 1575*x**3*sin(pi*b**2*x**2/2)*gamma(1/4)/(512*pi**3*b**5*gamma(13/4)) + 4725*x*cos(pi*b**2*x**2/2)*gamma(1/4)/(512*pi**4*b**7*gamma(13/4)) - 4725*fresnelc(b*x)*gamma(1/4)/(512*pi**4*b**8*gamma(13/4))

3.111 $\int x^6 C(bx)\,dx$

Optimal. Leaf size=109

$$-\frac{x^6\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{48\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}+\frac{24x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}-\frac{6x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{1}{7}x^7C(bx)$$

```
[Out] 48/7*cos(1/2*b^2*Pi*x^2)/b^7/Pi^4-6/7*x^4*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/7*
x^7*FresnelC(b*x)+24/7*x^2*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-1/7*x^6*sin(1/2*b^2
*Pi*x^2)/b/Pi
```

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3380, 3296, 2638}

$$-\frac{x^6\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{24x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}-\frac{6x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{48\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}+\frac{1}{7}x^7\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelC[b*x],x]
```

```
[Out] (48*Cos[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x^4*Cos[(b^2*Pi*x^2)/2])/(7*b^3*
Pi^2) + (x^7*FresnelC[b*x])/7 + (24*x^2*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) -
 (x^6*Sin[(b^2*Pi*x^2)/2])/(7*b*Pi)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^6 C(bx)\,dx &= \frac{1}{7}x^7C(bx) - \frac{1}{7}b\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)dx\\
&= \frac{1}{7}x^7C(bx) - \frac{1}{14}b\,\text{Subst}\left(\int x^3\cos\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)\\
&= \frac{1}{7}x^7C(bx) - \frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{3\,\text{Subst}\left(\int x^2\sin\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{7b\pi}\\
&= -\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{1}{7}x^7C(bx) - \frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{12\,\text{Subst}\left(\int x\cos\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{7b^3\pi^2}\\
&= -\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{1}{7}x^7C(bx) + \frac{24x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} - \frac{24\,\text{Subst}\left(\int \sin\left(\frac{1}{2}b^2\right.\right.}{7b^5\pi^3}\\
&= \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{1}{7}x^7C(bx) + \frac{24x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 83, normalized size = 0.76

$$-\frac{6\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7} - \frac{x^2\left(\pi^2b^4x^4-24\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5} + \frac{1}{7}x^7C(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelC[b*x],x]
```

```
[Out] (-6*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) + (x^7*FresnelC[b
*x])/7 - (x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(x^6\text{fresnelc}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^6*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnelc(b*x), x)
```

maple [A] time = 0.00, size = 107, normalized size = 0.98

$$\frac{\frac{b^7x^7\,\mathrm{FresnelC}(bx)}{7}-\frac{b^6x^6\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{7\pi}+\frac{-\frac{6b^4x^4\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{7\pi}+\frac{6\left(\frac{4b^2x^2\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi^2}\right)}{7\pi}}{\pi}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x),x)
```

```
[Out] 1/b^7*(1/7*b^7*x^7*FresnelC(b*x)-1/7/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)+6/7/Pi*
(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2
/Pi^2*cos(1/2*b^2*Pi*x^2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\,\mathrm{FresnelC}\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x),x)
```

[Out] int(x^6*FresnelC(b*x), x)

sympy [A] time = 2.23, size = 153, normalized size = 1.40

$$\frac{x^7 C(bx)\Gamma\left(\frac{1}{4}\right)}{28\Gamma\left(\frac{5}{4}\right)} - \frac{x^6\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{28\pi b\Gamma\left(\frac{5}{4}\right)} - \frac{3x^4\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{14\pi^2b^3\Gamma\left(\frac{5}{4}\right)} + \frac{6x^2\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{7\pi^3b^5\Gamma\left(\frac{5}{4}\right)} + \frac{12\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{7\pi^4b^7\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnelc(b*x),x)
```

```
[Out] x**7*fresnelc(b*x)*gamma(1/4)/(28*gamma(5/4)) - x**6*sin(pi*b**2*x**2/2)*ga
mma(1/4)/(28*pi*b*gamma(5/4)) - 3*x**4*cos(pi*b**2*x**2/2)*gamma(1/4)/(14*p
i**2*b**3*gamma(5/4)) + 6*x**2*sin(pi*b**2*x**2/2)*gamma(1/4)/(7*pi**3*b**5
*gamma(5/4)) + 12*cos(pi*b**2*x**2/2)*gamma(1/4)/(7*pi**4*b**7*gamma(5/4))
```

3.112 $\int x^5 C(bx)\,dx$

Optimal. Leaf size=99

$$-\frac{5S(bx)}{2\pi^3b^6}-\frac{x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi b}+\frac{5x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3b^5}-\frac{5x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi^2b^3}+\frac{1}{6}x^6C(bx)$$

```
[Out] -5/6*x^3*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/6*x^6*FresnelC(b*x)-5/2*FresnelS(b*
x)/b^6/Pi^3+5/2*x*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-1/6*x^5*sin(1/2*b^2*Pi*x^2)/
b/Pi
```

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3386, 3385, 3351}

$$-\frac{5S(bx)}{2\pi^3b^6}-\frac{x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi b}+\frac{5x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3b^5}-\frac{5x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi^2b^3}+\frac{1}{6}x^6\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelC[b*x],x]
```

```
[Out] (-5*x^3*Cos[(b^2*Pi*x^2)/2])/(6*b^3*Pi^2) + (x^6*FresnelC[b*x])/6 - (5*Fres
nelS[b*x])/(2*b^6*Pi^3) + (5*x*Sin[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) - (x^5*Sin
[(b^2*Pi*x^2)/2])/(6*b*Pi)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^5 C(bx)\,dx &= \frac{1}{6}x^6 C(bx) - \frac{1}{6}b\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)dx\\ &= \frac{1}{6}x^6 C(bx) - \frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5\int x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{6b\pi}\\ &= -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6 C(bx) - \frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{2b^3\pi^2}\\ &= -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6 C(bx) + \frac{5x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} - \frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} - \frac{5\int \sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{2b^5\pi^3}\\ &= -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6 C(bx) - \frac{5S(bx)}{2b^6\pi^3} + \frac{5x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} - \frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi}\end{aligned}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.81

$$\frac{\pi^3 b^6 x^6 C(bx) + bx\left(15 - \pi^2 b^4 x^4\right)\sin\left(\frac{1}{2}\pi b^2 x^2\right) - 5\pi b^3 x^3\cos\left(\frac{1}{2}\pi b^2 x^2\right) - 15S(bx)}{6\pi^3 b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelC[b*x],x]
```

```
[Out] (-5*b^3*Pi*x^3*Cos[(b^2*Pi*x^2)/2] + b^6*Pi^3*x^6*FresnelC[b*x] - 15*Fresne
lS[b*x] + b*x*(15 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(6*b^6*Pi^3)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(x^5\text{fresnelc}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^5*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnelc(b*x), x)
```

maple [A] time = 0.00, size = 97, normalized size = 0.98

$$\frac{\frac{b^6x^6\,\mathrm{FresnelC}(bx)}{6}-\frac{b^5x^5\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{6\pi}+\frac{-\frac{5b^3x^3\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{6\pi}+\frac{5\left(\frac{3bx\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi}-\frac{3\,\mathrm{S}(bx)}{\pi}\right)}{6\pi}}{\pi}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x),x)
```

```
[Out] 1/b^6*(1/6*b^6*x^6*FresnelC(b*x)-1/6/Pi*b^5*x^5*sin(1/2*b^2*Pi*x^2)+5/6/Pi*
(-1/Pi*b^3*x^3*cos(1/2*b^2*Pi*x^2)+3/Pi*(1/Pi*b*x*sin(1/2*b^2*Pi*x^2)-1/Pi*
FresnelS(b*x))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,\mathrm{FresnelC}\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x),x)
```

```
[Out] int(x^5*FresnelC(b*x), x)
```

<u>**sympy**</u> **[A]** time = 0.97, size = 49, normalized size = 0.49

$$\frac{bx^7\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{7}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\\frac{1}{2},\frac{5}{4},\frac{11}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnelc(b*x),x)
```

```
[Out] b*x**7*gamma(1/4)*gamma(7/4)*hyper((1/4, 7/4), (1/2, 5/4, 11/4), -pi**2*b**
4*x**4/16)/(16*gamma(5/4)*gamma(11/4))
```

3.113 $\int x^4 C(bx)\, dx$

Optimal. Leaf size=84

$$-\frac{x^4 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi b}+\frac{8 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5}-\frac{4x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^2 b^3}+\frac{1}{5}x^5 C(bx)$$

```
[Out] -4/5*x^2*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/5*x^5*FresnelC(b*x)+8/5*sin(1/2*b^2
*Pi*x^2)/b^5/Pi^3-1/5*x^4*sin(1/2*b^2*Pi*x^2)/b/Pi
```

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3380, 3296, 2637}

$$-\frac{x^4 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi b}+\frac{8 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5}-\frac{4x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^2 b^3}+\frac{1}{5}x^5 \text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelC[b*x],x]
```

```
[Out] (-4*x^2*Cos[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x])/5 + (8*Sin[
(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) - (x^4*Sin[(b^2*Pi*x^2)/2])/(5*b*Pi)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m + 
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^4 C(bx)\,dx &= \frac{1}{5}x^5C(bx)-\frac{1}{5}b\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)dx\\
&= \frac{1}{5}x^5C(bx)-\frac{1}{10}b\,\text{Subst}\left(\int x^2\cos\left(\frac{1}{2}b^2\pi x\right)dx,x,x^2\right)\\
&= \frac{1}{5}x^5C(bx)-\frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi}+\frac{2\,\text{Subst}\left(\int x\sin\left(\frac{1}{2}b^2\pi x\right)dx,x,x^2\right)}{5b\pi}\\
&= -\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2}+\frac{1}{5}x^5C(bx)-\frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi}+\frac{4\,\text{Subst}\left(\int\cos\left(\frac{1}{2}b^2\pi x\right)dx,x,x^2\right)}{5b^3\pi^2}\\
&= -\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2}+\frac{1}{5}x^5C(bx)+\frac{8\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3}-\frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.85

$$-\frac{4x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}-\frac{\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}+\frac{1}{5}x^5C(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelC[b*x],x]
```

```
[Out] (-4*x^2*Cos[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x])/5 - ((-8 + 
b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(x^4\text{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^4*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \mathrm{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnelc(b*x), x)
```

maple [A] time = 0.00, size = 81, normalized size = 0.96

$$\frac{\frac{b^5x^5\,\mathrm{FresnelC}(bx)}{5}-\frac{b^4x^4\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{5\pi}+\frac{-\frac{4b^2x^2\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{5\pi}+\frac{8\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{5\pi^2}}{\pi}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x),x)
```

```
[Out] 1/b^5*(1/5*b^5*x^5*FresnelC(b*x)-1/5/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)+4/5/Pi*
(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \mathrm{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{FresnelC}\,(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x),x)
```

```
[Out] int(x^4*FresnelC(b*x), x)
```

sympy [A] time = 1.20, size = 116, normalized size = 1.38

$$\frac{x^{5}C\left(bx\right)\Gamma\left(\frac{1}{4}\right)}{20\Gamma\left(\frac{5}{4}\right)}-\frac{x^{4}\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)\Gamma\left(\frac{1}{4}\right)}{20\pi b\Gamma\left(\frac{5}{4}\right)}-\frac{x^{2}\cos\left(\frac{\pi b^{2}x^{2}}{2}\right)\Gamma\left(\frac{1}{4}\right)}{5\pi^{2}b^{3}\Gamma\left(\frac{5}{4}\right)}+\frac{2\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)\Gamma\left(\frac{1}{4}\right)}{5\pi^{3}b^{5}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnelc(b*x),x)
```

```
[Out] x**5*fresnelc(b*x)*gamma(1/4)/(20*gamma(5/4)) - x**4*sin(pi*b**2*x**2/2)*ga
mma(1/4)/(20*pi*b*gamma(5/4)) - x**2*cos(pi*b**2*x**2/2)*gamma(1/4)/(5*pi**
2*b**3*gamma(5/4)) + 2*sin(pi*b**2*x**2/2)*gamma(1/4)/(5*pi**3*b**5*gamma(5
/4))
```

3.114 $\int x^3 C(bx)\,dx$

Optimal. Leaf size=74

$$\frac{3C(bx)}{4\pi^2 b^4} - \frac{x^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} - \frac{3x\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{1}{4}x^4 C(bx)$$

```
[Out] -3/4*x*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+3/4*FresnelC(b*x)/b^4/Pi^2+1/4*x^4*Fres
nelC(b*x)-1/4*x^3*sin(1/2*b^2*Pi*x^2)/b/Pi
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3386, 3385, 3352}

$$\frac{3\text{FresnelC}(bx)}{4\pi^2 b^4} - \frac{x^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi b} - \frac{3x\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{1}{4}x^4 \text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelC[b*x],x]
```

```
[Out] (-3*x*Cos[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (3*FresnelC[b*x])/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x])/4 - (x^3*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi)
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^3 C(bx)\,dx &= \frac{1}{4}x^4 C(bx) - \frac{1}{4}b\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)dx\\
&= \frac{1}{4}x^4 C(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{4b\pi}\\
&= -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{1}{4}x^4 C(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3\int\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{4b^3\pi^2}\\
&= -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3C(bx)}{4b^4\pi^2} + \frac{1}{4}x^4 C(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 74, normalized size = 1.00

$$\frac{3C(bx)}{4\pi^2 b^4} - \frac{x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b} - \frac{3x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2 b^3} + \frac{1}{4}x^4 C(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[b*x],x]
[Out] (-3*x*Cos[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (3*FresnelC[b*x])/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x])/4 - (x^3*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi)
```

fricas [F] time = 0.38, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^3\mathrm{fresnelc}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x),x, algorithm="fricas")
[Out] integral(x^3*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\mathrm{fresnelc}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnelc(b*x), x)
```

maple [A] time = 0.01, size = 70, normalized size = 0.95

$$\frac{\frac{b^4x^4 \operatorname{FresnelC}(bx)}{4} - \frac{b^3x^3 \sin\left(\frac{b^2\pi x^2}{2}\right)}{4\pi} + \frac{-\frac{3bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{4\pi} + \frac{3\operatorname{FresnelC}(bx)}{4\pi}}{\pi}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x),x)
```

```
[Out] 1/b^4*(1/4*b^4*x^4*FresnelC(b*x)-1/4/Pi*b^3*x^3*sin(1/2*b^2*Pi*x^2)+3/4/Pi*
(-1/Pi*b*x*cos(1/2*b^2*Pi*x^2)+1/Pi*FresnelC(b*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{fresnelc}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{FresnelC}(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x),x)
```

```
[Out] int(x^3*FresnelC(b*x), x)
```

sympy [A] time = 1.01, size = 112, normalized size = 1.51

$$\frac{5x^4 C(bx)\Gamma\left(\frac{1}{4}\right)}{64\Gamma\left(\frac{9}{4}\right)} - \frac{5x^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{64\pi b\Gamma\left(\frac{9}{4}\right)} - \frac{15x\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{64\pi^2b^3\Gamma\left(\frac{9}{4}\right)} + \frac{15C(bx)\Gamma\left(\frac{1}{4}\right)}{64\pi^2b^4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnelc(b*x),x)
```

```
[Out] 5*x**4*fresnelc(b*x)*gamma(1/4)/(64*gamma(9/4)) - 5*x**3*sin(pi*b**2*x**2/2
)*gamma(1/4)/(64*pi*b*gamma(9/4)) - 15*x*cos(pi*b**2*x**2/2)*gamma(1/4)/(64
*pi**2*b**3*gamma(9/4)) + 15*fresnelc(b*x)*gamma(1/4)/(64*pi**2*b**4*gamma(
9/4))
```

3.115 $\int x^2 C(bx)\,dx$

Optimal. Leaf size=59

$$-\frac{x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}-\frac{2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{1}{3}x^3C(bx)$$

```
[Out] -2/3*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/3*x^3*FresnelC(b*x)-1/3*x^2*sin(1/2*b^2
*Pi*x^2)/b/Pi
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3380, 3296, 2638}

$$-\frac{x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}-\frac{2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{1}{3}x^3\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[b*x],x]
```

```
[Out] (-2*Cos[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x])/3 - (x^2*Sin[(b
^2*Pi*x^2)/2])/(3*b*Pi)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 C(bx)\,dx &= \frac{1}{3}x^3 C(bx) - \frac{1}{3}b\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)dx\\
&= \frac{1}{3}x^3 C(bx) - \frac{1}{6}b\,\text{Subst}\left(\int x\cos\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)\\
&= \frac{1}{3}x^3 C(bx) - \frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{\text{Subst}\left(\int\sin\left(\frac{1}{2}b^2\pi x\right)dx, x, x^2\right)}{3b\pi}\\
&= -\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{1}{3}x^3 C(bx) - \frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.00

$$-\frac{x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b} - \frac{2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3} + \frac{1}{3}x^3C(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[b*x],x]
```

```
[Out] (-2*Cos[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x])/3 - (x^2*Sin[(b
^2*Pi*x^2)/2])/(3*b*Pi)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\text{fresnelc}\,(bx)\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnelc(b*x), x)
```

maple [A] time = 0.01, size = 54, normalized size = 0.92

$$\frac{\frac{b^3x^3\,\mathrm{FresnelC}(bx)}{3}-\frac{b^2x^2\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{3\pi}-\frac{2\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{3\pi^2}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x),x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*FresnelC(b*x)-1/3/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)-2/3/Pi^
2*cos(1/2*b^2*Pi*x^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\mathrm{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\mathrm{FresnelC}\,(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x),x)
```

```
[Out] int(x^2*FresnelC(b*x), x)
```

sympy [A] time = 1.11, size = 80, normalized size = 1.36

$$\frac{x^3C\,(bx)\,\Gamma\left(\frac{1}{4}\right)}{12\Gamma\left(\frac{5}{4}\right)}-\frac{x^2\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi b\Gamma\left(\frac{5}{4}\right)}-\frac{\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{6\pi^2b^3\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(b*x),x)
```

```
[Out] x**3*fresnelc(b*x)*gamma(1/4)/(12*gamma(5/4)) - x**2*sin(pi*b**2*x**2/2)*ga
mma(1/4)/(12*pi*b*gamma(5/4)) - cos(pi*b**2*x**2/2)*gamma(1/4)/(6*pi**2*b**
3*gamma(5/4))
```

3.116 $\int xC(bx)\,dx$

Optimal. Leaf size=49

$$\frac{S(bx)}{2\pi b^2} - \frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b} + \frac{1}{2}x^2C(bx)$$

```
[Out] 1/2*x^2*FresnelC(b*x)+1/2*FresnelS(b*x)/b^2/Pi-1/2*x*sin(1/2*b^2*Pi*x^2)/b/
Pi
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3386, 3351}

$$\frac{S(bx)}{2\pi b^2} - \frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b} + \frac{1}{2}x^2\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[b*x],x]
```

```
[Out] (x^2*FresnelC[b*x])/2 + FresnelS[b*x]/(2*b^2*Pi) - (x*Sin[(b^2*Pi*x^2)/2])/
(2*b*Pi)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int xC(bx)\,dx = \frac{1}{2}x^2C(bx) - \frac{1}{2}b\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)dx$$
$$= \frac{1}{2}x^2C(bx) - \frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{\int\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{2b\pi}$$
$$= \frac{1}{2}x^2C(bx) + \frac{S(bx)}{2b^2\pi} - \frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{S(bx)}{2\pi b^2} - \frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b} + \frac{1}{2}x^2C(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelC[b*x],x]
[Out] (x^2*FresnelC[b*x])/2 + FresnelS[b*x]/(2*b^2*Pi) - (x*Sin[(b^2*Pi*x^2)/2])/
(2*b*Pi)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(x\text{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x),x, algorithm="fricas")
[Out] integral(x*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x),x, algorithm="giac")
[Out] integrate(x*fresnelc(b*x), x)
```

maple [A] time = 0.01, size = 44, normalized size = 0.90

$$\frac{\frac{b^2x^2\,\mathrm{FresnelC}(bx)}{2}-\frac{bx\sin\left(\frac{b^2\pi x^2}{2}\right)}{2\pi}+\frac{\mathrm{S}(bx)}{2\pi}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x),x)
```

```
[Out] 1/b^2*(1/2*b^2*x^2*FresnelC(b*x)-1/2/Pi*b*x*sin(1/2*b^2*Pi*x^2)+1/2/Pi*Fres
nelS(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{FresnelC}\,(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x),x)
```

```
[Out] int(x*FresnelC(b*x), x)
```

sympy [A] time = 0.60, size = 49, normalized size = 1.00

$$\frac{bx^3\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x),x)
```

```
[Out] b*x**3*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -pi**2*b**4
*x**4/16)/(16*gamma(5/4)*gamma(7/4))
```

3.117 $\int C(bx)\,dx$

Optimal. Leaf size=27

$$xC(bx) - \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b}$$

[Out] x*FresnelC(b*x)-sin(1/2*b^2*Pi*x^2)/b/Pi

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6419}

$$x\text{FresnelC}(bx) - \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x],x]

[Out] x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)

Rule 6419

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelC[a +
 b*x])/b, x] - Simp[Sin[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int C(bx)\,dx = xC(bx) - \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$xC(bx) - \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[b*x],x]

[Out] x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x),x, algorithm="fricas")
[Out] integral(fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x),x, algorithm="giac")
[Out] integrate(fresnelc(b*x), x)
```

maple [A] time = 0.01, size = 28, normalized size = 1.04

$$\frac{bx\,\text{FresnelC}\left(bx\right)-\frac{\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x),x)
[Out] 1/b*(b*x*FresnelC(b*x)-sin(1/2*b^2*Pi*x^2)/Pi)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x),x, algorithm="maxima")
[Out] integrate(fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \text{FresnelC}\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x),x)
```

```
[Out] int(FresnelC(b*x), x)
```

sympy [B] time = 0.70, size = 44, normalized size = 1.63

$$\frac{xC\left(bx\right)\Gamma\left(\frac{1}{4}\right)}{4\Gamma\left(\frac{5}{4}\right)}-\frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{4\pi b\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x),x)
```

```
[Out] x*fresnelc(b*x)*gamma(1/4)/(4*gamma(5/4)) - sin(pi*b**2*x**2/2)*gamma(1/4)/
(4*pi*b*gamma(5/4))
```

3.118 $\int \frac{C(bx)}{x}\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/2*b*x*HypergeometricPFQ([1/2, 1/2],[3/2, 3/2],-1/2*I*b^2*Pi*x^2)+1/2*b*x*
HypergeometricPFQ([1/2, 1/2],[3/2, 3/2],1/2*I*b^2*Pi*x^2)
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6425, 6358, 6360}

$$\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x,x]
```

```
[Out] (b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-I/2)*b^2*Pi*x^2])/2 + (b*x
*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2])/2
```

Rule 6358

```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(2*b*x*HypergeometricPFQ[{1/2,
1/2}, {3/2, 3/2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[b, x]
```

Rule 6360

```
Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(2*b*x*HypergeometricPFQ[{1/2,
 1/2}, {3/2, 3/2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[b, x]
```

Rule 6425

```
Int[FresnelC[(b_.)*(x_)]/(x_), x_Symbol] :> Dist[(1 - I)/4, Int[Erf[(Sqrt[P
i]*(1 + I)*b*x)/2]/x, x], x] + Dist[(1 + I)/4, Int[Erf[(Sqrt[Pi]*(1 - I)*b*
x)/2]/x, x], x] /; FreeQ[b, x]
```

Rubi steps

$$\int \frac{C(bx)}{x}\,dx = \left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)}{x}\,dx + \left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)}{x}\,dx$$
$$= \frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{C(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]/x,x]
```

```
[Out] Integrate[FresnelC[b*x]/x, x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{fresnelc}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x, x)
```

maple [A] time = 0.04, size = 23, normalized size = 0.33

$$bx\,\operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{5}{4},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x,x)
```

```
[Out] b*x*hypergeom([1/4,1/4],[1/2,5/4,5/4],-1/16*x^4*Pi^2*b^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{fresnelc}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}\left(b\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x,x)
```

```
[Out] int(FresnelC(b*x)/x, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x,x)
```

```
[Out] Exception raised: AttributeError
```

3.119 $\int \frac{C(bx)}{x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}b\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{C(bx)}{x}$$

```
[Out] 1/2*b*Ci(1/2*b^2*Pi*x^2)-FresnelC(b*x)/x
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6427, 3376}

$$\frac{1}{2}b\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right)-\frac{\text{FresnelC}(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^2,x]
```

```
[Out] (b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)}{x^2}\,dx &= -\frac{C(bx)}{x}+b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx\\ &= \frac{1}{2}b\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{C(bx)}{x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.00

$$\frac{1}{2}b\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{C(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^2,x]
[Out] (b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}(bx)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^2,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^2,x, algorithm="giac")
[Out] integrate(fresnelc(b*x)/x^2, x)
```

maple [A] time = 0.01, size = 28, normalized size = 1.04

$$b\left(-\frac{\text{FresnelC}(bx)}{bx}+\frac{\text{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^2,x)
[Out] b*(-FresnelC(b*x)/b/x+1/2*Ci(1/2*b^2*Pi*x^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{FresnelC}(b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^2,x)
```

```
[Out] int(FresnelC(b*x)/x^2, x)
```

sympy [B] time = 1.06, size = 53, normalized size = 1.96

$$-\frac{\pi^2 b^5 x^4 \Gamma\left(\frac{5}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{5}{4} \\ \frac{3}{2}, 2, 2, \frac{9}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{128\Gamma\left(\frac{9}{4}\right)} + \frac{b \log\left(b^4 x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**2,x)
```

```
[Out] -pi**2*b**5*x**4*gamma(5/4)*hyper((1, 1, 5/4), (3/2, 2, 2, 9/4), -pi**2*b**
4*x**4/16)/(128*gamma(9/4)) + b*log(b**4*x**4)/4
```

3.120 $\int \frac{C(bx)}{x^3} \, dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\pi b^2 S(bx) - \frac{b\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{C(bx)}{2x^2}$$

```
[Out] -1/2*b*cos(1/2*b^2*Pi*x^2)/x-1/2*FresnelC(b*x)/x^2-1/2*b^2*Pi*FresnelS(b*x)
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6427, 3388, 3351}

$$-\frac{1}{2}\pi b^2 S(bx) - \frac{b\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{\text{FresnelC}(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^3,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(2*x) - FresnelC[b*x]/(2*x^2) - (b^2*Pi*FresnelS[b
*x])/2
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)}{x^3}\,dx &= -\frac{C(bx)}{2x^2} + \frac{1}{2}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x} - \frac{C(bx)}{2x^2} - \frac{1}{2}\left(b^3\pi\right)\int \sin\left(\frac{1}{2}b^2\pi x^2\right)dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x} - \frac{C(bx)}{2x^2} - \frac{1}{2}b^2\pi S(bx)\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$-\frac{1}{2}\pi b^2 S(bx) - \frac{b\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{C(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^3,x]
```

```
[Out] -1/2*(b*Cos[(b^2*Pi*x^2)/2])/x - FresnelC[b*x]/(2*x^2) - (b^2*Pi*FresnelS[b
*x])/2
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^3, x)
```

maple [A] time = 0.01, size = 43, normalized size = 0.98

$$b^{2}\left(-\frac{\mathrm{FresnelC}\left(bx\right)}{2b^{2}x^{2}}-\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{2bx}-\frac{\pi\,\mathrm{S}\left(bx\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^3,x)
```

```
[Out] b^2*(-1/2*FresnelC(b*x)/b^2/x^2-1/2/b/x*cos(1/2*b^2*Pi*x^2)-1/2*Pi*FresnelS
(b*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\left(bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{FresnelC}\left(b\,x\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^3,x)
```

```
[Out] int(FresnelC(b*x)/x^3, x)
```

sympy [A] time = 0.70, size = 51, normalized size = 1.16

$$\frac{b\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_{2}F_{3}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4},\frac{5}{4}\end{matrix}\middle|-\frac{\pi^{2}b^{4}x^{4}}{16}\right)}{16x\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**3,x)
```

```
[Out] b*gamma(-1/4)*gamma(1/4)*hyper((-1/4, 1/4), (1/2, 3/4, 5/4), -pi**2*b**4*x*
*4/16)/(16*x*gamma(3/4)*gamma(5/4))
```

3.121 $\int \frac{C(bx)}{x^4} dx$

Optimal. Leaf size=52

$$-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^2}-\frac{1}{12}\pi b^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{C(bx)}{3x^3}$$

```
[Out] -1/6*b*cos(1/2*b^2*Pi*x^2)/x^2-1/3*FresnelC(b*x)/x^3-1/12*b^3*Pi*Si(1/2*b^2
*Pi*x^2)
```

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3380, 3297, 3299}

$$-\frac{1}{12}\pi b^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^2}-\frac{\text{FresnelC}(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^4,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(6*x^2) - FresnelC[b*x]/(3*x^3) - (b^3*Pi*SinInteg
ral[(b^2*Pi*x^2)/2])/12
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m + 
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)}{x^4}\,dx &= -\frac{C(bx)}{3x^3}+\frac{1}{3}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx\\
&= -\frac{C(bx)}{3x^3}+\frac{1}{6}b\,\mathrm{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}-\frac{C(bx)}{3x^3}-\frac{1}{12}\left(b^3\pi\right)\mathrm{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx,x,x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}-\frac{C(bx)}{3x^3}-\frac{1}{12}b^3\pi\mathrm{Si}\left(\frac{1}{2}b^2\pi x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 1.00

$$-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^2}-\frac{1}{12}\pi b^3\mathrm{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{C(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^4,x]
```

```
[Out] -1/6*(b*Cos[(b^2*Pi*x^2)/2])/x^2 - FresnelC[b*x]/(3*x^3) - (b^3*Pi*SinInteg
ral[(b^2*Pi*x^2)/2])/12
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\left(bx\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^4,x, algorithm="fricas")
```

[Out] integral(fresnelc(b*x)/x^4, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^4,x, algorithm="giac")

[Out] integrate(fresnelc(b*x)/x^4, x)

maple [A] time = 0.00, size = 49, normalized size = 0.94

$$b^3\left(-\frac{\mathrm{FresnelC}\,(bx)}{3b^3x^3}-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{6b^2x^2}-\frac{\pi\,\mathrm{Si}\left(\frac{b^2\pi\,x^2}{2}\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)/x^4,x)

[Out] b^3*(-1/3*FresnelC(b*x)/b^3/x^3-1/6/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/12*Pi*Si(
1/2*b^2*Pi*x^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x^4,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x)/x^4, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{FresnelC}\,(b\,x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)/x^4,x)

[Out] int(FresnelC(b*x)/x^4, x)

sympy [A] time = 0.81, size = 42, normalized size = 0.81

$$-\frac{b\Gamma\left(\frac{1}{4}\right){}_{2}F_{3}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2}, \frac{1}{2}, \frac{5}{4} \end{matrix}\middle| -\frac{\pi^{2}b^{4}x^{4}}{16}\right)}{8x^{2}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**4,x)
```

```
[Out] -b*gamma(1/4)*hyper((-1/2, 1/4), (1/2, 1/2, 5/4), -pi**2*b**4*x**4/16)/(8*x
**2*gamma(5/4))
```

3.122 $\int \frac{C(bx)}{x^5}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{12}\pi^2b^4C(bx)-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{12x^3}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{12x}-\frac{C(bx)}{4x^4}$$

```
[Out] -1/12*b*cos(1/2*b^2*Pi*x^2)/x^3-1/12*b^4*Pi^2*FresnelC(b*x)-1/4*FresnelC(b*
x)/x^4+1/12*b^3*Pi*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3388, 3387, 3352}

$$-\frac{1}{12}\pi^2b^4\text{FresnelC}(bx)+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{12x}-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{12x^3}-\frac{\text{FresnelC}(bx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^5,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(12*x^3) - (b^4*Pi^2*FresnelC[b*x])/12 - FresnelC[
b*x]/(4*x^4) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(12*x)
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)}{x^5}\,dx &= -\frac{C(bx)}{4x^4}+\frac{1}{4}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}-\frac{C(bx)}{4x^4}-\frac{1}{12}\left(b^3\pi\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}-\frac{C(bx)}{4x^4}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x}-\frac{1}{12}\left(b^5\pi^2\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}-\frac{1}{12}b^4\pi^2C(bx)-\frac{C(bx)}{4x^4}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 1.00

$$-\frac{1}{12}\pi^2b^4C(bx)-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{12x^3}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{12x}-\frac{C(bx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^5,x]
```

```
[Out] -1/12*(b*Cos[(b^2*Pi*x^2)/2])/x^3 - (b^4*Pi^2*FresnelC[b*x])/12 - FresnelC[
b*x]/(4*x^4) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(12*x)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^5, x)
```

maple [A] time = 0.00, size = 64, normalized size = 0.93

$$b^4\left(-\frac{\text{FresnelC}(bx)}{4b^4x^4}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{12b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\,\text{FresnelC}(bx)\right)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^5,x)
```

```
[Out] b^4*(-1/4*FresnelC(b*x)/b^4/x^4-1/12/b^3/x^3*cos(1/2*b^2*Pi*x^2)-1/12*Pi*(-
sin(1/2*b^2*Pi*x^2)/b/x+Pi*FresnelC(b*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{FresnelC}(b\,x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^5,x)
```

```
[Out] int(FresnelC(b*x)/x^5, x)
```

sympy [A] time = 1.35, size = 110, normalized size = 1.59

$$\frac{\pi^2 b^4 C(bx)\,\Gamma\left(-\frac{3}{4}\right)}{64\Gamma\left(\frac{5}{4}\right)}-\frac{\pi b^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{3}{4}\right)}{64x\Gamma\left(\frac{5}{4}\right)}+\frac{b\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{3}{4}\right)}{64x^3\Gamma\left(\frac{5}{4}\right)}+\frac{3C(bx)\,\Gamma\left(-\frac{3}{4}\right)}{64x^4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**5,x)
```

```
[Out] pi**2*b**4*fresnelc(b*x)*gamma(-3/4)/(64*gamma(5/4)) - pi*b**3*sin(pi*b**2*
x**2/2)*gamma(-3/4)/(64*x*gamma(5/4)) + b*cos(pi*b**2*x**2/2)*gamma(-3/4)/(
64*x**3*gamma(5/4)) + 3*fresnelc(b*x)*gamma(-3/4)/(64*x**4*gamma(5/4))
```

3.123 $\int \frac{C(bx)}{x^6}\,dx$

Optimal. Leaf size=77

$$-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{20x^4}-\frac{1}{80}\pi^2b^5\mathrm{Ci}\left(\frac{1}{2}b^2\pi x^2\right)+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{40x^2}-\frac{C(bx)}{5x^5}$$

```
[Out] -1/80*b^5*Pi^2*Ci(1/2*b^2*Pi*x^2)-1/20*b*cos(1/2*b^2*Pi*x^2)/x^4-1/5*Fresne
lC(b*x)/x^5+1/40*b^3*Pi*sin(1/2*b^2*Pi*x^2)/x^2
```

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3380, 3297, 3302}

$$-\frac{1}{80}\pi^2b^5\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right)+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{40x^2}-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{20x^4}-\frac{\text{FresnelC}(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^6,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*CosIntegral[(b^2*Pi*x^2)/2])/
80 - FresnelC[b*x]/(5*x^5) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(40*x^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)}{x^6}\,dx &= -\frac{C(bx)}{5x^5}+\frac{1}{5}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx\\
&= -\frac{C(bx)}{5x^5}+\frac{1}{10}b\,\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx,x,x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{C(bx)}{5x^5}-\frac{1}{40}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{C(bx)}{5x^5}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}-\frac{1}{80}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx,x,x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{1}{80}b^5\pi^2\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{C(bx)}{5x^5}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 1.00

$$-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{20x^4}-\frac{1}{80}\pi^2b^5\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{40x^2}-\frac{C(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^6,x]
```

```
[Out] -1/20*(b*Cos[(b^2*Pi*x^2)/2])/x^4 - (b^5*Pi^2*CosIntegral[(b^2*Pi*x^2)/2])/
80 - FresnelC[b*x]/(5*x^5) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(40*x^2)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^6, x)
```

maple [A] time = 0.01, size = 71, normalized size = 0.92

$$b^5\left(-\frac{\text{FresnelC}\,(bx)}{5b^5x^5}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{20b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi\,\text{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^6,x)
```

```
[Out] b^5*(-1/5*FresnelC(b*x)/b^5/x^5-1/20/b^4/x^4*cos(1/2*b^2*Pi*x^2)-1/20*Pi*(-
1/2*sin(1/2*b^2*Pi*x^2)/b^2/x^2+1/4*Pi*Ci(1/2*b^2*Pi*x^2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^6,x, algorithm="maxima")
```

[Out] integrate(fresnelc(b*x)/x^6, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}\left(b\,x\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)/x^6,x)

[Out] int(FresnelC(b*x)/x^6, x)

sympy [A] time = 1.76, size = 65, normalized size = 0.84

$$\frac{\pi^4 b^9 x^4 \Gamma\left(\frac{9}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{9}{4} \\ 2, \frac{5}{2}, 3, \frac{13}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{6144\Gamma\left(\frac{13}{4}\right)} - \frac{\pi^2 b^5 \log\left(b^4 x^4\right)}{160} - \frac{b}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)/x**6,x)

[Out] pi**4*b**9*x**4*gamma(9/4)*hyper((1, 1, 9/4), (2, 5/2, 3, 13/4), -pi**2*b**4*x**4/16)/(6144*gamma(13/4)) - pi**2*b**5*log(b**4*x**4)/160 - b/(4*x**4)

3.124 $\int \frac{C(bx)}{x^7} \, dx$

Optimal. Leaf size=94

$$\frac{1}{90}\pi^3 b^6 S(bx) - \frac{b\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{30x^5} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{90x} + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{90x^3} - \frac{C(bx)}{6x^6}$$

```
[Out] -1/30*b*cos(1/2*b^2*Pi*x^2)/x^5+1/90*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)/x-1/6*Fre
snelC(b*x)/x^6+1/90*b^6*Pi^3*FresnelS(b*x)+1/90*b^3*Pi*sin(1/2*b^2*Pi*x^2)/
x^3
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3388, 3387, 3351}

$$\frac{1}{90}\pi^3 b^6 S(bx) + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{90x^3} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{90x} - \frac{b\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{30x^5} - \frac{\text{FresnelC}(bx)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^7,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(30*x^5) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(90*x) -
 FresnelC[b*x]/(6*x^6) + (b^6*Pi^3*FresnelS[b*x])/90 + (b^3*Pi*Sin[(b^2*Pi*
x^2)/2])/(90*x^3)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{C(bx)}{x^7}\,dx &= -\frac{C(bx)}{6x^6}+\frac{1}{6}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}-\frac{C(bx)}{6x^6}-\frac{1}{30}\left(b^3\pi\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}-\frac{C(bx)}{6x^6}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}-\frac{1}{90}\left(b^5\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x}-\frac{C(bx)}{6x^6}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}+\frac{1}{90}\left(b^7\pi^3\right)\int \sin\left(\frac{1}{2}b^2\pi x\right.\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x}-\frac{C(bx)}{6x^6}+\frac{1}{90}b^6\pi^3S(bx)+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 74, normalized size = 0.79

$$
\frac{1}{90}\left(\pi^3b^6S(bx)+\frac{b\left(\pi^2b^4x^4-3\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^5}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^3}-\frac{15C(bx)}{x^6}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^7,x]
```

```
[Out] ((b*(-3 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^5 - (15*FresnelC[b*x])/x^6 +
 b^6*Pi^3*FresnelS[b*x] + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/x^3)/90
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\mathrm{fresnelc}\,(bx)}{x^7},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^7,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^7, x)
```

maple [A] time = 0.00, size = 87, normalized size = 0.93

$$b^6\left(-\frac{\text{FresnelC}\,(bx)}{6b^6x^6}-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{30b^5x^5}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{3b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{bx}-\pi\,\mathrm{S}(bx)\right)}{3}\right)}{30}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^7,x)
```

```
[Out] b^6*(-1/6*FresnelC(b*x)/b^6/x^6-1/30/b^5/x^5*cos(1/2*b^2*Pi*x^2)-1/30*Pi*(-
1/3*sin(1/2*b^2*Pi*x^2)/b^3/x^3+1/3*Pi*(-1/b/x*cos(1/2*b^2*Pi*x^2)-Pi*Fresn
elS(b*x))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}\left(b\,x\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^7,x)
[Out] int(FresnelC(b*x)/x^7, x)
```

sympy [A] time = 1.42, size = 56, normalized size = 0.60

$$\frac{b\Gamma\left(-\frac{5}{4}\right)\Gamma\left(\frac{1}{4}\right){}_{2}F_{3}\left(\left.\begin{matrix}-\frac{5}{4},\frac{1}{4}\\-\frac{1}{4},\frac{1}{2},\frac{5}{4}\end{matrix}\right|-\frac{\pi^2b^4x^4}{16}\right)}{16x^5\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**7,x)
[Out] b*gamma(-5/4)*gamma(1/4)*hyper((-5/4, 1/4), (-1/4, 1/2, 5/4), -pi**2*b**4*x
**4/16)/(16*x**5*gamma(-1/4)*gamma(5/4))
```

3.125 $\int \frac{C(bx)}{x^8}\,dx$

Optimal. Leaf size=102

$$-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{42x^6}+\frac{1}{672}\pi^3b^7\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)+\frac{\pi^2b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{336x^2}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{168x^4}-\frac{C(bx)}{7x^7}$$

```
[Out] -1/42*b*cos(1/2*b^2*Pi*x^2)/x^6+1/336*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)/x^2-1/7*
FresnelC(b*x)/x^7+1/672*b^7*Pi^3*Si(1/2*b^2*Pi*x^2)+1/168*b^3*Pi*sin(1/2*b^
2*Pi*x^2)/x^4
```

Rubi [A] time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3380, 3297, 3299}

$$\frac{1}{672}\pi^3b^7\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{168x^4}+\frac{\pi^2b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{336x^2}-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{42x^6}-\frac{\text{FresnelC}(bx)}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^8,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(42*x^6) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(336*x^2
) - FresnelC[b*x]/(7*x^7) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(168*x^4) + (b^7*P
i^3*SinIntegral[(b^2*Pi*x^2)/2])/672
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)}{x^8}\,dx &= -\frac{C(bx)}{7x^7} + \frac{1}{7}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx \\
&= -\frac{C(bx)}{7x^7} + \frac{1}{14}b\,\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx, x, x^2\right) \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} - \frac{C(bx)}{7x^7} - \frac{1}{84}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx, x, x^2\right) \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} - \frac{C(bx)}{7x^7} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{1}{336}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx, x, x^2\right) \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2} - \frac{C(bx)}{7x^7} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} + \frac{1}{672}\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\sin\right. \\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2} - \frac{C(bx)}{7x^7} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} + \frac{1}{672}b^7\pi^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.15, size = 84, normalized size = 0.82

$$\frac{1}{672}\left(\pi^3 b^7\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) + \frac{2b\left(\pi^2 b^4 x^4 - 8\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^6} + \frac{4\pi b^3\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x^4} - \frac{96C(bx)}{x^7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^8,x]
```

```
[Out] ((2*b*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^6 - (96*FresnelC[b*x])/x^7
 + (4*b^3*Pi*Sin[(b^2*Pi*x^2)/2])/x^4 + b^7*Pi^3*SinIntegral[(b^2*Pi*x^2)/2
])/672
```

fricas [F] time = 0.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^8, x)
```

maple [A] time = 0.00, size = 93, normalized size = 0.91

$$b^7\left(-\frac{\text{FresnelC}\,(bx)}{7b^7x^7}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{42b^6x^6}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi\,\text{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{42}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^8,x)
```

```
[Out] b^7*(-1/7*FresnelC(b*x)/b^7/x^7-1/42/b^6/x^6*cos(1/2*b^2*Pi*x^2)-1/42*Pi*(-
1/4*sin(1/2*b^2*Pi*x^2)/b^4/x^4+1/4*Pi*(-1/2/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/
4*Pi*Si(1/2*b^2*Pi*x^2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{FresnelC}(b\,x)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^8,x)
```

```
[Out] int(FresnelC(b*x)/x^8, x)
```

sympy [A] time = 1.90, size = 44, normalized size = 0.43

$$-\frac{b\Gamma\left(\frac{1}{4}\right){}_2F_3\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2}, \frac{1}{2}, \frac{5}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{24x^6\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**8,x)
```

```
[Out] -b*gamma(1/4)*hyper((-3/2, 1/4), (-1/2, 1/2, 5/4), -pi**2*b**4*x**4/16)/(24
*x**6*gamma(5/4))
```

3.126 $\int \frac{C(bx)}{x^9}\,dx$

Optimal. Leaf size=119

$$\frac{1}{840}\pi^4 b^8 C(bx) - \frac{b\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{56x^7} - \frac{\pi^3 b^7 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{840x} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{840x^3} + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{280x^5} - \frac{C(bx)}{8x^8}$$

```
[Out] -1/56*b*cos(1/2*b^2*Pi*x^2)/x^7+1/840*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)/x^3+1/84
0*b^8*Pi^4*FresnelC(b*x)-1/8*FresnelC(b*x)/x^8+1/280*b^3*Pi*sin(1/2*b^2*Pi*
x^2)/x^5-1/840*b^7*Pi^3*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3388, 3387, 3352}

$$\frac{1}{840}\pi^4 b^8 \text{FresnelC}(bx) - \frac{\pi^3 b^7 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{840x} + \frac{\pi b^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{280x^5} + \frac{\pi^2 b^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{840x^3} - \frac{b\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{56x^7} - \frac{\text{FresnelC(}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^9,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(56*x^7) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(840*x^3
) + (b^8*Pi^4*FresnelC[b*x])/840 - FresnelC[b*x]/(8*x^8) + (b^3*Pi*Sin[(b^2
*Pi*x^2)/2])/(280*x^5) - (b^7*Pi^3*Sin[(b^2*Pi*x^2)/2])/(840*x)
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{C(bx)}{x^9}\,dx = -\frac{C(bx)}{8x^8} + \frac{1}{8}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} - \frac{C(bx)}{8x^8} - \frac{1}{56}\left(b^3\pi\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} - \frac{C(bx)}{8x^8} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{1}{280}\left(b^5\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} - \frac{C(bx)}{8x^8} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} + \frac{1}{840}\left(b^7\pi^3\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi\right.}{x^2}$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} - \frac{C(bx)}{8x^8} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x} + \frac{1}{84}$$
$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} + \frac{1}{840}b^8\pi^4C(bx) - \frac{C(bx)}{8x^8} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{b^7\pi^3\,\mathrm{si}}{}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 0.71

$$\frac{\left(\pi^4b^8x^8-105\right)C(bx)+bx\left(\pi^2b^4x^4-15\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)+\pi b^3x^3\left(3-\pi^2b^4x^4\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{840x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^9,x]
```

```
[Out] (b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*Fresn
elC[b*x] + b^3*Pi*x^3*(3 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(840*x^8)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^9,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^9, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^9, x)
```

maple [A] time = 0.00, size = 108, normalized size = 0.91

$$b^8\left(-\frac{\text{FresnelC}\,(bx)}{8b^8x^8}-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{56b^7x^7}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{5b^5x^5}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{3b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{bx}+\pi\,\text{FresnelC}(bx)\right)}{3}\right)}{5}\right)}{56}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^9,x)
```

```
[Out] b^8*(-1/8*FresnelC(b*x)/b^8/x^8-1/56/b^7/x^7*cos(1/2*b^2*Pi*x^2)-1/56*Pi*(-
1/5*sin(1/2*b^2*Pi*x^2)/b^5/x^5+1/5*Pi*(-1/3/b^3/x^3*cos(1/2*b^2*Pi*x^2)-1/
3*Pi*(-sin(1/2*b^2*Pi*x^2)/b/x+Pi*FresnelC(b*x)))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}\,(b\,x)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^9,x)
```

```
[Out] int(FresnelC(b*x)/x^9, x)
```

sympy [A] time = 2.91, size = 185, normalized size = 1.55

$$\frac{\pi^4 b^8 C\,(bx)\,\Gamma\left(-\frac{7}{4}\right)}{2560\Gamma\left(\frac{5}{4}\right)} - \frac{\pi^3 b^7 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x\Gamma\left(\frac{5}{4}\right)} + \frac{\pi^2 b^5 \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x^3\Gamma\left(\frac{5}{4}\right)} + \frac{3\pi b^3 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x^5\Gamma\left(\frac{5}{4}\right)} - \frac{3b \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma}{512x^7\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**9,x)
```

```
[Out] pi**4*b**8*fresnelc(b*x)*gamma(-7/4)/(2560*gamma(5/4)) - pi**3*b**7*sin(pi*
b**2*x**2/2)*gamma(-7/4)/(2560*x*gamma(5/4)) + pi**2*b**5*cos(pi*b**2*x**2/
2)*gamma(-7/4)/(2560*x**3*gamma(5/4)) + 3*pi*b**3*sin(pi*b**2*x**2/2)*gamma
(-7/4)/(2560*x**5*gamma(5/4)) - 3*b*cos(pi*b**2*x**2/2)*gamma(-7/4)/(512*x*
*7*gamma(5/4)) - 21*fresnelc(b*x)*gamma(-7/4)/(512*x**8*gamma(5/4))
```

3.127 $\int \frac{C(bx)}{x^{10}}\,dx$

Optimal. Leaf size=127

$$-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{72x^8}+\frac{\pi^4b^9\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}-\frac{\pi^3b^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3456x^2}+\frac{\pi^2b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{1728x^4}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{432x^6}-\frac{C(bx)}{9x^9}$$

```
[Out] 1/6912*b^9*Pi^4*Ci(1/2*b^2*Pi*x^2)-1/72*b*cos(1/2*b^2*Pi*x^2)/x^8+1/1728*b^
5*Pi^2*cos(1/2*b^2*Pi*x^2)/x^4-1/9*FresnelC(b*x)/x^9+1/432*b^3*Pi*sin(1/2*b
^2*Pi*x^2)/x^6-1/3456*b^7*Pi^3*sin(1/2*b^2*Pi*x^2)/x^2
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6427, 3380, 3297, 3302}

$$\frac{\pi^4b^9\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right)}{6912}-\frac{\pi^3b^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3456x^2}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{432x^6}+\frac{\pi^2b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{1728x^4}-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{72x^8}-\text{Fres}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^10,x]
```

```
[Out] -(b*Cos[(b^2*Pi*x^2)/2])/(72*x^8) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(1728*x^
4) + (b^9*Pi^4*CosIntegral[(b^2*Pi*x^2)/2])/6912 - FresnelC[b*x]/(9*x^9) +
(b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(432*x^6) - (b^7*Pi^3*Sin[(b^2*Pi*x^2)/2])/(34
56*x^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6427

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m +
1)*FresnelC[b*x])/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi*b^2*x^2)/2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{C(bx)}{x^{10}}\,dx = -\frac{C(bx)}{9x^9}+\frac{1}{9}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

$$= -\frac{C(bx)}{9x^9}+\frac{1}{18}b\,\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^5}\,dx,x,x^2\right)$$

$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}-\frac{C(bx)}{9x^9}-\frac{1}{144}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx,x,x^2\right)$$

$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}-\frac{C(bx)}{9x^9}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}-\frac{1}{864}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx,x,x^2\right)$$

$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}-\frac{C(bx)}{9x^9}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}+\frac{\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi\right.}{x^2}\right.}{3456}$$

$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}-\frac{C(bx)}{9x^9}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}-\frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2}+\frac{(b^9}{}$$

$$= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}+\frac{b^9\pi^4\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}-\frac{C(bx)}{9x^9}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}-\frac{b^7\pi}{}$$

Mathematica [A] time = 0.11, size = 96, normalized size = 0.76

$$\frac{\pi^4b^9\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)+\frac{4b\left(\pi^2b^4x^4-24\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^8}-\frac{2\pi b^3\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^6}-\frac{768C(bx)}{x^9}}{6912}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^10,x]
```

```
[Out] ((4*b*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^8 + b^9*Pi^4*CosIntegral[
(b^2*Pi*x^2)/2] - (768*FresnelC[b*x])/x^9 - (2*b^3*Pi*(-8 + b^4*Pi^2*x^4)*S
in[(b^2*Pi*x^2)/2])/x^6)/6912
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\left(bx\right)}{x^{10}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)/x^10, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\left(bx\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)/x^10, x)
```

maple [A] time = 0.01, size = 115, normalized size = 0.91

$$b^{9}\left(-\frac{\mathrm{FresnelC}\left(bx\right)}{9b^{9}x^{9}}-\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{72b^{8}x^{8}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{6b^{6}x^{6}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{4b^{4}x^{4}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{2b^{2}x^{2}}+\frac{\pi\,\mathrm{Ci}\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^10,x)
```

```
[Out] b^9*(-1/9*FresnelC(b*x)/b^9/x^9-1/72*cos(1/2*b^2*Pi*x^2)/b^8/x^8-1/72*Pi*(-
1/6*sin(1/2*b^2*Pi*x^2)/b^6/x^6+1/6*Pi*(-1/4/b^4/x^4*cos(1/2*b^2*Pi*x^2)-1/
4*Pi*(-1/2*sin(1/2*b^2*Pi*x^2)/b^2/x^2+1/4*Pi*Ci(1/2*b^2*Pi*x^2)))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\left(bx\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)/x^10, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}\left(b\,x\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^10,x)
```

```
[Out] int(FresnelC(b*x)/x^10, x)
```

sympy [**A**] time = 3.82, size = 76, normalized size = 0.60

$$-\frac{\pi^6 b^{13} x^4 \Gamma\left(\frac{13}{4}\right) {}_3F_4\left(\begin{matrix} 1,1,\frac{13}{4} \\ 2,\frac{7}{2},4,\frac{17}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{737280\Gamma\left(\frac{17}{4}\right)} + \frac{\pi^4 b^9 \log\left(b^4 x^4\right)}{13824} + \frac{\pi^2 b^5}{160 x^4} - \frac{b}{8 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**10,x)
```

```
[Out] -pi**6*b**13*x**4*gamma(13/4)*hyper((1, 1, 13/4), (2, 7/2, 4, 17/4), -pi**2
*b**4*x**4/16)/(737280*gamma(17/4)) + pi**4*b**9*log(b**4*x**4)/13824 + pi*
*2*b**5/(160*x**4) - b/(8*x**8)
```

3.128 $\int (c+dx)^3 C(a+bx)\,dx$

Optimal. Leaf size=298

$$-\frac{d^2(a+bx)^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2 b^4}-\frac{(bc-ad)^4C(a+bx)}{4b^4d}+\frac{3d(bc-ad)^2S(}{2\pi b^4}$$

```
[Out] -2*d^2*(-a*d+b*c)*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi^2-3/4*d^3*(b*x+a)*cos(1/2*Pi
*(b*x+a)^2)/b^4/Pi^2-1/4*(-a*d+b*c)^4*FresnelC(b*x+a)/b^4/d+3/4*d^3*Fresnel
C(b*x+a)/b^4/Pi^2+1/4*(d*x+c)^4*FresnelC(b*x+a)/d+3/2*d*(-a*d+b*c)^2*Fresne
lS(b*x+a)/b^4/Pi-(-a*d+b*c)^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi-3/2*d*(-a*d+b*c)
^2*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi-d^2*(-a*d+b*c)*(b*x+a)^2*sin(1/2*Pi
*(b*x+a)^2)/b^4/Pi-1/4*d^3*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi
```

Rubi [A] time = 0.37, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351, 3296, 2638, 3385}

$$-\frac{d^2(a+bx)^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2 b^4}-\frac{(bc-ad)^4\text{FresnelC}(a+bx)}{4b^4d}+\frac{3d(bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*FresnelC[a + b*x],x]
```

```
[Out] (-2*d^2*(b*c - a*d)*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*d^3*(a + b*x)*
Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2) - ((b*c - a*d)^4*FresnelC[a + b*x])/(
4*b^4*d) + (3*d^3*FresnelC[a + b*x])/(4*b^4*Pi^2) + ((c + d*x)^4*FresnelC[a
 + b*x])/(4*d) + (3*d*(b*c - a*d)^2*FresnelS[a + b*x])/(2*b^4*Pi) - ((b*c -
 a*d)^3*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - (3*d*(b*c - a*d)^2*(a + b*x)*Si
n[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) - (d^2*(b*c - a*d)*(a + b*x)^2*Sin[(Pi*(a
 + b*x)^2)/2])/(b^4*Pi) - (d^3*(a + b*x)^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*
Pi)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
```

```
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 C(a+bx)\,dx = \frac{(c+dx)^4 C(a+bx)}{4d} - \frac{b\int (c+dx)^4 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{4d}$$

$$= \frac{(c+dx)^4 C(a+bx)}{4d} - \frac{\mathrm{Subst}\left(\int \left(b^4c^4\left(1+\frac{ad\left(-4b^3c^3+6ab^2c^2d-4a^2bcd^2+a^3d^3\right)}{b^4c^4}\right)\cos\left(\frac{\pi x^2}{2}\right)+\cdots\right.\right.}{\cdots}$$

$$= \frac{(c+dx)^4 C(a+bx)}{4d} - \frac{d^3\,\mathrm{Subst}\left(\int x^4\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{4b^4} - \frac{\left(d^2(bc-ad)\right)\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{(bc-ad)^4 C(a+bx)}{4b^4d} + \frac{(c+dx)^4 C(a+bx)}{4d} - \frac{3d(bc-ad)^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\cdots\right.}{2b^4\pi}$$

$$= -\frac{3d^3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{(bc-ad)^4 C(a+bx)}{4b^4d} + \frac{(c+dx)^4 C(a+bx)}{4d} + \frac{3d(b\cdots}{\cdots}$$

$$= -\frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} - \frac{3d^3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{(bc-ad)^4 C(a+\cdots}{4b^4d}$$

Mathematica [A] time = 0.95, size = 424, normalized size = 1.42

$$\frac{\pi a^3d^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right) - 4\pi a^2bcd^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right) - \pi a^2bd^3x\sin\left(\frac{1}{2}\pi(a+bx)^2\right) + C(a+bx)\left(d^3\left(-\pi^2a^4+\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*FresnelC[a + b*x],x]
```

```
[Out] (-8*b*c*d^2*Cos[(Pi*(a + b*x)^2)/2] + 5*a*d^3*Cos[(Pi*(a + b*x)^2)/2] - 3*b
*d^3*x*Cos[(Pi*(a + b*x)^2)/2] + (4*b^3*c^3*Pi^2*(a + b*x) + 6*b^2*c^2*d*Pi
^2*(-a^2 + b^2*x^2) + 4*b*c*d^2*Pi^2*(a^3 + b^3*x^3) + d^3*(3 - a^4*Pi^2 +
```

b^4*Pi^2*x^4))*FresnelC[a + b*x] + 6*d*(b*c - a*d)^2*Pi*FresnelS[a + b*x] - 4*b^3*c^3*Pi*Sin[(Pi*(a + b*x)^2)/2] + 6*a*b^2*c^2*d*Pi*Sin[(Pi*(a + b*x)^2)/2] - 4*a^2*b*c*d^2*Pi*Sin[(Pi*(a + b*x)^2)/2] + a^3*d^3*Pi*Sin[(Pi*(a + b*x)^2)/2] - 6*b^3*c^2*d*Pi*x*Sin[(Pi*(a + b*x)^2)/2] + 4*a*b^2*c*d^2*Pi*x*Sin[(Pi*(a + b*x)^2)/2] - a^2*b*d^3*Pi*x*Sin[(Pi*(a + b*x)^2)/2] - 4*b^3*c*d^2*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2] + a*b^2*d^3*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2] - b^3*d^3*Pi*x^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(d^3x^3 + 3\,cd^2x^2 + 3\,c^2dx + c^3\right)\mathrm{fresnelc}\left(bx + a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*fresnelc(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3\mathrm{fresnelc}\left(bx + a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*fresnelc(b*x + a), x)
```

maple [A] time = 0.02, size = 397, normalized size = 1.33

$$\frac{\frac{\mathrm{FresnelC}(bx+a)((bx+a)d-ad+bc)^4}{4b^3d} - \frac{\frac{d^4(bx+a)^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{3d^4\left(-\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{\mathrm{FresnelC}(bx+a)}{\pi}\right)}{\pi} + \frac{\left(-4a\,d^4+4bc\,d^3\right)(bx+a)^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{2\left(-4a\,d^4 \cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*FresnelC(b*x+a),x)
```

```
[Out] 1/b*(1/4*FresnelC(b*x+a)*((b*x+a)*d-a*d+b*c)^4/b^3/d-1/4/b^3/d*(d^4/Pi*(b*x
+a)^3*sin(1/2*Pi*(b*x+a)^2)-3*d^4/Pi*(-1/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+1
/Pi*FresnelC(b*x+a))+(-4*a*d^4+4*b*c*d^3)/Pi*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2
)+2*(-4*a*d^4+4*b*c*d^3)/Pi^2*cos(1/2*Pi*(b*x+a)^2)+(6*a^2*d^4-12*a*b*c*d^3
+6*b^2*c^2*d^2)/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-(6*a^2*d^4-12*a*b*c*d^3+6*
b^2*c^2*d^2)/Pi*FresnelS(b*x+a)+(-4*a^3*d^4+12*a^2*b*c*d^3-12*a*b^2*c^2*d^2
```

```
+4*b^3*c^3*d)/Pi*sin(1/2*Pi*(b*x+a)^2)+a^4*d^4*FresnelC(b*x+a)-4*a^3*b*c*d^
3*FresnelC(b*x+a)+6*a^2*b^2*c^2*d^2*FresnelC(b*x+a)-4*a*b^3*c^3*d*FresnelC(
b*x+a)+b^4*c^4*FresnelC(b*x+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \operatorname{fresnelc}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnelc(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^3*fresnelc(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{FresnelC}(a+b\,x)\,(c+d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)*(c + d*x)^3,x)
[Out] int(FresnelC(a + b*x)*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3\, C(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*fresnelc(b*x+a),x)
[Out] Integral((c + d*x)**3*fresnelc(a + b*x), x)
```

3.129 $\int (c+dx)^2 C(a+bx)\,dx$

Optimal. Leaf size=194

$$-\frac{(bc-ad)^3C(a+bx)}{3b^3d}+\frac{d(bc-ad)S(a+bx)}{\pi b^3}-\frac{(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{d(a+bx)(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-$$

```
[Out] -2/3*d^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi^2-1/3*(-a*d+b*c)^3*FresnelC(b*x+a)/b^
3/d+1/3*(d*x+c)^3*FresnelC(b*x+a)/d+d*(-a*d+b*c)*FresnelS(b*x+a)/b^3/Pi-(-a
*d+b*c)^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-d*(-a*d+b*c)*(b*x+a)*sin(1/2*Pi*(b*x
+a)^2)/b^3/Pi-1/3*d^2*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi
```

Rubi [A] time = 0.20, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351, 3296, 2638}

$$-\frac{(bc-ad)^3\text{FresnelC}(a+bx)}{3b^3d}+\frac{d(bc-ad)S(a+bx)}{\pi b^3}-\frac{(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{d(a+bx)(bc-ad)\sin\left(\frac{1}{2}\pi(a\right.}{\pi b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelC[a + b*x],x]
```

```
[Out] (-2*d^2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2) - ((b*c - a*d)^3*FresnelC[a +
 b*x])/(3*b^3*d) + ((c + d*x)^3*FresnelC[a + b*x])/(3*d) + (d*(b*c - a*d)*F
resnelS[a + b*x])/(b^3*Pi) - ((b*c - a*d)^2*Sin[(Pi*(a + b*x)^2)/2])/(b^3*P
i) - (d*(b*c - a*d)*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (d^2*(a +
 b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2 C(a+bx)\,dx = \frac{(c+dx)^3 C(a+bx)}{3d} - \frac{b\int (c+dx)^3 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{3d}$$

$$= \frac{(c+dx)^3 C(a+bx)}{3d} - \frac{\text{Subst}\left(\int \left(b^3c^3\left(1-\frac{ad(3b^2c^2-3abcd+a^2d^2)}{b^3c^3}\right)\cos\left(\frac{\pi x^2}{2}\right)+3b^2c^2d\left(1+\right.\right.\right.}{}$$

$$= \frac{(c+dx)^3 C(a+bx)}{3d} - \frac{d^2\,\text{Subst}\left(\int x^3\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{3b^3} - \frac{(d(bc-ad))\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(bc-ad)^3 C(a+bx)}{3b^3d} + \frac{(c+dx)^3 C(a+bx)}{3d} - \frac{d(bc-ad)(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} -$$

$$= -\frac{(bc-ad)^3 C(a+bx)}{3b^3d} + \frac{(c+dx)^3 C(a+bx)}{3d} + \frac{d(bc-ad)S(a+bx)}{b^3\pi} - \frac{(bc-ad)^2\sin\left(\frac{1}{2}\right.}{b^3\pi}$$

$$= -\frac{2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2} - \frac{(bc-ad)^3 C(a+bx)}{3b^3d} + \frac{(c+dx)^3 C(a+bx)}{3d} + \frac{d(bc-ad)S(a}{b^3\pi}$$

Mathematica [A] time = 0.54, size = 237, normalized size = 1.22

$$\frac{-\pi a^2d^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi^2 C(a+bx)\left(a^3d^2-3a^2bcd+3ab^2c^2+b^3x\left(3c^2+3cdx+d^2x^2\right)\right)-3\pi b^2c^2\sin\left(\frac{1}{2}\pi(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*FresnelC[a + b*x],x]
```

```
[Out] (-2*d^2*Cos[(Pi*(a + b*x)^2)/2] + Pi^2*(3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2
 + b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*FresnelC[a + b*x] + 3*d*(b*c - a*d)*P
i*FresnelS[a + b*x] - 3*b^2*c^2*Pi*Sin[(Pi*(a + b*x)^2)/2] + 3*a*b*c*d*Pi*S
in[(Pi*(a + b*x)^2)/2] - a^2*d^2*Pi*Sin[(Pi*(a + b*x)^2)/2] - 3*b^2*c*d*Pi*
x*Sin[(Pi*(a + b*x)^2)/2] + a*b*d^2*Pi*x*Sin[(Pi*(a + b*x)^2)/2] - b^2*d^2*
Pi*x^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2x^2 + 2\,cdx + c^2\right)\text{fresnelc}\left(bx+a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*fresnelc(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \text{fresnelc}\,(bx + a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnelc(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^2*fresnelc(b*x + a), x)
```

maple [A] time = 0.02, size = 249, normalized size = 1.28

$$\frac{\frac{\mathrm{FresnelC}(bx+a)((bx+a)d-ad+bc)^3}{3b^2d} - \frac{\frac{d^3(bx+a)^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{2d^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2} + \frac{\left(-3a\,d^3+3bc\,d^2\right)(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{\left(-3a\,d^3+3bc\,d^2\right)\mathrm{S}(bx+a)}{\pi} + \frac{\left(3a^2d^3-6a \cdots\right.}{\cdots}}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*FresnelC(b*x+a),x)
[Out] 1/b*(1/3*FresnelC(b*x+a)*((b*x+a)*d-a*d+b*c)^3/b^2/d-1/3/b^2/d*(d^3/Pi*(b*x
+a)^2*sin(1/2*Pi*(b*x+a)^2)+2*d^3/Pi^2*cos(1/2*Pi*(b*x+a)^2)+(-3*a*d^3+3*b*
c*d^2)/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-(-3*a*d^3+3*b*c*d^2)/Pi*FresnelS(b*
x+a)+(3*a^2*d^3-6*a*b*c*d^2+3*b^2*c^2*d)/Pi*sin(1/2*Pi*(b*x+a)^2)-a^3*d^3*F
resnelC(b*x+a)+3*a^2*b*c*d^2*FresnelC(b*x+a)-3*a*b^2*c^2*d*FresnelC(b*x+a)+
b^3*c^3*FresnelC(b*x+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \text{fresnelc}\,(bx + a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnelc(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^2*fresnelc(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{FresnelC}\,(a + b\,x)\ (c + d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)*(c + d*x)^2,x)
```

```
[Out] int(FresnelC(a + b*x)*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 C(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnelc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*fresnelc(a + b*x), x)
```

3.130 $\int (c+dx)C(a+bx)\,dx$

Optimal. Leaf size=122

$$-\frac{(bc-ad)^2C(a+bx)}{2b^2d}-\frac{(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}+\frac{dS(a+bx)}{2\pi b^2}-\frac{d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{(c+dx)^2C(a+b}{2d}$$

```
[Out] -1/2*(-a*d+b*c)^2*FresnelC(b*x+a)/b^2/d+1/2*(d*x+c)^2*FresnelC(b*x+a)/d+1/2
*d*FresnelS(b*x+a)/b^2/Pi-(-a*d+b*c)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi-1/2*d*(b*
x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi
```

Rubi [A] time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351}

$$-\frac{(bc-ad)^2\text{FresnelC}(a+bx)}{2b^2d}-\frac{(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}+\frac{dS(a+bx)}{2\pi b^2}-\frac{d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{(c+dx)^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelC[a + b*x],x]
```

```
[Out] -((b*c - a*d)^2*FresnelC[a + b*x])/(2*b^2*d) + ((c + d*x)^2*FresnelC[a + b*
x])/(2*d) + (d*FresnelS[a + b*x])/(2*b^2*Pi) - ((b*c - a*d)*Sin[(Pi*(a + b*
x)^2)/2])/(b^2*Pi) - (d*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)C(a+bx)\,dx = \frac{(c+dx)^2C(a+bx)}{2d} - \frac{b\int (c+dx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{2d}$$

$$= \frac{(c+dx)^2C(a+bx)}{2d} - \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)\cos\left(\frac{\pi x^2}{2}\right)+2bcd\left(1-\frac{ad}{bc}\right)x\cos\left(\right.\right.\right.}{2b^2d}$$

$$= \frac{(c+dx)^2C(a+bx)}{2d} - \frac{d\,\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{2b^2} - \frac{(bc-ad)\,\text{Subst}\left(\int x\cos\right.}{b^2}$$

$$= -\frac{(bc-ad)^2C(a+bx)}{2b^2d} + \frac{(c+dx)^2C(a+bx)}{2d} - \frac{d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{(bc-ad)\,}{}$$

$$= -\frac{(bc-ad)^2C(a+bx)}{2b^2d} + \frac{(c+dx)^2C(a+bx)}{2d} + \frac{dS(a+bx)}{2b^2\pi} - \frac{(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right.}{b^2\pi}$$

Mathematica [A] time = 0.28, size = 74, normalized size = 0.61

$$\frac{-\pi(a+bx)C(a+bx)(ad-b(2c+dx))+\sin\left(\frac{1}{2}\pi(a+bx)^2\right)(ad-2bc-bdx)+dS(a+bx)}{2\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*FresnelC[a + b*x],x]
```

```
[Out] (-(Pi*(a + b*x)*(a*d - b*(2*c + d*x))*FresnelC[a + b*x]) + d*FresnelS[a + b
*x] + (-2*b*c + a*d - b*d*x)*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)\text{fresnelc}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*fresnelc(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\text{fresnelc}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*fresnelc(b*x + a), x)
```

maple [A] time = 0.02, size = 107, normalized size = 0.88

$$\frac{\frac{\text{FresnelC}(bx+a)\left(\frac{(bx+a)^2d}{2}-ad(bx+a)+bc(bx+a)\right)}{b}-\frac{\frac{d(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{d\,\mathrm{S}(bx+a)}{\pi}+\frac{(-2ad+2bc)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*FresnelC(b*x+a),x)
```

```
[Out] 1/b*(FresnelC(b*x+a)/b*(1/2*(b*x+a)^2*d-a*d*(b*x+a)+b*c*(b*x+a))-1/2/b*(d/P
i*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-d/Pi*FresnelS(b*x+a)+(-2*a*d+2*b*c)/Pi*sin(
1/2*Pi*(b*x+a)^2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\text{fresnelc}\,(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*fresnelc(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \text{FresnelC}\,(a+b\,x)\ (c+d\,x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)*(c + d*x),x)
```

```
[Out] int(FresnelC(a + b*x)*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\,C\,(a+bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a),x)
```

```
[Out] Integral((c + d*x)*fresnelc(a + b*x), x)
```

3.131 $\int C(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{(a+bx)C(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[Out] (b*x+a)*FresnelC(b*x+a)/b-sin(1/2*Pi*(b*x+a)^2)/b/Pi
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6419}

$$\frac{(a+bx)\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[a + b*x],x]
```

```
[Out] ((a + b*x)*FresnelC[a + b*x])/b - Sin[(Pi*(a + b*x)^2)/2]/(b*Pi)
```

Rule 6419

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelC[a +
 b*x])/b, x] - Simp[Sin[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int C(a+bx)\,dx=\frac{(a+bx)C(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

Mathematica [B] time = 0.04, size = 90, normalized size = 2.43

$$-\frac{\sin\left(\frac{\pi a^2}{2}\right)\cos\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}-\frac{\cos\left(\frac{\pi a^2}{2}\right)\sin\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xC(a+bx)+\frac{aC(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[a + b*x],x]
```

```
[Out] (a*FresnelC[a + b*x])/b + x*FresnelC[a + b*x] - (Cos[a*b*Pi*x + (b^2*Pi*x^2
)/2]*Sin[(a^2*Pi)/2])/(b*Pi) - (Cos[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnelc}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnelc}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a), x)
```

maple [A] time = 0.00, size = 34, normalized size = 0.92

$$\frac{\left(bx+a\right)\text{FresnelC}\left(bx+a\right)-\frac{\sin\left(\frac{\pi\left(bx+a\right)^{2}}{2}\right)}{\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*FresnelC(b*x+a)-sin(1/2*Pi*(b*x+a)^2)/Pi)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnelc}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{FresnelC}\,(a + b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x),x)
[Out] int(FresnelC(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int C\,(a + bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x)
[Out] Integral(fresnelc(a + b*x), x)
```

3.132 $\int \frac{C(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\frac{C(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\mathrm{FresnelC}(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{C(a+bx)}{c+dx} \, dx = \int \frac{C(a+bx)}{c+dx} \, dx$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{C(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[FresnelC[a + b*x]/(c + d*x), x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\left(bx+a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\left(bx + a\right)}{dx + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(fresnelc(b*x + a)/(d*x + c), x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx + a\right)}{dx + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)/(d*x+c),x)
[Out] int(FresnelC(b*x+a)/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\left(bx + a\right)}{dx + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate(fresnelc(b*x + a)/(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\text{FresnelC}\left(a + b\,x\right)}{c + d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)/(c + d*x),x)
[Out] int(FresnelC(a + b*x)/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C\left(a + bx\right)}{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(fresnelc(a + b*x)/(c + d*x), x)
```

3.133 $\int \frac{C(a+bx)}{(c+dx)^2} \, dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{C(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{C(a+bx)}{(c+dx)^2} \, dx = \int \frac{C(a+bx)}{(c+dx)^2} \, dx$$

Mathematica [A] time = 2.54, size = 0, normalized size = 0.00

$$\int \frac{C(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[FresnelC[a + b*x]/(c + d*x)^2, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(fresnelc(b*x + a)/(d*x + c)^2, x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)/(d*x+c)^2,x)
[Out] int(FresnelC(b*x+a)/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(fresnelc(b*x + a)/(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\text{FresnelC}\,(a+b\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)/(c + d*x)^2,x)
[Out] int(FresnelC(a + b*x)/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C\,(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(fresnelc(a + b*x)/(c + d*x)**2, x)
```

3.134 $\int x^3 C(a+bx)\,dx$

Optimal. Leaf size=227

$$-\frac{a^4C(a+bx)}{4b^4}+\frac{a^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}+\frac{3a^2S(a+bx)}{2\pi b^4}-\frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}+\frac{3C(a+bx)}{4\pi^2b^4}+\frac{a(a+bx)^2\sin}{\pi}$$

```
[Out] 2*a*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi^2-3/4*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi
^2-1/4*a^4*FresnelC(b*x+a)/b^4+3/4*FresnelC(b*x+a)/b^4/Pi^2+1/4*x^4*Fresnel
C(b*x+a)+3/2*a^2*FresnelS(b*x+a)/b^4/Pi+a^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi-3/
2*a^2*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi+a*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2
)/b^4/Pi-1/4*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi
```

Rubi [A] time = 0.19, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351, 3296, 2638, 3385}

$$-\frac{a^4\text{FresnelC}(a+bx)}{4b^4}+\frac{3a^2S(a+bx)}{2\pi b^4}+\frac{a^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}+\frac{3\text{FresnelC}(a+bx)}{4\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelC[a + b*x],x]
```

```
[Out] (2*a*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*(a + b*x)*Cos[(Pi*(a + b*x)^2
)/2])/(4*b^4*Pi^2) - (a^4*FresnelC[a + b*x])/(4*b^4) + (3*FresnelC[a + b*x]
)/(4*b^4*Pi^2) + (x^4*FresnelC[a + b*x])/4 + (3*a^2*FresnelS[a + b*x])/(2*b
^4*Pi) + (a^3*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - (3*a^2*(a + b*x)*Sin[(Pi*
(a + b*x)^2)/2])/(2*b^4*Pi) + (a*(a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(b^4*
Pi) - ((a + b*x)^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^3 C(a+bx)\,dx = \frac{1}{4}x^4 C(a+bx) - \frac{1}{4}b\int x^4\cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx$$
$$= \frac{1}{4}x^4 C(a+bx) - \frac{\text{Subst}\left(\int\left(a^4\cos\left(\frac{\pi x^2}{2}\right) - 4a^3x\cos\left(\frac{\pi x^2}{2}\right) + 6a^2x^2\cos\left(\frac{\pi x^2}{2}\right) - 4ax^3\cos\left(\frac{\pi x^2}{2}\right)\right.}{4b^4}$$
$$= \frac{1}{4}x^4 C(a+bx) - \frac{\text{Subst}\left(\int x^4\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{4b^4} + \frac{a\,\text{Subst}\left(\int x^3\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right.}{b^4}$$
$$= -\frac{a^4C(a+bx)}{4b^4} + \frac{1}{4}x^4C(a+bx) - \frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right.}{4b^4\pi}$$
$$= -\frac{3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{a^4C(a+bx)}{4b^4} + \frac{1}{4}x^4C(a+bx) + \frac{3a^2S(a+bx)}{2b^4\pi} + \frac{a^3\sin\left(\frac{1}{2}\pi(\right.}{b^4\pi}$$
$$= \frac{2a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} - \frac{3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{a^4C(a+bx)}{4b^4} + \frac{3C(a+bx)}{4b^4\pi^2} + \frac{1}{4}x^4C(a$$

Mathematica [A] time = 0.37, size = 166, normalized size = 0.73

$$\frac{\left(-\pi^2a^4+\pi^2b^4x^4+3\right)C(a+bx)+\pi a^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+6\pi a^2S(a+bx)-\pi a^2bx\sin\left(\frac{1}{2}\pi(a+bx)^2\right)-\pi b^3x^3\sin}{4\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[a + b*x],x]
```

```
[Out] (5*a*Cos[(Pi*(a + b*x)^2)/2] - 3*b*x*Cos[(Pi*(a + b*x)^2)/2] + (3 - a^4*Pi^
2 + b^4*Pi^2*x^4)*FresnelC[a + b*x] + 6*a^2*Pi*FresnelS[a + b*x] + a^3*Pi*S
in[(Pi*(a + b*x)^2)/2] - a^2*b*Pi*x*Sin[(Pi*(a + b*x)^2)/2] + a*b^2*Pi*x^2*
Sin[(Pi*(a + b*x)^2)/2] - b^3*Pi*x^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\text{fresnelc}\,(bx+a)\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnelc(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \mathrm{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnelc(b*x + a), x)
```

maple [A] time = 0.02, size = 187, normalized size = 0.82

$$\frac{\frac{\mathrm{FresnelC}(bx+a)b^4x^4}{4} - \frac{(bx+a)^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{4\pi} + \frac{-\frac{3(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{4\pi}+\frac{3\,\mathrm{FresnelC}(bx+a)}{4\pi}}{\pi} + \frac{a(bx+a)^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{2a\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2} - 3a^2(}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x+a),x)
```

```
[Out] 1/b^4*(1/4*FresnelC(b*x+a)*b^4*x^4-1/4/Pi*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)+3
/4/Pi*(-1/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+1/Pi*FresnelC(b*x+a))+a/Pi*(b*x+
a)^2*sin(1/2*Pi*(b*x+a)^2)+2*a/Pi^2*cos(1/2*Pi*(b*x+a)^2)-3/2*a^2/Pi*(b*x+a
)*sin(1/2*Pi*(b*x+a)^2)+3/2*a^2/Pi*FresnelS(b*x+a)+a^3/Pi*sin(1/2*Pi*(b*x+a
)^2)-1/4*a^4*FresnelC(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \mathrm{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnelc(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\mathrm{FresnelC}\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(a + b*x),x)
```

```
[Out] int(x^3*FresnelC(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 C(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnelc(b*x+a),x)
```

```
[Out] Integral(x**3*fresnelc(a + b*x), x)
```

3.135 $\int x^2 C(a+bx)\,dx$

Optimal. Leaf size=148

$$\frac{a^3 C(a+bx)}{3b^3}-\frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{aS(a+bx)}{\pi b^3}+\frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3}-\frac{2\,\mathrm{c}}{}$$

```
[Out] -2/3*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi^2+1/3*a^3*FresnelC(b*x+a)/b^3+1/3*x^3*Fre
snelC(b*x+a)-a*FresnelS(b*x+a)/b^3/Pi-a^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi+a*(b
*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-1/3*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)/b^3/
Pi
```

Rubi [A] time = 0.12, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351, 3296, 2638}

$$\frac{a^3\text{FresnelC}(a+bx)}{3b^3}-\frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{aS(a+bx)}{\pi b^3}+\frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx\right.}{3\pi b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[a + b*x],x]
```

```
[Out] (-2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2) + (a^3*FresnelC[a + b*x])/(3*b^3)
 + (x^3*FresnelC[a + b*x])/3 - (a*FresnelS[a + b*x])/(b^3*Pi) - (a^2*Sin[(P
i*(a + b*x)^2)/2])/(b^3*Pi) + (a*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi
) - ((a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2 C(a+bx)\,dx = \frac{1}{3}x^3 C(a+bx) - \frac{1}{3}b\int x^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx$$
$$= \frac{1}{3}x^3 C(a+bx) - \frac{\text{Subst}\left(\int\left(-a^3\cos\left(\frac{\pi x^2}{2}\right)+3a^2x\cos\left(\frac{\pi x^2}{2}\right)-3ax^2\cos\left(\frac{\pi x^2}{2}\right)+x^3\cos\left(\frac{\pi x^2}{2}\right)\right.\right.}{3b^3}$$
$$= \frac{1}{3}x^3 C(a+bx) - \frac{\text{Subst}\left(\int x^3\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{3b^3} + \frac{a\,\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+b\right.}{b^3}$$
$$= \frac{a^3 C(a+bx)}{3b^3} + \frac{1}{3}x^3 C(a+bx) + \frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{\text{Subst}\left(\int x\cos\left(\frac{\pi x}{2}\right)dx, x, (\right.}{6b^3}$$
$$= \frac{a^3 C(a+bx)}{3b^3} + \frac{1}{3}x^3 C(a+bx) - \frac{aS(a+bx)}{b^3\pi} - \frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+\right.}{b^3\pi}$$
$$= -\frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2} + \frac{a^3 C(a+bx)}{3b^3} + \frac{1}{3}x^3 C(a+bx) - \frac{aS(a+bx)}{b^3\pi} - \frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}$$

Mathematica [A] time = 0.33, size = 116, normalized size = 0.78

$$-\frac{-\pi^2\left(a^3+b^3x^3\right)C(a+bx)+\pi a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi b^2x^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+3\pi aS(a+bx)-\pi abx\sin\left(\frac{1}{2}\right.}{3\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[a + b*x],x]
```

```
[Out] -1/3*(2*Cos[(Pi*(a + b*x)^2)/2] - Pi^2*(a^3 + b^3*x^3)*FresnelC[a + b*x] +
3*a*Pi*FresnelS[a + b*x] + a^2*Pi*Sin[(Pi*(a + b*x)^2)/2] - a*b*Pi*x*Sin[(P
i*(a + b*x)^2)/2] + b^2*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi^2)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\text{fresnelc}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnelc(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{fresnelc}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnelc(b*x + a), x)
```

maple [A] time = 0.02, size = 122, normalized size = 0.82

$$\frac{\frac{b^3x^3\,\mathrm{FresnelC}(bx+a)}{3} - \frac{(bx+a)^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{3\pi} - \frac{2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{3\pi^2} + \frac{a(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{a\,\mathrm{S}(bx+a)}{\pi} - \frac{a^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{a^3\,\mathrm{FresnelC}(bx+a)}{3}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x+a),x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*FresnelC(b*x+a)-1/3/Pi*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)-2
/3/Pi^2*cos(1/2*Pi*(b*x+a)^2)+a/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-a/Pi*Fresn
elS(b*x+a)-a^2/Pi*sin(1/2*Pi*(b*x+a)^2)+1/3*a^3*FresnelC(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{fresnelc}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnelc(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{FresnelC}\,(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(a + b*x),x)
```

```
[Out] int(x^2*FresnelC(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 C\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(b*x+a),x)
```

```
[Out] Integral(x**2*fresnelc(a + b*x), x)
```

3.136 $\int xC(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{a^2C(a+bx)}{2b^2}+\frac{S(a+bx)}{2\pi b^2}+\frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}-\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{1}{2}x^2C(a+bx)$$

```
[Out] -1/2*a^2*FresnelC(b*x+a)/b^2+1/2*x^2*FresnelC(b*x+a)+1/2*FresnelS(b*x+a)/b^
2/Pi+a*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi-1/2*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/P
i
```

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6429, 3434, 3352, 3380, 2637, 3386, 3351}

$$-\frac{a^2\text{FresnelC}(a+bx)}{2b^2}+\frac{S(a+bx)}{2\pi b^2}+\frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}-\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{1}{2}x^2\text{FresnelC}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[a + b*x],x]
```

```
[Out] -(a^2*FresnelC[a + b*x])/(2*b^2) + (x^2*FresnelC[a + b*x])/2 + FresnelS[a +
 b*x]/(2*b^2*Pi) + (a*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi) - ((a + b*x)*Sin[(P
i*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6429

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[((c + d*x)^(m + 1)*FresnelC[a + b*x])/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi*(a + b*x)^2)/2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int xC(a+bx)\,dx = \frac{1}{2}x^2C(a+bx) - \frac{1}{2}b\int x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx$$

$$= \frac{1}{2}x^2C(a+bx) - \frac{\text{Subst}\left(\int\left(a^2\cos\left(\frac{\pi x^2}{2}\right) - 2ax\cos\left(\frac{\pi x^2}{2}\right) + x^2\cos\left(\frac{\pi x^2}{2}\right)\right)dx, x, a+bx\right)}{2b^2}$$

$$= \frac{1}{2}x^2C(a+bx) - \frac{\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{2b^2} + \frac{a\,\text{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)dx, x, a+bx\right)}{b^2}$$

$$= -\frac{a^2C(a+bx)}{2b^2} + \frac{1}{2}x^2C(a+bx) - \frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} + \frac{a\,\text{Subst}\left(\int\cos\left(\frac{\pi x}{2}\right)dx, x, (a+\right.}{2b^2}$$

$$= -\frac{a^2C(a+bx)}{2b^2} + \frac{1}{2}x^2C(a+bx) + \frac{S(a+bx)}{2b^2\pi} + \frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} - \frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx\right.}{2b^2\pi}$$

Mathematica [A] time = 0.18, size = 59, normalized size = 0.62

$$\frac{\left(\pi b^2x^2-\pi a^2\right)C(a+bx)+S(a+bx)+(a-bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelC[a + b*x],x]
```

```
[Out] ((-(a^2*Pi) + b^2*Pi*x^2)*FresnelC[a + b*x] + FresnelS[a + b*x] + (a - b*x)
*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x\mathrm{fresnelc}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*fresnelc(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{fresnelc}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*fresnelc(b*x + a), x)
```

maple [A] time = 0.02, size = 79, normalized size = 0.83

$$\frac{\mathrm{FresnelC}\left(bx+a\right)\left(\frac{(bx+a)^2}{2}-a\left(bx+a\right)\right)-\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}+\frac{\mathrm{S}(bx+a)}{2\pi}+\frac{a\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x+a),x)
```

```
[Out] 1/b^2*(FresnelC(b*x+a)*(1/2*(b*x+a)^2-a*(b*x+a))-1/2/Pi*(b*x+a)*sin(1/2*Pi*
(b*x+a)^2)+1/2/Pi*FresnelS(b*x+a)+a/Pi*sin(1/2*Pi*(b*x+a)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{fresnelc}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x+a),x, algorithm="maxima")
[Out] integrate(x*fresnelc(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{FresnelC}\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(a + b*x),x)
[Out] int(x*FresnelC(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xC\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x+a),x)
[Out] Integral(x*fresnelc(a + b*x), x)
```

3.137 $\int C(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{(a+bx)C(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

```
[Out] (b*x+a)*FresnelC(b*x+a)/b-sin(1/2*Pi*(b*x+a)^2)/b/Pi
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6419}

$$\frac{(a+bx)\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[a + b*x],x]
```

```
[Out] ((a + b*x)*FresnelC[a + b*x])/b - Sin[(Pi*(a + b*x)^2)/2]/(b*Pi)
```

Rule 6419

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelC[a +
 b*x])/b, x] - Simp[Sin[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int C(a+bx)\,dx=\frac{(a+bx)C(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

Mathematica [B] time = 0.04, size = 90, normalized size = 2.43

$$-\frac{\sin\left(\frac{\pi a^2}{2}\right)\cos\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}-\frac{\cos\left(\frac{\pi a^2}{2}\right)\sin\left(\pi abx+\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xC(a+bx)+\frac{aC(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[a + b*x],x]
```

```
[Out] (a*FresnelC[a + b*x])/b + x*FresnelC[a + b*x] - (Cos[a*b*Pi*x + (b^2*Pi*x^2
)/2]*Sin[(a^2*Pi)/2])/(b*Pi) - (Cos[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnelc}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnelc}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x + a), x)
```

maple [A] time = 0.00, size = 34, normalized size = 0.92

$$\frac{(bx+a)\,\text{FresnelC}\left(bx+a\right)-\frac{\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*FresnelC(b*x+a)-sin(1/2*Pi*(b*x+a)^2)/Pi)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnelc}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \text{FresnelC}\,(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x),x)
```

```
[Out] int(FresnelC(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int C\,(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x)
```

```
[Out] Integral(fresnelc(a + b*x), x)
```

3.138 $\int \frac{C(a+bx)}{x} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{C(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)/x,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[a + b*x]/x,x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{C(a+bx)}{x} \, dx = \int \frac{C(a+bx)}{x} \, dx$$

Mathematica [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{C(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]/x,x]
```

```
[Out] Integrate[FresnelC[a + b*x]/x, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x,x, algorithm="giac")
[Out] integrate(fresnelc(b*x + a)/x, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)/x,x)
[Out] int(FresnelC(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(fresnelc(b*x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\text{FresnelC}\,(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)/x,x)
[Out] int(FresnelC(a + b*x)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C\,(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x,x)
```

```
[Out] Integral(fresnelc(a + b*x)/x, x)
```

3.139 $\int \frac{C(a+bx)}{x^2} dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{C(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)/x^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]/x^2, x]
```

Rubi steps

$$\int \frac{C(a+bx)}{x^2}\,dx = \int \frac{C(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 1.57, size = 0, normalized size = 0.00

$$\int \frac{C(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]/x^2,x]
```

```
[Out] Integrate[FresnelC[a + b*x]/x^2, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}(bx+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(fresnelc(b*x + a)/x^2, x)
```

maple [A] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)/x^2,x)
[Out] int(FresnelC(b*x+a)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x^2,x, algorithm="maxima")
[Out] integrate(fresnelc(b*x + a)/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\text{FresnelC}\,(a+b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)/x^2,x)
[Out] int(FresnelC(a + b*x)/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C\,(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x**2,x)
```

```
[Out] Integral(fresnelc(a + b*x)/x**2, x)
```

3.140 $\int x^7 C(bx)^2\,dx$

Optimal. Leaf size=253

$$-\frac{105C(bx)^2}{8\pi^4b^8}-\frac{105x^2}{16\pi^4b^6}-\frac{x^7C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b}+\frac{7x^6}{48\pi^2b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{16\pi^2b^2}-\frac{10\sin\left(\pi b^2x^2\right)}{\pi^5b^8}+\frac{105xC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^4b^7}$$

```
[Out] -105/16*x^2/b^6/Pi^4+7/48*x^6/b^2/Pi^2+55/16*x^2*cos(b^2*Pi*x^2)/b^6/Pi^4-1
/16*x^6*cos(b^2*Pi*x^2)/b^2/Pi^2+105/4*x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/
b^7/Pi^4-7/4*x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^3/Pi^2-105/8*FresnelC(
b*x)^2/b^8/Pi^4+1/8*x^8*FresnelC(b*x)^2+35/4*x^3*FresnelC(b*x)*sin(1/2*b^2*
Pi*x^2)/b^5/Pi^3-1/4*x^7*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi-10*sin(b^2*
Pi*x^2)/b^8/Pi^5+5/8*x^4*sin(b^2*Pi*x^2)/b^4/Pi^3
```

Rubi [A] time = 0.42, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6431, 6455, 6463, 6441, 30, 3380, 2634, 3379, 3296, 2637, 3309}

$$-\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b}+\frac{35x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^3b^5}-\frac{7x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2b^3}+\frac{105x\text{Fresne}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelC[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) + (105*x*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(4*b^7*Pi^4) - (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*
x])/(4*b^3*Pi^2) - (105*FresnelC[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelC[b*x]^
2)/8 + (35*x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^5*Pi^3) - (x^7*Fresn
elC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi) - (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) +
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\int x^7 C(bx)^2\,dx = \frac{1}{8}x^8C(bx)^2 - \frac{1}{4}b\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$

$$= \frac{1}{8}x^8C(bx)^2 - \frac{x^7C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{\int x^7\sin\left(b^2\pi x^2\right)\,dx}{8\pi} + \frac{7\int x^6C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b\pi}$$

$$= -\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^3\pi^2} + \frac{1}{8}x^8C(bx)^2 - \frac{x^7C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^3\pi^2}$$

$$= -\frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^3\pi^2} + \frac{1}{8}x^8C(bx)^2 + \frac{35x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^5\pi^3} - \frac{x^7C(bx)}{}$$

$$= -\frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^3\pi^2} + \frac{1}{8}x^8C(bx)^2 + \frac{35x^3C}{}$$

$$= \frac{7x^6}{48b^2\pi^2} + \frac{41x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}$$

$$= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} + \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^7\pi^4} - \frac{7x^5\,\mathrm{co}}{}$$

$$= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} + \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^7\pi^4} - \frac{7x^5\,\mathrm{co}}{}$$

Mathematica [A] time = 0.03, size = 253, normalized size = 1.00

$$-\frac{105C(bx)^2}{8\pi^4b^8}-\frac{105x^2}{16\pi^4b^6}-\frac{x^7C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b}+\frac{7x^6}{48\pi^2b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{16\pi^2b^2}-\frac{10\sin\left(\pi b^2x^2\right)}{\pi^5b^8}+\frac{105xC(bx)\cos\left(\frac{1}{2}\pi b^2\right.}{4\pi^4b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelC[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) + (105*x*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(4*b^7*Pi^4) - (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*
x])/(4*b^3*Pi^2) - (105*FresnelC[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelC[b*x]^
2)/8 + (35*x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^5*Pi^3) - (x^7*Fresn
elC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi) - (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) +
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(x^7\text{fresnelc}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^7*fresnelc(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\text{fresnelc}\,(bx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnelc(b*x)^2, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^7\,\text{FresnelC}\,(bx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x)^2,x)
```

```
[Out] int(x^7*FresnelC(b*x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \text{fresnelc}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnelc(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7\,\text{FresnelC}\,(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x)^2,x)
```

```
[Out] int(x^7*FresnelC(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 C^2\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**7*fresnelc(b*x)**2, x)
```

3.141 $\int x^6 C(bx)^2\,dx$

Optimal. Leaf size=239

$$-\frac{531C\left(\sqrt{2}\,bx\right)}{56\sqrt{2}\,\pi^4b^7}-\frac{48x}{7\pi^4b^6}-\frac{2x^6C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{6x^5}{35\pi^2b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{14\pi^2b^2}+\frac{96C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}+\frac{21x\cos\left(\pi\right.}{8\pi^4b^{6}}$$

```
[Out] -48/7*x/b^6/Pi^4+6/35*x^5/b^2/Pi^2+21/8*x*cos(b^2*Pi*x^2)/b^6/Pi^4-1/14*x^5
*cos(b^2*Pi*x^2)/b^2/Pi^2+96/7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^7/Pi^4-1
2/7*x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^3/Pi^2+1/7*x^7*FresnelC(b*x)^2+
48/7*x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-2/7*x^6*FresnelC(b*x)*s
in(1/2*b^2*Pi*x^2)/b/Pi+17/28*x^3*sin(b^2*Pi*x^2)/b^4/Pi^3-531/112*FresnelC
(b*x*2^(1/2))/b^7/Pi^4*2^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6431, 6455, 6463, 6461, 3358, 3352, 3385, 3392, 30, 3386}

$$-\frac{2x^6\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{48x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}-\frac{12x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\text{96Fresn}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelC[b*x]^2,x]
```

```
[Out] (-48*x)/(7*b^6*Pi^4) + (6*x^5)/(35*b^2*Pi^2) + (21*x*Cos[b^2*Pi*x^2])/(8*b^
6*Pi^4) - (x^5*Cos[b^2*Pi*x^2])/(14*b^2*Pi^2) + (96*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(7*b^7*Pi^4) - (12*x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*b^
3*Pi^2) + (x^7*FresnelC[b*x]^2)/7 - (531*FresnelC[Sqrt[2]*b*x])/(56*Sqrt[2]
*b^7*Pi^4) + (48*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) - (2*x
^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b*Pi) + (17*x^3*Sin[b^2*Pi*x^2])/(
28*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
```

```
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\int x^6 C(bx)^2\,dx = \frac{1}{7}x^7C(bx)^2 - \frac{1}{7}(2b)\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$

$$= \frac{1}{7}x^7C(bx)^2 - \frac{2x^6C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{\int x^6\sin\left(b^2\pi x^2\right)\,dx}{7\pi} + \frac{12\int x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{7b\pi}$$

$$= -\frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^3\pi^2} + \frac{1}{7}x^7C(bx)^2 - \frac{2x^6C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{48\int x}{}$$

$$= -\frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^3\pi^2} + \frac{1}{7}x^7C(bx)^2 + \frac{48x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \frac{2x^6C}{}$$

$$= \frac{6x^5}{35b^2\pi^2} + \frac{111x\cos\left(b^2\pi x^2\right)}{56b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^7\pi^4} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2}$$

$$= \frac{6x^5}{35b^2\pi^2} + \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^7\pi^4} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C}{7b^3\pi^2}$$

$$= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} + \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^7\pi^4} - \frac{12x^4\cos}{}$$

$$= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} + \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^7\pi^4} - \frac{12x^4\cos}{}$$

Mathematica [A] time = 0.27, size = 170, normalized size = 0.71

$$\frac{80\pi^4b^7x^7C(bx)^2 - 160C(bx)\left(\pi b^2x^2\left(\pi^2b^4x^4-24\right)\sin\left(\frac{1}{2}\pi b^2x^2\right) + 6\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)\right) + 2bx\left(2\left(24\pi\right.\right.}{560\pi^4b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelC[b*x]^2,x]
```

```
[Out] (80*b^7*Pi^4*x^7*FresnelC[b*x]^2 - 2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 160
*FresnelC[b*x]*(6*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24
 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*((735 - 20*b^4*Pi^2*x^4)*Cos[
b^2*Pi*x^2] + 2*(-960 + 24*b^4*Pi^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/
(560*b^7*Pi^4)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(x^6\text{fresnelc}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^6*fresnelc(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6\text{fresnelc}\,(bx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnelc(b*x)^2, x)
```

maple [A] time = 0.06, size = 324, normalized size = 1.36

$$\frac{\frac{b^7x^7\,\text{FresnelC}(bx)^2}{7}-2\,\text{FresnelC}\,(bx)\left(\frac{b^6x^6\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{7\pi}-\frac{6\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi^2}}{\pi}\right)}{7\pi}\right)+\frac{\frac{6}{35}\pi^2b^5x^5-\frac{48}{7}bx}{\pi^4}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x)^2,x)
```

```
[Out] 1/b^7*(1/7*b^7*x^7*FresnelC(b*x)^2-2*FresnelC(b*x)*(1/7/Pi*b^6*x^6*sin(1/2*
b^2*Pi*x^2)-6/7/Pi*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*si
n(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2))))+6/7/Pi^4*(1/5*Pi^2*b^5*x^5-
```

```
8*b*x)+6/7/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2
*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/
2)))+1/7/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b
^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*
2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \text{fresnelc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnelc(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6 \operatorname{FresnelC}(b\,x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x)^2,x)
```

```
[Out] int(x^6*FresnelC(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 C^2(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**6*fresnelc(b*x)**2, x)
```

3.142 $\int x^5 C(bx)^2\, dx$

Optimal. Leaf size=265

$$-\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}+\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}-\frac{5C(bx)S(bx)}{2\pi^3b^6}-\frac{x^5C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}+\frac{5x^4}{24\pi^2b^2}-\frac{x^4}{}$$

```
[Out] 5/24*x^4/b^2/Pi^2+11/6*cos(b^2*Pi*x^2)/b^6/Pi^4-1/12*x^4*cos(b^2*Pi*x^2)/b^
2/Pi^2-5/3*x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^3/Pi^2+1/6*x^6*FresnelC(
b*x)^2-5/2*FresnelC(b*x)*FresnelS(b*x)/b^6/Pi^3-5/8*I*x^2*HypergeometricPFQ
([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^4/Pi^3+5/8*I*x^2*HypergeometricPFQ([1
, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^4/Pi^3+5*x*FresnelC(b*x)*sin(1/2*b^2*Pi*x
^2)/b^5/Pi^3-1/3*x^5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+7/12*x^2*sin(b^
2*Pi*x^2)/b^4/Pi^3
```

Rubi [A] time = 0.29, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6431, 6455, 6463, 6447, 3379, 2638, 3380, 3309, 30, 3296}

$$-\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}+\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}-\frac{5\text{FresnelC}(bx)S(bx)}{2\pi^3b^6}-\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x\right.}{3\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelC[b*x]^2,x]
```

```
[Out] (5*x^4)/(24*b^2*Pi^2) + (11*Cos[b^2*Pi*x^2])/(6*b^6*Pi^4) - (x^4*Cos[b^2*Pi
*x^2])/(12*b^2*Pi^2) - (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(3*b^3*Pi^
2) + (x^6*FresnelC[b*x]^2)/6 - (5*FresnelC[b*x]*FresnelS[b*x])/(2*b^6*Pi^3)
 - (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(
b^4*Pi^3) + (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi
*x^2])/(b^4*Pi^3) + (5*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^5*Pi^3) - (x
^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b*Pi) + (7*x^2*Sin[b^2*Pi*x^2])/(1
2*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
```

```
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$
\begin{aligned}
\int x^5 C(bx)^2\,dx &= \frac{1}{6}x^6 C(bx)^2 - \frac{1}{3}b\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx \\
&= \frac{1}{6}x^6C(bx)^2 - \frac{x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{\int x^5\sin\left(b^2\pi x^2\right)\,dx}{6\pi} + \frac{5\int x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{3b\pi} \\
&= -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6C(bx)^2 - \frac{x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{5\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,d}{b^3\pi^2} \\
&= -\frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6C(bx)^2 + \frac{5xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^5\pi^3} - \frac{x^5C(bx)\,\mathrm{si}}{3} \\
&= -\frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6C(bx)^2 - \frac{5C(bx)S(bx)}{2b^6\pi^3} - \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\right.}{8b^4\pi^3} \\
&= \frac{5x^4}{24b^2\pi^2} + \frac{17\cos\left(b^2\pi x^2\right)}{12b^6\pi^4} - \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6C(bx)^2 - \frac{5C(bx)S}{2b^6\pi} \\
&= \frac{5x^4}{24b^2\pi^2} + \frac{11\cos\left(b^2\pi x^2\right)}{6b^6\pi^4} - \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6C(bx)^2 - \frac{5C(bx)S}{2b^6\pi}
\end{aligned}
$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int x^5C(bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^5*FresnelC[b*x]^2,x]
```

```
[Out] Integrate[x^5*FresnelC[b*x]^2, x]
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(x^{5}\text{fresnelc}\left(bx\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)^2,x, algorithm="fricas")
[Out] integral(x^5*fresnelc(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5}\text{fresnelc}\left(bx\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)^2,x, algorithm="giac")
[Out] integrate(x^5*fresnelc(b*x)^2, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x^{5}\,\text{FresnelC}\left(bx\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x)^2,x)
[Out] int(x^5*FresnelC(b*x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5}\text{fresnelc}\left(bx\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^5*fresnelc(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{5}\,\text{FresnelC}\left(b\,x\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x)^2,x)
```

```
[Out] int(x^5*FresnelC(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 C^2(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**5*fresnelc(b*x)**2, x)
```

3.143 $\int x^4 C(bx)^2\,dx$

Optimal. Leaf size=177

$$-\frac{43S\left(\sqrt{2}\,bx\right)}{20\sqrt{2}\,\pi^3 b^5}-\frac{2x^4C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}+\frac{4x^3}{15\pi^2b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{10\pi^2b^2}+\frac{16C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}+\frac{11x\sin\left(\pi b^2x^2\right)}{20\pi^3b^4}-\frac{8x}{}$$

```
[Out] 4/15*x^3/b^2/Pi^2-1/10*x^3*cos(b^2*Pi*x^2)/b^2/Pi^2-8/5*x^2*cos(1/2*b^2*Pi*
x^2)*FresnelC(b*x)/b^3/Pi^2+1/5*x^5*FresnelC(b*x)^2+16/5*FresnelC(b*x)*sin(
1/2*b^2*Pi*x^2)/b^5/Pi^3-2/5*x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+11/
20*x*sin(b^2*Pi*x^2)/b^4/Pi^3-43/40*FresnelS(b*x*2^(1/2))/b^5/Pi^3*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6431, 6455, 6463, 6453, 3351, 3392, 30, 3386, 3385}

$$-\frac{2x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}+\frac{16\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}-\frac{8x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}-\frac{43S\left(\sqrt{2}\,bx\right.}{20\sqrt{2}\,\pi^3b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelC[b*x]^2,x]
```

```
[Out] (4*x^3)/(15*b^2*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(10*b^2*Pi^2) - (8*x^2*Cos[(b
^2*Pi*x^2)/2]*FresnelC[b*x])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x]^2)/5 - (43*F
resnelS[Sqrt[2]*b*x])/(20*Sqrt[2]*b^5*Pi^3) + (16*FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(5*b^5*Pi^3) - (2*x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b*Pi)
 + (11*x*Sin[b^2*Pi*x^2])/(20*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
```

```
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\int x^4 C(bx)^2\,dx = \frac{1}{5}x^5C(bx)^2 - \frac{1}{5}(2b)\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$

$$= \frac{1}{5}x^5C(bx)^2 - \frac{2x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{\int x^4\sin\left(b^2\pi x^2\right)\,dx}{5\pi} + \frac{8\int x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{5b\pi}$$

$$= -\frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5b^3\pi^2} + \frac{1}{5}x^5C(bx)^2 - \frac{2x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{16\int x\,\mathrm{c}$$

$$= -\frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5b^3\pi^2} + \frac{1}{5}x^5C(bx)^2 + \frac{16C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} - \frac{2x^4C(bx}{}$$

$$= \frac{4x^3}{15b^2\pi^2} - \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5b^3\pi^2} + \frac{1}{5}x^5C(bx)^2 - \frac{3S\left(\sqrt{2}\,bx\right)}{20\sqrt{2}\,b^5\pi^3} - \frac{4\sqrt{2}\,S\left(\sqrt{2}\right.}{5b^5\pi^3}$$

$$= \frac{4x^3}{15b^2\pi^2} - \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5b^3\pi^2} + \frac{1}{5}x^5C(bx)^2 - \frac{3S\left(\sqrt{2}\,bx\right)}{20\sqrt{2}\,b^5\pi^3} - \frac{\sqrt{2}\,S\left(\sqrt{2}\,b\right.}{b^5\pi^3}$$

Mathematica [A] time = 0.16, size = 137, normalized size = 0.77

$$\frac{24\pi^3b^5x^5C(bx)^2 + 32\pi b^3x^3 + 66bx\sin\left(\pi b^2x^2\right) - 48C(bx)\left(4\pi b^2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right) + \left(\pi^2b^4x^4 - 8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\right)}{120\pi^3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelC[b*x]^2,x]
```

```
[Out] (32*b^3*Pi*x^3 - 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 24*b^5*Pi^3*x^5*FresnelC[b
*x]^2 - 129*Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 48*FresnelC[b*x]*(4*b^2*Pi*x^2*
Cos[(b^2*Pi*x^2)/2] + (-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 66*b*x*Sin
[b^2*Pi*x^2])/(120*b^5*Pi^3)
```

fricas [F] time = 0.38, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^4\mathrm{fresnelc}\left(bx\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*fresnelc(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \text{fresnelc}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnelc(b*x)^2, x)
```

maple [A] time = 0.05, size = 209, normalized size = 1.18

$$\frac{\frac{b^5x^5\,\text{FresnelC}(bx)^2}{5} - 2\,\text{FresnelC}\,(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{5\pi} - \frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi} + \frac{2\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi^2}\right)}{5\pi}\right) + \frac{4b^3x^3}{15\pi^2} + \frac{\frac{2bx\sin\left(b^2\pi\,x^2\right)}{5\pi} - \frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{5\pi}}{\pi^2} + }{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x)^2,x)
```

```
[Out] 1/b^5*(1/5*b^5*x^5*FresnelC(b*x)^2-2*FresnelC(b*x)*(1/5/Pi*b^4*x^4*sin(1/2*
b^2*Pi*x^2)-4/5/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi
*x^2)))+4/15/Pi^2*b^3*x^3+4/5/Pi^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/
2)*FresnelS(b*x*2^(1/2)))+1/5/Pi^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*
(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)
*FresnelS(b*x*2^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \text{fresnelc}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4*fresnelc(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\text{FresnelC}\,(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x)^2,x)
```

```
[Out] int(x^4*FresnelC(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 C^2 (bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**4*fresnelc(b*x)**2, x)
```

3.144 $\int x^3 C(bx)^2\,dx$

Optimal. Leaf size=140

$$\frac{3C(bx)^2}{4\pi^2 b^4}-\frac{x^3 C(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi b}+\frac{3x^2}{8\pi^2 b^2}-\frac{x^2\cos\left(\pi b^2 x^2\right)}{8\pi^2 b^2}+\frac{\sin\left(\pi b^2 x^2\right)}{2\pi^3 b^4}-\frac{3xC(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi^2 b^3}+\frac{1}{4}x^4 C(bx)^2$$

```
[Out] 3/8*x^2/b^2/Pi^2-1/8*x^2*cos(b^2*Pi*x^2)/b^2/Pi^2-3/2*x*cos(1/2*b^2*Pi*x^2)
*FresnelC(b*x)/b^3/Pi^2+3/4*FresnelC(b*x)^2/b^4/Pi^2+1/4*x^4*FresnelC(b*x)^
2-1/2*x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+1/2*sin(b^2*Pi*x^2)/b^4/Pi
^3
```

Rubi [A] time = 0.16, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6431, 6455, 6463, 6441, 30, 3380, 2634, 3379, 3296, 2637}

$$-\frac{x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi b}-\frac{3x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{2\pi^2 b^3}+\frac{3\text{FresnelC}(bx)^2}{4\pi^2 b^4}+\frac{3x^2}{8\pi^2 b^2}+\frac{\sin\left(\pi b^2 x^2\right)}{2\pi^3 b^4}-\frac{x^2\cos}{8\pi}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelC[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) - (3*x*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(2*b^3*Pi^2) + (3*FresnelC[b*x]^2)/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x]^2)/4 - (x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b*Pi)
+ Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
```

```
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\int x^3 C(bx)^2\,dx = \frac{1}{4}x^4C(bx)^2 - \frac{1}{2}b\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$
$$= \frac{1}{4}x^4C(bx)^2 - \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{\int x^3\sin\left(b^2\pi x^2\right)\,dx}{4\pi} + \frac{3\int x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b\pi}$$
$$= -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2b^3\pi^2} + \frac{1}{4}x^4C(bx)^2 - \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{3\int\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{2b^3\pi^2} + \ldots$$
$$= -\frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} - \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2b^3\pi^2} + \frac{1}{4}x^4C(bx)^2 - \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{3\,\mathrm{Subst}(\int \ldots}{2b \ldots}$$
$$= \frac{3x^2}{8b^2\pi^2} - \frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} - \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2b^3\pi^2} + \frac{3C(bx)^2}{4b^4\pi^2} + \frac{1}{4}x^4C(bx)^2 - \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2 \ldots\right.}{2b\pi}$$

Mathematica [A] time = 0.01, size = 140, normalized size = 1.00

$$\frac{3C(bx)^2}{4\pi^2b^4} - \frac{x^3C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b} + \frac{3x^2}{8\pi^2b^2} - \frac{x^2\cos\left(\pi b^2x^2\right)}{8\pi^2b^2} + \frac{\sin\left(\pi b^2x^2\right)}{2\pi^3b^4} - \frac{3xC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3} + \frac{1}{4}x^4C(bx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) - (3*x*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(2*b^3*Pi^2) + (3*FresnelC[b*x]^2)/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x]^2)/4 - (x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b*Pi)
+ Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^3\mathrm{fresnelc}\,(bx)^2\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnelc(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \text{fresnelc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnelc(b*x)^2, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x^3 \,\text{FresnelC}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x)^2,x)
```

```
[Out] int(x^3*FresnelC(b*x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \text{fresnelc}(bx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnelc(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \,\text{FresnelC}(b\,x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x)^2,x)
```

```
[Out] int(x^3*FresnelC(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 C^2(bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**3*fresnelc(b*x)**2, x)
```

3.145 $\int x^2 C(bx)^2\,dx$

Optimal. Leaf size=124

$$\frac{5C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,\pi^2b^3}-\frac{2x^2C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}-\frac{x\cos\left(\pi b^2x^2\right)}{6\pi^2b^2}+\frac{2x}{3\pi^2b^2}-\frac{4C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{1}{3}x^3C(bx)^2$$

```
[Out] 2/3*x/b^2/Pi^2-1/6*x*cos(b^2*Pi*x^2)/b^2/Pi^2-4/3*cos(1/2*b^2*Pi*x^2)*Fresn
elC(b*x)/b^3/Pi^2+1/3*x^3*FresnelC(b*x)^2-2/3*x^2*FresnelC(b*x)*sin(1/2*b^2
*Pi*x^2)/b/Pi+5/12*FresnelC(b*x*2^(1/2))/b^3/Pi^2*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6431, 6455, 6461, 3358, 3352, 3385}

$$-\frac{2x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}-\frac{4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{5\text{FresnelC}\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,\pi^2b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{6\pi^2b^2}+\frac{2x}{3\pi^2b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[b*x]^2,x]
```

```
[Out] (2*x)/(3*b^2*Pi^2) - (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (4*Cos[(b^2*Pi*x^2)
/2]*FresnelC[b*x])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x]^2)/3 + (5*FresnelC[Sqr
t[2]*b*x])/(6*Sqrt[2]*b^3*Pi^2) - (2*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])
/(3*b*Pi)
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
```

```
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int x^2 C(bx)^2\,dx = \frac{1}{3}x^3C(bx)^2 - \frac{1}{3}(2b)\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$

$$= \frac{1}{3}x^3C(bx)^2 - \frac{2x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{\int x^2\sin\left(b^2\pi x^2\right)\,dx}{3\pi} + \frac{4\int xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{3b\pi}$$

$$= -\frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{3}x^3C(bx)^2 - \frac{2x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{\int\cos\left(b^2\pi x\right.}{6b^2\pi^2}$$

$$= -\frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{3}x^3C(bx)^2 + \frac{C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2} - \frac{2x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} +$$

$$= \frac{2x}{3b^2\pi^2} - \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{3}x^3C(bx)^2 + \frac{C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2} - \frac{2x^2C(bx)\sin\left(\frac{1}{2}b\right.}{3b\pi}$$

$$= \frac{2x}{3b^2\pi^2} - \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{3}x^3C(bx)^2 + \frac{C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2} + \frac{\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{3b^3\pi^2} - $$

Mathematica [A] time = 0.09, size = 100, normalized size = 0.81

$$\frac{4\pi^2b^3x^3C(bx)^2-8C(bx)\left(\pi b^2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)+2\cos\left(\frac{1}{2}\pi b^2x^2\right)\right)-2bx\left(\cos\left(\pi b^2x^2\right)-4\right)+5\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{12\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[b*x]^2,x]
```

```
[Out] (-2*b*x*(-4 + Cos[b^2*Pi*x^2]) + 4*b^3*Pi^2*x^3*FresnelC[b*x]^2 + 5*Sqrt[2]
*FresnelC[Sqrt[2]*b*x] - 8*FresnelC[b*x]*(2*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^
2*Sin[(b^2*Pi*x^2)/2]))/(12*b^3*Pi^2)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\text{fresnelc}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnelc(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{fresnelc}\,(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnelc(b*x)^2, x)
```

maple [A] time = 0.05, size = 122, normalized size = 0.98

$$\frac{\frac{b^3x^3\,\text{FresnelC}(bx)^2}{3}-2\,\text{FresnelC}\,(bx)\left(\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}\right)+\frac{2bx}{3\pi^2}+\frac{\sqrt{2}\;\text{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2}+\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi}+\frac{\sqrt{2}\;\text{Fresne}}{4\pi}}{3\pi}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x)^2,x)
```

```
[Out] 1/b^3*(1/3*b^3*x^3*FresnelC(b*x)^2-2*FresnelC(b*x)*(1/3/Pi*b^2*x^2*sin(1/2*
b^2*Pi*x^2)+2/3/Pi^2*cos(1/2*b^2*Pi*x^2))+2/3/Pi^2*b*x+1/3/Pi^2*2^(1/2)*Fre
```

```
snelC(b*x*2^(1/2))+1/3/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fresn
elC(b*x*2^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{fresnelc}\left(bx\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnelc(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\, \mathrm{FresnelC}\left(b\, x\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x)^2,x)
```

```
[Out] int(x^2*FresnelC(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 C^2\left(bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**2*fresnelc(b*x)**2, x)
```

3.146 $\int xC(bx)^2\,dx$

Optimal. Leaf size=144

$$\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi}+\frac{C(bx)S(bx)}{2\pi b^2}-\frac{xC(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}-\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^2}+\frac{1}{2}x$$

```
[Out] -1/4*cos(b^2*Pi*x^2)/b^2/Pi^2+1/2*x^2*FresnelC(b*x)^2+1/2*FresnelC(b*x)*Fre
snelS(b*x)/b^2/Pi+1/8*I*x^2*HypergeometricPFQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi
*x^2)/Pi-1/8*I*x^2*HypergeometricPFQ([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/Pi-x
*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi
```

Rubi [A] time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6431, 6455, 6447, 3379, 2638}

$$\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi}+\frac{\text{FresnelC}(bx)S(bx)}{2\pi b^2}-\frac{x\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}-$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[b*x]^2,x]
```

```
[Out] -Cos[b^2*Pi*x^2]/(4*b^2*Pi^2) + (x^2*FresnelC[b*x]^2)/2 + (FresnelC[b*x]*Fr
esnelS[b*x])/(2*b^2*Pi) + ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-
I/2)*b^2*Pi*x^2])/Pi - ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)
*b^2*Pi*x^2])/Pi - (x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
```

```
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$
\begin{aligned}
\int xC(bx)^2\,dx &= \frac{1}{2}x^2C(bx)^2 - b\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx \\
&= \frac{1}{2}x^2C(bx)^2 - \frac{xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + \frac{\int x\sin\left(b^2\pi x^2\right)\,dx}{2\pi} + \frac{\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\
&= \frac{1}{2}x^2C(bx)^2 + \frac{C(bx)S(bx)}{2b^2\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{xC(bx)
\end{aligned}
$$

$$
= -\frac{\cos\left(b^2\pi x^2\right)}{4b^2\pi^2} + \frac{1}{2}x^2C(bx)^2 + \frac{C(bx)S(bx)}{2b^2\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\right.}{8\pi}
$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int xC(bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*FresnelC[b*x]^2,x]
[Out] Integrate[x*FresnelC[b*x]^2, x]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(x\text{fresnelc}\,(bx)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)^2,x, algorithm="fricas")
[Out] integral(x*fresnelc(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnelc}\,(bx)^2\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)^2,x, algorithm="giac")
[Out] integrate(x*fresnelc(b*x)^2, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x\,\text{FresnelC}\,(bx)^2\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x)^2,x)
[Out] int(x*FresnelC(b*x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnelc}\,(bx)^2\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)^2,x, algorithm="maxima")
[Out] integrate(x*fresnelc(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\text{FresnelC}\,(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x)^2,x)
```

```
[Out] int(x*FresnelC(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xC^2\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x*fresnelc(b*x)**2, x)
```

3.147 $\int C(bx)^2\,dx$

Optimal. Leaf size=54

$$-\frac{2C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xC(bx)^2+\frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,\pi b}$$

```
[Out] x*FresnelC(b*x)^2-2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+1/2*FresnelS(b*x
*2^(1/2))/b/Pi*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6421, 12, 6453, 3351}

$$-\frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+x\text{FresnelC}(bx)^2+\frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]^2,x]
```

```
[Out] x*FresnelC[b*x]^2 + FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi) - (2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6421

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelC[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
```

```
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int C(bx)^2\,dx &= xC(bx)^2 - 2\int bx\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx\\ &= xC(bx)^2 - (2b)\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx\\ &= xC(bx)^2 - \frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + \frac{\int\sin\left(b^2\pi x^2\right)\,dx}{\pi}\\ &= xC(bx)^2 + \frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b\pi} - \frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$-\frac{2C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b} + xC(bx)^2 + \frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,\pi b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]^2,x]
```

```
[Out] x*FresnelC[b*x]^2 + FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi) - (2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\mathrm{fresnelc}\,(bx)^2\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{fresnelc}\,(bx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2, x)
```

maple [A] time = 0.00, size = 49, normalized size = 0.91

$$\frac{bx\,\mathrm{FresnelC}\,(bx)^2-\frac{2\,\mathrm{FresnelC}(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi}+\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{2\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2,x)
```

```
[Out] 1/b*(b*x*FresnelC(b*x)^2-2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/Pi+1/2/Pi*2^(1
/2)*FresnelS(b*x*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{fresnelc}\,(bx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{FresnelC}\,(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2,x)
```

```
[Out] int(FresnelC(b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int C^2\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2,x)
```

```
[Out] Integral(fresnelc(b*x)**2, x)
```

3.148 $\int \frac{C(bx)^2}{x} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{C(bx)^2}{x}, x\right)$$

```
[Out] Unintegrable(FresnelC(b*x)^2/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x,x]
[Out] Defer[Int][FresnelC[b*x]^2/x, x]
```

Rubi steps

$$\int \frac{C(bx)^2}{x} \, dx = \int \frac{C(bx)^2}{x} \, dx$$

Mathematica [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{C(bx)^2}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x,x]
[Out] Integrate[FresnelC[b*x]^2/x, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x,x, algorithm="giac")
[Out] integrate(fresnelc(b*x)^2/x, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x,x)
[Out] int(FresnelC(b*x)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x,x, algorithm="maxima")
[Out] integrate(fresnelc(b*x)^2/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\text{FresnelC}\,(b\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x,x)
[Out] int(FresnelC(b*x)^2/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x, x)
```

3.149 $\int \frac{C(bx)^2}{x^2} \, dx$

Optimal. Leaf size=38

$$2b\text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{C(bx)^2}{x}$$

```
[Out] -FresnelC(b*x)^2/x+2*b*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^2,x]
```

```
[Out] -(FresnelC[b*x]^2/x) + 2*b*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x
, x]
```

Rubi steps

$$\int \frac{C(bx)^2}{x^2} \, dx = -\frac{C(bx)^2}{x} + (2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x} \, dx$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{C(bx)^2}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^2,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^2, x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^2, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^2,x)
```

```
[Out] int(FresnelC(b*x)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\text{FresnelC}\,(b\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^2,x)
```

```
[Out] int(FresnelC(b*x)^2/x^2, x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2\left(bx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**2,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**2, x)
```

3.150 $\int \frac{C(bx)^2}{x^3}\,dx$

Optimal. Leaf size=39

$$b\text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)-\frac{C(bx)^2}{2x^2}$$

```
[Out] -1/2*FresnelC(b*x)^2/x^2+b*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x
^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^3,x]
```

```
[Out] -FresnelC[b*x]^2/(2*x^2) + b*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/x^2, x]
```

Rubi steps

$$\int \frac{C(bx)^2}{x^3}\,dx=-\frac{C(bx)^2}{2x^2}+b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx$$

Mathematica [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{C(bx)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^3,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^3, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^3,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)^2/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^3,x, algorithm="giac")
[Out] integrate(fresnelc(b*x)^2/x^3, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^3,x)
[Out] int(FresnelC(b*x)^2/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^3,x, algorithm="maxima")
[Out] integrate(fresnelc(b*x)^2/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\text{FresnelC}\,(b\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^3,x)
```

```
[Out] int(FresnelC(b*x)^2/x^3, x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**3,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**3, x)
```

3.151 $\int \frac{C(bx)^2}{x^4}\,dx$

Optimal. Leaf size=120

$$-\frac{1}{3}\pi b^3\text{Int}\left(\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{\pi b^3 S\left(\sqrt{2}\,bx\right)}{3\sqrt{2}}-\frac{bC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^2}-\frac{b^2\cos\left(\pi b^2x^2\right)}{6x}-\frac{b^2}{6x}-\frac{C(bx)^2}{3x^3}$$

```
[Out] -1/6*b^2/x-1/6*b^2*cos(b^2*Pi*x^2)/x-1/3*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/x^2-1/3*FresnelC(b*x)^2/x^3-1/6*b^3*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)-1/3*
b^3*Pi*Unintegrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^4,x]
```

```
[Out] -b^2/(6*x) - (b^2*Cos[b^2*Pi*x^2])/(6*x) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^2) - FresnelC[b*x]^2/(3*x^3) - (b^3*Pi*FresnelS[Sqrt[2]*b*x])/(3
*Sqrt[2]) - (b^3*Pi*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/3
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)^2}{x^4}\,dx &= -\frac{C(bx)^2}{3x^3}+\frac{1}{3}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^3}\,dx\\ &= -\frac{b^2}{6x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^2}-\frac{C(bx)^2}{3x^3}+\frac{1}{6}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{3}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x\right.}{x}\\ &= -\frac{b^2}{6x}-\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^2}-\frac{C(bx)^2}{3x^3}-\frac{1}{3}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx-\\ &= -\frac{b^2}{6x}-\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^2}-\frac{C(bx)^2}{3x^3}-\frac{b^3\pi S\left(\sqrt{2}\,bx\right)}{3\sqrt{2}}-\frac{1}{3}\left(b^3\pi\right)\int \frac{C(bx)\,\text{si}}{}\end{aligned}$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{C(bx)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^4,x]
[Out] Integrate[FresnelC[b*x]^2/x^4, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\,(bx)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^4,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)^2/x^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\,(bx)^2}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^4,x, algorithm="giac")
[Out] integrate(fresnelc(b*x)^2/x^4, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{FresnelC}\,(bx)^2}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^4,x)
[Out] int(FresnelC(b*x)^2/x^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\,(bx)^2}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}\,(b\,x)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^4,x)
```

```
[Out] int(FresnelC(b*x)^2/x^4, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**4,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**4, x)
```

3.152 $\int \frac{C(bx)^2}{x^5}\,dx$

Optimal. Leaf size=127

$$-\frac{1}{12}\pi^2b^4C(bx)^2-\frac{bC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3}-\frac{b^2}{24x^2}-\frac{b^2\cos\left(\pi b^2x^2\right)}{24x^2}-\frac{1}{12}\pi b^4\text{Si}\left(b^2\pi x^2\right)+\frac{\pi b^3C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x}-\frac{C(bx}{4x^4}$$

```
[Out] -1/24*b^2/x^2-1/24*b^2*cos(b^2*Pi*x^2)/x^2-1/6*b*cos(1/2*b^2*Pi*x^2)*Fresne
lC(b*x)/x^3-1/12*b^4*Pi^2*FresnelC(b*x)^2-1/4*FresnelC(b*x)^2/x^4-1/12*b^4*
Pi*Si(b^2*Pi*x^2)+1/6*b^3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6431, 6457, 6465, 6441, 30, 3375, 3380, 3297, 3299}

$$\frac{\pi b^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x}-\frac{b\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3}-\frac{1}{12}\pi^2b^4\text{FresnelC}(bx)^2-\frac{1}{12}\pi b^4\text{Si}\left(b^2\pi x^2\right)-\frac{b^2}{24x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]^2/x^5,x]
```

```
[Out] -b^2/(24*x^2) - (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(6*x^3) - (b^4*Pi^2*FresnelC[b*x]^2)/12 - FresnelC[b*x]^2/(4*x^
4) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x) - (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]
```

Rubi steps

$$\int \frac{C(bx)^2}{x^5}\,dx = -\frac{C(bx)^2}{4x^4} + \frac{1}{2}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^4}\,dx$$

$$= -\frac{b^2}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^3} - \frac{C(bx)^2}{4x^4} + \frac{1}{12}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx - \frac{1}{6}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\right.}{x^2}$$

$$= -\frac{b^2}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^3} - \frac{C(bx)^2}{4x^4} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x} + \frac{1}{24}b^2\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi x\right.}{x^2}\right.$$

$$= -\frac{b^2}{24x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^3} - \frac{C(bx)^2}{4x^4} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x} - \frac{1}{24}b^4\pi\mathrm{Si}$$

$$= -\frac{b^2}{24x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^3} - \frac{1}{12}b^4\pi^2C(bx)^2 - \frac{C(bx)^2}{4x^4} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\right.}{6x}$$

Mathematica [A] time = 0.01, size = 127, normalized size = 1.00

$$-\frac{1}{12}\pi^2b^4C(bx)^2 - \frac{bC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3} - \frac{b^2}{24x^2} - \frac{b^2\cos\left(\pi b^2x^2\right)}{24x^2} - \frac{1}{12}\pi b^4\mathrm{Si}\left(b^2\pi x^2\right) + \frac{\pi b^3C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x} - \frac{C(bx}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]^2/x^5,x]
```

```
[Out] -1/24*b^2/x^2 - (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(6*x^3) - (b^4*Pi^2*FresnelC[b*x]^2)/12 - FresnelC[b*x]^2/(4*x^
4) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x) - (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\left(bx\right)^2}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\left(bx\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^5, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{FresnelC}\,(bx)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^5,x)
```

```
[Out] int(FresnelC(b*x)^2/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\,(bx)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}\,(b\,x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^5,x)
```

```
[Out] int(FresnelC(b*x)^2/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2\,(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**5,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**5, x)
```

3.153 $\int \frac{C(bx)^2}{x^6}\,dx$

Optimal. Leaf size=171

$$-\frac{1}{20}\pi^2 b^5 \text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{7\pi^2b^5C\left(\sqrt{2}\,bx\right)}{60\sqrt{2}}-\frac{bC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{10x^4}-\frac{b^2}{60x^3}-\frac{b^2\cos\left(\pi b^2x^2\right)}{60x^3}+\frac{7\pi b^4\sin}{12(}$$

```
[Out] -1/60*b^2/x^3-1/60*b^2*cos(b^2*Pi*x^2)/x^3-1/10*b*cos(1/2*b^2*Pi*x^2)*Fresn
elC(b*x)/x^4-1/5*FresnelC(b*x)^2/x^5+1/20*b^3*Pi*FresnelC(b*x)*sin(1/2*b^2*
Pi*x^2)/x^2+7/120*b^4*Pi*sin(b^2*Pi*x^2)/x-7/120*b^5*Pi^2*FresnelC(b*x*2^(1
/2))*2^(1/2)-1/20*b^5*Pi^2*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x
,x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^6,x]
```

```
[Out] -b^2/(60*x^3) - (b^2*Cos[b^2*Pi*x^2])/(60*x^3) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(10*x^4) - FresnelC[b*x]^2/(5*x^5) - (7*b^5*Pi^2*FresnelC[Sqrt[
2]*b*x])/(60*Sqrt[2]) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(20*x^2)
 + (7*b^4*Pi*Sin[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*Defer[Int][(Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/x, x])/20
```

Rubi steps

$$\int \frac{C(bx)^2}{x^6}\,dx = -\frac{C(bx)^2}{5x^5} + \frac{1}{5}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^5}\,dx$$

$$= -\frac{b^2}{60x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{10x^4} - \frac{C(bx)^2}{5x^5} + \frac{1}{20}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx - \frac{1}{10}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}\right.}{x^3}$$

$$= -\frac{b^2}{60x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{10x^4} - \frac{C(bx)^2}{5x^5} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2} - \frac{1}{40}\left(b^4\right.$$

$$= -\frac{b^2}{60x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{10x^4} - \frac{C(bx)^2}{5x^5} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2} + \frac{7b^4\pi\,\mathrm{si}}{1}$$

$$= -\frac{b^2}{60x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{10x^4} - \frac{C(bx)^2}{5x^5} - \frac{7b^5\pi^2 C\left(\sqrt{2}\,bx\right)}{60\sqrt{2}} + \frac{b^3\pi C(bx)\sin\left(\right.}{20x^2}$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{C(bx)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^6,x]
[Out] Integrate[FresnelC[b*x]^2/x^6, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\,(bx)^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^6,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)^2/x^6, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\,(bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^6,x, algorithm="giac")
[Out] integrate(fresnelc(b*x)^2/x^6, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{FresnelC}\left(bx\right)^{2}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^6,x)
[Out] int(FresnelC(b*x)^2/x^6,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\left(bx\right)^{2}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^6,x, algorithm="maxima")
[Out] integrate(fresnelc(b*x)^2/x^6, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}\left(b\,x\right)^{2}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^6,x)
[Out] int(FresnelC(b*x)^2/x^6, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^{2}\left(bx\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**6,x)
[Out] Integral(fresnelc(b*x)**2/x**6, x)
```

3.154 $\int \frac{C(bx)^2}{x^7}\,dx$

Optimal. Leaf size=166

$$-\frac{1}{45}\pi^2 b^5 \text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)-\frac{bC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{15x^5}-\frac{b^2}{120x^4}-\frac{b^2\cos\left(\pi b^2x^2\right)}{120x^4}-\frac{1}{72}\pi^2b^6\text{Ci}\left(b^2\pi x^2\right)+\frac{\pi b^4}{}$$

```
[Out] -1/120*b^2/x^4-1/72*b^6*Pi^2*Ci(b^2*Pi*x^2)-1/120*b^2*cos(b^2*Pi*x^2)/x^4-1
/15*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5-1/6*FresnelC(b*x)^2/x^6+1/45*b^
3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3+1/72*b^4*Pi*sin(b^2*Pi*x^2)/x^2-
1/45*b^5*Pi^2*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Rubi [A] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^7,x]
```

```
[Out] -b^2/(120*x^4) - (b^2*Cos[b^2*Pi*x^2])/(120*x^4) - (b^6*Pi^2*CosIntegral[b^
2*Pi*x^2])/72 - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(15*x^5) - FresnelC[b
*x]^2/(6*x^6) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(45*x^3) + (b^4*
Pi*Sin[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*Fr
esnelC[b*x])/x^2, x])/45
```

Rubi steps

$$\int \frac{C(bx)^2}{x^7}\,dx = -\frac{C(bx)^2}{6x^6} + \frac{1}{3}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^6}\,dx$$

$$= -\frac{b^2}{120x^4} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5} - \frac{C(bx)^2}{6x^6} + \frac{1}{30}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx - \frac{1}{15}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}\right.}{x^4}$$

$$= -\frac{b^2}{120x^4} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5} - \frac{C(bx)^2}{6x^6} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3} + \frac{1}{60}b^2\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi\right.}{x^3}\right.$$

$$= -\frac{b^2}{120x^4} - \frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5} - \frac{C(bx)^2}{6x^6} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3} - \frac{1}{180}\left(b^4\right.$$

$$= -\frac{b^2}{120x^4} - \frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5} - \frac{C(bx)^2}{6x^6} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3} + \frac{b^4\pi\sin}{72}$$

$$= -\frac{b^2}{120x^4} - \frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4} - \frac{1}{72}b^6\pi^2\mathrm{Ci}\left(b^2\pi x^2\right) - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5} - \frac{C(bx)^2}{6x^6} + \frac{b^3\pi C(bx)\sin}{45x}$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{C(bx)^2}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^7,x]
[Out] Integrate[FresnelC[b*x]^2/x^7, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\,(bx)^2}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^7,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)^2/x^7, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\,(bx)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^7, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^7,x)
```

```
[Out] int(FresnelC(b*x)^2/x^7,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^7,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^7, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{FresnelC}\,(b\,x)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^7,x)
```

```
[Out] int(FresnelC(b*x)^2/x^7, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2\,(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**7,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**7, x)
```

3.155 $\int \frac{C(bx)^2}{x^8}\,dx$

Optimal. Leaf size=259

$$\frac{1}{168}\pi^3 b^7 \text{Int}\left(\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{2}{315}\sqrt{2}\,\pi^3b^7S\left(\sqrt{2}\,bx\right)+\frac{\pi^3b^7S\left(\sqrt{2}\,bx\right)}{72\sqrt{2}}+\frac{\pi^2b^6}{336x}-\frac{bC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{21x^6}-\frac{b^2}{210}$$

```
[Out] -1/210*b^2/x^5+1/336*b^6*Pi^2/x-1/210*b^2*cos(b^2*Pi*x^2)/x^5+67/5040*b^6*P
i^2*cos(b^2*Pi*x^2)/x-1/21*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6+1/168*b^
5*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2-1/7*FresnelC(b*x)^2/x^7+1/84*b
^3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4+13/2520*b^4*Pi*sin(b^2*Pi*x^2)/
x^3+67/5040*b^7*Pi^3*FresnelS(b*x*2^(1/2))*2^(1/2)+1/168*b^7*Pi^3*Unintegra
ble(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^8,x]
```

```
[Out] -b^2/(210*x^5) + (b^6*Pi^2)/(336*x) - (b^2*Cos[b^2*Pi*x^2])/(210*x^5) + (67
*b^6*Pi^2*Cos[b^2*Pi*x^2])/(5040*x) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/(21*x^6) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(168*x^2) - Fresne
lC[b*x]^2/(7*x^7) + (b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/(72*Sqrt[2]) + (2*Sqrt
[2]*b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/315 + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(84*x^4) + (13*b^4*Pi*Sin[b^2*Pi*x^2])/(2520*x^3) + (b^7*Pi^3*Def
er[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/168
```

Rubi steps

$$\int \frac{C(bx)^2}{x^8}\,dx = -\frac{C(bx)^2}{7x^7} + \frac{1}{7}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^7}\,dx$$

$$= -\frac{b^2}{210x^5} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} - \frac{C(bx)^2}{7x^7} + \frac{1}{42}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx - \frac{1}{21}\left(b^3\pi\right)\int \frac{C(bx)\sin\Big(}{x^5}$$

$$= -\frac{b^2}{210x^5} - \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} - \frac{C(bx)^2}{7x^7} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{84x^4} - \frac{1}{168}\Big(b$$

$$= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{168x^2} - \frac{C(bx)}{7x^7}$$

$$= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} + \frac{67b^6\pi^2\cos\left(b^2\pi x^2\right)}{5040x} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} + \frac{b^5\pi^2\cos\Big(}{1}$$

$$= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} + \frac{67b^6\pi^2\cos\left(b^2\pi x^2\right)}{5040x} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} + \frac{b^5\pi^2\cos\Big(}{1}$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{C(bx)^2}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^8,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^8, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\,(bx)^2}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^8,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^8, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\,(bx)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^8, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{FresnelC}\,(bx)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^8,x)
```

```
[Out] int(FresnelC(b*x)^2/x^8,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\,(bx)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^8, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}\,(b\,x)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^8,x)
```

```
[Out] int(FresnelC(b*x)^2/x^8, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**8,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**8, x)
```

3.156 $\int \frac{C(bx)^2}{x^9} dx$

Optimal. Leaf size=242

$$\frac{1}{840}\pi^4 b^8 C(bx)^2+\frac{\pi^2 b^6}{1680x^2}-\frac{bC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{28x^7}-\frac{b^2}{336x^6}-\frac{b^2\cos\left(\pi b^2x^2\right)}{336x^6}+\frac{1}{280}\pi^3 b^8\text{Si}\left(b^2\pi x^2\right)-\frac{\pi^3 b^7 C(bx)\sin\left(\right.}{420x}$$

```
[Out] -1/336*b^2/x^6+1/1680*b^6*Pi^2/x^2-1/336*b^2*cos(b^2*Pi*x^2)/x^6+1/336*b^6*
Pi^2*cos(b^2*Pi*x^2)/x^2-1/28*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7+1/420
*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3+1/840*b^8*Pi^4*FresnelC(b*x
)^2-1/8*FresnelC(b*x)^2/x^8+1/280*b^8*Pi^3*Si(b^2*Pi*x^2)+1/140*b^3*Pi*Fres
nelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/420*b^7*Pi^3*FresnelC(b*x)*sin(1/2*b^2*
Pi*x^2)/x+1/420*b^4*Pi*sin(b^2*Pi*x^2)/x^4
```

Rubi [A] time = 0.39, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6431, 6457, 6465, 6441, 30, 3375, 3380, 3297, 3299, 3379}

$$-\frac{\pi^3 b^7 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{420x}+\frac{\pi b^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{140x^5}+\frac{\pi^2 b^5 \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{420x^3}-\frac{b\text{Fresn}}{}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]^2/x^9,x]
```

```
[Out] -b^2/(336*x^6) + (b^6*Pi^2)/(1680*x^2) - (b^2*Cos[b^2*Pi*x^2])/(336*x^6) +
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(28*x^7) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(420*x^3) + (b^8*
Pi^4*FresnelC[b*x]^2)/840 - FresnelC[b*x]^2/(8*x^8) + (b^3*Pi*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(140*x^5) - (b^7*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
2])/(420*x) + (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) + (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
```

```
)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]
```

Rubi steps

$$\int \frac{C(bx)^2}{x^9}\,dx = -\frac{C(bx)^2}{8x^8} + \frac{1}{4}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^8}\,dx$$

$$= -\frac{b^2}{336x^6} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} - \frac{C(bx)^2}{8x^8} + \frac{1}{56}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx - \frac{1}{28}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\right.}{x^6}$$

$$= -\frac{b^2}{336x^6} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} - \frac{C(bx)^2}{8x^8} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5} + \frac{1}{112}b^2\,\text{Subst}\left(\int \frac{\cos\left(b\right.}{x^4}\right.$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{420x^3} - \frac{C(b}{8x}$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{420x^3} - \frac{C(b}{8x}$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} + \frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}\right.}{42}$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} + \frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}\right.}{42}$$

Mathematica [A] time = 0.02, size = 242, normalized size = 1.00

$$\frac{1}{840}\pi^4b^8C(bx)^2 + \frac{\pi^2b^6}{1680x^2} - \frac{bC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{28x^7} - \frac{b^2}{336x^6} - \frac{b^2\cos\left(\pi b^2x^2\right)}{336x^6} + \frac{1}{280}\pi^3b^8\,\text{Si}\left(b^2\pi x^2\right) - \frac{\pi^3b^7C(bx)\sin\left(\right.}{420x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]^2/x^9,x]
```

```
[Out] -1/336*b^2/x^6 + (b^6*Pi^2)/(1680*x^2) - (b^2*Cos[b^2*Pi*x^2])/(336*x^6) +
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(28*x^7) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(420*x^3) + (b^8*
Pi^4*FresnelC[b*x]^2)/840 - FresnelC[b*x]^2/(8*x^8) + (b^3*Pi*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(140*x^5) - (b^7*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
2])/(420*x) + (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) + (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)^2}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^9,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)^2/x^9, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^9, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^9,x)
```

```
[Out] int(FresnelC(b*x)^2/x^9,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^9,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\text{FresnelC}\left(b\,x\right)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^9,x)
```

```
[Out] int(FresnelC(b*x)^2/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^{2}\left(bx\right)}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**9,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**9, x)
```

3.157 $\int \frac{C(bx)^2}{x^{10}} dx$

Optimal. Leaf size=286

$$\frac{\pi^4 b^9 \text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)}{1728}+\frac{853\pi^4 b^9 C\left(\sqrt{2}\,bx\right)}{181440\sqrt{2}}+\frac{\pi^2 b^6}{5184x^3}-\frac{bC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{36x^8}-\frac{b^2}{504x^7}-\frac{b^2\cos\left(\pi b^2x^2\right)}{504x^7}-\frac{853\pi}{}$$

```
[Out] -1/504*b^2/x^7+1/5184*b^6*Pi^2/x^3-1/504*b^2*cos(b^2*Pi*x^2)/x^7+187/181440
*b^6*Pi^2*cos(b^2*Pi*x^2)/x^3-1/36*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8+
1/864*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4-1/9*FresnelC(b*x)^2/x^
9+1/216*b^3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6-1/1728*b^7*Pi^3*Fresne
lC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+19/15120*b^4*Pi*sin(b^2*Pi*x^2)/x^5-853/362
880*b^8*Pi^3*sin(b^2*Pi*x^2)/x+853/362880*b^9*Pi^4*FresnelC(b*x*2^(1/2))*2^
(1/2)+1/1728*b^9*Pi^4*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [A] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^10,x]
```

```
[Out] -b^2/(504*x^7) + (b^6*Pi^2)/(5184*x^3) - (b^2*Cos[b^2*Pi*x^2])/(504*x^7) +
(187*b^6*Pi^2*Cos[b^2*Pi*x^2])/(181440*x^3) - (b*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(36*x^8) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(864*x^4)
- FresnelC[b*x]^2/(9*x^9) + (853*b^9*Pi^4*FresnelC[Sqrt[2]*b*x])/(181440*Sq
rt[2]) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(216*x^6) - (b^7*Pi^3*F
resnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(1728*x^2) + (19*b^4*Pi*Sin[b^2*Pi*x^2])/
(15120*x^5) - (853*b^8*Pi^3*Sin[b^2*Pi*x^2])/(362880*x) + (b^9*Pi^4*Defer[I
nt][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])/1728
```

Rubi steps

$$\int \frac{C(bx)^2}{x^{10}}\,dx = -\frac{C(bx)^2}{9x^9} + \frac{1}{9}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^9}\,dx$$

$$= -\frac{b^2}{504x^7} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} - \frac{C(bx)^2}{9x^9} + \frac{1}{72}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx - \frac{1}{36}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\right.}{x^7}$$

$$= -\frac{b^2}{504x^7} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} - \frac{C(bx)^2}{9x^9} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{216x^6} - \frac{1}{432}\left(\right.$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{864x^4} - \frac{C(b}{9x}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} + \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} + \frac{b^5\pi^2\text{co}}{}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} + \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} + \frac{b^5\pi^2\text{co}}{}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} + \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} + \frac{b^5\pi^2\text{co}}{}$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{C(bx)^2}{x^{10}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^10,x]
[Out] Integrate[FresnelC[b*x]^2/x^10, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\left(bx\right)^2}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^10,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)^2/x^10, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}(bx)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)^2/x^10, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^10,x)
```

```
[Out] int(FresnelC(b*x)^2/x^10,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}(bx)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^2/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^2/x^10, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\text{FresnelC}(b\,x)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^10,x)
```

```
[Out] int(FresnelC(b*x)^2/x^10, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**10,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**10, x)
```

3.158 $\int (c+dx)^2 C(a+bx)^2\,dx$

Optimal. Leaf size=495

$$\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}+\frac{d(bc-ad)C}{}$$

```
[Out] 2/3*d^2*x/b^2/Pi^2-1/2*d*(-a*d+b*c)*cos(Pi*(b*x+a)^2)/b^3/Pi^2-1/6*d^2*(b*x
+a)*cos(Pi*(b*x+a)^2)/b^3/Pi^2-4/3*d^2*cos(1/2*Pi*(b*x+a)^2)*FresnelC(b*x+a
)/b^3/Pi^2+(-a*d+b*c)^2*(b*x+a)*FresnelC(b*x+a)^2/b^3+d*(-a*d+b*c)*(b*x+a)^
2*FresnelC(b*x+a)^2/b^3+1/3*d^2*(b*x+a)^3*FresnelC(b*x+a)^2/b^3+d*(-a*d+b*c
)*FresnelC(b*x+a)*FresnelS(b*x+a)/b^3/Pi+1/4*I*d*(-a*d+b*c)*(b*x+a)^2*Hyper
geometricPFQ([1, 1],[3/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^3/Pi-1/4*I*d*(-a*d+b*c)
*(b*x+a)^2*HypergeometricPFQ([1, 1],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^3/Pi-2*(
-a*d+b*c)^2*FresnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-2*d*(-a*d+b*c)*(b*
x+a)*FresnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-2/3*d^2*(b*x+a)^2*Fresnel
C(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi+5/12*d^2*FresnelC((b*x+a)*2^(1/2))/b^
3/Pi^2*2^(1/2)+1/2*(-a*d+b*c)^2*FresnelS((b*x+a)*2^(1/2))/b^3/Pi*2^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {6433, 6421, 6453, 3351, 6431, 6455, 6447, 3379, 2638, 6461, 3358, 3352, 3385}

$$\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}+\frac{d(bc-ad)\text{F}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelC[a + b*x]^2,x]
```

```
[Out] (2*d^2*x)/(3*b^2*Pi^2) - (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) -
 (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (4*d^2*Cos[(Pi*(a + b*x
)^2)/2]*FresnelC[a + b*x])/(3*b^3*Pi^2) + ((b*c - a*d)^2*(a + b*x)*FresnelC
[a + b*x]^2)/b^3 + (d*(b*c - a*d)*(a + b*x)^2*FresnelC[a + b*x]^2)/b^3 + (d
^2*(a + b*x)^3*FresnelC[a + b*x]^2)/(3*b^3) + (5*d^2*FresnelC[Sqrt[2]*(a +
b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (d*(b*c - a*d)*FresnelC[a + b*x]*FresnelS[a +
 b*x])/(b^3*Pi) + ((b*c - a*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*b^3*
Pi) + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2},
(-I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - ((I/4)*d*(b*c - a*d)*(a + b*x)^2*Hyperge
ometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - (2*(b*c - a*
d)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (2*d*(b*c - a*d)
*(a + b*x)*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (2*d^2*(a
+ b*x)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6421

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelC[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6433

```
Int[FresnelC[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelC[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int (c+dx)^2 C(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)C(x)^2+2bcd\left(1-\frac{ad}{bc}\right)xC(x)^2+d^2x^2C(x)^2\right)dx, x, a}{b^3}$$

$$= \frac{d^2\,\text{Subst}\left(\int x^2C(x)^2\,dx, x, a+bx\right)}{b^3} + \frac{(2d(bc-ad))\,\text{Subst}\left(\int xC(x)^2\,dx, x, a+bx\right)}{b^3} +$$

$$= \frac{(bc-ad)^2(a+bx)C(a+bx)^2}{b^3} + \frac{d(bc-ad)(a+bx)^2C(a+bx)^2}{b^3} + \frac{d^2(a+bx)^3C(a+}{3b^3}$$

$$= \frac{(bc-ad)^2(a+bx)C(a+bx)^2}{b^3} + \frac{d(bc-ad)(a+bx)^2C(a+bx)^2}{b^3} + \frac{d^2(a+bx)^3C(a+}{3b^3}$$

$$= -\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} - \frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)C(a+bx)}{3b^3\pi^2} + \frac{(bc-ad)^2(a+b}{b^3}$$

$$= -\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2} - \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} - \frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right.}{3b^3\pi^2}$$

$$= \frac{2d^2x}{3b^2\pi^2} - \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2} - \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} - \frac{4d^2\cos\left(\frac{1}{2}\pi(\right.}{}$$

$$= \frac{2d^2x}{3b^2\pi^2} - \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2} - \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} - \frac{4d^2\cos\left(\frac{1}{2}\pi(\right.}{}$$

Mathematica [F] time = 0.82, size = 0, normalized size = 0.00

$$\int (c+dx)^2 C(a+bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^2*FresnelC[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^2*FresnelC[a + b*x]^2, x]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2x^2 + 2\,cdx + c^2\right)\text{fresnelc}\,(bx+a)^2\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnelc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*fresnelc(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \text{fresnelc}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnelc(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*fresnelc(b*x + a)^2, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \operatorname{FresnelC}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*FresnelC(b*x+a)^2,x)
[Out] int((d*x+c)^2*FresnelC(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \text{fresnelc}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnelc(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^2*fresnelc(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{FresnelC}(a+b\,x)^2\,(c+d\,x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)^2*(c + d*x)^2,x)
[Out] int(FresnelC(a + b*x)^2*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 C^2(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnelc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*fresnelc(a + b*x)**2, x)
```

3.159 $\int (c+dx)C(a+bx)^2\,dx$

Optimal. Leaf size=279

$$\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}-\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}+\frac{(a+bx)(bc-ad)C(a+bx)^2}{b^2}$$

```
[Out] -1/4*d*cos(Pi*(b*x+a)^2)/b^2/Pi^2+(-a*d+b*c)*(b*x+a)*FresnelC(b*x+a)^2/b^2+
1/2*d*(b*x+a)^2*FresnelC(b*x+a)^2/b^2+1/2*d*FresnelC(b*x+a)*FresnelS(b*x+a)
/b^2/Pi+1/8*I*d*(b*x+a)^2*HypergeometricPFQ([1, 1],[3/2, 2],-1/2*I*Pi*(b*x+
a)^2)/b^2/Pi-1/8*I*d*(b*x+a)^2*HypergeometricPFQ([1, 1],[3/2, 2],1/2*I*Pi*(
b*x+a)^2)/b^2/Pi-2*(-a*d+b*c)*FresnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi-
d*(b*x+a)*FresnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi+1/2*(-a*d+b*c)*Fresn
elS((b*x+a)*2^(1/2))/b^2/Pi*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6433, 6421, 6453, 3351, 6431, 6455, 6447, 3379, 2638}

$$\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}-\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}+\frac{(a+bx)(bc-ad)\text{FresnelC}(a}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelC[a + b*x]^2,x]
```

```
[Out] -(d*Cos[Pi*(a + b*x)^2])/(4*b^2*Pi^2) + ((b*c - a*d)*(a + b*x)*FresnelC[a +
 b*x]^2)/b^2 + (d*(a + b*x)^2*FresnelC[a + b*x]^2)/(2*b^2) + (d*FresnelC[a
+ b*x]*FresnelS[a + b*x])/(2*b^2*Pi) + ((b*c - a*d)*FresnelS[Sqrt[2]*(a + b
*x)])/(Sqrt[2]*b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, (-I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*Hypergeometr
icPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - (2*(b*c - a*d)*Fr
esnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi) - (d*(a + b*x)*FresnelC[a
 + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6421

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelC[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6431

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*Fresnel
C[b*x]^2)/(m + 1), x] - Dist[(2*b)/(m + 1), Int[x^(m + 1)*Cos[(Pi*b^2*x^2)/
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6433

```
Int[FresnelC[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelC[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
```

]

Rubi steps

$$\int (c+dx)C(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(bc\left(1-\frac{ad}{bc}\right)C(x)^2+dxC(x)^2\right)dx,x,a+bx\right)}{b^2}$$
$$= \frac{d\,\text{Subst}\left(\int xC(x)^2\,dx,x,a+bx\right)}{b^2}+\frac{(bc-ad)\,\text{Subst}\left(\int C(x)^2\,dx,x,a+bx\right)}{b^2}$$
$$= \frac{(bc-ad)(a+bx)C(a+bx)^2}{b^2}+\frac{d(a+bx)^2C(a+bx)^2}{2b^2}-\frac{d\,\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)C(x)\,d\right.}{b^2}$$
$$= \frac{(bc-ad)(a+bx)C(a+bx)^2}{b^2}+\frac{d(a+bx)^2C(a+bx)^2}{2b^2}-\frac{2(bc-ad)C(a+bx)\sin\left(\frac{1}{2}\pi(a\right.}{b^2\pi}$$
$$= \frac{(bc-ad)(a+bx)C(a+bx)^2}{b^2}+\frac{d(a+bx)^2C(a+bx)^2}{2b^2}+\frac{dC(a+bx)S(a+bx)}{2b^2\pi}+\frac{(bc-a}{}$$
$$= -\frac{d\cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2}+\frac{(bc-ad)(a+bx)C(a+bx)^2}{b^2}+\frac{d(a+bx)^2C(a+bx)^2}{2b^2}+\frac{dC(a+}{}$$

Mathematica [F] time = 0.68, size = 0, normalized size = 0.00

$$\int (c+dx)C(a+bx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)*FresnelC[a + b*x]^2,x]
[Out] Integrate[(c + d*x)*FresnelC[a + b*x]^2, x]
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)\text{fresnelc}\,(bx+a)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)*fresnelc(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\text{fresnelc}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)*fresnelc(b*x + a)^2, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int (dx+c)\,\text{FresnelC}\,(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*FresnelC(b*x+a)^2,x)
[Out] int((d*x+c)*FresnelC(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\text{fresnelc}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)*fresnelc(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \text{FresnelC}\,(a+b\,x)^2\,(c+d\,x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)^2*(c + d*x),x)
[Out] int(FresnelC(a + b*x)^2*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\,C^2\,(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a)**2,x)
[Out] Integral((c + d*x)*fresnelc(a + b*x)**2, x)
```

3.160 $\int C(a+bx)^2\,dx$

Optimal. Leaf size=69

$$\frac{(a+bx)C(a+bx)^2}{b}-\frac{2C(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}+\frac{S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,\pi b}$$

```
[Out] (b*x+a)*FresnelC(b*x+a)^2/b-2*FresnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b/Pi+1/
2*FresnelS((b*x+a)*2^(1/2))/b/Pi*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6421, 6453, 3351}

$$\frac{(a+bx)\text{FresnelC}(a+bx)^2}{b}-\frac{2\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}+\frac{S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*FresnelC[a + b*x]^2)/b + FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*
Pi) - (2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b*Pi)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6421

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[((a + b*x)*FresnelC[a
 + b*x]^2)/b, x] - Dist[2, Int[(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int C(a+bx)^2\,dx=\frac{(a+bx)C(a+bx)^2}{b}-2\int (a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)C(a+bx)\,dx$$
$$=\frac{(a+bx)C(a+bx)^2}{b}-\frac{2\,\mathrm{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)C(x)\,dx,x,a+bx\right)}{b}$$
$$=\frac{(a+bx)C(a+bx)^2}{b}-\frac{2C(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{\mathrm{Subst}\left(\int \sin\left(\pi x^2\right)\,dx,x,a+bx\right)}{b\pi}$$
$$=\frac{(a+bx)C(a+bx)^2}{b}+\frac{S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,b\pi}-\frac{2C(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.96

$$\frac{2\pi(a+bx)C(a+bx)^2-4C(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+\sqrt{2}\,S\left(\sqrt{2}\,(a+bx)\right)}{2\pi b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[a + b*x]^2,x]
```

```
[Out] (2*Pi*(a + b*x)*FresnelC[a + b*x]^2 + Sqrt[2]*FresnelS[Sqrt[2]*(a + b*x)] -
 4*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(2*b*Pi)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\mathrm{fresnelc}\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{fresnelc}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2,x, algorithm="giac")
```

[Out] integrate(fresnelc(b*x + a)^2, x)

maple [A] time = 0.00, size = 60, normalized size = 0.87

$$\frac{(bx+a)\,\mathrm{FresnelC}\,(bx+a)^{2}-\frac{2\,\mathrm{FresnelC}(bx+a)\sin\left(\frac{\pi(bx+a)^{2}}{2}\right)}{\pi}+\frac{\sqrt{2}\,\mathrm{S}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x+a)^2,x)

[Out] 1/b*((b*x+a)*FresnelC(b*x+a)^2-2*FresnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/Pi+1
/2/Pi*2^(1/2)*FresnelS((b*x+a)*2^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{fresnelc}\,(bx+a)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)^2,x, algorithm="maxima")

[Out] integrate(fresnelc(b*x + a)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{FresnelC}\,(a+b\,x)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(a + b*x)^2,x)

[Out] int(FresnelC(a + b*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int C^{2}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x+a)**2,x)

[Out] Integral(fresnelc(a + b*x)**2, x)

3.161 $\int \frac{C(a+bx)^2}{c+dx}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{C(a+bx)^2}{c+dx},x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)^2/(d*x+c),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{C(a+bx)^2}{c+dx}\,dx = \int \frac{C(a+bx)^2}{c+dx}\,dx$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{C(a+bx)^2}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[FresnelC[a + b*x]^2/(c + d*x), x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(fresnelc(b*x + a)^2/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\left(bx+a\right)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(fresnelc(b*x + a)^2/(d*x + c), x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)^2/(d*x+c),x)
[Out] int(FresnelC(b*x+a)^2/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\left(bx+a\right)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] integrate(fresnelc(b*x + a)^2/(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\text{FresnelC}\left(a+b\,x\right)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)^2/(c + d*x),x)
[Out] int(FresnelC(a + b*x)^2/(c + d*x), x)
```

<u>**sympy**</u> **[A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(fresnelc(a + b*x)**2/(c + d*x), x)
```

3.162 $\int \frac{C(a+bx)^2}{(c+dx)^2} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{C(a+bx)^2}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{C(a+bx)^2}{(c+dx)^2} dx = \int \frac{C(a+bx)^2}{(c+dx)^2} dx$$

Mathematica [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{C(a+bx)^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[FresnelC[a + b*x]^2/(c + d*x)^2, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx+a)^2}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(fresnelc(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(fresnelc(b*x + a)^2/(d*x + c)^2, x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)^2/(d*x+c)^2,x)
[Out] int(FresnelC(b*x+a)^2/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(fresnelc(b*x + a)^2/(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\text{FresnelC}(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)^2/(c + d*x)^2,x)
[Out] int(FresnelC(a + b*x)^2/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2 (a + bx)}{(c + dx)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(fresnelc(a + b*x)**2/(c + d*x)**2, x)
```

3.163 $\int x^2 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=231

$$\left(\frac{1}{12}+\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{3}{n}\right)}{\sqrt{\pi}\,bd}\right)-\left(\frac{1}{12}+\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}-}$$

```
[Out] (1/12+1/12*I)*exp(-3*a/b/n+9/2*I/b^2/d^2/n^2/Pi)*x^3*erf((1/2+1/2*I)*(3/n+I
*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))-(1/12+1/1
2*I)*exp(-3*a/b/n-9/2*I/b^2/d^2/n^2/Pi)*x^3*erfi((1/2+1/2*I)*(3/n-I*a*b*d^2
*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))+1/3*x^3*FresnelC(
d*(a+b*ln(c*x^n)))
```

Rubi [A] time = 0.52, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6472, 4618, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{12}+\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{3}{n}\right)}{\sqrt{\pi}\,bd}\right)-\left(\frac{1}{12}+\frac{i}{12}\right)x^3\left(cx^n\right)^{-3/n}e^{-\frac{3a}{bn}-}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/12 + I/12)*E^((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erf[((1/2
+ I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*
x^n)^(3/n) - ((1/12 + I/12)*E^((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))*x
^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(c*x^n)^(3/n) + (x^3*FresnelC[d*(a + b*Log[c*x^n])])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 4618

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6472

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelC[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Cos[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx = \frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{3}(bdn)\int x^2\cos\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)dx$$

$$=\frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}(bdn)\int e^{-\frac{1}{2}id^2\pi\left(a+b\log(cx^n)\right)^2}x^2\,dx-\frac{1}{6}(bdn)\int e$$

$$=\frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}(bdn)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-iabd^2\pi\log\left(cx^n\right)-\frac{1}{2}\right.$$

$$=\frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}(bdn)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)$$

$$=\frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-\right.$$

$$=\frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}\left(bdx^3\left(cx^n\right)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}\right.\right.$$

$$=\frac{1}{3}x^3 C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{6}\left(bde^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3\left(cx^n\right)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\text{Subst}$$

$$=\left(\frac{1}{12}+\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3\left(cx^n\right)^{-3/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx\right.\right.}{bd\sqrt{\pi}}\right.$$

Mathematica [A] time = 7.68, size = 318, normalized size = 1.38

$$\frac{1}{12}x^3\left(4C\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\sqrt[4]{-1}\sqrt{2}\left(cx^n\right)^{-3/n}\left(ie^{\frac{9i}{\pi b^2d^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\pi abd^2n+\pi b^2d^2n\log\left(cx^n\right)-3i\right)}{\sqrt{\pi}\,bdn}\right)+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*(4*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^(((-6*a)/(b*
n) - (9*I)/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - L
og[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*(I*E^((9*I)/(b^2
*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])] + Erfi[((-1)^(3/4)*(3*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(3/n)))/12
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\text{fresnelc}\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(x^2*fresnelc(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{fresnelc}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x^2*fresnelc((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{FresnelC}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(d*(a+b*ln(c*x^n))),x)
[Out] int(x^2*FresnelC(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{fresnelc}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(x^2*fresnelc((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{FresnelC}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x^2*FresnelC(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 C\left(ad + bd\log\left(cx^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**2*fresnelc(a*d + b*d*log(c*x**n)), x)
```

3.164 $\int xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=227

$$\left(\frac{1}{8}+\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\mathrm{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{-\frac{2\left(\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\mathrm{erf}$$

```
[Out] (1/8+1/8*I)*exp((2*I-2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x^2*erf((1/2+1/2*I)*(2
/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(2/n))-(1/8+
1/8*I)*x^2*erfi((1/2+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/P
i^(1/2))/exp(2*(I+a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/((c*x^n)^(2/n))+1/2*x^2*Fre
snelC(d*(a+b*ln(c*x^n)))
```

Rubi [A] time = 0.41, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6472, 4618, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{8}+\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\mathrm{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)x^2\left(cx^n\right)^{-2/n}e^{-\frac{2\left(\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\mathrm{Er}$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/8 + I/8)*E^((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*Erf[((1/2 + I/
2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)
^(2/n) - ((1/8 + I/8)*x^2*Erfi[((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*(c*x^n)^(2/n)) + (x^2*FresnelC[d*(a + b*Log[c*x^n])])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 4618

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6472

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelC[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Cos[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx=\frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bdn)\int x\cos\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)dx$$

$$=\frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}(bdn)\int e^{-\frac{1}{2}id^2\pi\left(a+b\log(cx^n)\right)^2}x\,dx-\frac{1}{4}(bdn)\int e^{\frac{1}{2}id^2}$$

$$=\frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}(bdn)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-iabd^2\pi\log\left(cx^n\right)-\frac{1}{2}ib\right.$$

$$=\frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}(bdn)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)x\,($$

$$=\frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}i\right.$$

$$=\frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(bdx^2\left(cx^n\right)^{-iabd^2\pi-\frac{2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2\right.\right.$$

$$=\frac{1}{2}x^2C\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{4}\left(bde^{-\frac{2\left(i+abd^2n\pi\right)}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-iabd^2\pi-\frac{2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int e\right.$$

$$=\left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-2/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)-\Big($$

Mathematica [**A**] time = 7.54, size = 318, normalized size = 1.40

$$\frac{1}{8}x^2\left(4C\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\sqrt[4]{-1}\sqrt{2}\left(cx^n\right)^{-2/n}\left(ie^{\frac{4i}{\pi b^2d^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\pi abd^2n+\pi b^2d^2n\log\left(cx^n\right)-2i\right)}{\sqrt{\pi}bdn}\right)+\text{erf}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*(4*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^((-2*a)/(b*n
) - (2*I)/(b^2*d^2*n^2*Pi) - (I/2)*a^2*d^2*Pi + I*a*b*d^2*Pi*(n*Log[x] - Lo
g[c*x^n]) - (I/2)*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)*(I*E^((4*I)/(b^2
*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])] + Erfi[((-1)^(3/4)*(2*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(2/n)))/8
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(x\text{fresnelc}\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(x*fresnelc(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnelc}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x*fresnelc((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int x\,\text{FresnelC}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(d*(a+b*ln(c*x^n))),x)
[Out] int(x*FresnelC(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnelc}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(x*fresnelc((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\text{FresnelC}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x*FresnelC(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xC\left(ad + bd\log\left(cx^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x*fresnelc(a*d + b*d*log(c*x**n)), x)
```

3.165 $\int C\left(d\left(a+b\log\left(cx^n\right)\right)\right)dx$

Optimal. Leaf size=214

$$\left(\frac{1}{4}+\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)-\left(\frac{1}{4}+\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{erfi}\left(\right.$$

```
[Out] (1/4+1/4*I)*x*erf((1/2+1/2*I)*(1/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d
/Pi^(1/2))/exp(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))-(1/4+1/4*I)*
x*erfi((1/2+1/2*I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/
exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))+x*FresnelC(d*(a+b*ln(c*
x^n)))
```

Rubi [A] time = 0.26, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {6469, 4616, 2277, 2274, 15, 2276, 2234, 2204, 2205}

$$\left(\frac{1}{4}+\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log\left(cx^n\right)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)-\left(\frac{1}{4}+\frac{i}{4}\right)x\left(cx^n\right)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{Erfi}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/4 + I/4)*x*Erf[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c
*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^
(-1)) - ((1/4 + I/4)*x*Erfi[((1/2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(
c*x^n)^n^(-1)) + x*FresnelC[d*(a + b*Log[c*x^n])]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2277

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)), x_Symbol] :> Int[
F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]^2), x] /; FreeQ[{F, a, b,
c, d, n}, x]
```

Rule 4616

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)], x_Symbol] :> Dist[1
/2, Int[E^(-(I*d*(a + b*Log[c*x^n])^2)), x], x] + Dist[1/2, Int[E^(I*d*(a +
 b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 6469

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Sim
p[x*FresnelC[d*(a + b*Log[c*x^n])], x] - Dist[b*d*n, Int[Cos[(Pi*(d*(a + b*
Log[c*x^n]))^2)/2], x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rubi steps

$$\int C\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx = xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-(bdn)\int\cos\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)\,dx$$

$$= xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bdn)\int e^{-\frac{1}{2}id^2\pi\left(a+b\log(cx^n)\right)^2}\,dx-\frac{1}{2}(bdn)\int e^{\frac{1}{2}id^2\pi\left(a+\right.}$$

$$= xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bdn)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-iabd^2\pi\log\left(cx^n\right)-\frac{1}{2}ib^2d^2\right.$$

$$= xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}(bdn)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)\left(cx^n\right.$$

$$= xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2\right.$$

$$= xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}\left(bdx\left(cx^n\right)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\right.\right.$$

$$= xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)-\frac{1}{2}\left(bde^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x\left(cx^n\right)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\right.$$

$$= \left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x\left(cx^n\right)^{-1/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)-\left(\right.$$

Mathematica [A] time = 7.46, size = 315, normalized size = 1.47

$$xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)+\frac{\sqrt[4]{-1}\,x\left(cx^n\right)^{-1/n}\left(ie^{\frac{i}{\pi b^2d^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\pi abd^2n+\pi b^2d^2n\log(cx^n)-i\right)}{\sqrt{\pi}\,bdn}\right)+\text{erfi}\left(\frac{(-1)^{3/4}\left(\pi abd^2n+\pi b^2d^2n\right.}{\sqrt{2\pi}\,bdn}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*E^(((-2*a)/(b*n) - I/(b^2*d^
2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b^2
*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*x*(I*E^(I/(b^2*d^2*n^2*Pi))*Erfi[(
(1/2 + I/2)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])
] + Erfi[((-1)^(3/4)*(I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*S
qrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2] + I*Sin[(d^
2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(2*Sqrt[2]*(c*x^n)^n^(-1))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnelc}\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(fresnelc(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnelc}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \text{FresnelC}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a+b*ln(c*x^n))),x)
[Out] int(FresnelC(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnelc}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \text{FresnelC}\left(d\ \left(a+b\ \ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(FresnelC(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int C\left(d\left(a+b\log\left(cx^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(fresnelc(d*(a + b*log(c*x**n))), x)
```

3.166 $\int \frac{C\left(d\left(a+b\log(cx^n)\right)\right)}{x}\,dx$

Optimal. Leaf size=66

$$\frac{\left(a+b\log\left(cx^n\right)\right)C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{bn}-\frac{\sin\left(\frac{1}{2}\pi d^2\left(a+b\log\left(cx^n\right)\right)^2\right)}{\pi bdn}$$

```
[Out] FresnelC(d*(a+b*ln(c*x^n)))*(a+b*ln(c*x^n))/b/n-sin(1/2*d^2*Pi*(a+b*ln(c*x^
n))^2)/b/d/n/Pi
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6419}

$$\frac{\left(a+b\log\left(cx^n\right)\right)\text{FresnelC}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{bn}-\frac{\sin\left(\frac{1}{2}\pi d^2\left(a+b\log\left(cx^n\right)\right)^2\right)}{\pi bdn}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (FresnelC[d*(a + b*Log[c*x^n])]*(a + b*Log[c*x^n]))/(b*n) - Sin[(d^2*Pi*(a
+ b*Log[c*x^n])^2)/2]/(b*d*n*Pi)
```

Rule 6419

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*FresnelC[a +
 b*x])/b, x] - Simp[Sin[(Pi*(a + b*x)^2)/2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int C(d(a+bx))\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{\text{Subst}\left(\int C(x)\,dx,x,ad+bd\log\left(cx^n\right)\right)}{bdn}\\ &= \frac{C\left(ad+bd\log\left(cx^n\right)\right)\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\sin\left(\frac{1}{2}\pi\left(ad+bd\log\left(cx^n\right)\right)^2\right)}{bdn\pi}\end{aligned}$$

Mathematica [B] time = 0.10, size = 165, normalized size = 2.50

$$-\frac{\sin\left(\frac{1}{2}\pi a^2d^2\right)\cos\left(\pi abd^2\log\left(cx^n\right)+\frac{1}{2}\pi b^2d^2\log^2\left(cx^n\right)\right)}{\pi bdn}-\frac{\cos\left(\frac{1}{2}\pi a^2d^2\right)\sin\left(\pi abd^2\log\left(cx^n\right)+\frac{1}{2}\pi b^2d^2\log^2\left(cx\right.\right.}{\pi bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (a*FresnelC[d*(a + b*Log[c*x^n])])/(b*n) + (FresnelC[d*(a + b*Log[c*x^n])]*
Log[c*x^n])/n - (Cos[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2]*S
in[(a^2*d^2*Pi)/2])/(b*d*n*Pi) - (Cos[(a^2*d^2*Pi)/2]*Sin[a*b*d^2*Pi*Log[c*
x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2])/(b*d*n*Pi)
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\left(bd\log\left(cx^{n}\right)+ad\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*d*log(c*x^n) + a*d)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\text{fresnelc}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x, x)
```

maple [A] time = 0.02, size = 81, normalized size = 1.23

$$\frac{\ln\left(c\,x^{n}\right)\text{FresnelC}\left(ad+bd\ln\left(c\,x^{n}\right)\right)}{n}+\frac{\text{FresnelC}\left(ad+bd\ln\left(c\,x^{n}\right)\right)a}{nb}-\frac{\sin\left(\frac{\pi\left(ad+bd\ln\left(c\,x^{n}\right)\right)^{2}}{2}\right)}{nbd\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x,x)
```

```
[Out] 1/n*ln(c*x^n)*FresnelC(a*d+b*d*ln(c*x^n))+1/n/b*FresnelC(a*d+b*d*ln(c*x^n))
*a-1/n/b/d*sin(1/2*Pi*(a*d+b*d*ln(c*x^n))^2)/Pi
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\text{fresnelc}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\text{FresnelC}\left(d\ \left(a+b\ \ln\left(c\,x^n\right)\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a + b*log(c*x^n)))/x,x)
```

```
[Out] int(FresnelC(d*(a + b*log(c*x^n)))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C\left(ad + bd\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x, x)
```

3.167 $\int \frac{C\left(d\left(a+b\log(cx^n)\right)\right)}{x^2}\,dx$

Optimal. Leaf size=217

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{\frac{1}{n}} e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}} \operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{\frac{1}{n}} e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}} \operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(c\right.}{\sqrt{\pi}\,bd}\right.}{x}$$

```
[Out] (1/4+1/4*I)*exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erf((1/2+1/2*
I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x-(1/4+1/4*I)*ex
p(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erfi((1/2+1/2*I)*(1/n+I*a*b
*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x-FresnelC(d*(a+b*ln(c*x^n)))
/x
```

Rubi [A] time = 0.49, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6472, 4618, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{\frac{1}{n}} e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}} \operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{\frac{1}{n}} e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}} \operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(c\right.}{\sqrt{\pi}\,bd}\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] ((1/4 + I/4)*E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Erf[((1/
2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])]
)/x - ((1/4 + I/4)*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Er
fi[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqr
t[Pi])])/x - FresnelC[d*(a + b*Log[c*x^n])]/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 4618

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6472

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelC[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Cos[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx = -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + (bdn)\int \frac{\cos\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)}{x^2}\,dx$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + \frac{1}{2}(bdn)\int \frac{e^{-\frac{1}{2}id^2\pi\left(a+b\log\left(cx^n\right)\right)^2}}{x^2}\,dx + \frac{1}{2}(bdn)\int \frac{e^{\frac{1}{2}id^2\pi(}}{}$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + \frac{1}{2}(bdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log\left(cx^n\right) - \frac{1}{2}ib^2d\right.}{x^2}$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + \frac{1}{2}(bdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)\left(cx^n\right.}{x^2}$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + \frac{1}{2}\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib\right.$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + \frac{\left(bd\left(cx^n\right)^{-iabd^2\pi-\frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi - \right.\right.}{2x}$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x} + \frac{\left(bde^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}\left(cx^n\right)^{-iabd^2\pi-\frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i\left(-\right.}{}\right.\right.}{2x}$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}\left(cx^n\right)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}\left(c\right.}{}$$

Mathematica [A] time = 4.62, size = 194, normalized size = 0.89

$$-\frac{4C\left(d\left(a+b\log\left(cx^n\right)\right)\right) + \sqrt[4]{-1}\sqrt{2}\left(cx^n\right)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\left(ie^{\frac{i}{\pi b^2d^2n^2}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\pi abd^2n+\pi b^2d^2n\log\left(cx^n\right)+i\right)}{\sqrt{\pi}\,bdn}\right) + \operatorname{erfi}\left(\frac{(-1)^{3/4}(\pi}{}\right.\right.}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[2]*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(
-1)*(Erfi[((-1)^(3/4)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n
*Sqrt[2*Pi])] + I*E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(I + a*b*d^2*n*P
i + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]) + 4*FresnelC[d*(a + b*Log[
c*x^n])])/x
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\left(bd\log\left(cx^{n}\right)+ad\right)}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
[Out] integral(fresnelc(b*d*log(c*x^n) + a*d)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\text{fresnelc}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x^2, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int\frac{\text{FresnelC}\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x^2,x)
[Out] int(FresnelC(d*(a+b*ln(c*x^n)))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\text{fresnelc}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\text{FresnelC}\left(d\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int(FresnelC(d*(a + b*log(c*x^n)))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C\left(ad + bd\log\left(cx^{n}\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x**2, x)
```

3.168 $\int \frac{C\left(d\left(a+b\log(cx^n)\right)\right)}{x^3}\,dx$

Optimal. Leaf size=228

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(cx^n)^{2/n}\,e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x^2}-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(cx^n)^{2/n}\,e^{-\frac{2\left(-\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\right.}{\sqrt{\pi}\,bd}\right.}{x^2}$$

```
[Out] (1/8+1/8*I)*exp((2*I+2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*(c*x^n)^(2/n)*erf((1/2
+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x^2-(1/8+1/
8*I)*(c*x^n)^(2/n)*erfi((1/2+1/2*I)*(2/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n
))/b/d/Pi^(1/2))/exp(2*(I-a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/x^2-1/2*FresnelC(d*
(a+b*ln(c*x^n)))/x^2
```

Rubi [A] time = 0.49, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {6472, 4618, 2278, 2274, 15, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(cx^n)^{2/n}\,e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x^2}-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(cx^n)^{2/n}\,e^{-\frac{2\left(-\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\right.}{\sqrt{\pi}\,bd}\right.}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] ((1/8 + I/8)*E^((2*I + 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*(c*x^n)^(2/n)*Erf[
((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])
])/x^2 - ((1/8 + I/8)*(c*x^n)^(2/n)*Erfi[((1/2 + I/2)*(2/n + I*a*b*d^2*Pi +
 I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I - a*b*d^2*n*Pi))/(b^2
*d^2*n^2*Pi))*x^2) - FresnelC[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 4618

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6472

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelC[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Cos[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx = -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2} + \frac{1}{2}(bdn)\int \frac{\cos\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right)}{x^3}\,dx$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2} + \frac{1}{4}(bdn)\int \frac{e^{-\frac{1}{2}id^2\pi\left(a+b\log(cx^n)\right)^2}}{x^3}\,dx + \frac{1}{4}(bdn)\int \frac{e^{\frac{1}{2}id^2\pi\left(a+\right.}}{}$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2} + \frac{1}{4}(bdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log\left(cx^n\right) - \frac{1}{2}ib^2d^2\right.}{x^3}$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2} + \frac{1}{4}(bdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2\left(cx^n\right)\right)\left(cx^n\right)^{}}{x^3}$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2} + \frac{1}{4}\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2\right.$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2} + \frac{\left(bd\left(cx^n\right)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\pi + \right.\right.}{4x^2}$$

$$= -\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2} + \frac{\left(bde^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}\left(cx^n\right)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(\frac{i(}{}\right.\right.}{4x^2}$$

$$= \frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}\left(cx^n\right)^{2/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} - \frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}}{}$$

Mathematica [A] time = 4.58, size = 199, normalized size = 0.87

$$-\frac{C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{2x^2} - \frac{\sqrt[4]{-1}\left(ie^{\frac{4i}{\pi b^2d^2n^2}}\text{erfi}\left(\frac{\sqrt[4]{-1}\left(\pi abd^2n+\pi b^2d^2n\log(cx^n)+2i\right)}{\sqrt{2\pi}\,bdn}\right) + \text{erfi}\left(\frac{(-1)^{3/4}\left(\pi abd^2n+\pi b^2d^2n\log(cx^n)-2i\right)}{\sqrt{2\pi}\,bdn}\right)\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((2*((a*n)/b - I/(b^2*d^2*Pi) + n*(-(n*Log[x]) + Log[c*x
^n])))/n^2)*(Erfi[((-1)^(3/4)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n
]))/(b*d*n*Sqrt[2*Pi])] + I*E^((4*I)/(b^2*d^2*n^2*Pi))*Erfi[((-1)^(1/4)*(2*
I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])]))/Sqrt[2]
- FresnelC[d*(a + b*Log[c*x^n])]/(2*x^2)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\left(bd\log\left(cx^{n}\right)+ad\right)}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
[Out] integral(fresnelc(b*d*log(c*x^n) + a*d)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x^3, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x^3,x)
[Out] int(FresnelC(d*(a+b*ln(c*x^n)))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
[Out] integrate(fresnelc((b*log(c*x^n) + a)*d)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\text{FresnelC}\left(d\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int(FresnelC(d*(a + b*log(c*x^n)))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C\left(ad + bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x**3, x)
```

3.169 $\int (ex)^m C\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=280

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-\frac{m+1}{n}}\exp\left(\frac{i(m+1)\left(2i\pi abd^2n+m+1\right)}{2\pi b^2d^2n^2}\right)\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2n+i\pi b^2d^2n\log\left(cx^n\right)+m+1\right)}{\sqrt{\pi}\,bdn}\right)}{m+1}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(ex)^m\left(cx^n\right)}{}$$

```
[Out] (1/4+1/4*I)*exp(1/2*I*(1+m)*(1+m+2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x*(e*x)^
m*erf((1/2+1/2*I)*(1+m+I*a*b*d^2*n*Pi+I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1
/2))/(1+m)/((c*x^n)^((1+m)/n))-(1/4+1/4*I)*x*(e*x)^m*erfi((1/2+1/2*I)*(1+m-
I*a*b*d^2*n*Pi-I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1/2))/exp(1/2*I*(1+m)*(1
+m-2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/(1+m)/((c*x^n)^((1+m)/n))+(e*x)^(1+m)*
FresnelC(d*(a+b*ln(c*x^n)))/e/(1+m)
```

Rubi [A] time = 0.61, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {6472, 4618, 2278, 2274, 15, 20, 2276, 2234, 2204, 2205}

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-\frac{m+1}{n}}\exp\left(\frac{i(m+1)\left(2i\pi abd^2n+m+1\right)}{2\pi b^2d^2n^2}\right)\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2n+i\pi b^2d^2n\log\left(cx^n\right)+m+1\right)}{\sqrt{\pi}\,bdn}\right)}{m+1}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(ex)^m\left(cx^n\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/4 + I/4)*E^(((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*x*(e*x)^m*Erf[((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log
[c*x^n]))/(b*d*n*Sqrt[Pi])])/((1 + m)*(c*x^n)^((1 + m)/n)) - ((1/4 + I/4)*x
*(e*x)^m*Erfi[((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])])/(E^(((I/2)*(1 + m)*(1 + m - (2*I)*a*b*d^2*n*Pi))/(
b^2*d^2*n^2*Pi))*(1 + m)*(c*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*FresnelC[d*(
a + b*Log[c*x^n])])/(e*(1 + m))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
```

```
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 2276

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]^2*(b_.))*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(a
*d*Log[F] + ((m + 1)*x)/n + b*d*Log[F]*x^2), x], x, Log[c*x^n]], x] /; Free
Q[{F, a, b, c, d, e, m, n}, x]
```

Rule 2278

```
Int[(F_)^(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.))*((e_.)*(x_))^(m_.)
, x_Symbol] :> Int[(e*x)^m*F^(a^2*d + 2*a*b*d*Log[c*x^n] + b^2*d*Log[c*x^n]
^2), x] /; FreeQ[{F, a, b, c, d, e, m, n}, x]
```

Rule 4618

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6472

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((e*x)^(m + 1)*FresnelC[d*(a + b*Log[c*x^n])])/(e*(m +
 1)), x] - Dist[(b*d*n)/(m + 1), Int[(e*x)^m*Cos[(Pi*(d*(a + b*Log[c*x^n]))
^2)/2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (ex)^m C\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx = \frac{(ex)^{1+m}C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{(bdn)\int (ex)^m\cos\left(\frac{1}{2}d^2\pi\left(a+b\log\left(cx^n\right)\right)^2\right.}{1+m}$$

$$= \frac{(ex)^{1+m}C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{(bdn)\int e^{-\frac{1}{2}id^2\pi\left(a+b\log(cx^n)\right)^2}(ex)^m\,dx}{2(1+m)} - \frac{(bd}{}$$

$$= \frac{(ex)^{1+m}C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{(bdn)\int\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log\left(cx^n\right)\right.}{2(1+m)}$$

$$= \frac{(ex)^{1+m}C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{(bdn)\int\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2\left(c\right.\right.}{2(1+m)}$$

$$= \frac{(ex)^{1+m}C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{\left(bdnx^{iabd^2n\pi}\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(-\frac{1}{2}ia^2d^2\right.}{2(1+}$$

$$= \frac{(ex)^{1+m}C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{\left(bdnx^{-m+iabd^2n\pi}(ex)^m\left(cx^n\right)^{-iabd^2\pi}\right)\int\exp\left(\right.}{2(}$$

$$= \frac{(ex)^{1+m}C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{\left(bdx(ex)^m\left(cx^n\right)^{-iabd^2\pi-\frac{1+m-iabd^2n\pi}{n}}\right)\text{Subst}\left(\right.}{}$$

$$= \frac{(ex)^{1+m}C\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(1+m)} - \frac{\left(bd\exp\left(-\frac{i(1+m)\left(1+m-2iabd^2n\pi\right)}{2b^2d^2n^2\pi}\right)x(ex)^m\left(cx\right.\right.}{}$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\exp\left(\frac{i(1+m)\left(1+m+2iabd^2n\pi\right)}{2b^2d^2n^2\pi}\right)x(ex)^m\left(cx^n\right)^{-\frac{1+m}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+m+iabd^2n\pi+i\right.}{bdn\sqrt{\pi}}\right.}{1+m}$$

Mathematica [A] time = 6.05, size = 244, normalized size = 0.87

$$\frac{(ex)^m\left(4xC\left(d\left(a+b\log\left(cx^n\right)\right)\right)+(-1)^{3/4}\sqrt{2}\,x^{-m}\exp\left(-\frac{(m+1)\left(2\pi abd^2n+2\pi b^2d^2n\left(\log(cx^n)-n\log(x)\right)+im+i\right)}{2\pi b^2d^2n^2}\right)\left(\mathrm{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(\pi ab\right.\right.\right.}{4(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^m*(((-1)^(3/4)*Sqrt[2]*(Erf[((1/2 + I/2)*(I + I*m + a*b*d^2*n*Pi + b
^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])] - E^((I*(1 + m)^2)/(b^2*d^2*n^2*
Pi))*Erfi[((-1)^(3/4)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x^n]))
/(b*d*n*Sqrt[2*Pi])]))/(E^(((1 + m)*(I + I*m + 2*a*b*d^2*n*Pi + 2*b^2*d^2*n
*Pi*(-(n*Log[x]) + Log[c*x^n])))/(2*b^2*d^2*n^2*Pi))*x^m) + 4*x*FresnelC[d*
(a + b*Log[c*x^n])]))/(4*(1 + m))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((ex)^m\ \mathrm{fresnelc}\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*fresnelc(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m\ \mathrm{fresnelc}\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*fresnelc((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (ex)^m\ \mathrm{FresnelC}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*FresnelC(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*FresnelC(d*(a+b*ln(c*x^n))),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \operatorname{fresnelc}\left(\left(b \log\left(cx^n\right) + a\right)d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnelc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] integrate((e*x)^m*fresnelc((b*log(c*x^n) + a)*d), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{FresnelC}\left(d\ \left(a + b\ \ln\left(c\,x^n\right)\right)\right)\ \left(e\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a + b*log(c*x^n)))*(e*x)^m,x)
[Out] int(FresnelC(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m C\left(ad + bd \log\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*fresnelc(d*(a+b*ln(c*x**n))),x)
[Out] Integral((e*x)**m*fresnelc(a*d + b*d*log(c*x**n)), x)
```

3.170 $\int e^{c+\frac{1}{2}ib^2\pi x^2} C(bx)\,dx$

Optimal. Leaf size=64

$$\frac{1}{4}be^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}\,bx\right)^2}{8b}$$

[Out] 1/8*I*exp(c)*erf((1/2-1/2*I)*b*x*Pi^(1/2))^2/b+1/4*b*exp(c)*x^2*HypergeometricPFQ([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)

Rubi [A] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6437, 6376, 6375, 30}

$$\frac{1}{4}be^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\operatorname{Erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}\,bx\right)^2}{8b}$$

Antiderivative was successfully verified.

[In] Int[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]

[Out] ((-I/8)*E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/4

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 6375

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n}, x] && EqQ[d, b^2]
```

Rule 6376

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2])/Sqrt[Pi], x] /; FreeQ[{b, c, d}, x] && EqQ[d, b^2]
```

Rule 6437

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelC[(b_.)*(x_)], x_Symbol] :> Dist[(1 - I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 + I)*b*x)/2], x], x] + Dist[(1 + I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 - I)*b*x)/2], x], x] /; FreeQ[{b, c,
 d}, x] && EqQ[d^2, -((Pi^2*b^4)/4)]
```

Rubi steps

$$\begin{aligned}\int e^{c+\frac{1}{2}ib^2\pi x^2}C(bx)\,dx &= \left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)dx+\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\right.\\ &= \frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{(ie^c)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)\right)}{4b}\\ &= -\frac{ie^c\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)^2}{8b}+\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)\end{aligned}$$

Mathematica [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}C(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
[Out] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(e^{\left(\frac{1}{2}i\pi b^2x^2+c\right)}\operatorname{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
[Out] integral(e^(1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(\frac{1}{2}i\pi b^2x^2+c\right)}\operatorname{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int e^{c+\frac{i b^2 \pi x^2}{2}} \operatorname{FresnelC}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(\frac{1}{2} i \pi b^2 x^2 + c\right)} \operatorname{fresnelc}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{\frac{1\mathrm{i}\, \Pi\, b^2\, x^2}{2}+c} \operatorname{FresnelC}(b\, x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelC(b*x),x)
[Out] int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelC(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{c} \int e^{\frac{i \pi b^2 x^2}{2}} C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] exp(c)*Integral(exp(I*pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.171 $\int e^{c-\frac{1}{2}ib^2\pi x^2} C(bx)\,dx$

Optimal. Leaf size=64

$$\frac{1}{4}be^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}\,bx\right)^2}{8b}$$

```
[Out] -1/8*I*exp(c)*erf((1/2+1/2*I)*b*x*Pi^(1/2))^2/b+1/4*b*exp(c)*x^2*Hypergeome
tricPFQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)
```

Rubi [A] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6437, 6373, 30, 6378}

$$\frac{1}{4}be^c x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\text{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}\,bx\right)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
```

```
[Out] ((-I/8)*E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6373

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(E^c*
Sqrt[Pi])/(2*b), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 6378

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[(b*E^c*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, -(b^2*x^2)])/Sqrt[Pi], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 6437

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelC[(b_.)*(x_)], x_Symbol] :> Dist[(1 - I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 + I)*b*x)/2], x], x] + Dist[(1 + I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]*(1 - I)*b*x)/2], x], x] /; FreeQ[{b, c,
 d}, x] && EqQ[d^2, -((Pi^2*b^4)/4)]
```

Rubi steps

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}C(bx)\,dx = \left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)dx + \left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right.$$

$$= \frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) - \frac{(ie^c)\,\text{Subst}\left(\int x\,dx, x, \text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)\right)}{4b}$$

$$= -\frac{ie^c\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)^2}{8b} + \frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.04, size = 0, normalized size = 0.00

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}C(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
[Out] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)}\text{fresnelc}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
[Out] integral(e^(-1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)}\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{c-\frac{ib^2\pi x^2}{2}} \mathrm{FresnelC}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(-\frac{1}{2}i\pi b^2x^2+c\right)} \mathrm{fresnelc}(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{c-\frac{\Pi b^2 x^2 1\mathrm{i}}{2}} \mathrm{FresnelC}(b\, x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelC(b*x),x)
```

```
[Out] int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelC(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^c \int e^{-\frac{i\pi b^2x^2}{2}} C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] exp(c)*Integral(exp(-I*pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.172 $\int C(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=101

$$\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\cos(c)C(bx)S(bx)}{2b}+\frac{\sin(c)C(bx)^2}{2b}$$

```
[Out] 1/2*cos(c)*FresnelC(b*x)*FresnelS(b*x)/b+1/8*I*b*x^2*cos(c)*HypergeometricP
FQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)-1/8*I*b*x^2*cos(c)*HypergeometricPFQ(
[1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)+1/2*FresnelC(b*x)^2*sin(c)/b
```

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6449, 6441, 30, 6447}

$$\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\cos(c)\text{FresnelC}(bx)S(bx)}{2b}+\frac{\sin(c)\text{F}}{}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```

```
[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*Cos[c]*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2] - (I/8)*b*x^2*Cos[c]*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] + (FresnelC[b*x]^2*Sin[c])/(2*b
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6449

```
Int[FresnelC[(b_.)*(x_)]*Sin[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[Sin[c]
, Int[Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[Cos[c], Int[Sin[d*x^2]*Fresne
lC[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int C(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)dx=\cos(c)\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx+\sin(c)\int\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$
$$=\frac{\cos(c)C(bx)S(bx)}{2b}+\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2$$
$$=\frac{\cos(c)C(bx)S(bx)}{2b}+\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2$$

Mathematica [F] time = 0.04, size = 0, normalized size = 0.00

$$\int C(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
[Out] Integrate[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

maple **[F]** time = 0.03, size = 0, normalized size = 0.00

$$\int \text{FresnelC}\left(bx\right)\sin\left(c+\frac{b^{2}\pi\,x^{2}}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

```
[Out] int(FresnelC(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{\Pi\,b^{2}\,x^{2}}{2}+c\right)\text{FresnelC}\left(b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + (Pi*b^2*x^2)/2)*FresnelC(b*x),x)
```

```
[Out] int(sin(c + (Pi*b^2*x^2)/2)*FresnelC(b*x), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{\pi b^{2}x^{2}}{2}+c\right)C\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(c+1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2 + c)*fresnelc(b*x), x)
```

3.173 $\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$

Optimal. Leaf size=101

$$-\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{\sin(c)C(bx)S(bx)}{2b}+\frac{\cos(c)C(bx)^2}{2b}$$

```
[Out] 1/2*cos(c)*FresnelC(b*x)^2/b-1/2*FresnelC(b*x)*FresnelS(b*x)*sin(c)/b-1/8*I
*b*x^2*HypergeometricPFQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)*sin(c)+1/8*I*b*
x^2*HypergeometricPFQ([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)*sin(c)
```

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6443, 6441, 30, 6447}

$$-\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{\sin(c)\text{FresnelC}(bx)S(bx)}{2b}+\frac{\cos(c)\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (Cos[c]*FresnelC[b*x]^2)/(2*b) - (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
 - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2]*Sin[c
] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin[c
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6443

```
Int[Cos[(c_) + (d_.)*(x_)^2]*FresnelC[(b_.)*(x_)], x_Symbol] :> Dist[Cos[c]
, Int[Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[Sin[c], Int[Sin[d*x^2]*Fresne
lC[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx = \cos(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx - \sin(c)\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

$$= -\frac{C(bx)S(bx)\sin(c)}{2b} - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c) + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2}\right.$$

$$= \frac{\cos(c)C(bx)^2}{2b} - \frac{C(bx)S(bx)\sin(c)}{2b} - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c) + \frac{1}{8}i$$

Mathematica [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x],x]
[Out] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\cos\left(\frac{1}{2}\,\pi b^2x^2+c\right)\mathrm{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}\,\pi b^2x^2+c\right)\mathrm{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \cos\left(c+\frac{b^2\pi\, x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c+1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(cos(c+1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}\,\pi b^2 x^2+c\right)\mathrm{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(\frac{\Pi\, b^2\, x^2}{2}+c\right)\,\mathrm{FresnelC}\left(b\, x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + (Pi*b^2*x^2)/2)*FresnelC(b*x),x)
[Out] int(cos(c + (Pi*b^2*x^2)/2)*FresnelC(b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{\pi b^2 x^2}{2}+c\right)C\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Integral(cos(pi*b**2*x**2/2 + c)*fresnelc(b*x), x)
```

3.174 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)^2\, dx$

Optimal. Leaf size=13

$$\frac{C(bx)^3}{3b}$$

[Out] 1/3*FresnelC(b*x)^3/b

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6441, 30}

$$\frac{\text{FresnelC}(bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^2,x]

[Out] FresnelC[b*x]^3/(3*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned} \int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)^2\, dx &= \frac{\text{Subst}\left(\int x^2\, dx, x, C(bx)\right)}{b} \\ &= \frac{C(bx)^3}{3b} \end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{C(bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^2,x]
```

```
[Out] FresnelC[b*x]^3/(3*b)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\left(bx\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)^2, x)
```

maple [A] time = 0.02, size = 12, normalized size = 0.92

$$\frac{\text{FresnelC}\left(bx\right)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)^2,x)
```

```
[Out] 1/3*FresnelC(b*x)^3/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \text{FresnelC}\,(b\,x)^2\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelC(b*x)^2*cos((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 1.18, size = 10, normalized size = 0.77

$$\begin{cases} \frac{C^3(bx)}{3b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)**2,x)
[Out] Piecewise((fresnelc(b*x)**3/(3*b), Ne(b, 0)), (0, True))
```

3.175 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\, dx$

Optimal. Leaf size=13

$$\frac{C(bx)^2}{2b}$$

```
[Out] 1/2*FresnelC(b*x)^2/b
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6441, 30}

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] FresnelC[b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\, dx &= \frac{\text{Subst}\left(\int x\, dx, x, C(bx)\right)}{b}\\ &= \frac{C(bx)^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{C(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] FresnelC[b*x]^2/(2*b)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnelc}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnelc}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

maple [A] time = 0.02, size = 12, normalized size = 0.92

$$\frac{\text{FresnelC}\,(bx)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] 1/2*FresnelC(b*x)^2/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnelc}\,(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \text{FresnelC}\,(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 0.39, size = 10, normalized size = 0.77

$$\begin{cases} \frac{C^2(bx)}{2b} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Piecewise((fresnelc(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

3.176 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)}\,dx$

Optimal. Leaf size=9

$$\frac{\log(C(bx))}{b}$$

```
[Out] ln(FresnelC(b*x))/b
```

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6441, 29}

$$\frac{\log(\text{FresnelC}(bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x],x]
```

```
[Out] Log[FresnelC[b*x]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, C(bx)\right)}{b}\\ &= \frac{\log(C(bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{\log(C(bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x],x]
```

```
[Out] Log[FresnelC[b*x]]/b
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{fresnelc}\,(bx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)/fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{fresnelc}\,(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x), x)
```

maple [A] time = 0.08, size = 10, normalized size = 1.11

$$\frac{\ln\left(\text{FresnelC}\,(bx)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x),x)
```

```
[Out] ln(FresnelC(b*x))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{fresnelc}\,(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{\cos\left(\frac{\Pi\, b^2\, x^2}{2}\right)}{\mathrm{FresnelC}\left(b\, x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x),x)
```

```
[Out] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x), x)
```

sympy [A] time = 0.24, size = 10, normalized size = 1.11

$$\begin{cases} \frac{\log\left(C\left(bx\right)\right)}{b} & \text{for}\ b \neq 0 \\ \tilde{\infty} x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x),x)
```

```
[Out] Piecewise((log(fresnelc(b*x))/b, Ne(b, 0)), (zoo*x, True))
```

3.177 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)^2}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{bC(bx)}$$

```
[Out] -1/b/FresnelC(b*x)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6441, 30}

$$-\frac{1}{b\text{FresnelC}(bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^2,x]
```

```
[Out] -(1/(b*FresnelC[b*x]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, C(bx)\right)}{b}\\ &= -\frac{1}{bC(bx)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{1}{bC(bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^2,x]
```

```
[Out] -(1/(b*FresnelC[b*x]))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\text{fresnelc}(bx)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\text{fresnelc}(bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^2, x)
```

maple [A] time = 0.02, size = 12, normalized size = 1.09

$$-\frac{1}{b\,\text{FresnelC}(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x)^2,x)
```

```
[Out] -1/b/FresnelC(b*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\text{fresnelc}(bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{\cos\left(\frac{\Pi b^2 x^2}{2}\right)}{\mathrm{FresnelC}\left(b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^2,x)
```

```
[Out] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^2, x)
```

sympy [A] time = 0.75, size = 12, normalized size = 1.09

$$\begin{cases} -\frac{1}{bC(bx)} & \text{for}\ b \neq 0 \\ \tilde{\infty} x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x)**2,x)
```

```
[Out] Piecewise((-1/(b*fresnelc(b*x)), Ne(b, 0)), (zoo*x, True))
```

3.178 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2bC(bx)^2}$$

```
[Out] -1/2/b/FresnelC(b*x)^2
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6441, 30}

$$-\frac{1}{2b\text{FresnelC}(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^3,x]
```

```
[Out] -1/(2*b*FresnelC[b*x]^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, C(bx)\right)}{b}\\ &= -\frac{1}{2bC(bx)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$-\frac{1}{2bC(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^3,x]
[Out] -1/2*1/(b*FresnelC[b*x]^2)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{\mathrm{fresnelc}\left(bx\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^3,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{\mathrm{fresnelc}\left(bx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^3,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^3, x)
```

maple [A] time = 0.02, size = 12, normalized size = 0.92

$$-\frac{1}{2b\,\mathrm{FresnelC}\left(bx\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x)^3,x)
[Out] -1/2/b/FresnelC(b*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{\mathrm{fresnelc}\left(bx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnelc(b*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnelc(b*x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\cos\left(\frac{\Pi b^2 x^2}{2}\right)}{\mathrm{FresnelC}\,(b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^3,x)
```

```
[Out] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^3, x)
```

sympy [A] time = 1.66, size = 15, normalized size = 1.15

$$\begin{cases} -\frac{1}{2bC^2(bx)} & \text{for } b \neq 0 \\ \tilde{\infty} x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x)**3,x)
```

```
[Out] Piecewise((-1/(2*b*fresnelc(b*x)**2), Ne(b, 0)), (zoo*x, True))
```

3.179 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)^n\,dx$

Optimal. Leaf size=17

$$\frac{C(bx)^{n+1}}{b(n+1)}$$

```
[Out] FresnelC(b*x)^(1+n)/b/(1+n)
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6441, 30}

$$\frac{\text{FresnelC}(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^n,x]
```

```
[Out] FresnelC[b*x]^(1 + n)/(b*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)^n\,dx &= \frac{\text{Subst}\left(\int x^n\,dx, x, C(bx)\right)}{b}\\ &= \frac{C(bx)^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{C(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^n,x]
[Out] FresnelC[b*x]^(1 + n)/(b*(1 + n))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnelc}\,(bx)^n\cos\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^n,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnelc}\,(bx)^n\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^n,x, algorithm="giac")
[Out] integrate(fresnelc(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.02, size = 18, normalized size = 1.06

$$\frac{\text{FresnelC}\,(bx)^{1+n}}{b\,(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)^n,x)
[Out] FresnelC(b*x)^(1+n)/b/(1+n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnelc}\,(bx)^n\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \operatorname{FresnelC}(b\,x)^n\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^n*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(FresnelC(b*x)^n*cos((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 3.59, size = 34, normalized size = 2.00

$$\begin{cases} \tilde{\infty}x & \text{for } b = 0 \wedge n = -1 \\ 0^n x & \text{for } b = 0 \\ \frac{\log(C(bx))}{b} & \text{for } n = -1 \\ \frac{C(bx)C^n(bx)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)**n,x)
```

```
[Out] Piecewise((zoo*x, Eq(b, 0) & Eq(n, -1)), (0**n*x, Eq(b, 0)), (log(fresnelc(
b*x))/b, Eq(n, -1)), (fresnelc(b*x)*fresnelc(b*x)**n/(b*n + b), True))
```

3.180 $\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\, dx$

Optimal. Leaf size=231

$$\frac{105C(bx)^2}{2\pi^4b^9}+\frac{105x^2}{4\pi^4b^7}-\frac{7x^6}{12\pi^2b^3}+\frac{x^7C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{40\sin\left(\pi b^2x^2\right)}{\pi^5b^9}-\frac{105xC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{55x^2\cos\left(\pi b^2x^2\right)}{4\pi^4b^7}$$

```
[Out] 105/4*x^2/b^7/Pi^4-7/12*x^6/b^3/Pi^2-55/4*x^2*cos(b^2*Pi*x^2)/b^7/Pi^4+1/4*
x^6*cos(b^2*Pi*x^2)/b^3/Pi^2-105*x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^8/Pi
^4+7*x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^4/Pi^2+105/2*FresnelC(b*x)^2/b
^9/Pi^4-35*x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^7*FresnelC(b*x)
*sin(1/2*b^2*Pi*x^2)/b^2/Pi+40*sin(b^2*Pi*x^2)/b^9/Pi^5-5/2*x^4*sin(b^2*Pi*
x^2)/b^5/Pi^3
```

Rubi [A] time = 0.38, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6455, 6463, 6441, 30, 3380, 2634, 3379, 3296, 2637, 3309}

$$\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{35x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{7x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{105x\text{FresnelC}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (105*x*Cos[(b^2*Pi*x^2)/
2]*FresnelC[b*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b
^4*Pi^2) + (105*FresnelC[b*x]^2)/(2*b^9*Pi^4) - (35*x^3*FresnelC[b*x]*Sin[(
b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*P
i) + (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) - (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx = \frac{x^7C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{7\int x^6C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^7\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^7C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,d}{b^4\pi^2}$$

$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{35x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7C(bx)\,\mathrm{s}}{}$$

$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{35x^3C(b}{}$$

$$= -\frac{7x^6}{12b^3\pi^2} - \frac{41x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{7x}{}$$

$$= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(b}{b^8\pi^4}$$

$$= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(b}{b^8\pi^4}$$

Mathematica [A] time = 0.01, size = 231, normalized size = 1.00

$$\frac{105C(bx)^2}{2\pi^4b^9} + \frac{105x^2}{4\pi^4b^7} - \frac{7x^6}{12\pi^2b^3} + \frac{x^7C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{40\sin\left(\pi b^2x^2\right)}{\pi^5b^9} - \frac{105xC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8} - \frac{55x^2\cos\left(\pi b^2x\right.}{4\pi^4b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (105*x*Cos[(b^2*Pi*x^2)/
```

```
2]*FresnelC[b*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b
^4*Pi^2) + (105*FresnelC[b*x]^2)/(2*b^9*Pi^4) - (35*x^3*FresnelC[b*x]*Sin[(
b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*P
i) + (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) - (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(x^8 \cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{fresnelc}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
[Out] integral(x^8*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{fresnelc}\left(bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^8 \cos\left(\frac{b^2 \pi x^2}{2}\right)\text{FresnelC}\left(bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{fresnelc}\left(bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^8 \operatorname{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^8*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Timed out
```

3.181 $\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\, dx$

Optimal. Leaf size=215

$$\frac{531C\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,\pi^4b^8}+\frac{24x}{\pi^4b^7}-\frac{3x^5}{5\pi^2b^3}+\frac{x^6C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{48C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{147x\cos\left(\pi b^2x^2\right)}{16\pi^4b^7}-\frac{24x^2C(bx)\sin}{\pi^3 b}$$

```
[Out] 24*x/b^7/Pi^4-3/5*x^5/b^3/Pi^2-147/16*x*cos(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^5*co
s(b^2*Pi*x^2)/b^3/Pi^2-48*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^8/Pi^4+6*x^4*
cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^4/Pi^2-24*x^2*FresnelC(b*x)*sin(1/2*b^2
*Pi*x^2)/b^6/Pi^3+x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-17/8*x^3*sin
(b^2*Pi*x^2)/b^5/Pi^3+531/32*FresnelC(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6455, 6463, 6461, 3358, 3352, 3385, 3392, 30, 3386}

$$\frac{x^6\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{24x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{6x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{48\text{FresnelC}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (24*x)/(b^7*Pi^4) - (3*x^5)/(5*b^3*Pi^2) - (147*x*Cos[b^2*Pi*x^2])/(16*b^7*
Pi^4) + (x^5*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (48*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(b^8*Pi^4) + (6*x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2)
+ (531*FresnelC[Sqrt[2]*b*x])/(16*Sqrt[2]*b^8*Pi^4) - (24*x^2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/
(b^2*Pi) - (17*x^3*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
```

]

Rubi steps

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx = \frac{x^6 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{6\int x^5 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^6 \sin\left(b^2\pi x^2\right)dx}{2b\pi}$$

$$= \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^6 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{24\int x^3 \text{co}}{}$$

$$= \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{24x^2 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^6C(bx}{}$$

$$= -\frac{3x^5}{5b^3\pi^2} - \frac{111x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{6x^4\text{ c}}{}$$

$$= -\frac{3x^5}{5b^3\pi^2} - \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{6x^4\text{ c}}{}$$

$$= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} - \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4}$$

$$= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} - \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4}$$

Mathematica [A] time = 0.27, size = 154, normalized size = 0.72

$$\frac{160C(bx)\left(\pi b^2x^2\left(\pi^2b^4x^4-24\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)+6\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)\right)+2bx\left(5\left(4\pi^2b^4x^4-147\right)\cos\left(\pi b\right.\right.}{160\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 160*FresnelC[b*x]*(6*(-8 + b^4*Pi^2*x
^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/
2]) + 2*b*x*(5*(-147 + 4*b^4*Pi^2*x^4)*Cos[b^2*Pi*x^2] - 2*(-960 + 24*b^4*P
i^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/(160*b^8*Pi^4)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(x^7\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^7*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

maple [A] time = 0.02, size = 317, normalized size = 1.47

$$\frac{\mathrm{FresnelC}(bx)\left(\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{6\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{\pi}\right)}{b^7}-\frac{\frac{\frac{3}{5}\pi^2b^5x^5-24bx}{\pi^4}+\frac{\frac{3\pi b^3x^3\sin\left(b^2\pi x^2\right)}{2}-\frac{9\pi\left(-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi}+\frac{\sqrt{2}\ \mathrm{F}\cdots}{\cdots}\right)}{2}}{\pi^4}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] (FresnelC(b*x)/b^7*(1/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)-6/Pi*(-1/Pi*b^4*x^4*co
s(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2
*Pi*x^2))))-1/b^7*(3/Pi^4*(1/5*Pi^2*b^5*x^5-8*b*x)+3/Pi^4*(1/2*Pi*b^3*x^3*s
in(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(
b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))+1/2/Pi^3*(-1/2*Pi*b^5*x^5*co
s(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*co
s(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x
^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \operatorname{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^7*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Integral(x**7*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.182 $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx$

Optimal. Leaf size=247

$$\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}-\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}+\frac{15C(bx)S(bx)}{2\pi^3b^7}-\frac{5x^4}{8\pi^2b^3}+\frac{x^5C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-11$$

```
[Out] -5/8*x^4/b^3/Pi^2-11/2*cos(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^4*cos(b^2*Pi*x^2)/b^3
/Pi^2+5*x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^4/Pi^2+15/2*FresnelC(b*x)*F
resnelS(b*x)/b^7/Pi^3+15/8*I*x^2*HypergeometricPFQ([1, 1],[3/2, 2],-1/2*I*b
^2*Pi*x^2)/b^5/Pi^3-15/8*I*x^2*HypergeometricPFQ([1, 1],[3/2, 2],1/2*I*b^2*
Pi*x^2)/b^5/Pi^3-15*x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^5*Fresne
lC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-7/4*x^2*sin(b^2*Pi*x^2)/b^5/Pi^3
```

Rubi [A] time = 0.25, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6455, 6463, 6447, 3379, 2638, 3380, 3309, 30, 3296}

$$\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}-\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}+\frac{15\text{FresnelC}(bx)S(bx)}{2\pi^3b^7}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b\right.}{\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-5*x^4)/(8*b^3*Pi^2) - (11*Cos[b^2*Pi*x^2])/(2*b^7*Pi^4) + (x^4*Cos[b^2*Pi
*x^2])/(4*b^3*Pi^2) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2)
+ (15*FresnelC[b*x]*FresnelS[b*x])/(2*b^7*Pi^3) + (((15*I)/8)*x^2*Hypergeom
etricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (((15*I)/8)*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (15*x*
FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^5*FresnelC[b*x]*Sin[(b^2
*Pi*x^2)/2])/(b^2*Pi) - (7*x^2*Sin[b^2*Pi*x^2])/(4*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
```

```
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx = \frac{x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{5\int x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^2\pi} - \frac{\int x^5\sin\left(b^2\pi x^2\right)dx}{2b\pi}$$

$$= \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{15\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,d}{b^4\pi^2}$$

$$= \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{15xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5C(bx)\,\mathrm{si}}{b}$$

$$= \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{15C(bx)S(bx)}{2b^7\pi^3} + \frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\right.}{8b^5\pi^3}$$

$$= -\frac{5x^4}{8b^3\pi^2} - \frac{17\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{15C(bx}{2b^7}$$

$$= -\frac{5x^4}{8b^3\pi^2} - \frac{11\cos\left(b^2\pi x^2\right)}{2b^7\pi^4} + \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{15C(bx}{2b^7}$$

Mathematica [F] time = 0.44, size = 0, normalized size = 0.00

$$\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
[Out] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^6\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnelc}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
[Out] integral(x^6*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{fresnelc}\,(bx)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^6 \cos\left(\frac{b^2 \pi\, x^2}{2}\right) \mathrm{FresnelC}\,(bx)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{fresnelc}\,(bx)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\,\mathrm{FresnelC}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^6*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x**6*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.183 $\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\, dx$

Optimal. Leaf size=157

$$\frac{43S\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,\pi^3b^6}-\frac{2x^3}{3\pi^2b^3}+\frac{x^4C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{8C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}-\frac{11x\sin\left(\pi b^2x^2\right)}{8\pi^3b^5}+\frac{4x^2C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+$$

```
[Out] -2/3*x^3/b^3/Pi^2+1/4*x^3*cos(b^2*Pi*x^2)/b^3/Pi^2+4*x^2*cos(1/2*b^2*Pi*x^2
)*FresnelC(b*x)/b^4/Pi^2-8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^4*F
resnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-11/8*x*sin(b^2*Pi*x^2)/b^5/Pi^3+43/
16*FresnelS(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6455, 6463, 6453, 3351, 3392, 30, 3386, 3385}

$$\frac{x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{8\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{4x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{43S\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,\pi^3b^6}-$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-2*x^3)/(3*b^3*Pi^2) + (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (4*x^2*Cos[(b^
2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) + (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt
[2]*b^6*Pi^3) - (8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^4*Fre
snelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi
^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
```

```
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx = \frac{x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{4\int x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^4\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{8\int x\cos\left(\right.}{}$$

$$= \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{8C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^4C(bx)\sin}{b^2}$$

$$= -\frac{2x^3}{3b^3\pi^2} + \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{3S\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,b^6\pi^3} + \frac{2\sqrt{2}\,S\left(\sqrt{}\right.}{b^6\pi^3}$$

$$= -\frac{2x^3}{3b^3\pi^2} + \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{11S\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,b^6\pi^3} + \frac{2\sqrt{2}\,S\left(\right.}{b^6\pi}$$

Mathematica [A] time = 0.14, size = 120, normalized size = 0.76

$$\frac{-32\pi b^3x^3 - 66bx\sin\left(\pi b^2x^2\right) + 48C(bx)\left(4\pi b^2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right) + \left(\pi^2b^4x^4 - 8\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\right) + 12\pi b^3x^3\cos\left(\right.}{48\pi^3b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 129*Sqrt[2]*FresnelS[Sqrt
[2]*b*x] + 48*FresnelC[b*x]*(4*b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] + (-8 + b^4*P
i^2*x^4)*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin[b^2*Pi*x^2])/(48*b^6*Pi^3)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(x^5\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\left(bx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^5*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")

[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)

maple [A] time = 0.02, size = 202, normalized size = 1.29

$$\frac{\frac{\mathrm{FresnelC}(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{b^5}-\frac{\frac{2b^3x^3}{3\pi^2}+\frac{\frac{bx\sin\left(b^2\pi x^2\right)}{\pi}-\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{2\pi}}{\pi^2}+\frac{-\frac{\pi b^3x^3\cos\left(b^2\pi x^2\right)}{2}+\frac{3\pi\left(\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}-\frac{\sqrt{2}\,\mathrm{S}(\cdots}{4\cdots}\right.}{2}\cdots}{2\pi^3}\cdots}{b^5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)

[Out] (FresnelC(b*x)/b^5*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*co
s(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))-1/b^5*(2/3/Pi^2*b^3*x^3+2/Pi
^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))+1/2/Pi
^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/
Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")

[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,\mathrm{FresnelC}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)

[Out] int(x^5*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.184 $\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx$

Optimal. Leaf size=120

$$-\frac{3C(bx)^2}{2\pi^2b^5}-\frac{3x^2}{4\pi^2b^3}+\frac{x^3C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\sin\left(\pi b^2x^2\right)}{\pi^3b^5}+\frac{3xC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^2\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

```
[Out] -3/4*x^2/b^3/Pi^2+1/4*x^2*cos(b^2*Pi*x^2)/b^3/Pi^2+3*x*cos(1/2*b^2*Pi*x^2)*
FresnelC(b*x)/b^4/Pi^2-3/2*FresnelC(b*x)^2/b^5/Pi^2+x^3*FresnelC(b*x)*sin(1
/2*b^2*Pi*x^2)/b^2/Pi-sin(b^2*Pi*x^2)/b^5/Pi^3
```

Rubi [A] time = 0.12, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6455, 6463, 6441, 30, 3380, 2634, 3379, 3296, 2637}

$$\frac{x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{3x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{3\text{FresnelC}(bx)^2}{2\pi^2b^5}-\frac{3x^2}{4\pi^2b^3}-\frac{\sin\left(\pi b^2x^2\right)}{\pi^3b^5}+\frac{x^2\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (3*x*Cos[(b^2*
Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]^2)/(2*b^5*Pi^2) + (
x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx = \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3\int x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^3\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3\int\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^4\pi^2} - \frac{3}{}$$

$$= \frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3\,\mathrm{Subst}(\int x}{b^5}$$

$$= -\frac{3x^2}{4b^3\pi^2} + \frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{3C(bx)^2}{2b^5\pi^2} + \frac{x^3C(bx)\sin\left(\frac{1}{2}\right.}{b^2\pi}$$

Mathematica [A] time = 0.01, size = 120, normalized size = 1.00

$$-\frac{3C(bx)^2}{2\pi^2b^5}-\frac{3x^2}{4\pi^2b^3}+\frac{x^3C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\sin\left(\pi b^2x^2\right)}{\pi^3b^5}+\frac{3xC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^2\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (3*x*Cos[(b^2*
Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]^2)/(2*b^5*Pi^2) + (
x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^4\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnelc}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^4*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^4 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\text{FresnelC}\,(b\,x)\;\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^4*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 18.87, size = 151, normalized size = 1.26

$$\begin{cases} \frac{x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi b^2} - \frac{x^2 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^2 b^3} - \frac{x^2 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} + \frac{3x \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi^2 b^4} - \frac{2 \sin\left(\frac{\pi b^2 x^2}{2}\right) \cos\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^3 b^5} - \frac{3C^2(bx)}{2\pi^2 b^5} & \text{for}\; b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Piecewise((x**3*sin(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) - x**2*sin(pi*b
**2*x**2/2)**2/(pi**2*b**3) - x**2*cos(pi*b**2*x**2/2)**2/(2*pi**2*b**3) +
3*x*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi**2*b**4) - 2*sin(pi*b**2*x**2/2)*
cos(pi*b**2*x**2/2)/(pi**3*b**5) - 3*fresnelc(b*x)**2/(2*pi**2*b**5), Ne(b,
 0)), (0, True))
```

3.185 $\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx$

Optimal. Leaf size=104

$$-\frac{5C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,\pi^2 b^4}-\frac{x}{\pi^2 b^3}+\frac{x^2 C(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}+\frac{2C(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4}+\frac{x\cos\left(\pi b^2 x^2\right)}{4\pi^2 b^3}$$

```
[Out] -x/b^3/Pi^2+1/4*x*cos(b^2*Pi*x^2)/b^3/Pi^2+2*cos(1/2*b^2*Pi*x^2)*FresnelC(b
*x)/b^4/Pi^2+x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-5/8*FresnelC(b*x*
2^(1/2))/b^4/Pi^2*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6455, 6461, 3358, 3352, 3385}

$$\frac{x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}+\frac{2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4}-\frac{5\text{FresnelC}\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,\pi^2 b^4}+\frac{x\cos\left(\pi b^2 x^2\right)}{4\pi^2 b^3}-\frac{x}{\pi^2 b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] -(x/(b^3*Pi^2)) + (x*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (2*Cos[(b^2*Pi*x^2)/2]
*FresnelC[b*x])/(b^4*Pi^2) - (5*FresnelC[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2)
 + (x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx = \frac{x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{2\int xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^2\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

$$= \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2} + \frac{x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{2}{}$$

$$= -\frac{x}{b^3\pi^2} + \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2} + \frac{x^2C(bx)\sin\left(\frac{1}{2}b^2\right.}{b^2\pi}$$

$$= -\frac{x}{b^3\pi^2} + \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{5C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2} + \frac{x^2C(bx)\sin\left(\frac{1}{2}b\right.}{b^2\pi}$$

Mathematica [A] time = 0.10, size = 83, normalized size = 0.80

$$\frac{8C(bx)\left(\pi b^2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)+2\cos\left(\frac{1}{2}\pi b^2x^2\right)\right)+2bx\left(\cos\left(\pi b^2x^2\right)-4\right)-5\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{8\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (2*b*x*(-4 + Cos[b^2*Pi*x^2]) - 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 8*Fresnel
C[b*x]*(2*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]))/(8*b^4*Pi^
2)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

maple [A] time = 0.02, size = 114, normalized size = 1.10

$$\frac{\frac{\text{FresnelC}(bx)\left(\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{b^3}-\frac{\frac{bx}{\pi^2}+\frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{2\pi^2}+\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi}+\frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] (FresnelC(b*x)/b^3*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*
x^2))-1/b^3*(1/Pi^2*b*x+1/2/Pi^2*2^(1/2)*FresnelC(b*x*2^(1/2))+1/2/Pi*(-1/2
/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^3*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x**3*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.186 $\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx$

Optimal. Leaf size=136

$$-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}-\frac{C(bx)S(bx)}{2\pi b^3}+\frac{xC(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

```
[Out] 1/4*cos(b^2*Pi*x^2)/b^3/Pi^2-1/2*FresnelC(b*x)*FresnelS(b*x)/b^3/Pi-1/8*I*x
^2*HypergeometricPFQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b/Pi+1/8*I*x^2*Hype
rgeometricPFQ([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b/Pi+x*FresnelC(b*x)*sin(1/
2*b^2*Pi*x^2)/b^2/Pi
```

Rubi [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6455, 6447, 3379, 2638}

$$-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}-\frac{\text{FresnelC}(bx)S(bx)}{2\pi b^3}+\frac{x\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] Cos[b^2*Pi*x^2]/(4*b^3*Pi^2) - (FresnelC[b*x]*FresnelS[b*x])/(2*b^3*Pi) - (
(I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b*Pi) +
((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b*Pi) +
(x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
```

```
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx=\frac{xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi}-\frac{\int x\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$=-\frac{C(bx)S(bx)}{2b^3\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}+\frac{xC(\ldots}{\ldots}$$

$$=\frac{\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{C(bx)S(bx)}{2b^3\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib\ldots\right)}{8b\pi}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
[Out] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \mathrm{fresnelc}\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right) \mathrm{FresnelC}\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \mathrm{fresnelc}\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{FresnelC}\,(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^2*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cos\left(\frac{\pi b^2 x^2}{2}\right) C\,(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x**2*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.187 $\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx$

Optimal. Leaf size=48

$$\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,\pi b^2}$$

```
[Out] FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-1/4*FresnelS(b*x*2^(1/2))/b^2/Pi*2
^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6453, 3351}

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] -FresnelS[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*Pi) + (FresnelC[b*x]*Sin[(b^2*Pi*x^2)
/2])/(b^2*Pi)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}
\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx &= \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\
&= -\frac{S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,b^2\pi} + \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.92

$$-\frac{\sqrt{2}\,S\left(\sqrt{2}\,bx\right)-4C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] -1/4*(Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/
(b^2*Pi)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\,(bx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(x*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

maple [A] time = 0.02, size = 45, normalized size = 0.94

$$\frac{\frac{\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi b}-\frac{\text{S}\left(bx\sqrt{2}\right)\sqrt{2}}{4b\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] (FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/Pi/b-1/4*FresnelS(b*x*2^(1/2))/b/Pi*2^(1
/2))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{FresnelC}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(\frac{\pi b^2x^2}{2}\right)C\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Integral(x*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.188 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx$

Optimal. Leaf size=13

$$\frac{C(bx)^2}{2b}$$

[Out] 1/2*FresnelC(b*x)^2/b

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6441, 30}

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]

[Out] FresnelC[b*x]^2/(2*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, C(bx)\right)}{b}\\ &= \frac{C(bx)^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{C(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] FresnelC[b*x]^2/(2*b)
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\left(bx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{\text{FresnelC}\left(bx\right)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

```
[Out] 1/2*FresnelC(b*x)^2/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x),x, algorithm="maxima")
```

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \text{FresnelC}\,(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)

[Out] int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)

sympy [A] time = 0.38, size = 10, normalized size = 0.77

$$\begin{cases} \frac{C^{2}\left(bx\right)}{2b} & \text{for}\: b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)

[Out] Piecewise((fresnelc(b*x)**2/(2*b), Ne(b, 0)), (0, True))

3.189 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)$$

```
[Out] Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x,x]
[Out] Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,dx$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}(bx)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnelc}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2 \pi\, x^2}{2}\right) \text{FresnelC}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnelc}\,(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\text{FresnelC}\,(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x,x)
```

```
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C\left(bx\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x, x)
```

3.190 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)$$

```
[Out] Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2,x]
```

```
[Out] Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}(bx)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^2,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnelc}\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^2,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^2, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelC}\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{fresnelc}\,(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^2,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\text{FresnelC}\,(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^2,x)
```

```
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C\left(bx\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**2,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**2, x)
```

3.191 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^3}\,dx$

Optimal. Leaf size=102

$$-\frac{1}{2}\pi b^2\text{Int}\left(\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{\pi b^2S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{b\cos\left(\pi b^2x^2\right)}{4x}-\frac{b}{4x}$$

```
[Out] -1/4*b/x-1/4*b*cos(b^2*Pi*x^2)/x-1/2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2-
1/4*b^2*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)-1/2*b^2*Pi*Unintegrable(FresnelC(b
*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3,x]
```

```
[Out] -b/(4*x) - (b*Cos[b^2*Pi*x^2])/(4*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/
(2*x^2) - (b^2*Pi*FresnelS[Sqrt[2]*b*x])/(2*Sqrt[2]) - (b^2*Pi*Defer[Int][(
FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/2
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^3}\,dx=-\frac{b}{4x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2x^2}+\frac{1}{4}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{2}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{x}$$

$$=-\frac{b}{4x}-\frac{b\cos\left(b^2\pi x^2\right)}{4x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2x^2}-\frac{1}{2}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx-$$

$$=-\frac{b}{4x}-\frac{b\cos\left(b^2\pi x^2\right)}{4x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2x^2}-\frac{b^2\pi S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{1}{2}\left(b^2\pi\right)\int \frac{C(bx)\,\mathrm{s}}{}$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{fresnelc}(bx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^3,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{fresnelc}(bx)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^3,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^3, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2 \pi x^2}{2}\right)\text{FresnelC}(bx)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{fresnelc}(bx)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^3,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{FresnelC}\,(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^3,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C\left(bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**3,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**3, x)
```

3.192 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^4}\,dx$

Optimal. Leaf size=109

$$-\frac{1}{6}\pi^2b^3C(bx)^2+\frac{\pi b^2C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x}-\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b\cos\left(\pi b^2x^2\right)}{12x^2}-\frac{1}{6}\pi b^3\text{Si}\left(b^2\pi x^2\right)-\frac{b}{12x^2}$$

```
[Out] -1/12*b/x^2-1/12*b*cos(b^2*Pi*x^2)/x^2-1/3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/x^3-1/6*b^3*Pi^2*FresnelC(b*x)^2-1/6*b^3*Pi*Si(b^2*Pi*x^2)+1/3*b^2*Pi*Fre
snelC(b*x)*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A] time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6457, 6465, 6441, 30, 3375, 3380, 3297, 3299}

$$\frac{\pi b^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{6}\pi^2b^3\text{FresnelC}(bx)^2-\frac{1}{6}\pi b^3\text{Si}\left(b^2\pi x^2\right)-\frac{b\cos\left(\pi b\right.}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^4,x]
```

```
[Out] -b/(12*x^2) - (b*Cos[b^2*Pi*x^2])/(12*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^3) - (b^3*Pi^2*FresnelC[b*x]^2)/6 + (b^2*Pi*FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/(3*x) - (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^4}\,dx = -\frac{b}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^3} + \frac{1}{6}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx - \frac{1}{3}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{x^2}$$

$$= -\frac{b}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^3} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x} + \frac{1}{12}b\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi x\right.}{x^2}\right.$$

$$= -\frac{b}{12x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^3} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x} - \frac{1}{12}b^3\pi\mathrm{Si}$$

$$= -\frac{b}{12x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^3} - \frac{1}{6}b^3\pi^2C(bx)^2 + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\right.}{3x}$$

Mathematica [A] time = 0.01, size = 109, normalized size = 1.00

$$-\frac{1}{6}\pi^2b^3C(bx)^2+\frac{\pi b^2C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x}-\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b\cos\left(\pi b^2x^2\right)}{12x^2}-\frac{1}{6}\pi b^3\mathrm{Si}\left(b^2\pi x^2\right)-\frac{b}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^4,x]
```

```
[Out] -1/12*b/x^2 - (b*Cos[b^2*Pi*x^2])/(12*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^3) - (b^3*Pi^2*FresnelC[b*x]^2)/6 + (b^2*Pi*FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/(3*x) - (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnelc}\left(bx\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnelc}\left(bx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^4,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^4, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnelc}\left(bx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^4,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^4,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C\left(bx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**4,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**4, x)
```

3.193 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^5}\,dx$

Optimal. Leaf size=153

$$-\frac{1}{8}\pi^2 b^4 \text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)-\frac{7\pi^2 b^4 C\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}+\frac{\pi b^2 C(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^2}-\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{4x^4}-\frac{b\cos\left(\pi b\right.}{24x^3}$$

```
[Out] -1/24*b/x^3-1/24*b*cos(b^2*Pi*x^2)/x^3-1/4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/x^4+1/8*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+7/48*b^3*Pi*sin(b^2*
Pi*x^2)/x-7/48*b^4*Pi^2*FresnelC(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Unintegr
able(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5,x]
```

```
[Out] -b/(24*x^3) - (b*Cos[b^2*Pi*x^2])/(24*x^3) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(4*x^4) - (7*b^4*Pi^2*FresnelC[Sqrt[2]*b*x])/(24*Sqrt[2]) + (b^2*Pi*F
resnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^2) + (7*b^3*Pi*Sin[b^2*Pi*x^2])/(48*
x) - (b^4*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])/8
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^5}\,dx &= -\frac{b}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4x^4}+\frac{1}{8}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx-\frac{1}{4}\left(b^2\pi\right)\int\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{x^3}\\
&= -\frac{b}{24x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4x^4}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}-\frac{1}{16}\left(b^3\pi\right.\\
&= -\frac{b}{24x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4x^4}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}+\frac{7b^3\pi\sin}{4}\\
&= -\frac{b}{24x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4x^4}-\frac{7b^4\pi^2 C\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}+\frac{b^2\pi C(bx)\sin\left(\right.}{8x^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnelc}\,(bx)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^5,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^5, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnelc}\,(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^5,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^5, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\,(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{fresnelc}\,(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^5,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^5, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{FresnelC}\,(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^5,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^5, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C\,(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**5,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**5, x)
```

3.194 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^6}\,dx$

Optimal. Leaf size=148

$$-\frac{1}{15}\pi^2 b^4 \text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2},x\right)-\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5}+\frac{\pi b^2 C(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{15x^3}-\frac{b\cos\left(\pi b^2 x^2\right)}{40x^4}-\frac{1}{24}\pi^2 b^5 \text{C}$$

```
[Out] -1/40*b/x^4-1/24*b^5*Pi^2*Ci(b^2*Pi*x^2)-1/40*b*cos(b^2*Pi*x^2)/x^4-1/5*cos
(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5+1/15*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi
*x^2)/x^3+1/24*b^3*Pi*sin(b^2*Pi*x^2)/x^2-1/15*b^4*Pi^2*Unintegrable(cos(1/
2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6,x]
```

```
[Out] -b/(40*x^4) - (b*Cos[b^2*Pi*x^2])/(40*x^4) - (b^5*Pi^2*CosIntegral[b^2*Pi*x
^2])/24 - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(5*x^5) + (b^2*Pi*FresnelC[b*
x]*Sin[(b^2*Pi*x^2)/2])/(15*x^3) + (b^3*Pi*Sin[b^2*Pi*x^2])/(24*x^2) - (b^4
*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x])/15
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^6}\,dx = -\frac{b}{40x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5} + \frac{1}{10}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx - \frac{1}{5}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\right.}{x^4}$$

$$= -\frac{b}{40x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} + \frac{1}{20}b\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi x\right.}{x^3}\right.$$

$$= -\frac{b}{40x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{40x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} - \frac{1}{60}\left(b^3\pi\right.$$

$$= -\frac{b}{40x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{40x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} + \frac{b^3\pi\sin}{24}$$

$$= -\frac{b}{40x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{40x^4} - \frac{1}{24}b^5\pi^2\mathrm{Ci}\left(b^2\pi x^2\right) - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5} + \frac{b^2\pi C(bx)\sin}{15x}$$

Mathematica [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{fresnelc}\left(bx\right)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^6,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^6, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{fresnelc}\left(bx\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^6,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^6, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{fresnelc}\left(bx\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^6,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^6, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^6,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^6, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C\left(bx\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**6,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**6, x)
```

3.195 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^7}\,dx$

Optimal. Leaf size=241

$$\frac{1}{48}\pi^3 b^6 \text{Int}\left(\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)+\frac{1}{45}\sqrt{2}\,\pi^3 b^6 S\left(\sqrt{2}\,bx\right)+\frac{7\pi^3 b^6 S\left(\sqrt{2}\,bx\right)}{144\sqrt{2}}+\frac{\pi^2 b^5}{96x}-\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^6}+\frac{\pi b^2 C}{}$$

```
[Out] -1/60*b/x^5+1/96*b^5*Pi^2/x-1/60*b*cos(b^2*Pi*x^2)/x^5+67/1440*b^5*Pi^2*cos
(b^2*Pi*x^2)/x-1/6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6+1/48*b^4*Pi^2*cos(
1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2+1/24*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*
x^2)/x^4+13/720*b^3*Pi*sin(b^2*Pi*x^2)/x^3+67/1440*b^6*Pi^3*FresnelS(b*x*2^
(1/2))*2^(1/2)+1/48*b^6*Pi^3*Unintegrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)
/x,x)
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7,x]
```

```
[Out] -b/(60*x^5) + (b^5*Pi^2)/(96*x) - (b*Cos[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi
^2*Cos[b^2*Pi*x^2])/(1440*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(6*x^6)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(48*x^2) + (7*b^6*Pi^3*Fresn
elS[Sqrt[2]*b*x])/(144*Sqrt[2]) + (Sqrt[2]*b^6*Pi^3*FresnelS[Sqrt[2]*b*x])/
45 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(24*x^4) + (13*b^3*Pi*Sin[b
^2*Pi*x^2])/(720*x^3) + (b^6*Pi^3*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2
)/2])/x, x])/48
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^7}\,dx = -\frac{b}{60x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6} + \frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx - \frac{1}{6}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b\right.}{x^5}$$

$$= -\frac{b}{60x^5} - \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} - \frac{1}{48}\left(b^3\right.$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} - \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{48x^2}$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} - \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6} + \frac{b^4}{}$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} - \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6} + \frac{b^4}{}$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}(bx)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^7,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^7, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^7,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^7, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{fresnelc}\left(bx\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^7,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^7, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^7,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^7, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C\left(bx\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**7,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**7, x)
```

3.196 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^8}\,dx$

Optimal. Leaf size=224

$$\frac{1}{210}\pi^4b^7C(bx)^2+\frac{\pi^2b^5}{420x^2}-\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{\pi b^2C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}-\frac{b\cos\left(\pi b^2x^2\right)}{84x^6}+\frac{1}{70}\pi^3b^7\text{Si}\left(b^2\pi x^2\right)-\frac{\pi^3 b}{}$$

```
[Out] -1/84*b/x^6+1/420*b^5*Pi^2/x^2-1/84*b*cos(b^2*Pi*x^2)/x^6+1/84*b^5*Pi^2*cos
(b^2*Pi*x^2)/x^2-1/7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7+1/105*b^4*Pi^2*c
os(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3+1/210*b^7*Pi^4*FresnelC(b*x)^2+1/70*b^
7*Pi^3*Si(b^2*Pi*x^2)+1/35*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/1
05*b^6*Pi^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x+1/105*b^3*Pi*sin(b^2*Pi*x^2
)/x^4
```

Rubi [A] time = 0.35, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6457, 6465, 6441, 30, 3375, 3380, 3297, 3299, 3379}

$$-\frac{\pi^3b^6\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{105x}+\frac{\pi b^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}+\frac{\pi^2b^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{105x^3}-\frac{\text{Fresne}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^8,x]
```

```
[Out] -b/(84*x^6) + (b^5*Pi^2)/(420*x^2) - (b*Cos[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*x^7)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(105*x^3) + (b^7*Pi^4*Fresne
lC[b*x]^2)/210 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(35*x^5) - (b^6
*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x) + (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) + (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
```

```
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^8}\,dx = -\frac{b}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{1}{14}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx - \frac{1}{7}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}\right.}{x^6}$$

$$= -\frac{b}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} + \frac{1}{28}b\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\pi\right.}{x^4}\right.$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx}{105x^3}$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx}{105x^3}$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} + \frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^4\pi}{}$$

$$= -\frac{b}{84x^6} + \frac{b^5\pi^2}{420x^2} - \frac{b\cos\left(b^2\pi x^2\right)}{84x^6} + \frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7} + \frac{b^4\pi}{}$$

Mathematica [A] time = 0.01, size = 224, normalized size = 1.00

$$\frac{1}{210}\pi^4b^7C(bx)^2+\frac{\pi^2b^5}{420x^2}-\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{\pi b^2C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}-\frac{b\cos\left(\pi b^2x^2\right)}{84x^6}+\frac{1}{70}\pi^3b^7\mathrm{Si}\left(b^2\pi x^2\right)-\frac{\pi^3 b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^8,x]
```

```
[Out] -1/84*b/x^6 + (b^5*Pi^2)/(420*x^2) - (b*Cos[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*x^7)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(105*x^3) + (b^7*Pi^4*Fresne
lC[b*x]^2)/210 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(35*x^5) - (b^6
*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x) + (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) + (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\,(bx)}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^8,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^8,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^8, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{fresnelc}\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^8,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\text{FresnelC}\,(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^8,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C\,(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**8,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**8, x)
```

3.197 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^9}\,dx$

Optimal. Leaf size=268

$$\frac{1}{384}\pi^4 b^8 \text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)+\frac{853\pi^4 b^8 C\left(\sqrt{2}\,bx\right)}{40320\sqrt{2}}+\frac{\pi^2 b^5}{1152x^3}-\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8}+\frac{\pi b^2 C(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{48x^6}$$

```
[Out] -1/112*b/x^7+1/1152*b^5*Pi^2/x^3-1/112*b*cos(b^2*Pi*x^2)/x^7+187/40320*b^5*
Pi^2*cos(b^2*Pi*x^2)/x^3-1/8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8+1/192*b^
4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4+1/48*b^2*Pi*FresnelC(b*x)*sin(
1/2*b^2*Pi*x^2)/x^6-1/384*b^6*Pi^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+19
/3360*b^3*Pi*sin(b^2*Pi*x^2)/x^5-853/80640*b^7*Pi^3*sin(b^2*Pi*x^2)/x+853/8
0640*b^8*Pi^4*FresnelC(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrable(cos
(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [A] time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9,x]
```

```
[Out] -b/(112*x^7) + (b^5*Pi^2)/(1152*x^3) - (b*Cos[b^2*Pi*x^2])/(112*x^7) + (187
*b^5*Pi^2*Cos[b^2*Pi*x^2])/(40320*x^3) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(8*x^8) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(192*x^4) + (853*b
^8*Pi^4*FresnelC[Sqrt[2]*b*x])/(40320*Sqrt[2]) + (b^2*Pi*FresnelC[b*x]*Sin[
(b^2*Pi*x^2)/2])/(48*x^6) - (b^6*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3
84*x^2) + (19*b^3*Pi*Sin[b^2*Pi*x^2])/(3360*x^5) - (853*b^7*Pi^3*Sin[b^2*Pi
*x^2])/(80640*x) + (b^8*Pi^4*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/x, x])/384
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^9}\,dx = -\frac{b}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{1}{16}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx - \frac{1}{8}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}\right.}{x^7}$$

$$= -\frac{b}{112x^7} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} - \frac{1}{96}\left(b\right.$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(}{192x^4}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnelc}\left(bx\right)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^9,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^9, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\operatorname{fresnelc}\left(bx\right)}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^9,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^9, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)\operatorname{FresnelC}\left(bx\right)}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^9,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^9,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\operatorname{fresnelc}\left(bx\right)}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^9,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^9, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{FresnelC}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^{2}\,x^{2}}{2}\right)}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^9,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^9, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C\left(bx\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**9,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**9, x)
```

3.198 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^{10}}\,dx$

Optimal. Leaf size=263

$$\frac{1}{945}\pi^4 b^8 \text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{x^2},x\right)+\frac{\pi^2 b^5}{2520x^4}-\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+\frac{\pi b^2 C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{63x^7}-\frac{b\cos\left(\pi b^2x^2\right)}{144x^8}+-$$

```
[Out] -1/144*b/x^8+1/2520*b^5*Pi^2/x^4+5/2016*b^9*Pi^4*Ci(b^2*Pi*x^2)-1/144*b*cos
(b^2*Pi*x^2)/x^8+67/30240*b^5*Pi^2*cos(b^2*Pi*x^2)/x^4-1/9*cos(1/2*b^2*Pi*x
^2)*FresnelC(b*x)/x^9+1/315*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5+
1/63*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7-1/945*b^6*Pi^3*FresnelC(b
*x)*sin(1/2*b^2*Pi*x^2)/x^3+11/3024*b^3*Pi*sin(b^2*Pi*x^2)/x^6-5/2016*b^7*P
i^3*sin(b^2*Pi*x^2)/x^2+1/945*b^8*Pi^4*Unintegrable(cos(1/2*b^2*Pi*x^2)*Fre
snelC(b*x)/x^2,x)
```

Rubi [A] time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10,x]
```

```
[Out] -b/(144*x^8) + (b^5*Pi^2)/(2520*x^4) - (b*Cos[b^2*Pi*x^2])/(144*x^8) + (67*
b^5*Pi^2*Cos[b^2*Pi*x^2])/(30240*x^4) + (5*b^9*Pi^4*CosIntegral[b^2*Pi*x^2]
)/2016 - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(315*x^5) + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)
/2])/(63*x^7) - (b^6*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) + (1
1*b^3*Pi*Sin[b^2*Pi*x^2])/(3024*x^6) - (5*b^7*Pi^3*Sin[b^2*Pi*x^2])/(2016*x
^2) + (b^8*Pi^4*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x])/945
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^{10}}\,dx = -\frac{b}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{1}{18}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^9}\,dx - \frac{1}{9}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}\right.}{x^8}$$

$$= -\frac{b}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} + \frac{1}{36}b\,\mathrm{Subst}\left(\int \frac{\cos\left(b^2\right.}{x^5}\right.$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(}{315x^5}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(}{315x^5}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} -$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} -$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} + \frac{5b^9\pi^4\mathrm{Ci}\left(b^2\pi x^2\right)}{2016} - \frac{c}{}$$

Mathematica [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^{10}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{fresnelc}\left(bx\right)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^10,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^10, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{fresnelc}\left(bx\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^10,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^10, x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^10,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^10,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{fresnelc}\left(bx\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnelc(b*x)/x^10,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnelc(b*x)/x^10, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\text{FresnelC}\left(b\,x\right)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^10,x)
```

```
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^10, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C\left(bx\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**10,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**10, x)
```

3.199 $\int C(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\sin\left(\frac{1}{2}\pi b^2x^2\right)C(bx)^n, x\right)$$

```
[Out] Unintegrable(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] Defer[Int][FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]
```

Rubi steps

$$\int C(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)dx = \int C(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

Mathematica [A] time = 0.08, size = 0, normalized size = 0.00

$$\int C(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] Integrate[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnelc}\left(bx\right)^n \sin\left(\frac{1}{2}\,\pi b^2x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(fresnelc(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{fresnelc}(bx)^n \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnelc(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \operatorname{FresnelC}(bx)^n \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
[Out] int(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{fresnelc}(bx)^n \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(fresnelc(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \operatorname{FresnelC}(b\,x)^n\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^n*sin((Pi*b^2*x^2)/2),x)
[Out] int(FresnelC(b*x)^n*sin((Pi*b^2*x^2)/2), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{\pi b^2 x^2}{2}\right) C^n\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**n*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)**n, x)
```

3.200 $\int x^8 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=308

$$\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}-\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}+\frac{105C(bx)S(bx)}{2\pi^4b^9}-\frac{35x^4}{8\pi^3b^5}-\frac{x^7C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

```
[Out] -35/8*x^4/b^5/Pi^3+1/16*x^8/b/Pi-40*cos(b^2*Pi*x^2)/b^9/Pi^5+5/2*x^4*cos(b^
2*Pi*x^2)/b^5/Pi^3+35*x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^6/Pi^3-x^7*co
s(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+105/2*FresnelC(b*x)*FresnelS(b*x)/b^
9/Pi^4+105/8*I*x^2*HypergeometricPFQ([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^7
/Pi^4-105/8*I*x^2*HypergeometricPFQ([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^7/P
i^4-105*x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^8/Pi^4+7*x^5*FresnelC(b*x)*si
n(1/2*b^2*Pi*x^2)/b^4/Pi^2-55/4*x^2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^6*sin(b^
2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] time = 0.43, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6463, 6455, 6447, 3379, 2638, 3380, 3309, 30, 3296}

$$\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}-\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}+\frac{105\text{FresnelC}(bx)S(bx)}{2\pi^4b^9}+\frac{7x^5\text{FresnelC}(bx)\sin}{\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-35*x^4)/(8*b^5*Pi^3) + x^8/(16*b*Pi) - (40*Cos[b^2*Pi*x^2])/(b^9*Pi^5) +
(5*x^4*Cos[b^2*Pi*x^2])/(2*b^5*Pi^3) + (35*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC
[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + (105
*FresnelC[b*x]*FresnelS[b*x])/(2*b^9*Pi^4) + (((105*I)/8)*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (((105*I)/8)*x^2*Hy
pergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (105*x*Fr
esnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelC[b*x]*Sin[(b^2
*Pi*x^2)/2])/(b^4*Pi^2) - (55*x^2*Sin[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*Sin[
b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
```

```
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\int x^8 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi} + \frac{\int x^7\cos^2\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

$$= -\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{7x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{35\int x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}$$

$$= \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{7x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} -$$

$$= \frac{x^8}{16b\pi} + \frac{7x^4\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} -$$

$$= \frac{x^8}{16b\pi} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} +$$

$$= -\frac{35x^4}{8b^5\pi^3} + \frac{x^8}{16b\pi} - \frac{119\cos\left(b^2\pi x^2\right)}{4b^9\pi^5} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(b}{b^6\pi^3}$$

$$= -\frac{35x^4}{8b^5\pi^3} + \frac{x^8}{16b\pi} - \frac{40\cos\left(b^2\pi x^2\right)}{b^9\pi^5} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(b}{b^6\pi^3}$$

Mathematica [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x^8 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

[Out] Integrate[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(x^8\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")

[Out] integral(x^8*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")

[Out] integrate(x^8*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^8\,\text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)

[Out] int(x^8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")

[Out] integrate(x^8*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^8\,\text{FresnelC}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^8*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**8*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.201 $\int x^7 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=218

$$\frac{531S\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,\pi^4b^8}-\frac{4x^3}{\pi^3b^5}-\frac{x^6C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{48C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{147x\sin\left(\pi b^2x^2\right)}{16\pi^4b^7}+\frac{24x^2C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}$$

```
[Out] -4*x^3/b^5/Pi^3+1/14*x^7/b/Pi+17/8*x^3*cos(b^2*Pi*x^2)/b^5/Pi^3+24*x^2*cos(
1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^6/Pi^3-x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/b^2/Pi-48*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^8/Pi^4+6*x^4*FresnelC(b*x)*
sin(1/2*b^2*Pi*x^2)/b^4/Pi^2-147/16*x*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^5*sin(
b^2*Pi*x^2)/b^3/Pi^2+531/32*FresnelS(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6463, 6455, 6453, 3351, 3392, 30, 3386, 3385}

$$\frac{6x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{48\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{x^6\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{24x^2\text{FresnelC}(\ldots}{\pi\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-4*x^3)/(b^5*Pi^3) + x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3)
 + (24*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^6*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*
b^8*Pi^4) - (48*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (6*x^4*Fres
nelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7
*Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\int x^7 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi} + \frac{\int x^6\cos^2\left(\frac{1}{2}b^2\pi x^2\right)d}{b\pi}$$

$$= -\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{6x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{24\int x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}$$

$$= \frac{x^7}{14b\pi} + \frac{3x^3\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} +$$

$$= \frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} -$$

$$= -\frac{4x^3}{b^5\pi^3} + \frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right.}{b^2\pi}$$

$$= -\frac{4x^3}{b^5\pi^3} + \frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right.}{b^2\pi}$$

Mathematica [A] time = 0.18, size = 163, normalized size = 0.75

$$\frac{16\pi^3b^7x^7 - 896\pi b^3x^3 - 2058bx\sin\left(\pi b^2x^2\right) + 56\pi^2b^5x^5\sin\left(\pi b^2x^2\right) - 224C(bx)\left(\pi b^2x^2\left(\pi^2b^4x^4-24\right)\cos\left(\frac{1}{2}\pi b^2\right.\right.}{224\pi^4b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-896*b^3*Pi*x^3 + 16*b^7*Pi^3*x^7 + 476*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 3717*
Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 224*FresnelC[b*x]*(b^2*Pi*x^2*(-24 + b^4*Pi
^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) -
2058*b*x*Sin[b^2*Pi*x^2] + 56*b^5*Pi^2*x^5*Sin[b^2*Pi*x^2])/(224*b^8*Pi^4)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(x^7\text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^7*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.06, size = 322, normalized size = 1.48

$$\frac{\frac{\mathrm{FresnelC}(bx)\left(-\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{6b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{24\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}}{\pi}\right)}{b^7}-\frac{-\frac{\frac{1}{7}\pi^2b^7x^7-8b^3x^3}{2\pi^3}+\frac{-\frac{3\pi b^3x^3\cos\left(b^2\pi x^2\right)}{2}+\frac{9\pi\left(\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}\cdots\right)}{2}}{\pi^4}\cdots}{b^7}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (FresnelC(b*x)/b^7*(-1/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)+6/Pi*(1/Pi*b^4*x^4*si
n(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^
2*Pi*x^2))))-1/b^7*(-1/2/Pi^3*(1/7*Pi^2*b^7*x^7-8*b^3*x^3)+3/Pi^4*(-1/2*Pi*
b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*F
resnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^5
*x^5*sin(b^2*Pi*x^2)-5/2*Pi*(-1/2/Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2/Pi*(1/2/Pi
*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2))))-12/Pi*b*x*sin(b
^2*Pi*x^2)+6/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \operatorname{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^7*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**7*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.202 $\int x^6 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=185

$$-\frac{15C(bx)^2}{2\pi^3b^7}-\frac{15x^2}{4\pi^3b^5}-\frac{x^5C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{11\sin\left(\pi b^2x^2\right)}{2\pi^4b^7}+\frac{15xC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{7x^2\cos\left(\pi b^2x^2\right)}{4\pi^3b^5}+\frac{5x^3C(\cdots}{\cdots}$$

```
[Out] -15/4*x^2/b^5/Pi^3+1/12*x^6/b/Pi+7/4*x^2*cos(b^2*Pi*x^2)/b^5/Pi^3+15*x*cos(
1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^6/Pi^3-x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/b^2/Pi-15/2*FresnelC(b*x)^2/b^7/Pi^3+5*x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x
^2)/b^4/Pi^2-11/2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6463, 6455, 6441, 30, 3380, 2634, 3379, 3296, 2637, 3309}

$$\frac{5x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{15x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}-\frac{15\text{FresnelC}(\cdots}{2\pi^3b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-15*x^2)/(4*b^5*Pi^3) + x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^
3) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^5*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (15*FresnelC[b*x]^2)/(2*b^7*Pi^3) + (5*x
^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (11*Sin[b^2*Pi*x^2])/(2*
b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
```

```
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}\int x^6C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx &= -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi}+\frac{\int x^5\cos^2\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}\\
&= -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{5x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{15\int x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}\\
&= \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{5x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\\
&= \frac{x^6}{12b\pi}+\frac{5x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\\
&= -\frac{15x^2}{4b^5\pi^3}+\frac{x^6}{12b\pi}+\frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}\\
&= -\frac{15x^2}{4b^5\pi^3}+\frac{x^6}{12b\pi}+\frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}\end{aligned}$$

Mathematica [A] time = 0.01, size = 185, normalized size = 1.00

$$-\frac{15C(bx)^2}{2\pi^3b^7}-\frac{15x^2}{4\pi^3b^5}-\frac{x^5C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{11\sin\left(\pi b^2x^2\right)}{2\pi^4b^7}+\frac{15xC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{7x^2\cos\left(\pi b^2x^2\right)}{4\pi^3b^5}+\frac{5x^3C(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-15*x^2)/(4*b^5*Pi^3) + x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^
3) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^5*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (15*FresnelC[b*x]^2)/(2*b^7*Pi^3) + (5*x
^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (11*Sin[b^2*Pi*x^2])/(2*
b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(x^6\text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(x^6*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^6*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^6\,\text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^6*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\,\text{FresnelC}\,(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^6*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 77.91, size = 264, normalized size = 1.43

$$\begin{cases} \frac{x^6 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} + \frac{x^6 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} - \frac{x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi b^2} + \frac{x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) \cos\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} + \frac{5x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi^2 b^4} - \frac{11x^2 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^3 b^5} - \frac{2x^{\cdots}}{\cdots} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((x**6*sin(pi*b**2*x**2/2)**2/(12*pi*b) + x**6*cos(pi*b**2*x**2/2)
**2/(12*pi*b) - x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) + x**4*sin
(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + 5*x**3*sin(pi*b**2*x*
*2/2)*fresnelc(b*x)/(pi**2*b**4) - 11*x**2*sin(pi*b**2*x**2/2)**2/(2*pi**3*
b**5) - 2*x**2*cos(pi*b**2*x**2/2)**2/(pi**3*b**5) + 15*x*cos(pi*b**2*x**2/
2)*fresnelc(b*x)/(pi**3*b**6) - 11*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/
(pi**4*b**7) - 15*fresnelc(b*x)**2/(2*pi**3*b**7), Ne(b, 0)), (0, True))
```

3.203 $\int x^5 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=167

$$-\frac{43C\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,\pi^3b^6}-\frac{4x}{\pi^3b^5}-\frac{x^4C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{8C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{11x\cos\left(\pi b^2x^2\right)}{8\pi^3b^5}+\frac{4x^2C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+$$

```
[Out] -4*x/b^5/Pi^3+1/10*x^5/b/Pi+11/8*x*cos(b^2*Pi*x^2)/b^5/Pi^3+8*cos(1/2*b^2*P
i*x^2)*FresnelC(b*x)/b^6/Pi^3-x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+
4*x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+1/4*x^3*sin(b^2*Pi*x^2)/b^
3/Pi^2-43/16*FresnelC(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6463, 6455, 6461, 3358, 3352, 3385, 3392, 30, 3386}

$$\frac{4x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{8\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}-\frac{43\text{FresnelC}\left(\sqrt{2}\right.}{8\sqrt{2}\,\pi^3b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-4*x)/(b^5*Pi^3) + x^5/(10*b*Pi) + (11*x*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3) + (
8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^4*Cos[(b^2*Pi*x^2)/2]*
FresnelC[b*x])/(b^2*Pi) - (43*FresnelC[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) +
 (4*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x^3*Sin[b^2*Pi*x^2
])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\int x^5 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi} + \frac{\int x^4\cos^2\left(\frac{1}{2}b^2\pi x^2\right)d}{b\pi}$$

$$= -\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{4x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{8\int xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{b^4\pi^2}$$

$$= \frac{x^5}{10b\pi} + \frac{x\cos\left(b^2\pi x^2\right)}{b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{4x^2C(}{}$$

$$= \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} - \frac{C(}{\sqrt{2}}$$

$$= -\frac{4x}{b^5\pi^3} + \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(}{b^2\pi}$$

$$= -\frac{4x}{b^5\pi^3} + \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(}{b^2\pi}$$

Mathematica [A] time = 0.19, size = 126, normalized size = 0.75

$$\frac{-80C(bx)\left(\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)-4\pi b^2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)\right)+2bx\left(4\pi^2b^4x^4+10\pi b^2x^2\sin\left(\pi b^2x^2\right)+55\cos\left(\right.\right.}{80\pi^3b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-215*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 80*FresnelC[b*x]*((-8 + b^4*Pi^2*x^4)
*Cos[(b^2*Pi*x^2)/2] - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(-160 + 4*
b^4*Pi^2*x^4 + 55*Cos[b^2*Pi*x^2] + 10*b^2*Pi*x^2*Sin[b^2*Pi*x^2]))/(80*b^6
*Pi^3)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(x^5\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^5*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \text{fresnelc}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.05, size = 212, normalized size = 1.27

$$\frac{\frac{\text{FresnelC}(bx)\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{b^5}-\frac{-\frac{\frac{1}{5}\pi^2b^5x^5-8bx}{2\pi^3}+\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{\pi}+\frac{\sqrt{2}\ \text{FresnelC}\left(bx\sqrt{2}\right)}{2\pi}}{\pi^2}-\frac{\frac{\pi b^3x^3\sin\left(b^2\pi x^2\right)}{2}-3\pi\left(-\cdots\right.}{\cdots}}{b^5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (FresnelC(b*x)/b^5*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*si
n(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))-1/b^5*(-1/2/Pi^3*(1/5*Pi^2*b
^5*x^5-8*b*x)+2/Pi^2*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b
*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*c
os(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x
*2^(1/2)))))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \text{fresnelc}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,\mathrm{FresnelC}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)`

[Out] `int(x^5*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)`

[Out] `Integral(x**5*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)`

3.204 $\int x^4 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=196

$$-\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}+\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}-\frac{3C(bx)S(bx)}{2\pi^2b^5}-\frac{x^3C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x\right.}{\pi^3b^5}$$

```
[Out] 1/8*x^4/b/Pi+cos(b^2*Pi*x^2)/b^5/Pi^3-x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)
/b^2/Pi-3/2*FresnelC(b*x)*FresnelS(b*x)/b^5/Pi^2-3/8*I*x^2*HypergeometricPF
Q([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^3/Pi^2+3/8*I*x^2*HypergeometricPFQ([
1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^3/Pi^2+3*x*FresnelC(b*x)*sin(1/2*b^2*Pi*
x^2)/b^4/Pi^2+1/4*x^2*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] time = 0.16, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6463, 6455, 6447, 3379, 2638, 3380, 3309, 30, 3296}

$$-\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}+\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}-\frac{3\text{FresnelC}(bx)S(bx)}{2\pi^2b^5}+\frac{3x\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b\right.}{\pi^2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^4/(8*b*Pi) + Cos[b^2*Pi*x^2]/(b^5*Pi^3) - (x^3*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(b^2*Pi) - (3*FresnelC[b*x]*FresnelS[b*x])/(2*b^5*Pi^2) - (((3*I)/
8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-I/2)*b^2*Pi*x^2])/(b^3*Pi^2) +
 (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^3
*Pi^2) + (3*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x^2*Sin[b^2*
Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3309

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6455

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m - 1)*Sin[d*x^2]*FresnelC[b*x])/(2*d), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
```

]

Rubi steps

$$\int x^4 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi}+\frac{\int x^3\cos^2\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

$$= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{3xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{3\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^4\pi^2}$$

$$= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}-\frac{3C(bx)S(bx)}{2b^5\pi^2}-\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2}+\frac{3ix^2}{}$$

$$= \frac{x^4}{8b\pi}+\frac{3\cos\left(b^2\pi x^2\right)}{4b^5\pi^3}-\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}-\frac{3C(bx)S(bx)}{2b^5\pi^2}-\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2}\right.}{8b^3}$$

$$= \frac{x^4}{8b\pi}+\frac{\cos\left(b^2\pi x^2\right)}{b^5\pi^3}-\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}-\frac{3C(bx)S(bx)}{2b^5\pi^2}-\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},\right.}{8b^3\pi}$$

Mathematica [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x^4 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^4\mathrm{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(x^4*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\mathrm{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^4*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{fresnelc}(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^4*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \operatorname{FresnelC}(b\,x)\ \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^4*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**4*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.205 $\int x^3 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=109

$$-\frac{5S\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,\pi^2b^4}-\frac{x^2C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+\frac{x^3}{6\pi b}$$

```
[Out] 1/6*x^3/b/Pi-x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+2*FresnelC(b*x)*s
in(1/2*b^2*Pi*x^2)/b^4/Pi^2+1/4*x*sin(b^2*Pi*x^2)/b^3/Pi^2-5/8*FresnelS(b*x
*2^(1/2))/b^4/Pi^2*2^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6463, 6453, 3351, 3392, 30, 3386}

$$\frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5S\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,\pi^2b^4}+\frac{x\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+\frac{x^3}{6\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^3/(6*b*Pi) - (x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (5*Fresne
lS[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2) + (2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2
])/(b^4*Pi^2) + (x*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3392

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6453

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[(Sin[d*x
^2]*FresnelC[b*x])/(2*d), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$
\begin{aligned}
\int x^3 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx &= -\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi}+\frac{\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi} \\
&= -\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{\int\sin\left(b^2\pi x^2\right)\,dx}{b^3\pi^2}+\frac{\int x^2\,dx}{2b\pi}+ \\
&= \frac{x^3}{6b\pi}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}-\frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b^4\pi^2}+\frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{x\sin\left(b^2\pi x\right.}{4b^3\pi^2} \\
&= \frac{x^3}{6b\pi}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}-\frac{5S\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}+\frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{x\sin\left(b^2\pi\right.}{4b^3\pi^2}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 90, normalized size = 0.83

$$
\frac{4\pi b^3x^3-24C(bx)\left(\pi b^2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)-2\sin\left(\frac{1}{2}\pi b^2x^2\right)\right)+6bx\sin\left(\pi b^2x^2\right)-15\sqrt{2}\,S\left(\sqrt{2}\,bx\right)}{24\pi^2b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (4*b^3*Pi*x^3 - 15*Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 24*FresnelC[b*x]*(b^2*Pi
*x^2*Cos[(b^2*Pi*x^2)/2] - 2*Sin[(b^2*Pi*x^2)/2]) + 6*b*x*Sin[b^2*Pi*x^2])/
(24*b^4*Pi^2)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^3*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.05, size = 120, normalized size = 1.10

$$\frac{\frac{\text{FresnelC}(bx)\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{b^3}-\frac{\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{2\pi^2}-\frac{b^3x^3}{6\pi}-\frac{\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}-\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (FresnelC(b*x)/b^3*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi
*x^2))-1/b^3*(1/2/Pi^2*2^(1/2)*FresnelS(b*x*2^(1/2))-1/6/Pi*b^3*x^3-1/2/Pi*
(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^3*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**3*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.206 $\int x^2 C(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=74

$$\frac{C(bx)^2}{2\pi b^3} - \frac{xC(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{\sin\left(\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{x^2}{4\pi b}$$

```
[Out] 1/4*x^2/b/Pi-x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+1/2*FresnelC(b*x)^2
/b^3/Pi+1/4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6463, 6441, 30, 3380, 2634}

$$-\frac{x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{\text{FresnelC}(bx)^2}{2\pi b^3} + \frac{\sin\left(\pi b^2 x^2\right)}{4\pi^2 b^3} + \frac{x^2}{4\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^2/(4*b*Pi) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[b*
x]^2/(2*b^3*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6463

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(x
^(m - 1)*Cos[d*x^2]*FresnelC[b*x])/(2*d), x] + (Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos[d
*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && IGtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}\int x^2 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx &= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi}+\frac{\int x\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}\\ &= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{\text{Subst}(\int x\,dx,x,C(bx))}{b^3\pi}+\frac{\text{Subst}\left(\int\cos^2\left(\frac{1}{2}b^2\pi x\right)dx,x\right.}{2b\pi}\\ &= \frac{x^2}{4b\pi}-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{C(bx)^2}{2b^3\pi}+\frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 74, normalized size = 1.00

$$\frac{C(bx)^2}{2\pi b^3}-\frac{xC(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+\frac{x^2}{4\pi b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^2/(4*b*Pi) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[b*
x]^2/(2*b^3*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(x^2*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^2*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\,\text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^2*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\text{FresnelC}\,(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^2*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [A] time = 3.02, size = 114, normalized size = 1.54

$$\begin{cases} \frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} + \frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} - \frac{x\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi b^2} + \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} + \frac{C^2(bx)}{2\pi b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((x**2*sin(pi*b**2*x**2/2)**2/(4*pi*b) + x**2*cos(pi*b**2*x**2/2)*
*2/(4*pi*b) - x*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) + sin(pi*b**2*x
**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + fresnelc(b*x)**2/(2*pi*b**3), N
e(b, 0)), (0, True))
```

3.207 $\int xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=60

$$-\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{C\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,\pi b^2}+\frac{x}{2\pi b}$$

```
[Out] 1/2*x/b/Pi-cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+1/4*FresnelC(b*x*2^(1/2
))/b^2/Pi*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6461, 3358, 3352}

$$-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\text{FresnelC}\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,\pi b^2}+\frac{x}{2\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x/(2*b*Pi) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[Sqrt[2
]*b*x]/(2*Sqrt[2]*b^2*Pi)
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 6461

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> -Simp[(Cos[d*
x^2]*FresnelC[b*x])/(2*d), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{\int \cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{b\pi}$$
$$= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{\int\left(\frac{1}{2}+\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)dx}{b\pi}$$
$$= \frac{x}{2b\pi} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{\int\cos\left(b^2\pi x^2\right)dx}{2b\pi}$$
$$= \frac{x}{2b\pi} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{C\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,b^2\pi}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.80

$$\frac{-4C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)+\sqrt{2}\,C\left(\sqrt{2}\,bx\right)+2bx}{4\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (2*b*x - 4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x] + Sqrt[2]*FresnelC[Sqrt[2]*b*x
])/(4*b^2*Pi)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(x\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [A] time = 0.02, size = 52, normalized size = 0.87

$$\frac{-\frac{\mathrm{FresnelC}(bx)\cos\left(\frac{b^2\pi x^2}{2}\right)}{b\pi}+\frac{\frac{bx}{2}+\frac{\sqrt{2}\ \mathrm{FresnelC}\left(bx\sqrt{2}\right)}{4}}{b\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] (-FresnelC(b*x)/b/Pi*cos(1/2*b^2*Pi*x^2)+1/b/Pi*(1/2*b*x+1/4*2^(1/2)*Fresne
lC(b*x*2^(1/2))))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\mathrm{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{FresnelC}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin\left(\frac{\pi b^2x^2}{2}\right)C\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.208 $\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=80

$$\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{C(bx)S(bx)}{2b}$$

```
[Out] 1/2*FresnelC(b*x)*FresnelS(b*x)/b+1/8*I*b*x^2*HypergeometricPFQ([1, 1],[3/2
, 2],-1/2*I*b^2*Pi*x^2)-1/8*I*b*x^2*HypergeometricPFQ([1, 1],[3/2, 2],1/2*I
*b^2*Pi*x^2)
```

Rubi [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6447}

$$\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)S(bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-I/2)*b^2*Pi*x^2] - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2,
 2}, (I/2)*b^2*Pi*x^2]
```

Rule 6447

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(b*Pi*Fresnel
C[b*x]*FresnelS[b*x])/(4*d), x] + (Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, -(I*d*x^2)])/8, x] - Simp[(1*I*b*x^2*HypergeometricPFQ[{1, 1},
{3/2, 2}, I*d*x^2])/8, x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4]
```

Rubi steps

$$\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\frac{C(bx)S(bx)}{2b}+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F] time = 0.02, size = 0, normalized size = 0.00

$$\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \text{FresnelC}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{\pi b^2 x^2}{2}\right) C\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.209 $\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)$$

```
[Out] Unintegrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

```
[Out] Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx=\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

```
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\text{FresnelC}\,(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x,x)
```

```
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C\left(bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x, x)
```

3.210 $\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)+\frac{1}{2}\pi bC(bx)^2$$

```
[Out] 1/2*b*Pi*FresnelC(b*x)^2+1/4*b*Si(b^2*Pi*x^2)-FresnelC(b*x)*sin(1/2*b^2*Pi*
x^2)/x
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6465, 6441, 30, 3375}

$$-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)+\frac{1}{2}\pi b\text{FresnelC}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

```
[Out] (b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIn
tegral[b^2*Pi*x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
```

```
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}+\frac{1}{2}b\int \frac{\sin\left(b^2\pi x^2\right)}{x}\,dx+\left(b^2\pi\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx\\ &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}+\frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)+(b\pi)\,\mathrm{Subst}(\int x\,dx,x,C(bx))\\ &= \frac{1}{2}b\pi C(bx)^2-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}+\frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$-\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)+\frac{1}{2}\pi bC(bx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

```
[Out] (b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIn
tegral[b^2*Pi*x^2])/4
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="giac")
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{FresnelC}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="maxima")
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{FresnelC}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^{2}\,x^{2}}{2}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^2,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)C\left(bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**2,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**2, x)
```

3.211 $\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$

Optimal. Leaf size=94

$$\frac{1}{2}\pi b^2 \text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{\pi b^2C\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{b\sin\left(\pi b^2x^2\right)}{4x}$$

```
[Out] -1/2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-1/4*b*sin(b^2*Pi*x^2)/x+1/4*b^2*
Pi*FresnelC(b*x*2^(1/2))*2^(1/2)+1/2*b^2*Pi*Unintegrable(cos(1/2*b^2*Pi*x^2
)*FresnelC(b*x)/x,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (FresnelC[b*x]*Sin[(b^2*Pi*x^2
)/2])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) + (b^2*Pi*Defer[Int][(Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/x, x])/2
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}+\frac{1}{4}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,dx\\ &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,dx+\frac{1}{2}\left(b^3\right.\\ &= \frac{b^2\pi C\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C}{x}\end{aligned}$$

Mathematica [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="giac")
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^3,x)
```

```
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C\left(bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**3,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**3, x)
```

3.212 $\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$

Optimal. Leaf size=89

$$\frac{1}{3}\pi b^2\text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)-\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b\sin\left(\pi b^2x^2\right)}{12x^2}+\frac{1}{12}\pi b^3\text{Ci}\left(b^2\pi x^2\right)$$

```
[Out] 1/12*b^3*Pi*Ci(b^2*Pi*x^2)-1/3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-1/12*b
*sin(b^2*Pi*x^2)/x^2+1/3*b^2*Pi*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b
*x)/x^2,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(
3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) + (b^2*Pi*Defer[Int][(Cos[(b^2*Pi*x^2
)/2]*FresnelC[b*x])/x^2, x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}+\frac{1}{6}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^3}\,dx+\frac{1}{3}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx\\
&= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}+\frac{1}{12}b\,\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^2}\,dx,x,x^2\right)+\frac{1}{3}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi\right.}{x}\\
&= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}+\frac{1}{3}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx+\frac{1}{12}\left(b^3\right.\\
&= \frac{1}{12}b^3\pi\text{Ci}\left(b^2\pi x^2\right)-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}+\frac{1}{3}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right.}{x^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="giac")
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{FresnelC}\,(b\,x)\ \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^4,x)
```

```
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^4, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C\,(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**4,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**4, x)
```

3.213 $\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$

Optimal. Leaf size=156

$$-\frac{1}{8}\pi^2 b^4 \text{Int}\left(\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{x},x\right)-\frac{7\pi^2 b^4 S\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}-\frac{\pi b^3}{16x}-\frac{\pi b^2 C(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^2}-\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{4x^4}-\frac{b\,\mathrm{si}}{}$$

```
[Out] -1/16*b^3*Pi/x-7/48*b^3*Pi*cos(b^2*Pi*x^2)/x-1/8*b^2*Pi*cos(1/2*b^2*Pi*x^2)
*FresnelC(b*x)/x^2-1/4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-1/24*b*sin(b^2
*Pi*x^2)/x^3-7/48*b^4*Pi^2*FresnelS(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Unint
egrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

```
[Out] -(b^3*Pi)/(16*x) - (7*b^3*Pi*Cos[b^2*Pi*x^2])/(48*x) - (b^2*Pi*Cos[(b^2*Pi*
x^2)/2]*FresnelC[b*x])/(8*x^2) - (7*b^4*Pi^2*FresnelS[Sqrt[2]*b*x])/(24*Sqr
t[2]) - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*x^4) - (b*Sin[b^2*Pi*x^2])/(
24*x^3) - (b^4*Pi^2*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/8
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}+\frac{1}{8}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx+\frac{1}{4}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^3}\,dx\\ &= -\frac{b^3\pi}{16x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^2}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}+\frac{1}{16}\left(b^3\pi\right.\\ &= -\frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^2}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{b\,\mathrm{si}}{}\\ &= -\frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^2}-\frac{7b^4\pi^2 S\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}-\frac{C(bx)\,\mathrm{s}}{}\end{aligned}$$

Mathematica [A]   time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5, x]
```

fricas [A]   time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{fresnelc}(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

giac [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{fresnelc}(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="giac")
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

maple [A]   time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{FresnelC}(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="maxima")
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{FresnelC}\,(b\,x)\ \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^5,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^5, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C\,(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**5,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**5, x)
```

3.214 $\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

Optimal. Leaf size=163

$$-\frac{1}{30}\pi^3b^5C(bx)^2-\frac{\pi b^3}{60x^2}-\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{\pi b^2C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3}-\frac{b\sin\left(\pi b^2x^2\right)}{40x^4}-\frac{7}{120}\pi^2b^5\mathrm{Si}\left(b^2\pi x^2\right)+\frac{\pi^2b^4}{}$$

```
[Out] -1/60*b^3*Pi/x^2-1/24*b^3*Pi*cos(b^2*Pi*x^2)/x^2-1/15*b^2*Pi*cos(1/2*b^2*Pi
*x^2)*FresnelC(b*x)/x^3-1/30*b^5*Pi^3*FresnelC(b*x)^2-7/120*b^5*Pi^2*Si(b^2
*Pi*x^2)-1/5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5+1/15*b^4*Pi^2*FresnelC(b
*x)*sin(1/2*b^2*Pi*x^2)/x-1/40*b*sin(b^2*Pi*x^2)/x^4
```

Rubi [A] time = 0.22, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6465, 6457, 6441, 30, 3375, 3380, 3297, 3299, 3379}

$$\frac{\pi^2b^4\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{15x}-\frac{\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{\pi b^2\mathrm{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3}-\frac{1}{30}\pi^3b^5\mathrm{Fresnel}$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] -(b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (b^2*Pi*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(15*x^3) - (b^5*Pi^3*FresnelC[b*x]^2)/30 - (Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(15*x) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^
2*Pi*x^2])/120
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
```

```
LtQ[m, -1]
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{1}{10}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx + \frac{1}{5}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^4}\,dx$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^3} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{1}{20}b\,\mathrm{Subst}\left(\int \frac{\sin\left(b^2\pi x\right.}{x^3}\right.$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^3} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{b^4\pi^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x}$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^3} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{b^4\pi^2}{}$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^3} - \frac{1}{30}b^5\pi^3C(bx)^2 - \frac{C(bx)\sin\left(\right.}{5x^5}$$

Mathematica [A] time = 0.01, size = 163, normalized size = 1.00

$$-\frac{1}{30}\pi^3b^5C(bx)^2 - \frac{\pi b^3}{60x^2} - \frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5} - \frac{\pi b^2C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3} - \frac{b\sin\left(\pi b^2x^2\right)}{40x^4} - \frac{7}{120}\pi^2b^5\mathrm{Si}\left(b^2\pi x^2\right) + \frac{\pi^2b^4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] -1/60*(b^3*Pi)/x^2 - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (b^2*Pi*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(15*x^3) - (b^5*Pi^3*FresnelC[b*x]^2)/30 - (Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(15*x) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^
2*Pi*x^2])/120
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^6,x)
```

```
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C\left(bx\right)}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**6,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**6, x)
```

3.215 $\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$

Optimal. Leaf size=231

$$-\frac{1}{48}\pi^3 b^6 \text{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)-\frac{1}{45}\sqrt{2}\,\pi^3 b^6 C\left(\sqrt{2}\,bx\right)-\frac{7\pi^3 b^6 C\left(\sqrt{2}\,bx\right)}{144\sqrt{2}}-\frac{\pi b^3}{144x^3}-\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{\pi}{}$$

```
[Out] -1/144*b^3*Pi/x^3-13/720*b^3*Pi*cos(b^2*Pi*x^2)/x^3-1/24*b^2*Pi*cos(1/2*b^2
*Pi*x^2)*FresnelC(b*x)/x^4-1/6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6+1/48*b
^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-1/60*b*sin(b^2*Pi*x^2)/x^5+67
/1440*b^5*Pi^2*sin(b^2*Pi*x^2)/x-67/1440*b^6*Pi^3*FresnelC(b*x*2^(1/2))*2^(
1/2)-1/48*b^6*Pi^3*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

```
[Out] -(b^3*Pi)/(144*x^3) - (13*b^3*Pi*Cos[b^2*Pi*x^2])/(720*x^3) - (b^2*Pi*Cos[(
b^2*Pi*x^2)/2]*FresnelC[b*x])/(24*x^4) - (7*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])
/(144*Sqrt[2]) - (Sqrt[2]*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])/45 - (FresnelC[b*
x]*Sin[(b^2*Pi*x^2)/2])/(6*x^6) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
2])/(48*x^2) - (b*Sin[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])
/(1440*x) - (b^6*Pi^3*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])
/48
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx + \frac{1}{6}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^5}\,dx$$

$$= -\frac{b^3\pi}{144x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{24x^4} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} - \frac{b\sin\left(b^2\pi x^2\right)}{60x^5} + \frac{1}{48}\left(b^3\pi\right.$$

$$= -\frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{24x^4} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{b^4}{}$$

$$= -\frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{24x^4} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{b^4}{}$$

$$= -\frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{24x^4} - \frac{7b^6\pi^3C\left(\sqrt{2}\,bx\right)}{144\sqrt{2}} - \frac{1}{45}\sqrt{2}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="giac")
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{FresnelC}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="maxima")
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^{2}\,x^{2}}{2}\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^7,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^7, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)C\left(bx\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**7,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**7, x)
```

3.216 $\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

Optimal. Leaf size=202

$$-\frac{1}{105}\pi^3 b^6 \mathrm{Int}\left(\frac{C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2},x\right)-\frac{\pi b^3}{280x^4}-\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{\pi b^2C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}-\frac{b\sin\left(\pi b^2x^2\right)}{84x^6}-\frac{1}{84}$$

```
[Out] -1/280*b^3*Pi/x^4-1/84*b^7*Pi^3*Ci(b^2*Pi*x^2)-1/105*b^3*Pi*cos(b^2*Pi*x^2)
/x^4-1/35*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5-1/7*FresnelC(b*x)*si
n(1/2*b^2*Pi*x^2)/x^7+1/105*b^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-
1/84*b*sin(b^2*Pi*x^2)/x^6+1/84*b^5*Pi^2*sin(b^2*Pi*x^2)/x^2-1/105*b^6*Pi^3
*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Rubi [A] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

```
[Out] -(b^3*Pi)/(280*x^4) - (b^3*Pi*Cos[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*CosInt
egral[b^2*Pi*x^2])/84 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(35*x^5)
 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*Pi^2*FresnelC[b*x]*Si
n[(b^2*Pi*x^2)/2])/(105*x^3) - (b*Sin[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*Sin
[b^2*Pi*x^2])/(84*x^2) - (b^6*Pi^3*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC
[b*x])/x^2, x])/105
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{1}{14}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^7}\,dx + \frac{1}{7}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^6}\,d$$

$$= -\frac{b^3\pi}{280x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{1}{28}b\,\text{Subst}\left(\int \frac{\sin\left(b^2\right.}{x^4}\right.$$

$$= -\frac{b^3\pi}{280x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2C(bx)\sin\left(\frac{1}{2}b^2\pi x\right.}{105x^3}$$

$$= -\frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4}{}$$

$$= -\frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4}{}$$

$$= -\frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{1}{84}b^7\pi^3\text{Ci}\left(b^2\pi x^2\right) - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5} - \frac{C(b}{}$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^8,x)
```

```
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^8, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**8,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**8, x)
```

3.217 $\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

Optimal. Leaf size=271

$$\frac{1}{384}\pi^4 b^8 \mathrm{Int}\left(\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x},x\right)+\frac{853\pi^4b^8S\left(\sqrt{2}\,bx\right)}{40320\sqrt{2}}+\frac{\pi^3b^7}{768x}-\frac{\pi b^3}{480x^5}-\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{\pi b^2C(bx)\cos\left(\frac{1}{2}\pi\right.}{48x^6}$$

```
[Out] -1/480*b^3*Pi/x^5+1/768*b^7*Pi^3/x-19/3360*b^3*Pi*cos(b^2*Pi*x^2)/x^5+853/8
0640*b^7*Pi^3*cos(b^2*Pi*x^2)/x-1/48*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelC(b*
x)/x^6+1/384*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2-1/8*FresnelC(b*
x)*sin(1/2*b^2*Pi*x^2)/x^8+1/192*b^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)
/x^4-1/112*b*sin(b^2*Pi*x^2)/x^7+187/40320*b^5*Pi^2*sin(b^2*Pi*x^2)/x^3+853
/80640*b^8*Pi^4*FresnelS(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrable(F
resnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
```

```
[Out] -(b^3*Pi)/(480*x^5) + (b^7*Pi^3)/(768*x) - (19*b^3*Pi*Cos[b^2*Pi*x^2])/(336
0*x^5) + (853*b^7*Pi^3*Cos[b^2*Pi*x^2])/(80640*x) - (b^2*Pi*Cos[(b^2*Pi*x^2
)/2]*FresnelC[b*x])/(48*x^6) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/(384*x^2) + (853*b^8*Pi^4*FresnelS[Sqrt[2]*b*x])/(40320*Sqrt[2]) - (Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^8) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(192*x^4) - (b*Sin[b^2*Pi*x^2])/(112*x^7) + (187*b^5*Pi^2*Sin[b^2*
Pi*x^2])/(40320*x^3) + (b^8*Pi^4*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)
/2])/x, x])/384
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{1}{16}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}\,dx + \frac{1}{8}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^7}\,d$$
$$= -\frac{b^3\pi}{480x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{48x^6} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7} + \frac{1}{96}\left(b\right.$$
$$= -\frac{b^3\pi}{480x^5} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{48x^6} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} +$$
$$= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{48x^6} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\right.}{384x}$$
$$= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6}$$
$$= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9, x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{fresnelc}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="fricas")
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="giac")
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

maple [A] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="maxima")
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\text{FresnelC}\,(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^9,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^9, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C\left(bx\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**9,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**9, x)
```

3.218 $\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

Optimal. Leaf size=278

$$\frac{\pi^5 b^9 C(bx)^2}{1890}+\frac{\pi^3 b^7}{3780x^2}-\frac{\pi b^3}{756x^6}-\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{\pi b^2 C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{63x^7}-\frac{b\sin\left(\pi b^2x^2\right)}{144x^8}+\frac{83\pi^4 b^9\mathrm{Si}\left(b^2\pi x^2\right)}{30240}-$$

```
[Out] -1/756*b^3*Pi/x^6+1/3780*b^7*Pi^3/x^2-11/3024*b^3*Pi*cos(b^2*Pi*x^2)/x^6+5/
2016*b^7*Pi^3*cos(b^2*Pi*x^2)/x^2-1/63*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelC(
b*x)/x^7+1/945*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3+1/1890*b^9*Pi
^5*FresnelC(b*x)^2+83/30240*b^9*Pi^4*Si(b^2*Pi*x^2)-1/9*FresnelC(b*x)*sin(1
/2*b^2*Pi*x^2)/x^9+1/315*b^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/9
45*b^8*Pi^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x-1/144*b*sin(b^2*Pi*x^2)/x^8
+67/30240*b^5*Pi^2*sin(b^2*Pi*x^2)/x^4
```

Rubi [A] time = 0.52, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6465, 6457, 6441, 30, 3375, 3380, 3297, 3299, 3379}

$$-\frac{\pi^4 b^8 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{945x}+\frac{\pi^2 b^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{315x^5}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+\frac{\pi^3 b^6 \text{Fresnel}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

```
[Out] -(b^3*Pi)/(756*x^6) + (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(
3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (b^2*Pi*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(63*x^7) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x
])/(945*x^3) + (b^9*Pi^5*FresnelC[b*x]^2)/1890 - (FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(9*x^9) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5)
- (b^8*Pi^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x) - (b*Sin[b^2*Pi*x^2]
)/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinI
ntegral[b^2*Pi*x^2])/30240
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6441

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[(Pi*b)/
(2*d), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2*b^4)/4]
```

Rule 6457

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[(x^
(m + 1)*Cos[d*x^2]*FresnelC[b*x])/(m + 1), x] + (Dist[(2*d)/(m + 1), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[(b*x^(m + 2))/(2*(m + 1)*(m + 2)), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && ILtQ[m, -2]
```

Rule 6465

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(x^
(m + 1)*Sin[d*x^2]*FresnelC[b*x])/(m + 1), x] + (-Dist[(2*d)/(m + 1), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2*b^4)/4] && I
LtQ[m, -1]
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{1}{18}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^9}\,dx + \frac{1}{9}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^8}\,dx$$

$$= -\frac{b^3\pi}{756x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{1}{36}b\,\mathrm{Subst}\left(\int \frac{\sin\left(b^2\pi\right.}{x^5}\right.$$

$$= -\frac{b^3\pi}{756x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} + \frac{b^4\pi^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5}$$

$$= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\right.}{945x}$$

$$= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\right.}{945x}$$

$$= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} + \frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C}{63x^7}$$

$$= -\frac{b^3\pi}{756x^6} + \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} + \frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C}{63x^7}$$

Mathematica [A] time = 0.03, size = 278, normalized size = 1.00

$$\frac{\pi^5b^9C(bx)^2}{1890}+\frac{\pi^3b^7}{3780x^2}-\frac{\pi b^3}{756x^6}-\frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{\pi b^2C(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{63x^7}-\frac{b\sin\left(\pi b^2x^2\right)}{144x^8}+\frac{83\pi^4b^9\mathrm{Si}\left(b^2\pi x^2\right)}{30240}-$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

```
[Out] -1/756*(b^3*Pi)/x^6 + (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(
3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (b^2*Pi*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(63*x^7) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x
])/(945*x^3) + (b^9*Pi^5*FresnelC[b*x]^2)/1890 - (FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(9*x^9) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5)
- (b^8*Pi^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x) - (b*Sin[b^2*Pi*x^2]
)/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinI
ntegral[b^2*Pi*x^2])/30240
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx)\sin\left(\frac{b^2 \pi\, x^2}{2}\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{fresnelc}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnelc(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}\left(b\,x\right)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^10,x)
```

```
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^10, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C\left(bx\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**10,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**10, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```