Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.6-Inverse-hyperbolic-cosecant/7.6.2-Inverse-hyperbolic-cosecant-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [71]. This is test number [203].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (71)	% 0.00 (0)
Mathematica	% 100.00 (71)	% 0.00 (0)
Maple	% 74.65 (53)	% 25.35 (18)
Maxima	% 59.15 (42)	% 40.85 (29)
Fricas	% 69.01 (49)	% 30.99 (22)
Sympy	% 45.07 (32)	% 54.93 (39)
Giac	% 32.39 (23)	% 67.61 (48)
Mupad	% 57.75 (41)	% 42.25 (30)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

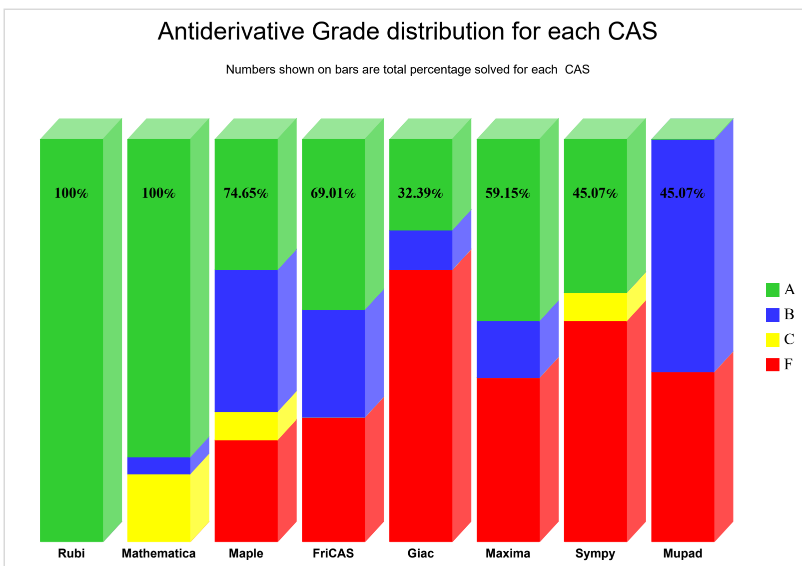
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

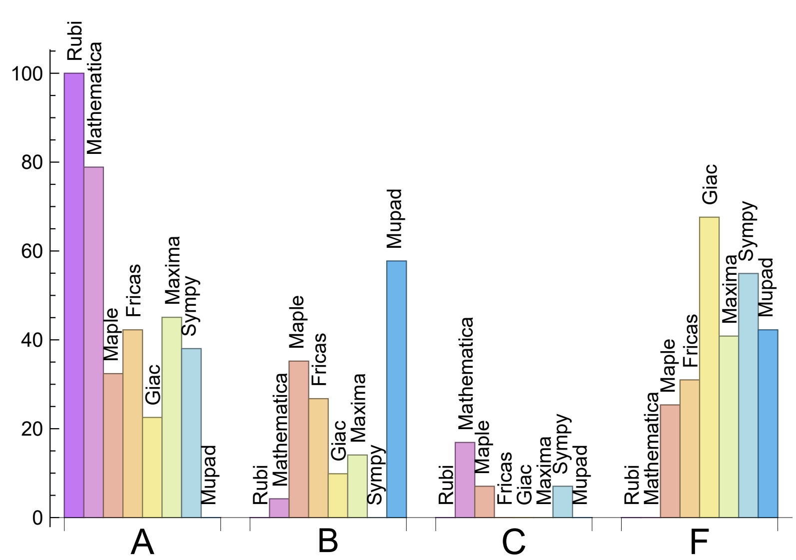
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	78.87	4.23	16.90	0.00
Maple	32.39	35.21	7.04	25.35
Maxima	45.07	14.08	0.00	40.85
Fricas	42.25	26.76	0.00	30.99
Sympy	38.03	0.00	7.04	54.93
Giac	22.54	9.86	0.00	67.61
Mupad	0.00	57.75	0.00	42.25

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	18	100.00 %	0.00 %	0.00 %
Maxima	29	93.10 %	0.00 %	6.90 %
Fricas	22	90.91 %	0.00 %	9.09 %
Sympy	39	84.62 %	5.13 %	10.26 %
Giac	48	68.75 %	0.00 %	31.25 %
Mupad	30	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

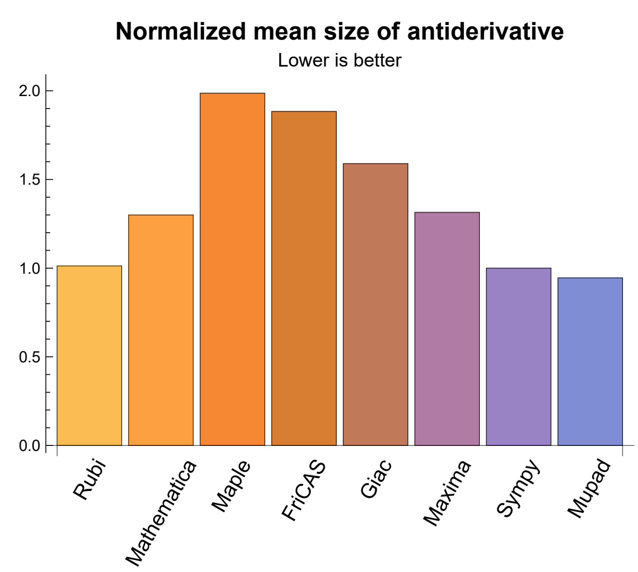
1.3 Performance

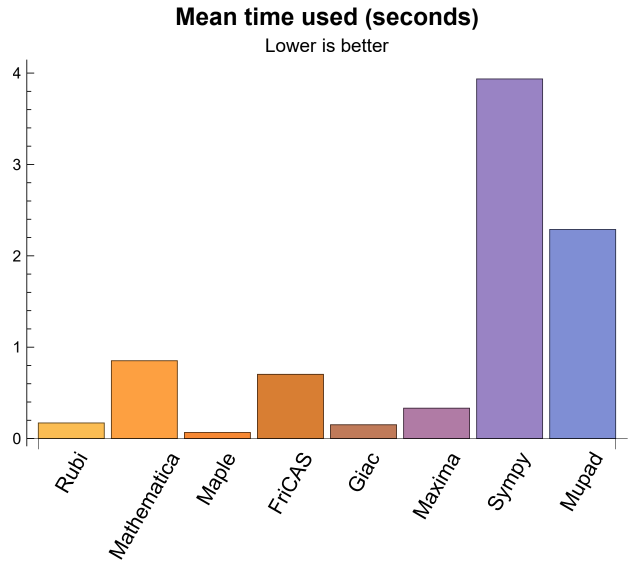
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.17	104.72	1.01	61.00	1.00
Mathematica	0.85	264.83	1.30	54.00	1.02
Maple	0.07	114.00	1.99	109.00	1.73
Maxima	0.33	69.24	1.31	59.50	1.21
Fricas	0.70	107.16	1.88	70.00	1.38
Sympy	3.94	54.91	1.00	50.00	1.06
Giac	0.15	71.30	1.59	69.00	1.43
Mupad	2.29	45.49	0.94	42.00	0.93

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {31}

Mathematica {4, 7, 8, 9, 10, 11, 12, 13, 18, 24, 28, 30, 31, 32, 33, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 51, 52, 53, 54, 56, 58, 59, 60, 62, 64, 65, 67, 69}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
```

```
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))
```

```
    except Exception as ee:
           leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

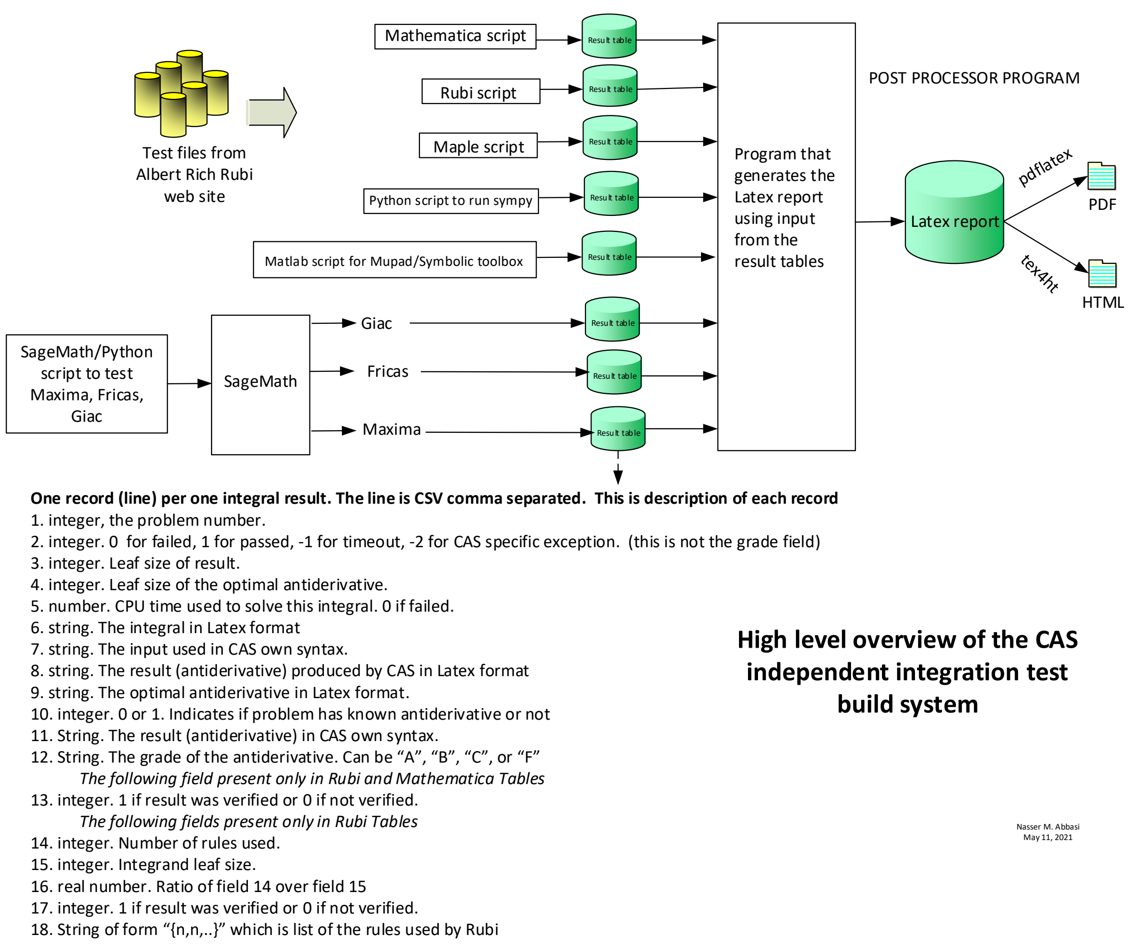
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 6, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 41, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71 }

B grade: { 5, 9, 25 }

C grade: { 4, 7, 8, 11, 12, 13, 23, 38, 40, 42, 44, 46 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 14, 15, 16, 17, 19, 20, 21, 22, 27, 28, 29, 34, 36, 47, 49, 50, 51, 57, 61, 70 }

B grade: { 5, 6, 30, 31, 32, 33, 35, 39, 41, 43, 45, 52, 53, 54, 55, 56, 58, 60, 62, 63, 64, 65, 66, 67, 68 }

C grade: { 38, 40, 42, 44, 46 }

F grade: { 4, 7, 8, 9, 10, 11, 12, 13, 18, 23, 24, 25, 26, 37, 48, 59, 69, 71 }

2.1.4 Maxima

A grade: { 14, 15, 16, 17, 19, 20, 21, 22, 27, 28, 29, 30, 31, 32, 34, 36, 39, 43, 47, 49, 50, 51, 52, 53, 55, 57, 61, 63, 66, 68, 70, 71 }

B grade: { 33, 35, 41, 45, 54, 56, 58, 60, 62, 64 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 18, 23, 24, 25, 26, 37, 38, 40, 42, 44, 46, 48, 59, 65, 67, 69 }

2.1.5 FriCAS

A grade: { 14, 15, 16, 17, 19, 20, 21, 22, 27, 28, 29, 30, 32, 34, 36, 39, 47, 49, 50, 51, 53, 57, 58, 60, 61, 62, 63, 64, 66, 68 }

B grade: { 1, 2, 3, 5, 6, 31, 33, 35, 41, 43, 45, 52, 54, 55, 56, 65, 67, 70, 71 }

C grade: { }

F grade: { 4, 7, 8, 9, 10, 11, 12, 13, 18, 23, 24, 25, 26, 37, 38, 40, 42, 44, 46, 48, 59, 69 }

2.1.6 Sympy

A grade: { 22, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 39, 41, 43, 45, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 68 }

B grade: { }

C grade: { 38, 40, 42, 44, 46 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 34, 47, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71 }

2.1.7 Giac

A grade: { 27, 28, 29, 30, 31, 35, 39, 41, 43, 60, 61, 62, 63, 64, 66, 68 }

B grade: { 33, 34, 36, 45, 47, 65, 67 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 32, 37, 38, 40, 42, 44, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 69, 70, 71 }

2.1.8 Mupad

A grade: { }

B grade: { 17, 19, 22, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 41, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 20, 21, 23, 24, 25, 26, 37, 38, 40, 46, 48, 59, 69 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	149	226	0	325	0	0	-1
normalized size	1	1.00	1.01	1.54	0.00	2.21	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.300	0.074	0.000	0.628	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	129	170	0	306	0	0	-1
normalized size	1	1.00	1.17	1.55	0.00	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.177	0.056	0.000	0.622	0.000	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	110	97	0	285	0	0	-1
normalized size	1	1.00	1.47	1.29	0.00	3.80	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.105	0.050	0.000	0.566	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	427	0	0	0	0	0	-1
normalized size	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.287	0.403	0.069	0.000	0.636	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	141	154	0	343	0	0	-1
normalized size	1	1.00	2.24	2.44	0.00	5.44	0.00	0.00	-0.02
time (sec)	N/A	0.102	0.165	0.074	0.000	0.487	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	220	453	0	461	0	0	-1
normalized size	1	1.00	1.93	3.97	0.00	4.04	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.477	0.061	0.000	0.512	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	1429	0	0	0	0	0	-1
normalized size	1	1.00	2.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.887	12.802	0.413	0.000	0.547	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	864	0	0	0	0	0	-1
normalized size	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.506	9.244	0.429	0.000	0.581	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	401	0	0	0	0	0	-1
normalized size	1	1.00	2.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.273	1.461	0.055	0.000	0.688	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	160	0	0	0	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.263	0.054	0.000	0.485	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	475	475	1008	0	0	0	0	0	-1
normalized size	1	1.00	2.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.073	2.980	0.537	0.000	0.492	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	2061	0	0	0	0	0	-1
normalized size	1	1.00	4.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.110	12.884	0.486	0.000	0.521	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1024	1024	8350	0	0	0	0	0	-1
normalized size	1	1.00	8.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.296	14.292	0.480	0.000	0.561	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	47	43	58	55	0	0	-1
normalized size	1	1.00	0.41	0.38	0.51	0.48	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.036	0.048	0.309	0.584	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	42	38	46	50	0	0	-1
normalized size	1	1.00	0.47	0.43	0.52	0.56	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.029	0.048	0.319	0.588	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	35	31	34	43	0	0	-1
normalized size	1	1.00	0.55	0.48	0.53	0.67	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.027	0.045	0.320	0.758	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	24	24	18	36	0	0	18
normalized size	1	1.00	0.77	0.77	0.58	1.16	0.00	0.00	0.58
time (sec)	N/A	0.008	0.011	0.050	0.319	0.525	0.000	0.000	2.571

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.097	0.037	0.073	0.000	0.666	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	42	45	65	44	0	0	33
normalized size	1	1.00	0.67	0.71	1.03	0.70	0.00	0.00	0.52
time (sec)	N/A	0.024	0.025	0.055	0.322	0.628	0.000	0.000	2.221

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	47	57	92	53	0	0	-1
normalized size	1	1.00	0.52	0.63	1.02	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.037	0.050	0.319	0.703	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	52	67	116	58	0	0	-1
normalized size	1	1.00	0.45	0.58	1.01	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.044	0.060	0.319	0.645	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	29	16	22	14	0	14
normalized size	1	1.00	1.12	1.81	1.00	1.38	0.88	0.00	0.88
time (sec)	N/A	0.006	0.003	0.049	0.312	0.729	0.116	0.000	0.074

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	64	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.108	0.104	0.238	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.101	0.045	0.089	0.000	0.630	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	236	0	0	0	0	0	-1
normalized size	1	1.00	3.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.603	0.269	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	54	0	0	0	51	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.98	0.00	-0.02
time (sec)	N/A	0.040	0.047	0.052	0.000	0.738	5.993	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	53	50	53	63	78	41
normalized size	1	1.00	0.91	0.98	0.93	0.98	1.17	1.44	0.76
time (sec)	N/A	0.029	0.051	0.086	0.327	0.395	3.014	0.142	2.177

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	76	109	107	79	73	69	61
normalized size	1	1.00	1.01	1.45	1.43	1.05	0.97	0.92	0.81
time (sec)	N/A	0.044	0.059	0.049	0.317	0.397	4.601	0.155	2.407

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	38	43	25	41	41	44	33
normalized size	1	1.00	1.23	1.39	0.81	1.32	1.32	1.42	1.06
time (sec)	N/A	0.022	0.039	0.044	0.311	0.560	2.712	0.142	2.166

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	85	78	64	29	52	39
normalized size	1	1.00	1.00	1.81	1.66	1.36	0.62	1.11	0.83
time (sec)	N/A	0.026	0.031	0.046	0.310	0.775	3.249	0.153	2.214

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	24	31	35	113	64	86	48	66	36
normalized size	1	1.29	1.46	4.71	2.67	3.58	2.00	2.75	1.50
time (sec)	N/A	0.015	0.017	0.046	0.314	0.448	1.143	0.127	2.253

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	107	54	64	41	0	34
normalized size	1	1.00	1.11	2.82	1.42	1.68	1.08	0.00	0.89
time (sec)	N/A	0.035	0.030	0.045	0.312	0.595	4.813	0.000	2.456

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	43	145	86	102	36	82	42
normalized size	1	1.00	1.08	3.62	2.15	2.55	0.90	2.05	1.05
time (sec)	N/A	0.029	0.030	0.046	0.323	0.462	3.233	0.144	2.556

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	37	42	25	47	0	69	42
normalized size	1	1.00	1.19	1.35	0.81	1.52	0.00	2.23	1.35
time (sec)	N/A	0.022	0.037	0.064	0.305	0.704	0.000	0.166	2.155

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	173	129	113	83	103	61
normalized size	1	1.00	0.82	2.66	1.98	1.74	1.28	1.58	0.94
time (sec)	N/A	0.043	0.051	0.056	0.323	0.678	4.254	0.157	2.543

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	52	41	58	65	124	61
normalized size	1	1.00	0.90	1.02	0.80	1.14	1.27	2.43	1.20
time (sec)	N/A	0.038	0.043	0.050	0.315	1.017	2.700	0.204	2.201

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	0	0	0	66	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	1.12	0.00	-0.02
time (sec)	N/A	0.042	0.063	0.072	0.000	0.666	6.029	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	112	150	0	0	48	0	-1
normalized size	1	1.00	0.55	0.74	0.00	0.00	0.24	0.00	-0.00
time (sec)	N/A	0.125	0.252	0.074	0.000	0.574	2.459	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	53	94	81	70	36	57	42
normalized size	1	1.00	1.02	1.81	1.56	1.35	0.69	1.10	0.81
time (sec)	N/A	0.038	0.052	0.168	0.328	0.649	3.684	0.153	2.610

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	113	104	0	0	41	0	-1
normalized size	1	1.00	1.31	1.21	0.00	0.00	0.48	0.00	-0.01
time (sec)	N/A	0.053	0.232	0.048	0.000	2.605	2.155	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	116	71	88	58	61	43
normalized size	1	1.00	1.05	2.90	1.78	2.20	1.45	1.52	1.08
time (sec)	N/A	0.036	0.031	0.153	0.366	0.614	6.793	0.145	2.971

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	96	146	0	0	42	0	24
normalized size	1	1.00	0.58	0.88	0.00	0.00	0.25	0.00	0.15
time (sec)	N/A	0.080	0.149	0.056	0.000	0.648	0.775	0.000	2.326

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	22	86	54	74	54	66	36
normalized size	1	1.00	0.48	1.87	1.17	1.61	1.17	1.43	0.78
time (sec)	N/A	0.033	0.047	0.160	0.306	0.871	10.694	0.163	2.380

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	96	111	0	0	42	0	27
normalized size	1	1.00	1.05	1.22	0.00	0.00	0.46	0.00	0.30
time (sec)	N/A	0.047	0.162	0.053	0.000	0.648	2.276	0.000	2.369

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	24	114	92	101	39	76	42
normalized size	1	1.00	0.57	2.71	2.19	2.40	0.93	1.81	1.00
time (sec)	N/A	0.039	0.048	0.148	0.311	0.721	3.971	0.145	2.829

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	114	171	0	0	44	0	-1
normalized size	1	1.00	0.63	0.94	0.00	0.00	0.24	0.00	-0.01
time (sec)	N/A	0.095	0.206	0.056	0.000	0.521	2.563	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	42	25	49	0	71	44
normalized size	1	1.00	1.26	1.35	0.81	1.58	0.00	2.29	1.42
time (sec)	N/A	0.022	0.054	0.135	0.321	0.563	0.000	0.144	2.163

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	0	0	0	71	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	1.11	0.00	-0.02
time (sec)	N/A	0.322	0.068	0.046	0.000	0.651	7.152	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	84	114	117	87	82	0	73
normalized size	1	1.00	0.99	1.34	1.38	1.02	0.96	0.00	0.86
time (sec)	N/A	0.254	0.070	0.041	0.312	0.665	4.721	0.000	2.171

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	44	61	32	49	51	0	40
normalized size	1	1.00	1.16	1.61	0.84	1.29	1.34	0.00	1.05
time (sec)	N/A	0.225	0.051	0.051	0.305	0.528	2.774	0.000	2.142

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	57	90	89	72	36	0	51
normalized size	1	1.00	1.10	1.73	1.71	1.38	0.69	0.00	0.98
time (sec)	N/A	0.240	0.037	0.051	0.322	0.768	3.333	0.000	2.128

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	44	120	75	99	63	0	52
normalized size	1	1.00	1.02	2.79	1.74	2.30	1.47	0.00	1.21
time (sec)	N/A	0.158	0.051	0.055	0.309	0.613	3.794	0.000	2.198

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	112	59	73	49	0	47
normalized size	1	1.00	1.11	2.38	1.26	1.55	1.04	0.00	1.00
time (sec)	N/A	0.072	0.038	0.053	0.312	0.937	3.631	0.000	2.260

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	39	150	93	112	34	0	44
normalized size	1	1.00	1.03	3.95	2.45	2.95	0.89	0.00	1.16
time (sec)	N/A	0.214	0.061	0.050	0.318	0.597	4.077	0.000	2.250

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	54	46	63	28	57	0	0	51
normalized size	1	1.59	1.35	1.85	0.82	1.68	0.00	0.00	1.50
time (sec)	N/A	0.204	0.044	0.050	0.307	0.587	0.000	0.000	2.208

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	176	139	121	92	0	68
normalized size	1	1.00	1.00	2.41	1.90	1.66	1.26	0.00	0.93
time (sec)	N/A	0.234	0.052	0.053	0.312	0.509	4.607	0.000	2.306

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	73	52	67	76	0	67
normalized size	1	1.00	0.93	1.26	0.90	1.16	1.31	0.00	1.16
time (sec)	N/A	0.233	0.056	0.050	0.304	0.734	3.073	0.000	2.263

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	74	209	180	131	114	0	89
normalized size	1	1.00	0.77	2.18	1.88	1.36	1.19	0.00	0.93
time (sec)	N/A	0.253	0.077	0.053	0.306	2.315	6.325	0.000	2.371

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	88	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.245	0.464	0.000	2.164	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	85	165	162	90	0	89	79
normalized size	1	1.00	0.92	1.79	1.76	0.98	0.00	0.97	0.86
time (sec)	N/A	0.108	0.201	0.119	0.402	1.642	0.000	0.147	2.434

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	120	49	58	0	85	61
normalized size	1	1.00	0.89	1.67	0.68	0.81	0.00	1.18	0.85
time (sec)	N/A	0.098	0.129	0.059	0.322	0.557	0.000	0.146	2.347

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	133	107	68	0	61	51
normalized size	1	1.00	0.92	2.25	1.81	1.15	0.00	1.03	0.86
time (sec)	N/A	0.090	0.131	0.059	0.492	0.888	0.000	0.138	2.339

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	89	31	38	0	44	31
normalized size	1	1.00	0.97	2.47	0.86	1.06	0.00	1.22	0.86
time (sec)	N/A	0.070	0.064	0.055	0.404	0.642	0.000	0.144	2.336

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	38	85	61	38	0	34	21
normalized size	1	1.00	1.41	3.15	2.26	1.41	0.00	1.26	0.78
time (sec)	N/A	0.059	0.042	0.056	0.413	0.658	0.000	0.137	2.359

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	37	172	0	80	0	70	38
normalized size	1	1.00	1.12	5.21	0.00	2.42	0.00	2.12	1.15
time (sec)	N/A	0.043	0.032	0.063	0.000	0.668	0.000	0.161	2.356

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	154	34	41	0	43	29
normalized size	1	1.00	1.00	5.13	1.13	1.37	0.00	1.43	0.97
time (sec)	N/A	0.071	0.095	0.066	0.427	0.543	0.000	0.136	2.146

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	210	0	130	0	114	61
normalized size	1	1.00	0.97	3.50	0.00	2.17	0.00	1.90	1.02
time (sec)	N/A	0.097	0.111	0.067	0.000	1.337	0.000	0.155	2.418

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	193	56	70	75	82	57
normalized size	1	1.00	0.89	3.16	0.92	1.15	1.23	1.34	0.93
time (sec)	N/A	0.095	0.141	0.071	0.426	0.551	5.231	0.165	2.218

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	52	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.098	0.060	0.546	0.000	0.523	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	74	63	57	266	0	0	42
normalized size	1	1.00	1.61	1.37	1.24	5.78	0.00	0.00	0.91
time (sec)	N/A	0.058	0.146	0.085	0.307	0.609	0.000	0.000	2.714

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	74	0	60	334	0	0	40
normalized size	1	1.00	1.61	0.00	1.30	7.26	0.00	0.00	0.87
time (sec)	N/A	0.070	0.181	0.171	0.315	0.655	0.000	0.000	2.214

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [42] had the largest ratio of [.8750]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	1.00	10	0.700
2	A	7	6	1.00	10	0.600
3	A	6	6	1.00	8	0.750
4	A	14	8	1.00	10	0.800
5	A	6	6	1.00	10	0.600
6	A	8	8	1.00	10	0.800
7	A	20	9	1.00	20	0.450
8	A	17	9	1.00	20	0.450
9	A	11	8	1.00	18	0.444
10	A	8	6	1.00	12	0.500
11	A	17	9	1.00	20	0.450
12	A	12	8	1.00	20	0.400
13	A	23	11	1.00	20	0.550
14	A	4	3	1.00	10	0.300
15	A	4	3	1.00	10	0.300
16	A	4	3	1.00	8	0.375
17	A	3	3	1.00	6	0.500
18	A	7	6	1.00	10	0.600
19	A	5	5	1.00	10	0.500
20	A	6	5	1.00	10	0.500
21	A	7	5	1.00	10	0.500
22	A	3	3	1.00	4	0.750
23	A	7	6	1.00	10	0.600
24	A	7	6	1.00	10	0.600
25	A	7	7	1.00	10	0.700
26	A	4	4	1.00	10	0.400
27	A	4	4	1.00	10	0.400

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	7	7	1.00	10	0.700
29	A	3	3	1.00	10	0.300
30	A	6	6	1.00	8	0.750
31	A	5	5	1.29	6	0.833
32	A	6	6	1.00	10	0.600
33	A	5	5	1.00	10	0.500
34	A	3	3	1.00	10	0.300
35	A	6	6	1.00	10	0.600
36	A	5	4	1.00	10	0.400
37	A	4	4	1.00	12	0.333
38	A	8	8	1.00	12	0.667
39	A	6	6	1.00	12	0.500
40	A	5	5	1.00	12	0.417
41	A	6	6	1.00	10	0.600
42	A	7	7	1.00	8	0.875
43	A	6	6	1.00	12	0.500
44	A	5	5	1.00	12	0.417
45	A	6	6	1.00	12	0.500
46	A	7	7	1.00	12	0.583
47	A	3	3	1.00	12	0.250
48	A	5	4	1.00	12	0.333
49	A	8	7	1.00	12	0.583
50	A	4	3	1.00	12	0.250
51	A	7	6	1.00	12	0.500
52	A	6	5	1.00	10	0.500
53	A	7	6	1.00	8	0.750
54	A	6	5	1.00	12	0.417
55	A	4	3	1.59	12	0.250
56	A	7	6	1.00	12	0.500
57	A	6	4	1.00	12	0.333
58	A	8	6	1.00	12	0.500
59	A	4	3	1.00	23	0.130
60	A	9	7	1.00	21	0.333
61	A	6	5	1.00	21	0.238
62	A	7	7	1.00	21	0.333
63	A	3	3	1.00	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	5	5	1.00	19	0.263
65	A	7	7	1.00	18	0.389
66	A	4	4	1.00	21	0.190
67	A	7	6	1.00	21	0.286
68	A	7	5	1.00	21	0.238
69	A	8	8	1.00	19	0.421
70	A	6	6	1.00	12	0.500
71	A	6	6	1.00	14	0.429

Chapter 3

Listing of integrals

3.1 $\int x^3 \operatorname{csch}^{-1}(a+bx)\,dx$

Optimal. Leaf size=147

$$-\frac{a^4\operatorname{csch}^{-1}(a+bx)}{4b^4}-\frac{\left(2-17a^2\right)(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}}{12b^4}+\frac{\left(1-2a^2\right)a\tanh^{-1}\left(\sqrt{\frac{1}{(a+bx)^2}+1}\right)}{2b^4}-\frac{a(a+bx)^2\sqrt{\frac{1}{(a+bx)^2}}}{3b^4}$$

```
[Out] -1/4*a^4*arccsch(b*x+a)/b^4+1/4*x^4*arccsch(b*x+a)+1/2*a*(-2*a^2+1)*arctanh
((1+1/(b*x+a)^2)^(1/2))/b^4-1/12*(-17*a^2+2)*(b*x+a)*(1+1/(b*x+a)^2)^(1/2)/
b^4+1/12*x^2*(b*x+a)*(1+1/(b*x+a)^2)^(1/2)/b^2-1/3*a*(b*x+a)^2*(1+1/(b*x+a)
^2)^(1/2)/b^4
```

Rubi [A] time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6322, 5469, 3782, 4048, 3770, 3767, 8}

$$-\frac{\left(2-17a^2\right)(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}}{12b^4}+\frac{\left(1-2a^2\right)a\tanh^{-1}\left(\sqrt{\frac{1}{(a+bx)^2}+1}\right)}{2b^4}-\frac{a^4\operatorname{csch}^{-1}(a+bx)}{4b^4}+\frac{x^2(a+bx)\sqrt{\frac{1}{(a+bx)^2}}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCsch[a + b*x],x]
```

```
[Out] -((2 - 17*a^2)*(a + b*x)*Sqrt[1 + (a + b*x)^(-2)])/(12*b^4) + (x^2*(a + b*x
)*Sqrt[1 + (a + b*x)^(-2)])/(12*b^2) - (a*(a + b*x)^2*Sqrt[1 + (a + b*x)^(-
2)])/(3*b^4) - (a^4*ArcCsch[a + b*x])/(4*b^4) + (x^4*ArcCsch[a + b*x])/4 +
(a*(1 - 2*a^2)*ArcTanh[Sqrt[1 + (a + b*x)^(-2)]])/(2*b^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 5469

```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Csch[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6322

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csch[x]*Co
th[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^3\text{csch}^{-1}(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x\coth(x)\text{csch}(x)(-a+\text{csch}(x))^3\,dx,x,\text{csch}^{-1}(a+bx)\right)}{b^4}\\
&= \frac{1}{4}x^4\text{csch}^{-1}(a+bx)-\frac{\text{Subst}\left(\int(-a+\text{csch}(x))^4\,dx,x,\text{csch}^{-1}(a+bx)\right)}{4b^4}\\
&= \frac{x^2(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{12b^2}+\frac{1}{4}x^4\text{csch}^{-1}(a+bx)-\frac{\text{Subst}\left(\int(-a+\text{csch}(x))\left(-3a^3-\left(2-9\right.\right.\right.}{}\\
&= \frac{x^2(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{12b^2}-\frac{a(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}}{3b^4}+\frac{1}{4}x^4\text{csch}^{-1}(a+bx)-\frac{\text{Subst}\left(\int\left(6a\right.\right.}{}\\
&= \frac{x^2(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{12b^2}-\frac{a(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}}{3b^4}-\frac{a^4\text{csch}^{-1}(a+bx)}{4b^4}+\frac{1}{4}x^4\text{csch}^{-1}(a\\
&= \frac{x^2(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{12b^2}-\frac{a(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}}{3b^4}-\frac{a^4\text{csch}^{-1}(a+bx)}{4b^4}+\frac{1}{4}x^4\text{csch}^{-1}(a\\
&= -\frac{\left(2-17a^2\right)(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{12b^4}+\frac{x^2(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{12b^2}-\frac{a(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}}{3b^4}\end{aligned}$$

Mathematica [A] time = 0.30, size = 149, normalized size = 1.01

$$\frac{-3a^4\sinh^{-1}\left(\frac{1}{a+bx}\right)+6\left(1-2a^2\right)a\log\left((a+bx)\left(\sqrt{\frac{a^2+2abx+b^2x^2+1}{(a+bx)^2}}+1\right)\right)+\sqrt{\frac{a^2+2abx+b^2x^2+1}{(a+bx)^2}}\left(13a^3+9a^2bx-3ab^2\right.}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCsch[a + b*x],x]
```

```
[Out] (Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(-2*a + 13*a^3 - 2*b*x + 9
*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3) + 3*b^4*x^4*ArcCsch[a + b*x] - 3*a^4*ArcS
inh[(a + b*x)^(-1)] + 6*a*(1 - 2*a^2)*Log[(a + b*x)*(1 + Sqrt[(1 + a^2 + 2*
a*b*x + b^2*x^2)/(a + b*x)^2])])/(12*b^4)
```

fricas [B] time = 0.63, size = 325, normalized size = 2.21

$$3\,b^4x^4\log\left(\frac{(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right)-3\,a^4\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+1}{b^2x^2+2\,abx+a^2}}-a+1\right)+3\,a^4\log\left(-bx+(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2
+ 2*a*b*x + a^2)) + 1)/(b*x + a)) - 3*a^4*log(-b*x + (b*x + a)*sqrt((b^2*x^
2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a + 1) + 3*a^4*log(-b*x
 + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2))
- a - 1) + 6*(2*a^3 - a)*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2
 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a) + (b^3*x^3 - 3*a*b^2*x^2 + 13*a^3 + (
9*a^2 - 2)*b*x - 2*a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x
 + a^2)))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,\operatorname{arcsch}(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccsch(b*x + a), x)
```

maple [A] time = 0.07, size = 226, normalized size = 1.54

$$\frac{\operatorname{arccsch}(bx+a)(bx+a)^4}{4}-\operatorname{arccsch}(bx+a)\,(bx+a)^3\,a+\frac{3\operatorname{arccsch}(bx+a)(bx+a)^2a^2}{2}-\operatorname{arccsch}(bx+a)\,(bx+a)\,a^3+\frac{\operatorname{arccsch}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccsch(b*x+a),x)
```

```
[Out] 1/b^4*(1/4*arccsch(b*x+a)*(b*x+a)^4-arccsch(b*x+a)*(b*x+a)^3*a+3/2*arccsch(
b*x+a)*(b*x+a)^2*a^2-arccsch(b*x+a)*(b*x+a)*a^3+1/4*arccsch(b*x+a)*a^4+1/12
*(1+(b*x+a)^2)^(1/2)*(-3*a^4*arctanh(1/(1+(b*x+a)^2)^(1/2))-12*arcsinh(b*x+
a)*a^3+(b*x+a)^2*(1+(b*x+a)^2)^(1/2)-6*a*(b*x+a)*(1+(b*x+a)^2)^(1/2)+18*a^2
*(1+(b*x+a)^2)^(1/2)+6*a*arcsinh(b*x+a)-2*(1+(b*x+a)^2)^(1/2))/((1+(b*x+a)^
2)/(b*x+a)^2)^(1/2)/(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(-i\,a^3+i\,a\right)\left(\log\left(\frac{i\left(b^2x+ab\right)}{b}+1\right)-\log\left(-\frac{i\left(b^2x+ab\right)}{b}+1\right)\right)}{2\,b^4}+\frac{2\,b^4x^4\log\left(\sqrt{b^2x^2+2\,abx+a^2+1}+1\right)+b^2x^2-6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*a^3 + I*a)*(log(I*(b^2*x + a*b)/b + 1) - log(-I*(b^2*x + a*b)/b +
1))/b^4 + 1/8*(2*b^4*x^4*log(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1) + b^2*x
^2 - 6*a*b*x - (a^4 - 6*a^2 + 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(b^4*
x^4 - a^4)*log(b*x + a))/b^4 + integrate(1/4*(b^2*x^5 + a*b*x^4)/(b^2*x^2 +
 2*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{asinh}\left(\frac{1}{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(1/(a + b*x)),x)
```

```
[Out] int(x^3*asinh(1/(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,\mathrm{acsch}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acsch(b*x+a),x)
```

```
[Out] Integral(x**3*acsch(a + b*x), x)
```

3.2 $\int x^2 \operatorname{csch}^{-1}(a+bx)\,dx$

Optimal. Leaf size=110

$$\frac{a^3\operatorname{csch}^{-1}(a+bx)}{3b^3}-\frac{(1-6a^2)\tanh^{-1}\left(\sqrt{\frac{1}{(a+bx)^2}+1}\right)}{6b^3}-\frac{5a(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}}{6b^3}+\frac{x(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}}{6b^2}+\frac{1}{3}x^3\operatorname{csc}$$

```
[Out] 1/3*a^3*arccsch(b*x+a)/b^3+1/3*x^3*arccsch(b*x+a)-1/6*(-6*a^2+1)*arctanh((1
+1/(b*x+a)^2)^(1/2))/b^3-5/6*a*(b*x+a)*(1+1/(b*x+a)^2)^(1/2)/b^3+1/6*x*(b*x
+a)*(1+1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A] time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6322, 5469, 3782, 3770, 3767, 8}

$$-\frac{(1-6a^2)\tanh^{-1}\left(\sqrt{\frac{1}{(a+bx)^2}+1}\right)}{6b^3}+\frac{a^3\operatorname{csch}^{-1}(a+bx)}{3b^3}-\frac{5a(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}}{6b^3}+\frac{x(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}}{6b^2}+\frac{1}{3}x^3\operatorname{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCsch[a + b*x],x]
```

```
[Out] (-5*a*(a + b*x)*Sqrt[1 + (a + b*x)^(-2)])/(6*b^3) + (x*(a + b*x)*Sqrt[1 + (
a + b*x)^(-2)])/(6*b^2) + (a^3*ArcCsch[a + b*x])/(3*b^3) + (x^3*ArcCsch[a +
 b*x])/3 - ((1 - 6*a^2)*ArcTanh[Sqrt[1 + (a + b*x)^(-2)]])/(6*b^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 5469

```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Csch[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6322

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csch[x]*Co
th[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^2\text{csch}^{-1}(a+bx)\,dx = -\frac{\text{Subst}\left(\int x\coth(x)\text{csch}(x)(-a+\text{csch}(x))^2\,dx, x, \text{csch}^{-1}(a+bx)\right)}{b^3}$$
$$= \frac{1}{3}x^3\text{csch}^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\text{csch}(x))^3\,dx, x, \text{csch}^{-1}(a+bx)\right)}{3b^3}$$
$$= \frac{x(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{6b^2} + \frac{1}{3}x^3\text{csch}^{-1}(a+bx) - \frac{\text{Subst}\left(\int\left(-2a^3-\left(1-6a^2\right)\text{csch}(x)-5 \right.\right.}{6b^3}$$
$$= \frac{x(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\text{csch}^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\text{csch}^{-1}(a+bx) + \frac{(5a)\,\text{Subst}\left(\int\text{csch}\right.}{}$$
$$= \frac{x(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\text{csch}^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\text{csch}^{-1}(a+bx) - \frac{\left(1-6a^2\right)\tanh^{-1}\left(\right.}{6b^3}$$
$$= -\frac{5a(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{6b^3} + \frac{x(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\text{csch}^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\text{csch}^{-1}(a$$

Mathematica [A] time = 0.18, size = 129, normalized size = 1.17

$$\frac{2a^3\sinh^{-1}\left(\frac{1}{a+bx}\right) + \left(-5a^2-4abx+b^2x^2\right)\sqrt{\frac{a^2+2abx+b^2x^2+1}{(a+bx)^2}} + \left(6a^2-1\right)\log\left((a+bx)\left(\sqrt{\frac{a^2+2abx+b^2x^2+1}{(a+bx)^2}}+1\right)\right) + 2}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCsch[a + b*x],x]
```

```
[Out] ((-5*a^2 - 4*a*b*x + b^2*x^2)*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^
2] + 2*b^3*x^3*ArcCsch[a + b*x] + 2*a^3*ArcSinh[(a + b*x)^(-1)] + (-1 + 6*a
^2)*Log[(a + b*x)*(1 + Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(
6*b^3)
```

fricas [B] time = 0.62, size = 306, normalized size = 2.78

$$2\,b^3x^3\log\left(\frac{(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right) + 2\,a^3\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+1}{b^2x^2+2\,abx+a^2}}-a+1\right) - 2\,a^3\log\left(-bx+(bx+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccsch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 +
 2*a*b*x + a^2)) + 1)/(b*x + a)) + 2*a^3*log(-b*x + (b*x + a)*sqrt((b^2*x^2
 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a + 1) - 2*a^3*log(-b*x
+ (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) -
 a - 1) - (6*a^2 - 1)*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 +
1)/(b^2*x^2 + 2*a*b*x + a^2)) - a) + (b^2*x^2 - 4*a*b*x - 5*a^2)*sqrt((b^2*
x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arcsch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccsch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccsch(b*x + a), x)
```

maple [A] time = 0.06, size = 170, normalized size = 1.55

$$\frac{\frac{\operatorname{arccsch}(bx+a)(bx+a)^3}{3} - \operatorname{arccsch}(bx+a)(bx+a)^2 a + \operatorname{arccsch}(bx+a)(bx+a)a^2 - \frac{\operatorname{arccsch}(bx+a)a^3}{3} + \frac{\sqrt{1+(bx+a)^2}\,\left(2a\right.}{}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccsch(b*x+a),x)
```

```
[Out] 1/b^3*(1/3*arccsch(b*x+a)*(b*x+a)^3-arccsch(b*x+a)*(b*x+a)^2*a+arccsch(b*x+
a)*(b*x+a)*a^2-1/3*arccsch(b*x+a)*a^3+1/6*(1+(b*x+a)^2)^(1/2)*(2*a^3*arctan
h(1/(1+(b*x+a)^2)^(1/2))+6*arcsinh(b*x+a)*a^2+(b*x+a)*(1+(b*x+a)^2)^(1/2)-6
*a*(1+(b*x+a)^2)^(1/2)-arcsinh(b*x+a))/((1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x
+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(3i\,a^2-i\right)\left(\log\left(\frac{i\left(b^2x+ab\right)}{b}+1\right)-\log\left(-\frac{i\left(b^2x+ab\right)}{b}+1\right)\right)}{6\,b^3}+\frac{2\,b^3x^3\log\left(\sqrt{b^2x^2+2\,abx+a^2+1}+1\right)+2\,bx+\left(a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccsch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(3*I*a^2 - I)*(log(I*(b^2*x + a*b)/b + 1) - log(-I*(b^2*x + a*b)/b + 1
))/b^3 + 1/6*(2*b^3*x^3*log(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1) + 2*b*x
+ (a^3 - 3*a)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(b^3*x^3 + a^3)*log(b*x
+ a))/b^3 + integrate(1/3*(b^2*x^4 + a*b*x^3)/(b^2*x^2 + 2*a*b*x + a^2 + (b
^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\operatorname{asinh}\left(\frac{1}{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(1/(a + b*x)),x)
```

```
[Out] int(x^2*asinh(1/(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acsch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acsch(b*x+a),x)
```

```
[Out] Integral(x**2*acsch(a + b*x), x)
```

3.3 $\int x \text{csch}^{-1}(a+bx)\,dx$

Optimal. Leaf size=75

$$-\frac{a^2\text{csch}^{-1}(a+bx)}{2b^2}+\frac{(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}}{2b^2}-\frac{a\tanh^{-1}\left(\sqrt{\frac{1}{(a+bx)^2}+1}\right)}{b^2}+\frac{1}{2}x^2\text{csch}^{-1}(a+bx)$$

```
[Out] -1/2*a^2*arccsch(b*x+a)/b^2+1/2*x^2*arccsch(b*x+a)-a*arctanh((1+1/(b*x+a)^2
)^(1/2))/b^2+1/2*(b*x+a)*(1+1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6322, 5469, 3773, 3770, 3767, 8}

$$-\frac{a^2\text{csch}^{-1}(a+bx)}{2b^2}+\frac{(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}}{2b^2}-\frac{a\tanh^{-1}\left(\sqrt{\frac{1}{(a+bx)^2}+1}\right)}{b^2}+\frac{1}{2}x^2\text{csch}^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCsch[a + b*x],x]
```

```
[Out] ((a + b*x)*Sqrt[1 + (a + b*x)^(-2)])/(2*b^2) - (a^2*ArcCsch[a + b*x])/(2*b^
2) + (x^2*ArcCsch[a + b*x])/2 - (a*ArcTanh[Sqrt[1 + (a + b*x)^(-2)]])/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3773

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 5469

```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Csch[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6322

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csch[x]*Co
th[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x\text{csch}^{-1}(a+bx)\,dx = -\frac{\text{Subst}\left(\int x\coth(x)\text{csch}(x)(-a+\text{csch}(x))\,dx, x, \text{csch}^{-1}(a+bx)\right)}{b^2}$$
$$= \frac{1}{2}x^2\text{csch}^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\text{csch}(x))^2\,dx, x, \text{csch}^{-1}(a+bx)\right)}{2b^2}$$
$$= -\frac{a^2\text{csch}^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\text{csch}^{-1}(a+bx) - \frac{\text{Subst}\left(\int\text{csch}^2(x)\,dx, x, \text{csch}^{-1}(a+bx)\right)}{2b^2}$$
$$= -\frac{a^2\text{csch}^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\text{csch}^{-1}(a+bx) - \frac{a\tanh^{-1}\left(\sqrt{1+\frac{1}{(a+bx)^2}}\right)}{b^2} + \frac{i\,\text{Subst}\left(\int 1\,d\right.}{}$$
$$= \frac{(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{2b^2} - \frac{a^2\text{csch}^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\text{csch}^{-1}(a+bx) - \frac{a\tanh^{-1}\left(\sqrt{1+\frac{}{(a+}}\right.}{b^2}$$

Mathematica [A] time = 0.11, size = 110, normalized size = 1.47

$$\frac{(a+bx)\sqrt{\frac{a^2+2abx+b^2x^2+1}{(a+bx)^2}} - 2a\log\left((a+bx)\left(\sqrt{\frac{a^2+2abx+b^2x^2+1}{(a+bx)^2}}+1\right)\right) + a^2\left(-\sinh^{-1}\left(\frac{1}{a+bx}\right)\right) + b^2x^2\text{csch}^{-1}(a+}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCsch[a + b*x],x]
```

```
[Out] ((a + b*x)*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2] + b^2*x^2*ArcCsc
h[a + b*x] - a^2*ArcSinh[(a + b*x)^(-1)] - 2*a*Log[(a + b*x)*(1 + Sqrt[(1 +
 a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]])])/(2*b^2)
```

fricas [B] time = 0.57, size = 285, normalized size = 3.80

$$\frac{b^2x^2\log\left(\frac{(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right) - a^2\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+1}{b^2x^2+2\,abx+a^2}}-a+1\right) + a^2\log\left(-bx+(bx+a)\right.}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccsch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2
*a*b*x + a^2)) + 1)/(b*x + a)) - a^2*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2
*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a + 1) + a^2*log(-b*x + (b*x
 + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a - 1
) + 2*a*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 +
2*a*b*x + a^2)) - a) + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^
2 + 2*a*b*x + a^2)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\text{arcsch}\,(bx+a)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccsch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*arccsch(b*x + a), x)
```

maple [A] time = 0.05, size = 97, normalized size = 1.29

$$\frac{\frac{\operatorname{arccsch}(bx+a)(bx+a)^2}{2} - \operatorname{arccsch}(bx+a)\,a\,(bx+a) - \frac{\sqrt{1+(bx+a)^2}\left(2a\operatorname{arcsinh}(bx+a)-\sqrt{1+(bx+a)^2}\right)}{2\sqrt{\frac{1+(bx+a)^2}{(bx+a)^2}}\,(bx+a)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccsch(b*x+a),x)
```

```
[Out] 1/b^2*(1/2*arccsch(b*x+a)*(b*x+a)^2-arccsch(b*x+a)*a*(b*x+a)-1/2*(1+(b*x+a)
^2)^(1/2)*(2*a*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/((1+(b*x+a)^2)/(b*x+a)^2
)^(1/2)/(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\,a\left(\log\left(\frac{i\left(b^2x+ab\right)}{b}+1\right)-\log\left(-\frac{i\left(b^2x+ab\right)}{b}+1\right)\right)}{2\,b^2}+\frac{2\,b^2x^2\log\left(\sqrt{b^2x^2+2\,abx+a^2+1}+1\right)-\left(a^2-1\right)\log\left(b^2x^2+2\,a\right.}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccsch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*I*a*(log(I*(b^2*x + a*b)/b + 1) - log(-I*(b^2*x + a*b)/b + 1))/b^2 + 1/
4*(2*b^2*x^2*log(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1) - (a^2 - 1)*log(b^2
*x^2 + 2*a*b*x + a^2 + 1) - 2*(b^2*x^2 - a^2)*log(b*x + a))/b^2 + integrate
(1/2*(b^2*x^3 + a*b*x^2)/(b^2*x^2 + 2*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^
2 + 1)^(3/2) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{asinh}\left(\frac{1}{a+b\,x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(1/(a + b*x)),x)
```

```
[Out] int(x*asinh(1/(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{acsch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acsch(b*x+a),x)
```

```
[Out] Integral(x*acsch(a + b*x), x)
```

3.4 $\int \frac{\operatorname{csch}^{-1}(a+bx)}{x}\,dx$

Optimal. Leaf size=162

$$\operatorname{Li}_2\left(\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{1-\sqrt{a^2+1}}\right)+\operatorname{Li}_2\left(\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{\sqrt{a^2+1}+1}\right)+\operatorname{csch}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{1-\sqrt{a^2+1}}\right)+\operatorname{csch}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{csch}^{-1}}}{\sqrt{a^2+}}\right.$$

```
[Out] -arccsch(b*x+a)*ln(1-(1/(b*x+a)+(1+1/(b*x+a)^2)^(1/2))^2)+arccsch(b*x+a)*ln
(1-a*(1/(b*x+a)+(1+1/(b*x+a)^2)^(1/2))/(1-(a^2+1)^(1/2)))+arccsch(b*x+a)*ln
(1-a*(1/(b*x+a)+(1+1/(b*x+a)^2)^(1/2))/(1+(a^2+1)^(1/2)))-1/2*polylog(2,(1/
(b*x+a)+(1+1/(b*x+a)^2)^(1/2))^2)+polylog(2,a*(1/(b*x+a)+(1+1/(b*x+a)^2)^(1
/2))/(1-(a^2+1)^(1/2)))+polylog(2,a*(1/(b*x+a)+(1+1/(b*x+a)^2)^(1/2))/(1+(a
^2+1)^(1/2)))
```

Rubi [A] time = 0.29, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6322, 5596, 5569, 3716, 2190, 2279, 2391, 5561}

$$\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{1-\sqrt{a^2+1}}\right)+\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{\sqrt{a^2+1}+1}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(a+bx)}\right)+\operatorname{csch}^{-1}(a+bx)\log\left(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsch[a + b*x]/x,x]
```

```
[Out] ArcCsch[a + b*x]*Log[1 - (a*E^ArcCsch[a + b*x])/(1 - Sqrt[1 + a^2])] + ArcC
sch[a + b*x]*Log[1 - (a*E^ArcCsch[a + b*x])/(1 + Sqrt[1 + a^2])] - ArcCsch[
a + b*x]*Log[1 - E^(2*ArcCsch[a + b*x])] + PolyLog[2, (a*E^ArcCsch[a + b*x]
)/(1 - Sqrt[1 + a^2])] + PolyLog[2, (a*E^ArcCsch[a + b*x])/(1 + Sqrt[1 + a^
2])] - PolyLog[2, E^(2*ArcCsch[a + b*x])]/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5596

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/(Csch[(c_.) + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> I
nt[((e + f*x)^m*Sinh[c + d*x]*F[c + d*x]^n*G[c + d*x]^p)/(b + a*Sinh[c + d*
x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G]
 && IntegersQ[m, n, p]
```

Rule 6322

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csch[x]*Co
th[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^{-1}(a+bx)}{x}\,dx &= -\operatorname{Subst}\left(\int \frac{x\coth(x)\operatorname{csch}(x)}{-a+\operatorname{csch}(x)}\,dx, x, \operatorname{csch}^{-1}(a+bx)\right)\\
&= -\operatorname{Subst}\left(\int \frac{x\coth(x)}{1-a\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(a+bx)\right)\\
&= -\left(a\operatorname{Subst}\left(\int \frac{x\cosh(x)}{1-a\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(a+bx)\right)\right) - \operatorname{Subst}\left(\int x\coth(x)\,dx, x, \operatorname{csch}^{-1}(a\right.\\
&= 2\operatorname{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \operatorname{csch}^{-1}(a+bx)\right) - a\operatorname{Subst}\left(\int \frac{e^x x}{1-\sqrt{1+a^2}-ae^x}\,dx, x, \operatorname{csch}^{-1}(\right.\\
&= \operatorname{csch}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{1-\sqrt{1+a^2}}\right) + \operatorname{csch}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{1+\sqrt{1+a^2}}\right) - \operatorname{csch}^{-1}(\\
&= \operatorname{csch}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{1-\sqrt{1+a^2}}\right) + \operatorname{csch}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{1+\sqrt{1+a^2}}\right) - \operatorname{csch}^{-1}(\\
&= \operatorname{csch}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{1-\sqrt{1+a^2}}\right) + \operatorname{csch}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{1+\sqrt{1+a^2}}\right) - \operatorname{csch}^{-1}(
\end{aligned}$$

Mathematica [C] time = 0.40, size = 427, normalized size = 2.64

$$\frac{1}{8}\left(8\operatorname{Li}_2\left(\frac{\left(\sqrt{a^2+1}-1\right)e^{\operatorname{csch}^{-1}(a+bx)}}{a}\right) + 8\operatorname{Li}_2\left(-\frac{\left(\sqrt{a^2+1}+1\right)e^{\operatorname{csch}^{-1}(a+bx)}}{a}\right) + 8\operatorname{csch}^{-1}(a+bx)\log\left(1-\frac{\left(\sqrt{a^2+1}\right.}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCsch[a + b*x]/x,x]
```

```
[Out] (Pi^2 - (4*I)*Pi*ArcCsch[a + b*x] - 8*ArcCsch[a + b*x]^2 - 32*ArcSin[Sqrt[(
-I + a)/a]/Sqrt[2]]*ArcTan[((1 - I*a)*Cot[(Pi + (2*I)*ArcCsch[a + b*x])/4])
/Sqrt[1 + a^2]] - 8*ArcCsch[a + b*x]*Log[1 - E^(-2*ArcCsch[a + b*x])] + (4*
I)*Pi*Log[1 - ((-1 + Sqrt[1 + a^2])*E^ArcCsch[a + b*x])/a] + 8*ArcCsch[a +
b*x]*Log[1 - ((-1 + Sqrt[1 + a^2])*E^ArcCsch[a + b*x])/a] + (16*I)*ArcSin[S
qrt[(-I + a)/a]/Sqrt[2]]*Log[1 - ((-1 + Sqrt[1 + a^2])*E^ArcCsch[a + b*x])/
a] + (4*I)*Pi*Log[1 + ((1 + Sqrt[1 + a^2])*E^ArcCsch[a + b*x])/a] + 8*ArcCs
ch[a + b*x]*Log[1 + ((1 + Sqrt[1 + a^2])*E^ArcCsch[a + b*x])/a] - (16*I)*Ar
cSin[Sqrt[(-I + a)/a]/Sqrt[2]]*Log[1 + ((1 + Sqrt[1 + a^2])*E^ArcCsch[a + b
*x])/a] - (4*I)*Pi*Log[(b*x)/(a + b*x)] + 4*PolyLog[2, E^(-2*ArcCsch[a + b*
x])] + 8*PolyLog[2, ((-1 + Sqrt[1 + a^2])*E^ArcCsch[a + b*x])/a] + 8*PolyLo
g[2, -(((1 + Sqrt[1 + a^2])*E^ArcCsch[a + b*x])/a)])/8
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcsch}\,(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arccsch(b*x + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcsch}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arccsch(b*x + a)/x, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\text{arccsch}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsch(b*x+a)/x,x)
```

```
[Out] int(arccsch(b*x+a)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcsch}\,(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(arccsch(b*x + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{asinh}\left(\frac{1}{a+b\,x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(1/(a + b*x))/x,x)
```

```
[Out] int(asinh(1/(a + b*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acsch}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsch(b*x+a)/x,x)
```

```
[Out] Integral(acsch(a + b*x)/x, x)
```

3.5 $\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=63

$$\frac{2b\tanh^{-1}\left(\frac{\tanh\left(\frac{1}{2}\operatorname{csch}^{-1}(a+bx)\right)+a}{\sqrt{a^2+1}}\right)}{a\sqrt{a^2+1}}-\frac{b\operatorname{csch}^{-1}(a+bx)}{a}-\frac{\operatorname{csch}^{-1}(a+bx)}{x}$$

```
[Out] -b*arccsch(b*x+a)/a-arccsch(b*x+a)/x+2*b*arctanh((a+tanh(1/2*arccsch(b*x+a)
))/(a^2+1)^(1/2))/a/(a^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6322, 5469, 3783, 2660, 618, 206}

$$\frac{2b\tanh^{-1}\left(\frac{\tanh\left(\frac{1}{2}\operatorname{csch}^{-1}(a+bx)\right)+a}{\sqrt{a^2+1}}\right)}{a\sqrt{a^2+1}}-\frac{b\operatorname{csch}^{-1}(a+bx)}{a}-\frac{\operatorname{csch}^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsch[a + b*x]/x^2,x]
```

```
[Out] -((b*ArcCsch[a + b*x])/a) - ArcCsch[a + b*x]/x + (2*b*ArcTanh[(a + Tanh[Arc
Csch[a + b*x]/2])/Sqrt[1 + a^2]])/(a*Sqrt[1 + a^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 5469

```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Csch[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6322

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csch[x]*Co
th[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^2}\,dx = -\left(b\operatorname{Subst}\left(\int \frac{x\coth(x)\operatorname{csch}(x)}{(-a+\operatorname{csch}(x))^2}\,dx, x, \operatorname{csch}^{-1}(a+bx)\right)\right)$$

$$= -\frac{\operatorname{csch}^{-1}(a+bx)}{x} + b\operatorname{Subst}\left(\int \frac{1}{-a+\operatorname{csch}(x)}\,dx, x, \operatorname{csch}^{-1}(a+bx)\right)$$

$$= -\frac{b\operatorname{csch}^{-1}(a+bx)}{a} - \frac{\operatorname{csch}^{-1}(a+bx)}{x} + \frac{b\operatorname{Subst}\left(\int \frac{1}{1-a\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(a+bx)\right)}{a}$$

$$= -\frac{b\operatorname{csch}^{-1}(a+bx)}{a} - \frac{\operatorname{csch}^{-1}(a+bx)}{x} + \frac{(2b)\operatorname{Subst}\left(\int \frac{1}{1-2ax-x^2}\,dx, x, \tanh\left(\frac{1}{2}\operatorname{csch}^{-1}(a+b\right.\right.}{a}$$

$$= -\frac{b\operatorname{csch}^{-1}(a+bx)}{a} - \frac{\operatorname{csch}^{-1}(a+bx)}{x} - \frac{(4b)\operatorname{Subst}\left(\int \frac{1}{4(1+a^2)-x^2}\,dx, x, -2a-2\tanh\left(\frac{1}{2}\operatorname{csc}\right.\right.}{a}$$

$$= -\frac{b\operatorname{csch}^{-1}(a+bx)}{a} - \frac{\operatorname{csch}^{-1}(a+bx)}{x} + \frac{2b\tanh^{-1}\left(\frac{a+\tanh\left(\frac{1}{2}\operatorname{csch}^{-1}(a+bx)\right)}{\sqrt{1+a^2}}\right)}{a\sqrt{1+a^2}}$$

Mathematica [B] time = 0.16, size = 141, normalized size = 2.24

$$-\frac{b\left(-\log\left(\sqrt{a^2+1}\,a\sqrt{\frac{a^2+2abx+b^2x^2+1}{(a+bx)^2}} + \sqrt{a^2+1}\,bx\sqrt{\frac{a^2+2abx+b^2x^2+1}{(a+bx)^2}} + a^2+abx+1\right) + \sqrt{a^2+1}\,\sinh^{-1}\left(\frac{1}{a+bx}\right) + 1\right.}{a\sqrt{a^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsch[a + b*x]/x^2,x]
```

```
[Out] -(ArcCsch[a + b*x]/x) - (b*(Sqrt[1 + a^2]*ArcSinh[(a + b*x)^(-1)] + Log[x]
- Log[1 + a^2 + a*b*x + a*Sqrt[1 + a^2]*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/
(a + b*x)^2] + Sqrt[1 + a^2]*b*x*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*
x)^2]]))/(a*Sqrt[1 + a^2])
```

fricas [B] time = 0.49, size = 343, normalized size = 5.44

$$-\frac{(a^2+1)bx\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2abx+a^2+1}{b^2x^2+2abx+a^2}} - a+1\right) - (a^2+1)bx\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2abx+a^2+1}{b^2x^2+2abx+a^2}} - a - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] -((a^2 + 1)*b*x*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^
2*x^2 + 2*a*b*x + a^2)) - a + 1) - (a^2 + 1)*b*x*log(-b*x + (b*x + a)*sqrt(
(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a - 1) - sqrt(a^
```

```
2 + 1)*b*x*log(-(a^2*b*x + a^3 + (a*b*x + a^2 + (a*b*x + a^2)*sqrt((b^2*x^2
 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)*sqrt(a^2 + 1) + (a^3
+ (a^2 + 1)*b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x
+ a^2)) + a)/x) + (a^3 + a)*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 +
1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)))/((a^3 + a)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsch}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccsch(b*x + a)/x^2, x)
```

maple [B] time = 0.07, size = 154, normalized size = 2.44

$$-\frac{\operatorname{arccsch}(bx+a)}{x}-\frac{b\sqrt{1+(bx+a)^2}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(bx+a)^2}}\right)}{\sqrt{\frac{1+(bx+a)^2}{(bx+a)^2}}\,(bx+a)\,a}+\frac{b\sqrt{1+(bx+a)^2}\,\ln\left(\frac{2\sqrt{a^2+1}\,\sqrt{1+(bx+a)^2}+2a(bx+a)+2}{bx}\right)}{\sqrt{\frac{1+(bx+a)^2}{(bx+a)^2}}\,(bx+a)\,a\sqrt{a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsch(b*x+a)/x^2,x)
```

```
[Out] -arccsch(b*x+a)/x-b*(1+(b*x+a)^2)^(1/2)/((1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*
x+a)/a*arctanh(1/(1+(b*x+a)^2)^(1/2))+b*(1+(b*x+a)^2)^(1/2)/((1+(b*x+a)^2)/
(b*x+a)^2)^(1/2)/(b*x+a)/a/(a^2+1)^(1/2)*ln(2*((a^2+1)^(1/2)*(1+(b*x+a)^2)^
(1/2)+a*(b*x+a)+1)/b/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\,b\left(\log\left(\frac{i\left(b^2x+ab\right)}{b}+1\right)-\log\left(-\frac{i\left(b^2x+ab\right)}{b}+1\right)\right)}{2\left(a^2+1\right)}-\frac{b\log(x)}{a^3+a}-\frac{a^2bx\log\left(b^2x^2+2\,abx+a^2+1\right)-2\left(a^3+\left(a^2b+b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*I*b*(log(I*(b^2*x + a*b)/b + 1) - log(-I*(b^2*x + a*b)/b + 1))/(a^2 +
1) - b*log(x)/(a^3 + a) - 1/2*(a^2*b*x*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2
*(a^3 + (a^2*b + b)*x + a)*log(b*x + a) + 2*(a^3 + a)*log(sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1) + 1))/((a^3 + a)*x) - integrate((b^2*x + a*b)/(b^2*x^3 + 2
*a*b*x^2 + (a^2 + 1)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 + 1)*x)*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(\frac{1}{a+b\,x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(1/(a + b*x))/x^2,x)
```

```
[Out] int(asinh(1/(a + b*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acsch}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsch(b*x+a)/x**2,x)
```

```
[Out] Integral(acsch(a + b*x)/x**2, x)
```

3.6 $\int \frac{\text{csch}^{-1}(a+bx)}{x^3} \, dx$

Optimal. Leaf size=114

$$\frac{b^2\text{csch}^{-1}(a+bx)}{2a^2}-\frac{\left(2a^2+1\right)b^2\tanh^{-1}\left(\frac{\tanh\left(\frac{1}{2}\text{csch}^{-1}(a+bx)\right)+a}{\sqrt{a^2+1}}\right)}{a^2\left(a^2+1\right)^{3/2}}+\frac{b(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}}{2a\left(a^2+1\right)x}-\frac{\text{csch}^{-1}(a+bx)}{2x^2}$$

```
[Out] 1/2*b^2*arccsch(b*x+a)/a^2-1/2*arccsch(b*x+a)/x^2-(2*a^2+1)*b^2*arctanh((a+
tanh(1/2*arccsch(b*x+a)))/(a^2+1)^(1/2))/a^2/(a^2+1)^(3/2)+1/2*b*(b*x+a)*(1
+1/(b*x+a)^2)^(1/2)/a/(a^2+1)/x
```

Rubi [A] time = 0.21, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6322, 5469, 3785, 3919, 3831, 2660, 618, 206}

$$\frac{b^2\text{csch}^{-1}(a+bx)}{2a^2}-\frac{\left(2a^2+1\right)b^2\tanh^{-1}\left(\frac{\tanh\left(\frac{1}{2}\text{csch}^{-1}(a+bx)\right)+a}{\sqrt{a^2+1}}\right)}{a^2\left(a^2+1\right)^{3/2}}+\frac{b(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}}{2a\left(a^2+1\right)x}-\frac{\text{csch}^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsch[a + b*x]/x^3,x]
```

```
[Out] (b*(a + b*x)*Sqrt[1 + (a + b*x)^(-2)])/(2*a*(1 + a^2)*x) + (b^2*ArcCsch[a +
 b*x])/(2*a^2) - ArcCsch[a + b*x]/(2*x^2) - ((1 + 2*a^2)*b^2*ArcTanh[(a + T
anh[ArcCsch[a + b*x]/2])/Sqrt[1 + a^2]])/(a^2*(1 + a^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 5469

```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Csch[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6322

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csch[x]*Co
th[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^3}\,dx = -\left(b^2\,\text{Subst}\left(\int \frac{x\coth(x)\operatorname{csch}(x)}{(-a+\operatorname{csch}(x))^3}\,dx, x, \operatorname{csch}^{-1}(a+bx)\right)\right)$$

$$= -\frac{\operatorname{csch}^{-1}(a+bx)}{2x^2} + \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{1}{(-a+\operatorname{csch}(x))^2}\,dx, x, \operatorname{csch}^{-1}(a+bx)\right)$$

$$= \frac{b(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{2a\left(1+a^2\right)x} - \frac{\operatorname{csch}^{-1}(a+bx)}{2x^2} + \frac{b^2\,\text{Subst}\left(\int \frac{-1-a^2-a\operatorname{csch}(x)}{-a+\operatorname{csch}(x)}\,dx, x, \operatorname{csch}^{-1}(a+bx\right.}{2a\left(1+a^2\right)}$$

$$= \frac{b(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{2a\left(1+a^2\right)x} + \frac{b^2\operatorname{csch}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{csch}^{-1}(a+bx)}{2x^2} - \frac{\left(\left(1+2a^2\right)b^2\right)\text{Subst}\left(\int \cdots\right.}{2a^2\cdots}$$

$$= \frac{b(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{2a\left(1+a^2\right)x} + \frac{b^2\operatorname{csch}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{csch}^{-1}(a+bx)}{2x^2} - \frac{\left(\left(1+2a^2\right)b^2\right)\text{Subst}\left(\int \cdots\right.}{2a^2\cdots}$$

$$= \frac{b(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{2a\left(1+a^2\right)x} + \frac{b^2\operatorname{csch}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{csch}^{-1}(a+bx)}{2x^2} - \frac{\left(\left(1+2a^2\right)b^2\right)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{b(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{2a\left(1+a^2\right)x} + \frac{b^2\operatorname{csch}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{csch}^{-1}(a+bx)}{2x^2} + \frac{\left(2\left(1+2a^2\right)b^2\right)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{b(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{2a\left(1+a^2\right)x} + \frac{b^2\operatorname{csch}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{csch}^{-1}(a+bx)}{2x^2} - \frac{\left(1+2a^2\right)b^2\tanh^{-1}\left(\frac{a+\cdots}{\cdots}\right.}{a^2\left(1+a^2\right)\cdots}$$

Mathematica [A] time = 0.48, size = 220, normalized size = 1.93

$$\frac{1}{2}\left(\frac{b(a+bx)\sqrt{\frac{a^2+2abx+b^2x^2+1}{(a+bx)^2}}}{a\left(a^2+1\right)x}-\frac{\left(2a^2+1\right)b^2\log\left(\sqrt{a^2+1}\,a\sqrt{\frac{a^2+2abx+b^2x^2+1}{(a+bx)^2}}+\sqrt{a^2+1}\,bx\sqrt{\frac{a^2+2abx+b^2x^2+1}{(a+bx)^2}}+a\right.}{a^2\left(a^2+1\right)^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsch[a + b*x]/x^3,x]
```

```
[Out] ((b*(a + b*x)*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])/(a*(1 + a^2)
*x) - ArcCsch[a + b*x]/x^2 + (b^2*ArcSinh[(a + b*x)^(-1)])/a^2 + ((1 + 2*a^
2)*b^2*Log[x])/(a^2*(1 + a^2)^(3/2)) - ((1 + 2*a^2)*b^2*Log[1 + a^2 + a*b*x
 + a*Sqrt[1 + a^2]*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2] + Sqrt[1
 + a^2]*b*x*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]])/(a^2*(1 + a^2
)^(3/2)))/2
```

fricas [B] time = 0.51, size = 461, normalized size = 4.04

$$\frac{\left(2\,a^2+1\right)\sqrt{a^2+1}\,b^2x^2\log\left(-\frac{a^2bx+a^3-\left(abx+a^2+\left(abx+a^2\right)\sqrt{\frac{b^2x^2+2\,abx+a^2+1}{b^2x^2+2\,abx+a^2}}+1\right)\sqrt{a^2+1}+\left(a^3+\left(a^2+1\right)bx+a\right)\sqrt{\frac{b^2x^2+2\,abx+a^2+1}{b^2x^2+2\,abx+a^2}}+a}{x}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a^2 + 1)*sqrt(a^2 + 1)*b^2*x^2*log(-(a^2*b*x + a^3 - (a*b*x + a^2 +
 (a*b*x + a^2)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)
) + 1)*sqrt(a^2 + 1) + (a^3 + (a^2 + 1)*b*x + a)*sqrt((b^2*x^2 + 2*a*b*x +
a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) + a)/x) + (a^4 + 2*a^2 + 1)*b^2*x^2*log
(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a
^2)) - a + 1) - (a^4 + 2*a^2 + 1)*b^2*x^2*log(-b*x + (b*x + a)*sqrt((b^2*x^
2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a - 1) + (a^3 + a)*b^2*
x^2 - (a^6 + 2*a^4 + a^2)*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)
/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)) + ((a^3 + a)*b^2*x^2 + (a^4 + a
^2)*b*x)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)))/((a
^6 + 2*a^4 + a^2)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsch}(bx+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccsch(b*x + a)/x^3, x)
```

maple [B] time = 0.06, size = 453, normalized size = 3.97

$$-\frac{\operatorname{arcsch}(bx+a)}{2x^2}+\frac{b^2\sqrt{1+(bx+a)^2}\;\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(bx+a)^2}}\right)}{2\sqrt{\frac{1+(bx+a)^2}{(bx+a)^2}}\;(bx+a)\left(a^2+1\right)}-\frac{b^2\sqrt{1+(bx+a)^2}\;a^2\ln\left(\frac{2\sqrt{a^2+1}\;\sqrt{1+(bx+a)^2}+2a(bx+}{bx}\right.}{\sqrt{\frac{1+(bx+a)^2}{(bx+a)^2}}\;(bx+a)\left(a^2+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsch(b*x+a)/x^3,x)
```

```
[Out] -1/2*arccsch(b*x+a)/x^2+1/2*b^2*(1+(b*x+a)^2)^(1/2)/((1+(b*x+a)^2)/(b*x+a)^
2)^(1/2)/(b*x+a)/(a^2+1)*arctanh(1/(1+(b*x+a)^2)^(1/2))-b^2*(1+(b*x+a)^2)^(
1/2)/((1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*a^2/(a^2+1)^(5/2)*ln(2*((a^2+1
)^(1/2)*(1+(b*x+a)^2)^(1/2)+a*(b*x+a)+1)/b/x)+1/2*b^2*(1+(b*x+a)^2)^(1/2)/(
(1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)/a^2/(a^2+1)*arctanh(1/(1+(b*x+a)^2)^
(1/2))+1/2*b*(1+(b*x+a)^2)/((1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)/a/(a^2+1
)/x-3/2*b^2*(1+(b*x+a)^2)^(1/2)/((1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)/(a^
2+1)^(5/2)*ln(2*((a^2+1)^(1/2)*(1+(b*x+a)^2)^(1/2)+a*(b*x+a)+1)/b/x)-1/2*b^
2*(1+(b*x+a)^2)^(1/2)/((1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)/a^2/(a^2+1)^(
5/2)*ln(2*((a^2+1)^(1/2)*(1+(b*x+a)^2)^(1/2)+a*(b*x+a)+1)/b/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\,ab^2\left(\log\left(\frac{i\left(b^2x+ab\right)}{b}+1\right)-\log\left(-\frac{i\left(b^2x+ab\right)}{b}+1\right)\right)}{2\left(a^4+2\,a^2+1\right)}+\frac{\left(3\,a^2b^2+b^2\right)\log(x)}{2\left(a^6+2\,a^4+a^2\right)}+\frac{\left(a^4b^2-a^2b^2\right)x^2\log\left(b^2x^2+2\,abx+a^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*I*a*b^2*(log(I*(b^2*x + a*b)/b + 1) - log(-I*(b^2*x + a*b)/b + 1))/(a^4
 + 2*a^2 + 1) + 1/2*(3*a^2*b^2 + b^2)*log(x)/(a^6 + 2*a^4 + a^2) + 1/4*((a^
4*b^2 - a^2*b^2)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(a^3*b + a*b)*x +
 2*(a^6 + 2*a^4 - (a^4*b^2 + 2*a^2*b^2 + b^2)*x^2 + a^2)*log(b*x + a) - 2*(
a^6 + 2*a^4 + a^2)*log(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1))/((a^6 + 2*a^
4 + a^2)*x^2) - integrate(1/2*(b^2*x + a*b)/(b^2*x^4 + 2*a*b*x^3 + (a^2 + 1
)*x^2 + (b^2*x^4 + 2*a*b*x^3 + (a^2 + 1)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}\left(\frac{1}{a+b\,x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(1/(a + b*x))/x^3,x)
```

```
[Out] int(asinh(1/(a + b*x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acsch}(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsch(b*x+a)/x**3,x)
```

```
[Out] Integral(acsch(a + b*x)/x**3, x)
```

3.7 $\int (e+fx)^3 \left(a+b\,\mathbf{csch}^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=501

$$\frac{bf^2(c+dx)^2\sqrt{\frac{1}{(c+dx)^2}+1}\,(de-cf)\left(a+b\text{csch}^{-1}(c+dx)\right)}{d^4}-\frac{2bf^2(de-cf)\tanh^{-1}\left(e^{\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csch}^{-1}(c\right.}{d^4}$$

```
[Out] b^2*f^2*(-c*f+d*e)*x/d^3+1/12*b^2*f^3*(d*x+c)^2/d^4-1/4*(-c*f+d*e)^4*(a+b*a
rccsch(d*x+c))^2/d^4/f+1/4*(f*x+e)^4*(a+b*arccsch(d*x+c))^2/f-2*b*f^2*(-c*f
+d*e)*(a+b*arccsch(d*x+c))*arctanh(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))/d^4+4*b
*(-c*f+d*e)^3*(a+b*arccsch(d*x+c))*arctanh(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))
/d^4-1/3*b^2*f^3*ln(d*x+c)/d^4+3*b^2*f*(-c*f+d*e)^2*ln(d*x+c)/d^4-b^2*f^2*(
-c*f+d*e)*polylog(2,-1/(d*x+c)-(1+1/(d*x+c)^2)^(1/2))/d^4+2*b^2*(-c*f+d*e)^
3*polylog(2,-1/(d*x+c)-(1+1/(d*x+c)^2)^(1/2))/d^4+b^2*f^2*(-c*f+d*e)*polylo
g(2,1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))/d^4-2*b^2*(-c*f+d*e)^3*polylog(2,1/(d*
x+c)+(1+1/(d*x+c)^2)^(1/2))/d^4-1/3*b*f^3*(d*x+c)*(a+b*arccsch(d*x+c))*(1+1
/(d*x+c)^2)^(1/2)/d^4+3*b*f*(-c*f+d*e)^2*(d*x+c)*(a+b*arccsch(d*x+c))*(1+1/
(d*x+c)^2)^(1/2)/d^4+b*f^2*(-c*f+d*e)*(d*x+c)^2*(a+b*arccsch(d*x+c))*(1+1/(
d*x+c)^2)^(1/2)/d^4+1/6*b*f^3*(d*x+c)^3*(a+b*arccsch(d*x+c))*(1+1/(d*x+c)^2
)^(1/2)/d^4
```

Rubi [A] time = 0.89, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6322, 5469, 4190, 4182, 2279, 2391, 4184, 3475, 4185}

$$-\frac{b^2f^2(de-cf)\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(c+dx)}\right)}{d^4}+\frac{b^2f^2(de-cf)\text{PolyLog}\left(2,e^{\text{csch}^{-1}(c+dx)}\right)}{d^4}+\frac{2b^2(de-cf)^3\text{PolyLog}\left(2\right.}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*(a + b*ArcCsch[c + d*x])^2,x]
```

```
[Out] (b^2*f^2*(d*e - c*f)*x)/d^3 + (b^2*f^3*(c + d*x)^2)/(12*d^4) - (b*f^3*(c +
d*x)*Sqrt[1 + (c + d*x)^(-2)]*(a + b*ArcCsch[c + d*x]))/(3*d^4) + (3*b*f*(d
*e - c*f)^2*(c + d*x)*Sqrt[1 + (c + d*x)^(-2)]*(a + b*ArcCsch[c + d*x]))/d^
4 + (b*f^2*(d*e - c*f)*(c + d*x)^2*Sqrt[1 + (c + d*x)^(-2)]*(a + b*ArcCsch[
c + d*x]))/d^4 + (b*f^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^(-2)]*(a + b*ArcCsch
[c + d*x]))/(6*d^4) - ((d*e - c*f)^4*(a + b*ArcCsch[c + d*x])^2)/(4*d^4*f)
+ ((e + f*x)^4*(a + b*ArcCsch[c + d*x])^2)/(4*f) - (2*b*f^2*(d*e - c*f)*(a
+ b*ArcCsch[c + d*x])*ArcTanh[E^ArcCsch[c + d*x]])/d^4 + (4*b*(d*e - c*f)^3
*(a + b*ArcCsch[c + d*x])*ArcTanh[E^ArcCsch[c + d*x]])/d^4 - (b^2*f^3*Log[c
 + d*x])/(3*d^4) + (3*b^2*f*(d*e - c*f)^2*Log[c + d*x])/d^4 - (b^2*f^2*(d*e
 - c*f)*PolyLog[2, -E^ArcCsch[c + d*x]])/d^4 + (2*b^2*(d*e - c*f)^3*PolyLog
[2, -E^ArcCsch[c + d*x]])/d^4 + (b^2*f^2*(d*e - c*f)*PolyLog[2, E^ArcCsch[c
 + d*x]])/d^4 - (2*b^2*(d*e - c*f)^3*PolyLog[2, E^ArcCsch[c + d*x]])/d^4
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5469

```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Csch[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6322

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csch[x]*Co
th[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e+fx)^3 \left(a+b\,\text{csch}^{-1}(c+dx)\right)^2 dx = -\frac{\text{Subst}\left(\int (a+bx)^2 \coth(x)\text{csch}(x)(de-cf+f\text{csch}(x))^3\,dx, x, \text{csch}\right.}{d^4}$$

$$= \frac{(e+fx)^4 \left(a+b\,\text{csch}^{-1}(c+dx)\right)^2}{4f} - \frac{b\,\text{Subst}\left(\int (a+bx)(de-cf+f\text{cs}\right.}{2d^4}$$

$$= \frac{(e+fx)^4 \left(a+b\,\text{csch}^{-1}(c+dx)\right)^2}{4f} - \frac{b\,\text{Subst}\left(\int \left(d^4e^4\left(1+\frac{cf\left(-4d^3e^3+6c\right.}{}\right.\right.\right.}{}$$

$$= -\frac{(de-cf)^4 \left(a+b\,\text{csch}^{-1}(c+dx)\right)^2}{4d^4f} + \frac{(e+fx)^4 \left(a+b\,\text{csch}^{-1}(c+dx)\right.}{4f}$$

$$= \frac{b^2f^2(de-cf)x}{d^3} + \frac{b^2f^3(c+dx)^2}{12d^4} + \frac{3bf(de-cf)^2(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}}{d^4}$$

$$= \frac{b^2f^2(de-cf)x}{d^3} + \frac{b^2f^3(c+dx)^2}{12d^4} - \frac{bf^3(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{cs}\right.}{3d^4}$$

$$= \frac{b^2f^2(de-cf)x}{d^3} + \frac{b^2f^3(c+dx)^2}{12d^4} - \frac{bf^3(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{cs}\right.}{3d^4}$$

$$= \frac{b^2f^2(de-cf)x}{d^3} + \frac{b^2f^3(c+dx)^2}{12d^4} - \frac{bf^3(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{cs}\right.}{3d^4}$$

Mathematica [C] time = 12.80, size = 1429, normalized size = 2.85

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^3*(a + b*ArcCsch[c + d*x])^2,x]
```

```
[Out] a^2*e^3*x + (3*a^2*e^2*f*x^2)/2 + a^2*e*f^2*x^3 + (a^2*f^3*x^4)/4 + (a*b*(3
*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCsch[c + d*x] + (f*(c + d
*x)*Sqrt[(1 + c^2 + 2*c*d*x + d^2*x^2)/(c + d*x)^2]*((-2 + 13*c^2)*f^2 - 2*
c*d*f*(15*e + 2*f*x) + d^2*(18*e^2 + 6*e*f*x + f^2*x^2)) - 3*c*(-4*d^3*e^3
+ 6*c*d^2*e^2*f - 4*c^2*d*e*f^2 + c^3*f^3)*ArcSinh[(c + d*x)^(-1)] + 6*(2*d
^3*e^3 - 6*c*d^2*e^2*f + (-1 + 6*c^2)*d*e*f^2 + c*(1 - 2*c^2)*f^3)*Log[(c +
 d*x)*(1 + Sqrt[(1 + c^2 + 2*c*d*x + d^2*x^2)/(c + d*x)^2])])/d^4))/6 - (b^
2*e^3*(-(ArcCsch[c + d*x]*((c + d*x)*ArcCsch[c + d*x] - 2*Log[1 - E^(-ArcCs
ch[c + d*x])] + 2*Log[1 + E^(-ArcCsch[c + d*x])])) + 2*PolyLog[2, -E^(-ArcC
sch[c + d*x])] - 2*PolyLog[2, E^(-ArcCsch[c + d*x])]))/d - (3*b^2*d*e^2*f*x
*(((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]*ArcCsch[c + d*x])/d^2 + ((c + d*x)^2*
ArcCsch[c + d*x]^2)/(2*d^2) - (c*ArcCsch[c + d*x]^2*Coth[ArcCsch[c + d*x]/2
])/(2*d^2) - Log[(c + d*x)^(-1)]/d^2 - ((2*I)*c*(I*ArcCsch[c + d*x]*(Log[1
- E^(-ArcCsch[c + d*x])] - Log[1 + E^(-ArcCsch[c + d*x])]) + I*(PolyLog[2,
-E^(-ArcCsch[c + d*x])] - PolyLog[2, E^(-ArcCsch[c + d*x])])))/d^2 + (c*Arc
Csch[c + d*x]^2*Tanh[ArcCsch[c + d*x]/2])/(2*d^2)))/((c + d*x)*(-1 + c/(c +
 d*x))) - (b^2*e*f^2*(2*(-2 + 12*c*ArcCsch[c + d*x] + ArcCsch[c + d*x]^2 -
6*c^2*ArcCsch[c + d*x]^2)*Coth[ArcCsch[c + d*x]/2] + 2*ArcCsch[c + d*x]*(-1
 + 3*c*ArcCsch[c + d*x])*Csch[ArcCsch[c + d*x]/2]^2 - (ArcCsch[c + d*x]^2*C
sch[ArcCsch[c + d*x]/2]^4)/(2*(c + d*x)) - 48*c*Log[(c + d*x)^(-1)] + 8*(-1
 + 6*c^2)*(ArcCsch[c + d*x]*(Log[1 - E^(-ArcCsch[c + d*x])] - Log[1 + E^(-A
rcCsch[c + d*x])]) + PolyLog[2, -E^(-ArcCsch[c + d*x])] - PolyLog[2, E^(-Ar
cCsch[c + d*x])]) - 2*ArcCsch[c + d*x]*(1 + 3*c*ArcCsch[c + d*x])*Sech[ArcC
sch[c + d*x]/2]^2 - 8*(c + d*x)^3*ArcCsch[c + d*x]^2*Sinh[ArcCsch[c + d*x]/
```

```
2]^4 + 2*(2 + 12*c*ArcCsch[c + d*x] - ArcCsch[c + d*x]^2 + 6*c^2*ArcCsch[c
+ d*x]^2)*Tanh[ArcCsch[c + d*x]/2]))/(8*d^3) - (b^2*f^3*x^3*(-16*(2*ArcCsch
[c + d*x] - 18*c^2*ArcCsch[c + d*x] + 6*c^3*ArcCsch[c + d*x]^2 - 3*c*(-2 +
ArcCsch[c + d*x]^2))*Coth[ArcCsch[c + d*x]/2] + 2*(2 - 24*c*ArcCsch[c + d*x
] - 3*ArcCsch[c + d*x]^2 + 36*c^2*ArcCsch[c + d*x]^2)*Csch[ArcCsch[c + d*x]
/2]^2 + 3*ArcCsch[c + d*x]^2*Csch[ArcCsch[c + d*x]/2]^4 - (2*ArcCsch[c + d*
x]*(-1 + 6*c*ArcCsch[c + d*x])*Csch[ArcCsch[c + d*x]/2]^4)/(c + d*x) - 64*(
-1 + 9*c^2)*Log[(c + d*x)^(-1)] + 192*c*(-1 + 2*c^2)*(ArcCsch[c + d*x]*(Log
[1 - E^(-ArcCsch[c + d*x])] - Log[1 + E^(-ArcCsch[c + d*x])]) + PolyLog[2,
-E^(-ArcCsch[c + d*x])] - PolyLog[2, E^(-ArcCsch[c + d*x])]) - 2*(2 + 24*c*
ArcCsch[c + d*x] - 3*ArcCsch[c + d*x]^2 + 36*c^2*ArcCsch[c + d*x]^2)*Sech[A
rcCsch[c + d*x]/2]^2 + 3*ArcCsch[c + d*x]^2*Sech[ArcCsch[c + d*x]/2]^4 - 32
*(c + d*x)^3*ArcCsch[c + d*x]*(1 + 6*c*ArcCsch[c + d*x])*Sinh[ArcCsch[c + d
*x]/2]^4 + 16*(-2*ArcCsch[c + d*x] + 18*c^2*ArcCsch[c + d*x] + 6*c^3*ArcCsc
h[c + d*x]^2 - 3*c*(-2 + ArcCsch[c + d*x]^2))*Tanh[ArcCsch[c + d*x]/2]))/(1
92*d*(c + d*x)^3*(-1 + c/(c + d*x))^3)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2 f^3 x^3 + 3\,a^2 e f^2 x^2 + 3\,a^2 e^2 f x + a^2 e^3 + \left(b^2 f^3 x^3 + 3\,b^2 e f^2 x^2 + 3\,b^2 e^2 f x + b^2 e^3\right) \operatorname{arcsch}\left(dx + c\right)^2 + 2\,\left(a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arccsch(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f^3*x^3 + 3*a^2*e*f^2*x^2 + 3*a^2*e^2*f*x + a^2*e^3 + (b^2*f^3
*x^3 + 3*b^2*e*f^2*x^2 + 3*b^2*e^2*f*x + b^2*e^3)*arccsch(d*x + c)^2 + 2*(a
*b*f^3*x^3 + 3*a*b*e*f^2*x^2 + 3*a*b*e^2*f*x + a*b*e^3)*arccsch(d*x + c), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx + e\right)^3 \left(b \operatorname{arcsch}\left(dx + c\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arccsch(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*(b*arccsch(d*x + c) + a)^2, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \left(fx + e\right)^3 \left(a + b \operatorname{arccsch}\left(dx + c\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(a+b*arccsch(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^3*(a+b*arccsch(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arccsch(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*f^3*x^4 + a^2*e*f^2*x^3 + 3/2*a^2*e^2*f*x^2 + a^2*e^3*x + (2*(d*x +
 c)*arccsch(d*x + c) + log(sqrt(1/(d*x + c)^2 + 1) + 1) - log(sqrt(1/(d*x +
 c)^2 + 1) - 1))*a*b*e^3/d + 1/4*(b^2*f^3*x^4 + 4*b^2*e*f^2*x^3 + 6*b^2*e^2
*f*x^2 + 4*b^2*e^3*x)*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)^2 - integr
```

```
ate(-1/2*(2*(b^2*d^2*f^3*x^5 + b^2*c^2*e^3 + b^2*e^3 + (3*b^2*d^2*e*f^2 + 2
*b^2*c*d*f^3)*x^4 + (6*b^2*c*d*e*f^2 + b^2*c^2*f^3 + (3*d^2*e^2*f + f^3)*b^
2)*x^3 + (6*b^2*c*d*e^2*f + 3*b^2*c^2*e*f^2 + (d^2*e^3 + 3*e*f^2)*b^2)*x^2
+ (2*b^2*c*d*e^3 + 3*b^2*c^2*e^2*f + 3*b^2*e^2*f)*x)*log(d*x + c)^2 - 4*(a*
b*d^2*f^3*x^5 + (3*a*b*d^2*e*f^2 + 2*a*b*c*d*f^3)*x^4 + (6*a*b*c*d*e*f^2 +
a*b*c^2*f^3 + (3*d^2*e^2*f + f^3)*a*b)*x^3 + 3*(2*a*b*c*d*e^2*f + a*b*c^2*e
*f^2 + a*b*e*f^2)*x^2 + 3*(a*b*c^2*e^2*f + a*b*e^2*f)*x)*log(d*x + c) + (4*
a*b*d^2*f^3*x^5 + 4*(3*a*b*d^2*e*f^2 + 2*a*b*c*d*f^3)*x^4 + 4*(6*a*b*c*d*e*
f^2 + a*b*c^2*f^3 + (3*d^2*e^2*f + f^3)*a*b)*x^3 + 12*(2*a*b*c*d*e^2*f + a*
b*c^2*e*f^2 + a*b*e*f^2)*x^2 + 12*(a*b*c^2*e^2*f + a*b*e^2*f)*x - 4*(b^2*d^
2*f^3*x^5 + b^2*c^2*e^3 + b^2*e^3 + (3*b^2*d^2*e*f^2 + 2*b^2*c*d*f^3)*x^4 +
 (6*b^2*c*d*e*f^2 + b^2*c^2*f^3 + (3*d^2*e^2*f + f^3)*b^2)*x^3 + (6*b^2*c*d
*e^2*f + 3*b^2*c^2*e*f^2 + (d^2*e^3 + 3*e*f^2)*b^2)*x^2 + (2*b^2*c*d*e^3 +
3*b^2*c^2*e^2*f + 3*b^2*e^2*f)*x)*log(d*x + c) + ((4*a*b*d^2*f^3 - b^2*d^2*
f^3)*x^5 + (12*a*b*d^2*e*f^2 - 4*b^2*d^2*e*f^2 + (8*a*b*d*f^3 - b^2*d*f^3)*
c)*x^4 - 2*(3*b^2*d^2*e^2*f - 2*a*b*c^2*f^3 - 2*(3*d^2*e^2*f + f^3)*a*b - 2
*(6*a*b*d*e*f^2 - b^2*d*e*f^2)*c)*x^3 - 2*(2*b^2*d^2*e^3 - 6*a*b*c^2*e*f^2
- 6*a*b*e*f^2 - 3*(4*a*b*d*e^2*f - b^2*d*e^2*f)*c)*x^2 - 4*(b^2*c*d*e^3 - 3
*a*b*c^2*e^2*f - 3*a*b*e^2*f)*x - 4*(b^2*d^2*f^3*x^5 + b^2*c^2*e^3 + b^2*e^
3 + (3*b^2*d^2*e*f^2 + 2*b^2*c*d*f^3)*x^4 + (6*b^2*c*d*e*f^2 + b^2*c^2*f^3
+ (3*d^2*e^2*f + f^3)*b^2)*x^3 + (6*b^2*c*d*e^2*f + 3*b^2*c^2*e*f^2 + (d^2*
e^3 + 3*e*f^2)*b^2)*x^2 + (2*b^2*c*d*e^3 + 3*b^2*c^2*e^2*f + 3*b^2*e^2*f)*x
)*log(d*x + c))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1) + 1) + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((b^2*d^2*f^3*x^5
+ b^2*c^2*e^3 + b^2*e^3 + (3*b^2*d^2*e*f^2 + 2*b^2*c*d*f^3)*x^4 + (6*b^2*c*
d*e*f^2 + b^2*c^2*f^3 + (3*d^2*e^2*f + f^3)*b^2)*x^3 + (6*b^2*c*d*e^2*f + 3
*b^2*c^2*e*f^2 + (d^2*e^3 + 3*e*f^2)*b^2)*x^2 + (2*b^2*c*d*e^3 + 3*b^2*c^2*
e^2*f + 3*b^2*e^2*f)*x)*log(d*x + c)^2 - 2*(a*b*d^2*f^3*x^5 + (3*a*b*d^2*e*
f^2 + 2*a*b*c*d*f^3)*x^4 + (6*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (3*d^2*e^2*f +
f^3)*a*b)*x^3 + 3*(2*a*b*c*d*e^2*f + a*b*c^2*e*f^2 + a*b*e*f^2)*x^2 + 3*(a*
b*c^2*e^2*f + a*b*e^2*f)*x)*log(d*x + c)))/(d^2*x^2 + 2*c*d*x + c^2 + (d^2*
x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c+d\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3*(a + b*asinh(1/(c + d*x)))^2,x)
```

```
[Out] int((e + f*x)^3*(a + b*asinh(1/(c + d*x)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{acsch}\,(c + dx))^2\,(e + fx)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(a+b*acsch(d*x+c))**2,x)
```

```
[Out] Integral((a + b*acsch(c + d*x))**2*(e + f*x)**3, x)
```

3.8 $\int (e+fx)^2 \left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=351

$$-\frac{(de-cf)^3\left(a+b\mathrm{csch}^{-1}(c+dx)\right)^2}{3d^3f}+\frac{2bf(c+dx)\sqrt{\frac{1}{(c+dx)^2}+1}\,(de-cf)\left(a+b\mathrm{csch}^{-1}(c+dx)\right)}{d^3}+\frac{4b(de-cf)^2\tanh}{}$$

```
[Out] 1/3*b^2*f^2*x/d^2-1/3*(-c*f+d*e)^3*(a+b*arccsch(d*x+c))^2/d^3/f+1/3*(f*x+e)
^3*(a+b*arccsch(d*x+c))^2/f-2/3*b*f^2*(a+b*arccsch(d*x+c))*arctanh(1/(d*x+c
)+(1+1/(d*x+c)^2)^(1/2))/d^3+4*b*(-c*f+d*e)^2*(a+b*arccsch(d*x+c))*arctanh(
1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))/d^3+2*b^2*f*(-c*f+d*e)*ln(d*x+c)/d^3-1/3*b
^2*f^2*polylog(2,-1/(d*x+c)-(1+1/(d*x+c)^2)^(1/2))/d^3+2*b^2*(-c*f+d*e)^2*p
olylog(2,-1/(d*x+c)-(1+1/(d*x+c)^2)^(1/2))/d^3+1/3*b^2*f^2*polylog(2,1/(d*x
+c)+(1+1/(d*x+c)^2)^(1/2))/d^3-2*b^2*(-c*f+d*e)^2*polylog(2,1/(d*x+c)+(1+1/
(d*x+c)^2)^(1/2))/d^3+2*b*f*(-c*f+d*e)*(d*x+c)*(a+b*arccsch(d*x+c))*(1+1/(d
*x+c)^2)^(1/2)/d^3+1/3*b*f^2*(d*x+c)^2*(a+b*arccsch(d*x+c))*(1+1/(d*x+c)^2)
^(1/2)/d^3
```

Rubi [A] time = 0.51, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6322, 5469, 4190, 4182, 2279, 2391, 4184, 3475, 4185}

$$\frac{2b^2(de-cf)^2\mathrm{PolyLog}\left(2,-e^{\mathrm{csch}^{-1}(c+dx)}\right)}{d^3}-\frac{2b^2(de-cf)^2\mathrm{PolyLog}\left(2,e^{\mathrm{csch}^{-1}(c+dx)}\right)}{d^3}-\frac{b^2f^2\mathrm{PolyLog}\left(2,-e^{\mathrm{csch}^{-1}(c+dx)}\right)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcCsch[c + d*x])^2,x]
```

```
[Out] (b^2*f^2*x)/(3*d^2) + (2*b*f*(d*e - c*f)*(c + d*x)*Sqrt[1 + (c + d*x)^(-2)]
*(a + b*ArcCsch[c + d*x]))/d^3 + (b*f^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^(-2)
]*(a + b*ArcCsch[c + d*x]))/(3*d^3) - ((d*e - c*f)^3*(a + b*ArcCsch[c + d*x
])^2)/(3*d^3*f) + ((e + f*x)^3*(a + b*ArcCsch[c + d*x])^2)/(3*f) - (2*b*f^2
*(a + b*ArcCsch[c + d*x])*ArcTanh[E^ArcCsch[c + d*x]])/(3*d^3) + (4*b*(d*e
- c*f)^2*(a + b*ArcCsch[c + d*x])*ArcTanh[E^ArcCsch[c + d*x]])/d^3 + (2*b^2
*f*(d*e - c*f)*Log[c + d*x])/d^3 - (b^2*f^2*PolyLog[2, -E^ArcCsch[c + d*x]]
)/(3*d^3) + (2*b^2*(d*e - c*f)^2*PolyLog[2, -E^ArcCsch[c + d*x]])/d^3 + (b^
2*f^2*PolyLog[2, E^ArcCsch[c + d*x]])/(3*d^3) - (2*b^2*(d*e - c*f)^2*PolyLo
g[2, E^ArcCsch[c + d*x]])/d^3
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5469

```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Csch[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6322

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csch[x]*Co
th[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 dx = -\frac{\text{Subst}\left(\int (a+bx)^2 \coth(x)\text{csch}(x)(de-cf+f\text{csch}(x))^2 dx, x, \text{csch}^{-1}(\right.}{d^3}$$

$$= \frac{(e+fx)^3 \left(a+b\text{csch}^{-1}(c+dx)\right)^2}{3f} - \frac{(2b)\,\text{Subst}\left(\int (a+bx)(de-cf+f\text{cs}\right.}{3d^3 f}$$

$$= \frac{(e+fx)^3 \left(a+b\text{csch}^{-1}(c+dx)\right)^2}{3f} - \frac{(2b)\,\text{Subst}\left(\int \left(d^3e^3\left(1-\frac{cf(3d^2e^2-3cd}{d^3e^3}\right.\right.\right.}{}$$

$$= -\frac{(de-cf)^3 \left(a+b\text{csch}^{-1}(c+dx)\right)^2}{3d^3 f} + \frac{(e+fx)^3 \left(a+b\text{csch}^{-1}(c+dx)\right)^2}{3f}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2bf(de-cf)(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)}{d^3} + \frac{bf^2}{}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2bf(de-cf)(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)}{d^3} + \frac{bf^2}{}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2bf(de-cf)(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)}{d^3} + \frac{bf^2}{}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2bf(de-cf)(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)}{d^3} + \frac{bf^2}{}$$

Mathematica [C] time = 9.24, size = 864, normalized size = 2.46

$$\frac{1}{3}a^2f^2x^3+a^2efx^2+a^2e^2x-\frac{2b^2def\left(\frac{(c+dx)^2\text{csch}^{-1}(c+dx)^2}{2d^2}-\frac{c\coth\left(\frac{1}{2}\text{csch}^{-1}(c+dx)\right)\text{csch}^{-1}(c+dx)^2}{2d^2}+\frac{c\tanh\left(\frac{1}{2}\text{csch}^{-1}(c+dx)\right)\text{csch}^{-1}(c+d}{2d^2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcCsch[c + d*x])^2,x]
```

```
[Out] a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(2*x*(3*e^2 + 3*e*f*x + f^
2*x^2)*ArcCsch[c + d*x] + (-(f*(c + d*x)*Sqrt[(1 + c^2 + 2*c*d*x + d^2*x^2)
/(c + d*x)^2]*(5*c*f - d*(6*e + f*x))) + 2*c*(3*d^2*e^2 - 3*c*d*e*f + c^2*f
^2)*ArcSinh[(c + d*x)^(-1)] + (6*d^2*e^2 - 12*c*d*e*f + (-1 + 6*c^2)*f^2)*L
og[(c + d*x)*(1 + Sqrt[(1 + c^2 + 2*c*d*x + d^2*x^2)/(c + d*x)^2])])/d^3))/
3 - (b^2*e^2*(-(ArcCsch[c + d*x]*((c + d*x)*ArcCsch[c + d*x] - 2*Log[1 - E^
(-ArcCsch[c + d*x])] + 2*Log[1 + E^(-ArcCsch[c + d*x])])) + 2*PolyLog[2, -E
^(-ArcCsch[c + d*x])] - 2*PolyLog[2, E^(-ArcCsch[c + d*x])]))/d - (2*b^2*d*
e*f*x*(((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]*ArcCsch[c + d*x])/d^2 + ((c + d*
x)^2*ArcCsch[c + d*x]^2)/(2*d^2) - (c*ArcCsch[c + d*x]^2*Coth[ArcCsch[c + d
*x]/2])/(2*d^2) - Log[(c + d*x)^(-1)]/d^2 - ((2*I)*c*(I*ArcCsch[c + d*x]*(L
og[1 - E^(-ArcCsch[c + d*x])] - Log[1 + E^(-ArcCsch[c + d*x])]) + I*(PolyLo
g[2, -E^(-ArcCsch[c + d*x])] - PolyLog[2, E^(-ArcCsch[c + d*x])])))/d^2 + (
c*ArcCsch[c + d*x]^2*Tanh[ArcCsch[c + d*x]/2])/(2*d^2)))/((c + d*x)*(-1 + c
/(c + d*x))) - (b^2*f^2*(2*(-2 + 12*c*ArcCsch[c + d*x] + ArcCsch[c + d*x]^2
 - 6*c^2*ArcCsch[c + d*x]^2)*Coth[ArcCsch[c + d*x]/2] + 2*ArcCsch[c + d*x]*
(-1 + 3*c*ArcCsch[c + d*x])*Csch[ArcCsch[c + d*x]/2]^2 - (ArcCsch[c + d*x]^
2*Csch[ArcCsch[c + d*x]/2]^4)/(2*(c + d*x)) - 48*c*Log[(c + d*x)^(-1)] + 8*
(-1 + 6*c^2)*(ArcCsch[c + d*x]*(Log[1 - E^(-ArcCsch[c + d*x])] - Log[1 + E^
```

```
(-ArcCsch[c + d*x])]) + PolyLog[2, -E^(-ArcCsch[c + d*x])] - PolyLog[2, E^(
-ArcCsch[c + d*x])]) - 2*ArcCsch[c + d*x]*(1 + 3*c*ArcCsch[c + d*x])*Sech[A
rcCsch[c + d*x]/2]^2 - 8*(c + d*x)^3*ArcCsch[c + d*x]^2*Sinh[ArcCsch[c + d*
x]/2]^4 + 2*(2 + 12*c*ArcCsch[c + d*x] - ArcCsch[c + d*x]^2 + 6*c^2*ArcCsch
[c + d*x]^2)*Tanh[ArcCsch[c + d*x]/2]))/(24*d^3)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2f^2x^2 + 2\,a^2efx + a^2e^2 + \left(b^2f^2x^2 + 2\,b^2efx + b^2e^2\right)\text{arcsch}\left(dx + c\right)^2 + 2\left(abf^2x^2 + 2\,abefx + abe^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccsch(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f^2*x^2 + 2*a^2*e*f*x + a^2*e^2 + (b^2*f^2*x^2 + 2*b^2*e*f*x +
 b^2*e^2)*arccsch(d*x + c)^2 + 2*(a*b*f^2*x^2 + 2*a*b*e*f*x + a*b*e^2)*arcc
sch(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx + e\right)^2 \left(b\,\text{arcsch}\left(dx + c\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccsch(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*(b*arccsch(d*x + c) + a)^2, x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \left(fx + e\right)^2 \left(a + b\,\text{arccsch}\left(dx + c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arccsch(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^2*(a+b*arccsch(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccsch(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*f^2*x^3 + a^2*e*f*x^2 + a^2*e^2*x + (2*(d*x + c)*arccsch(d*x + c) +
 log(sqrt(1/(d*x + c)^2 + 1) + 1) - log(sqrt(1/(d*x + c)^2 + 1) - 1))*a*b*e
^2/d + 1/3*(b^2*f^2*x^3 + 3*b^2*e*f*x^2 + 3*b^2*e^2*x)*log(sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1) + 1)^2 - integrate(-1/3*(3*(b^2*d^2*f^2*x^4 + b^2*c^2*e^2
 + b^2*e^2 + 2*(b^2*d^2*e*f + b^2*c*d*f^2)*x^3 + (4*b^2*c*d*e*f + b^2*c^2*f
^2 + (d^2*e^2 + f^2)*b^2)*x^2 + 2*(b^2*c*d*e^2 + b^2*c^2*e*f + b^2*e*f)*x)*
log(d*x + c)^2 - 6*(a*b*d^2*f^2*x^4 + 2*(a*b*d^2*e*f + a*b*c*d*f^2)*x^3 + (
4*a*b*c*d*e*f + a*b*c^2*f^2 + a*b*f^2)*x^2 + 2*(a*b*c^2*e*f + a*b*e*f)*x)*l
og(d*x + c) + 2*(3*a*b*d^2*f^2*x^4 + 6*(a*b*d^2*e*f + a*b*c*d*f^2)*x^3 + 3*
(4*a*b*c*d*e*f + a*b*c^2*f^2 + a*b*f^2)*x^2 + 6*(a*b*c^2*e*f + a*b*e*f)*x -
 3*(b^2*d^2*f^2*x^4 + b^2*c^2*e^2 + b^2*e^2 + 2*(b^2*d^2*e*f + b^2*c*d*f^2)
*x^3 + (4*b^2*c*d*e*f + b^2*c^2*f^2 + (d^2*e^2 + f^2)*b^2)*x^2 + 2*(b^2*c*d
*e^2 + b^2*c^2*e*f + b^2*e*f)*x)*log(d*x + c) + ((3*a*b*d^2*f^2 - b^2*d^2*f
^2)*x^4 + (6*a*b*d^2*e*f - 3*b^2*d^2*e*f + (6*a*b*d*f^2 - b^2*d*f^2)*c)*x^3
 - 3*(b^2*d^2*e^2 - a*b*c^2*f^2 - a*b*f^2 - (4*a*b*d*e*f - b^2*d*e*f)*c)*x^
2 - 3*(b^2*c*d*e^2 - 2*a*b*c^2*e*f - 2*a*b*e*f)*x - 3*(b^2*d^2*f^2*x^4 + b^
```

```
2*c^2*e^2 + b^2*e^2 + 2*(b^2*d^2*e*f + b^2*c*d*f^2)*x^3 + (4*b^2*c*d*e*f +
b^2*c^2*f^2 + (d^2*e^2 + f^2)*b^2)*x^2 + 2*(b^2*c*d*e^2 + b^2*c^2*e*f + b^2
*e*f)*x)*log(d*x + c))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1) + 1) + 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((b^2*d^2*f
^2*x^4 + b^2*c^2*e^2 + b^2*e^2 + 2*(b^2*d^2*e*f + b^2*c*d*f^2)*x^3 + (4*b^2
*c*d*e*f + b^2*c^2*f^2 + (d^2*e^2 + f^2)*b^2)*x^2 + 2*(b^2*c*d*e^2 + b^2*c^
2*e*f + b^2*e*f)*x)*log(d*x + c)^2 - 2*(a*b*d^2*f^2*x^4 + 2*(a*b*d^2*e*f +
a*b*c*d*f^2)*x^3 + (4*a*b*c*d*e*f + a*b*c^2*f^2 + a*b*f^2)*x^2 + 2*(a*b*c^2
*e*f + a*b*e*f)*x)*log(d*x + c)))/(d^2*x^2 + 2*c*d*x + c^2 + (d^2*x^2 + 2*c
*d*x + c^2 + 1)^(3/2) + 1), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e+f\,x\right)^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c+d\,x}\right)\right)^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*asinh(1/(c + d*x)))^2,x)
[Out] int((e + f*x)^2*(a + b*asinh(1/(c + d*x)))^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acsch}{\left(c + d x \right)}\right)^{2} \left(e + f x\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*acsch(d*x+c))**2,x)
[Out] Integral((a + b*acsch(c + d*x))**2*(e + f*x)**2, x)
```

3.9 $\int(e+fx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=194

$$-\frac{(de-cf)^2\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2d^2f}+\frac{4b(de-cf)\tanh^{-1}\left(e^{\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csch}^{-1}(c+dx)\right)}{d^2}+\frac{bf(c+dx)\sqrt{\frac{}{(c+}}}{}$$

```
[Out] -1/2*(-c*f+d*e)^2*(a+b*arccsch(d*x+c))^2/d^2/f+1/2*(f*x+e)^2*(a+b*arccsch(d
*x+c))^2/f+4*b*(-c*f+d*e)*(a+b*arccsch(d*x+c))*arctanh(1/(d*x+c)+(1+1/(d*x+
c)^2)^(1/2))/d^2+b^2*f*ln(d*x+c)/d^2+2*b^2*(-c*f+d*e)*polylog(2,-1/(d*x+c)-
(1+1/(d*x+c)^2)^(1/2))/d^2-2*b^2*(-c*f+d*e)*polylog(2,1/(d*x+c)+(1+1/(d*x+c
)^2)^(1/2))/d^2+b*f*(d*x+c)*(a+b*arccsch(d*x+c))*(1+1/(d*x+c)^2)^(1/2)/d^2
```

Rubi [A] time = 0.27, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6322, 5469, 4190, 4182, 2279, 2391, 4184, 3475}

$$\frac{2b^2(de-cf)\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(c+dx)}\right)}{d^2}-\frac{2b^2(de-cf)\text{PolyLog}\left(2,e^{\text{csch}^{-1}(c+dx)}\right)}{d^2}-\frac{(de-cf)^2\left(a+b\text{csch}^{-1}(c+d\right.}{2d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcCsch[c + d*x])^2,x]
```

```
[Out] (b*f*(c + d*x)*Sqrt[1 + (c + d*x)^(-2)]*(a + b*ArcCsch[c + d*x]))/d^2 - ((d
*e - c*f)^2*(a + b*ArcCsch[c + d*x])^2)/(2*d^2*f) + ((e + f*x)^2*(a + b*Arc
Csch[c + d*x])^2)/(2*f) + (4*b*(d*e - c*f)*(a + b*ArcCsch[c + d*x])*ArcTanh
[E^ArcCsch[c + d*x]])/d^2 + (b^2*f*Log[c + d*x])/d^2 + (2*b^2*(d*e - c*f)*P
olyLog[2, -E^ArcCsch[c + d*x]])/d^2 - (2*b^2*(d*e - c*f)*PolyLog[2, E^ArcCs
ch[c + d*x]])/d^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
```

```
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5469

```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Csch[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6322

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csch[x]*Co
th[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e+fx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx = -\frac{\text{Subst}\left(\int (a+bx)^2\coth(x)\text{csch}(x)(de-cf+f\text{csch}(x))\,dx, x, \text{csch}^{-1}(c \cdots\right.}{d^2}$$

$$= \frac{(e+fx)^2\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2f} - \frac{b\,\text{Subst}\left(\int (a+bx)(de-cf+f\text{csch}(\cdots\right.}{d^2 f}$$

$$= \frac{(e+fx)^2\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2f} - \frac{b\,\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)(a \cdots\right.\right.}{}$$

$$= -\frac{(de-cf)^2\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2d^2 f} + \frac{(e+fx)^2\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2f} - \cdots$$

$$= \frac{bf(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)}{d^2} - \frac{(de-cf)^2\left(a+b\text{csch}^{-1} \cdots\right.}{2d^2 f}$$

$$= \frac{bf(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)}{d^2} - \frac{(de-cf)^2\left(a+b\text{csch}^{-1} \cdots\right.}{2d^2 f}$$

$$= \frac{bf(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)}{d^2} - \frac{(de-cf)^2\left(a+b\text{csch}^{-1} \cdots\right.}{2d^2 f}$$

Mathematica [B] time = 1.46, size = 401, normalized size = 2.07

$$\frac{2a^2(c+dx)(de-cf)+a^2f(c+dx)^2+2abde\left(2(c+dx)\text{csch}^{-1}(c+dx)-2\log\left(2(c+dx)\sinh^2\left(\frac{1}{2}\text{csch}^{-1}(c+dx)\right)\cdots\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*(a + b*ArcCsch[c + d*x])^2,x]
```

```
[Out] (2*a^2*(d*e - c*f)*(c + d*x) + a^2*f*(c + d*x)^2 + 2*a*b*f*(c + d*x)*(Sqrt[
1 + (c + d*x)^(-2)] + (c + d*x)*ArcCsch[c + d*x]) + 2*b^2*f*((c + d*x)*Sqrt
```

```
[1 + (c + d*x)^(-2)]*ArcCsch[c + d*x] + ((c + d*x)^2*ArcCsch[c + d*x]^2)/2
- Log[(c + d*x)^(-1)]) + 2*a*b*d*e*(2*(c + d*x)*ArcCsch[c + d*x] - 2*Log[2*
(c + d*x)*Sinh[ArcCsch[c + d*x]/2]^2]) + 2*a*b*c*f*(-2*(c + d*x)*ArcCsch[c
+ d*x] + 2*Log[2*(c + d*x)*Sinh[ArcCsch[c + d*x]/2]^2]) + 2*b^2*d*e*(ArcCsc
h[c + d*x]*((c + d*x)*ArcCsch[c + d*x] - 2*Log[1 - E^(-ArcCsch[c + d*x])] +
 2*Log[1 + E^(-ArcCsch[c + d*x])]) - 2*PolyLog[2, -E^(-ArcCsch[c + d*x])] +
 2*PolyLog[2, E^(-ArcCsch[c + d*x])]) - 2*b^2*c*f*(ArcCsch[c + d*x]*((c + d
*x)*ArcCsch[c + d*x] - 2*Log[1 - E^(-ArcCsch[c + d*x])] + 2*Log[1 + E^(-Arc
Csch[c + d*x])]) - 2*PolyLog[2, -E^(-ArcCsch[c + d*x])] + 2*PolyLog[2, E^(-
ArcCsch[c + d*x])]))/(2*d^2)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(a^2fx + a^2e + \left(b^2fx + b^2e\right)\text{arcsch}\left(dx+c\right)^2 + 2\left(abfx + abe\right)\text{arcsch}\left(dx+c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccsch(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arccsch(d*x + c)^2 + 2*(a*b*f*
x + a*b*e)*arccsch(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(b\,\text{arcsch}\left(dx+c\right)+a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccsch(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(b*arccsch(d*x + c) + a)^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(a+b\,\text{arccsch}\left(dx+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arccsch(d*x+c))^2,x)
```

```
[Out] int((f*x+e)*(a+b*arccsch(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2fx^2+a^2ex+\frac{\left(2\left(dx+c\right)\text{arcsch}\left(dx+c\right)+\log\left(\sqrt{\frac{1}{(dx+c)^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{(dx+c)^2}+1}-1\right)\right)abe}{d}+\frac{1}{2}\left(b^2fx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccsch(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*f*x^2 + a^2*e*x + (2*(d*x + c)*arccsch(d*x + c) + log(sqrt(1/(d*x +
 c)^2 + 1) + 1) - log(sqrt(1/(d*x + c)^2 + 1) - 1))*a*b*e/d + 1/2*(b^2*f*x^
2 + 2*b^2*e*x)*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)^2 - integrate(-((
b^2*d^2*f*x^3 + b^2*c^2*e + b^2*e + (b^2*d^2*e + 2*b^2*c*d*f)*x^2 + (2*b^2*
c*d*e + b^2*c^2*f + b^2*f)*x)*log(d*x + c)^2 - 2*(a*b*d^2*f*x^3 + 2*a*b*c*d
*f*x^2 + (a*b*c^2*f + a*b*f)*x)*log(d*x + c) + (2*a*b*d^2*f*x^3 + 4*a*b*c*d
*f*x^2 + 2*(a*b*c^2*f + a*b*f)*x - 2*(b^2*d^2*f*x^3 + b^2*c^2*e + b^2*e + (
b^2*d^2*e + 2*b^2*c*d*f)*x^2 + (2*b^2*c*d*e + b^2*c^2*f + b^2*f)*x)*log(d*x
 + c) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((2*a*b*d^2*f - b^2*d^2*f)*x^3 -
```

```
(2*b^2*d^2*e - (4*a*b*d*f - b^2*d*f)*c)*x^2 - 2*(b^2*c*d*e - a*b*c^2*f - a*
b*f)*x - 2*(b^2*d^2*f*x^3 + b^2*c^2*e + b^2*e + (b^2*d^2*e + 2*b^2*c*d*f)*x
^2 + (2*b^2*c*d*e + b^2*c^2*f + b^2*f)*x)*log(d*x + c)))*log(sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1) + 1) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((b^2*d^2*f*x^
3 + b^2*c^2*e + b^2*e + (b^2*d^2*e + 2*b^2*c*d*f)*x^2 + (2*b^2*c*d*e + b^2*
c^2*f + b^2*f)*x)*log(d*x + c)^2 - 2*(a*b*d^2*f*x^3 + 2*a*b*c*d*f*x^2 + (a*
b*c^2*f + a*b*f)*x)*log(d*x + c)))/(d^2*x^2 + 2*c*d*x + c^2 + (d^2*x^2 + 2*
c*d*x + c^2 + 1)^(3/2) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(e+f\,x\right)\,{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c+d\,x}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*asinh(1/(c + d*x)))^2,x)
```

```
[Out] int((e + f*x)*(a + b*asinh(1/(c + d*x)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acsch}{\left(c + d x \right)}\right)^{2} \left(e + f x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*acsch(d*x+c))**2,x)
```

```
[Out] Integral((a + b*acsch(c + d*x))**2*(e + f*x), x)
```

3.10 $\int \left(a+b\,\text{csch}^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=85

$$\frac{(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{d}+\frac{4b\tanh^{-1}\left(e^{\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csch}^{-1}(c+dx)\right)}{d}+\frac{2b^2\text{Li}_2\left(-e^{\text{csch}^{-1}(c+dx)}\right)}{d}-\frac{2b^2\text{I}}{}$$

```
[Out] (d*x+c)*(a+b*arccsch(d*x+c))^2/d+4*b*(a+b*arccsch(d*x+c))*arctanh(1/(d*x+c)
+(1+1/(d*x+c)^2)^(1/2))/d+2*b^2*polylog(2,-1/(d*x+c)-(1+1/(d*x+c)^2)^(1/2))
/d-2*b^2*polylog(2,1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))/d
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6316, 6280, 5452, 4182, 2279, 2391}

$$\frac{2b^2\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(c+dx)}\right)}{d}-\frac{2b^2\text{PolyLog}\left(2,e^{\text{csch}^{-1}(c+dx)}\right)}{d}+\frac{(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{d}+\frac{4b\tanh^{-1}\left(e^{\text{c}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c + d*x])^2,x]
```

```
[Out] ((c + d*x)*(a + b*ArcCsch[c + d*x])^2)/d + (4*b*(a + b*ArcCsch[c + d*x])*Ar
cTanh[E^ArcCsch[c + d*x]])/d + (2*b^2*PolyLog[2, -E^ArcCsch[c + d*x]])/d -
(2*b^2*PolyLog[2, E^ArcCsch[c + d*x]])/d
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6280

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[c^(-1), Su
bst[Int[(a + b*x)^n*Csch[x]*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rule 6316

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCsch[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int \left(a+b\text{csch}^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=-\frac{\text{Subst}\left(\int (a+bx)^2\coth(x)\text{csch}(x)\,dx,x,\text{csch}^{-1}(c+dx)\right)}{d}$$
$$=\frac{(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{d}-\frac{(2b)\,\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx,x,\text{csch}^{-1}(c+\right.}{d}$$
$$=\frac{(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{d}+\frac{4b\left(a+b\text{csch}^{-1}(c+dx)\right)\tanh^{-1}\left(e^{\text{csch}^{-1}(c+dx)}\right)}{d}$$
$$=\frac{(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{d}+\frac{4b\left(a+b\text{csch}^{-1}(c+dx)\right)\tanh^{-1}\left(e^{\text{csch}^{-1}(c+dx)}\right)}{d}$$
$$=\frac{(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{d}+\frac{4b\left(a+b\text{csch}^{-1}(c+dx)\right)\tanh^{-1}\left(e^{\text{csch}^{-1}(c+dx)}\right)}{d}$$

Mathematica [A] time = 0.26, size = 160, normalized size = 1.88

$$a^2c+a^2dx+2ab(c+dx)\text{csch}^{-1}(c+dx)-2ab\log\left(\tanh\left(\tfrac{1}{2}\text{csch}^{-1}(c+dx)\right)\right)-2b^2\text{Li}_2\left(-e^{-\text{csch}^{-1}(c+dx)}\right)+2b^2\text{Li}_2\left(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c + d*x])^2,x]
```

```
[Out] (a^2*c + a^2*d*x + 2*a*b*(c + d*x)*ArcCsch[c + d*x] + b^2*c*ArcCsch[c + d*x
]^2 + b^2*d*x*ArcCsch[c + d*x]^2 - 2*b^2*ArcCsch[c + d*x]*Log[1 - E^(-ArcCs
ch[c + d*x])] + 2*b^2*ArcCsch[c + d*x]*Log[1 + E^(-ArcCsch[c + d*x])] - 2*a
*b*Log[Tanh[ArcCsch[c + d*x]/2]] - 2*b^2*PolyLog[2, -E^(-ArcCsch[c + d*x])]
 + 2*b^2*PolyLog[2, E^(-ArcCsch[c + d*x])])/d
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2\,\text{arcsch}\,(dx+c)^2+2\,ab\,\text{arcsch}\,(dx+c)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arccsch(d*x + c)^2 + 2*a*b*arccsch(d*x + c) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\text{arcsch}\,(dx+c)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(d*x + c) + a)^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arccsch}(dx + c))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(d*x+c))^2,x)
```

```
[Out] int((a+b*arccsch(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(x \log\left(\sqrt{d^2x^2 + 2\,cdx + c^2 + 1} + 1\right)^2 - \int -\frac{\left(d^2x^2 + 2\,cdx + c^2 + 1\right)^{\frac{3}{2}} \log(dx + c)^2 + \left(d^2x^2 + 2\,cdx + c^2 + 1\right) \mathrm{lo}}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (x*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)^2 - integrate(-((d^2*x^2 + 2*
c*d*x + c^2 + 1)^(3/2)*log(d*x + c)^2 + (d^2*x^2 + 2*c*d*x + c^2 + 1)*log(d
*x + c)^2 - 2*((d^2*x^2 + 2*c*d*x + c^2 + 1)*log(d*x + c) + sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1)*(d^2*x^2 + c*d*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)*log(d*x
 + c)))*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1))/(d^2*x^2 + 2*c*d*x + c^
2 + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 1), x))*b^2 + a^2*x + (2*(d*x + c
)*arccsch(d*x + c) + log(sqrt(1/(d*x + c)^2 + 1) + 1) - log(sqrt(1/(d*x + c
)^2 + 1) - 1))*a*b/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{asinh}\left(\frac{1}{c+d\,x}\right)\right)}^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c + d*x)))^2,x)
```

```
[Out] int((a + b*asinh(1/(c + d*x)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acsch}{\left(c + d x \right)}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(d*x+c))**2,x)
```

```
[Out] Integral((a + b*acsch(c + d*x))**2, x)
```

3.11 $\int \frac{\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)^2}{e+fx}\,dx$

Optimal. Leaf size=475

$$\frac{2b\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)\mathrm{Li}_2\left(-\frac{e^{\mathrm{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdfe+\left(c^2+1\right)f^2}}\right)}{f}+\frac{2b\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)\mathrm{Li}_2\left(-\frac{e^{\mathrm{csch}^{-1}(c+dx)}(de-cf)}{f+\sqrt{d^2e^2-2cdfe+\left(c^2+1\right)f^2}}\right)}{f}+\cdots$$

```
[Out] -(a+b*arccsch(d*x+c))^2*ln(1-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))^2)/f+(a+b*ar
ccsch(d*x+c))^2*ln(1+(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f-(d^2*e
^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/f+(a+b*arccsch(d*x+c))^2*ln(1+(1/(d*x+c)+
(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2))
)/f-b*(a+b*arccsch(d*x+c))*polylog(2,(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))^2)/f
+2*b*(a+b*arccsch(d*x+c))*polylog(2,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*
f+d*e)/(f-(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/f+2*b*(a+b*arccsch(d*x+c)
)*polylog(2,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*c*d
*e*f+(c^2+1)*f^2)^(1/2)))/f+1/2*b^2*polylog(3,(1/(d*x+c)+(1+1/(d*x+c)^2)^(1
/2))^2)/f-2*b^2*polylog(3,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f-
(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/f-2*b^2*polylog(3,-(1/(d*x+c)+(1+1/
(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/f
```

Rubi [A] time = 1.07, antiderivative size = 475, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6322, 5596, 5569, 3716, 2190, 2531, 2282, 6589, 5561}

$$\frac{2b\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)\mathrm{PolyLog}\left(2,-\frac{(de-cf)e^{\mathrm{csch}^{-1}(c+dx)}}{f-\sqrt{\left(c^2+1\right)f^2-2cdef+d^2e^2}}\right)}{f}+\frac{2b\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)\mathrm{PolyLog}\left(2,-\frac{(de-cf)\cdots}{\sqrt{\left(c^2+1\right)f^2\cdots}}\right)}{f}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c + d*x])^2/(e + f*x),x]
```

```
[Out] -(((a + b*ArcCsch[c + d*x])^2*Log[1 - E^(2*ArcCsch[c + d*x])])/f) + ((a + b
*ArcCsch[c + d*x])^2*Log[1 + (E^ArcCsch[c + d*x]*(d*e - c*f))/(f - Sqrt[d^2
*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])])/f + ((a + b*ArcCsch[c + d*x])^2*Log[1
+ (E^ArcCsch[c + d*x]*(d*e - c*f))/(f + Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2])])/f - (b*(a + b*ArcCsch[c + d*x])*PolyLog[2, E^(2*ArcCsch[c + d*x])
])/f + (2*b*(a + b*ArcCsch[c + d*x])*PolyLog[2, -((E^ArcCsch[c + d*x]*(d*e
- c*f))/(f - Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]))])/f + (2*b*(a + b*
ArcCsch[c + d*x])*PolyLog[2, -((E^ArcCsch[c + d*x]*(d*e - c*f))/(f + Sqrt[d
^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]))])/f + (b^2*PolyLog[3, E^(2*ArcCsch[c
+ d*x])])/(2*f) - (2*b^2*PolyLog[3, -((E^ArcCsch[c + d*x]*(d*e - c*f))/(f -
 Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]))])/f - (2*b^2*PolyLog[3, -((E^A
rcCsch[c + d*x]*(d*e - c*f))/(f + Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]
))])/f
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5596

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/(Csch[(c_.) + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> I
nt[((e + f*x)^m*Sinh[c + d*x]*F[c + d*x]^n*G[c + d*x]^p)/(b + a*Sinh[c + d*
x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G]
 && IntegersQ[m, n, p]
```

Rule 6322

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csch[x]*Co
th[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{e+fx}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^2\coth(x)\text{csch}(x)}{de-cf+f\text{csch}(x)}\,dx, x, \text{csch}^{-1}(c+dx)\right)$$

$$= -\text{Subst}\left(\int \frac{(a+bx)^2\coth(x)}{f+(de-cf)\sinh(x)}\,dx, x, \text{csch}^{-1}(c+dx)\right)$$

$$= -\frac{\text{Subst}\left(\int (a+bx)^2\coth(x)\,dx, x, \text{csch}^{-1}(c+dx)\right)}{f} + \frac{(de-cf)\,\text{Subst}\left(\int \frac{(a+bx)^2 \text{c}}{f+(de-cf)}\right.}{}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1-e^{2x}}\,dx, x, \text{csch}^{-1}(c+dx)\right)}{f} + \frac{(de-cf)\,\text{Subst}\left(\int \frac{e^x(a+b}{f+e^x(de-cf)-\sqrt{d^2e^2}}\right.}{}$$

$$= -\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2\log\left(1-e^{2\text{csch}^{-1}(c+dx)}\right)}{f} + \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2\log\left(1\right.}{f}$$

$$= -\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2\log\left(1-e^{2\text{csch}^{-1}(c+dx)}\right)}{f} + \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2\log\left(1\right.}{f}$$

$$= -\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2\log\left(1-e^{2\text{csch}^{-1}(c+dx)}\right)}{f} + \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2\log\left(1\right.}{f}$$

$$= -\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2\log\left(1-e^{2\text{csch}^{-1}(c+dx)}\right)}{f} + \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2\log\left(1\right.}{f}$$

Mathematica [C] time = 2.98, size = 1008, normalized size = 2.12

$$\frac{6\log(e+fx)a^2 + 6b\left(\frac{1}{4}\left(\pi-2i\text{csch}^{-1}(c+dx)\right)^2 - \text{csch}^{-1}(c+dx)^2 - 8\sin^{-1}\left(\sqrt{\frac{de-cf+if}{2de-2cf}}\right)\tan^{-1}\left(\frac{(ide-icf+f)\cot\left(\frac{1}{4}(2\right.}{\sqrt{f^2+(d}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c + d*x])^2/(e + f*x),x]
```

```
[Out] (6*a^2*Log[e + f*x] + 6*a*b*((Pi - (2*I)*ArcCsch[c + d*x])^2/4 - ArcCsch[c
+ d*x]^2 - 8*ArcSin[Sqrt[(d*e + I*f - c*f)/(2*d*e - 2*c*f)]]*ArcTan[((I*d*e
 + f - I*c*f)*Cot[(Pi + (2*I)*ArcCsch[c + d*x])/4])/Sqrt[f^2 + (d*e - c*f)^
2]] - 2*ArcCsch[c + d*x]*Log[1 - E^(-2*ArcCsch[c + d*x])] + (2*ArcCsch[c +
d*x] + I*(Pi + 4*ArcSin[Sqrt[(d*e + I*f - c*f)/(2*d*e - 2*c*f)]]))*Log[(d*e
 - c*f - E^ArcCsch[c + d*x]*f + E^ArcCsch[c + d*x]*Sqrt[f^2 + (d*e - c*f)^2
])/(d*e - c*f)] + (2*ArcCsch[c + d*x] + I*(Pi - 4*ArcSin[Sqrt[(d*e + I*f -
c*f)/(2*d*e - 2*c*f)]]))*Log[-((-(d*e) + c*f + E^ArcCsch[c + d*x]*f + E^Arc
Csch[c + d*x]*Sqrt[f^2 + (d*e - c*f)^2])/(d*e - c*f))] + 2*ArcCsch[c + d*x]
*Log[(d*(e + f*x))/(c + d*x)] - (I*Pi + 2*ArcCsch[c + d*x])*Log[(d*(e + f*x
))/(c + d*x)] + PolyLog[2, E^(-2*ArcCsch[c + d*x])] + 2*PolyLog[2, (E^ArcCs
ch[c + d*x]*(f - Sqrt[f^2 + (d*e - c*f)^2]))/(d*e - c*f)] + 2*PolyLog[2, (E
^ArcCsch[c + d*x]*(f + Sqrt[f^2 + (d*e - c*f)^2]))/(d*e - c*f)]) + b^2*((-I
)*Pi^3 - 2*ArcCsch[c + d*x]^3 - 6*ArcCsch[c + d*x]^2*Log[1 + E^(-ArcCsch[c
+ d*x])] - 6*ArcCsch[c + d*x]^2*Log[1 - E^ArcCsch[c + d*x]] + 6*ArcCsch[c +
 d*x]^2*Log[1 + (E^ArcCsch[c + d*x]*(-(d*e) + c*f))/(-f + Sqrt[d^2*e^2 - 2*
```

```
c*d*e*f + (1 + c^2)*f^2])] + 6*ArcCsch[c + d*x]^2*Log[1 + (E^ArcCsch[c + d*
x]*(d*e - c*f))/(f + Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])] + 12*ArcCs
ch[c + d*x]*PolyLog[2, -E^(-ArcCsch[c + d*x])] - 12*ArcCsch[c + d*x]*PolyLo
g[2, E^ArcCsch[c + d*x]] + 12*ArcCsch[c + d*x]*PolyLog[2, (E^ArcCsch[c + d*
x]*(d*e - c*f))/(-f + Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])] + 12*ArcC
sch[c + d*x]*PolyLog[2, (E^ArcCsch[c + d*x]*(-(d*e) + c*f))/(f + Sqrt[d^2*e
^2 - 2*c*d*e*f + (1 + c^2)*f^2])] + 12*PolyLog[3, -E^(-ArcCsch[c + d*x])] +
 12*PolyLog[3, E^ArcCsch[c + d*x]] - 12*PolyLog[3, (E^ArcCsch[c + d*x]*(d*e
 - c*f))/(-f + Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])] - 12*PolyLog[3,
(E^ArcCsch[c + d*x]*(-(d*e) + c*f))/(f + Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^
2)*f^2])]))/(6*f)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{arcsch}(dx+c)^2 + 2\,ab \operatorname{arcsch}(dx+c) + a^2}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccsch(d*x + c)^2 + 2*a*b*arccsch(d*x + c) + a^2)/(f*x + e),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(dx+c) + a)^2}{fx+e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(d*x + c) + a)^2/(f*x + e), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsch}(dx+c))^2}{fx+e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(d*x+c))^2/(f*x+e),x)
```

```
[Out] int((a+b*arccsch(d*x+c))^2/(f*x+e),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \log(fx+e)}{f} + \int \frac{b^2 \log\left(\sqrt{\frac{1}{(dx+c)^2}+1} + \frac{1}{dx+c}\right)^2}{fx+e} + \frac{2\,ab \log\left(\sqrt{\frac{1}{(dx+c)^2}+1} + \frac{1}{dx+c}\right)}{fx+e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^2*log(f*x + e)/f + integrate(b^2*log(sqrt(1/(d*x + c)^2 + 1) + 1/(d*x + c
))^2/(f*x + e) + 2*a*b*log(sqrt(1/(d*x + c)^2 + 1) + 1/(d*x + c))/(f*x + e)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c+d\,x}\right)\right)^2}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c + d*x)))^2/(e + f*x),x)
[Out] int((a + b*asinh(1/(c + d*x)))^2/(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acsch}\,(c+dx))^2}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(d*x+c))**2/(f*x+e),x)
[Out] Integral((a + b*acsch(c + d*x))**2/(e + f*x), x)
```

3.12 $\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^2} \, dx$

Optimal. Leaf size=448

$$-\frac{2bd\left(a+b\text{csch}^{-1}(c+dx)\right)\log\left(\frac{(de-cf)e^{\text{csch}^{-1}(c+dx)}}{f-\sqrt{(c^2+1)f^2-2cdef+d^2e^2}}+1\right)}{(de-cf)\sqrt{(c^2+1)f^2-2cdef+d^2e^2}}+\frac{2bd\left(a+b\text{csch}^{-1}(c+dx)\right)\log\left(\frac{(de-cf)e^{\text{csch}^{-1}(c+d}}{\sqrt{(c^2+1)f^2-2cdef+d^2}}\right.}{(de-cf)\sqrt{(c^2+1)f^2-2cdef+d^2e^2}}$$

```
[Out] d*(a+b*arccsch(d*x+c))^2/f/(-c*f+d*e)-(a+b*arccsch(d*x+c))^2/f/(f*x+e)-2*b*
d*(a+b*arccsch(d*x+c))*ln(1+(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f
-(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)/(d^2*e^2-2*c*d*e*f+(c^2
+1)*f^2)^(1/2)+2*b*d*(a+b*arccsch(d*x+c))*ln(1+(1/(d*x+c)+(1+1/(d*x+c)^2)^(
1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)/(d^2
*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)-2*b^2*d*polylog(2,-(1/(d*x+c)+(1+1/(d*x+c
)^2)^(1/2))*(-c*f+d*e)/(f-(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e
)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)+2*b^2*d*polylog(2,-(1/(d*x+c)+(1+1/
(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c
*f+d*e)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)
```

Rubi [A] time = 1.11, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6322, 5469, 4191, 3322, 2264, 2190, 2279, 2391}

$$-\frac{2b^2d\text{PolyLog}\left(2,-\frac{(de-cf)e^{\text{csch}^{-1}(c+dx)}}{f-\sqrt{(c^2+1)f^2-2cdef+d^2e^2}}\right)}{(de-cf)\sqrt{(c^2+1)f^2-2cdef+d^2e^2}}+\frac{2b^2d\text{PolyLog}\left(2,-\frac{(de-cf)e^{\text{csch}^{-1}(c+dx)}}{\sqrt{(c^2+1)f^2-2cdef+d^2e^2}+f}\right)}{(de-cf)\sqrt{(c^2+1)f^2-2cdef+d^2e^2}}-\frac{2bd\left(a+b\text{csch}^{-1}(c+\right.}{(de-cf)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (d*(a + b*ArcCsch[c + d*x])^2)/(f*(d*e - c*f)) - (a + b*ArcCsch[c + d*x])^2
/(f*(e + f*x)) - (2*b*d*(a + b*ArcCsch[c + d*x])*Log[1 + (E^ArcCsch[c + d*x
]*(d*e - c*f))/(f - Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])])/((d*e - c*
f)*Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]) + (2*b*d*(a + b*ArcCsch[c + d
*x])*Log[1 + (E^ArcCsch[c + d*x]*(d*e - c*f))/(f + Sqrt[d^2*e^2 - 2*c*d*e*f
 + (1 + c^2)*f^2])])/((d*e - c*f)*Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]
) - (2*b^2*d*PolyLog[2, -((E^ArcCsch[c + d*x]*(d*e - c*f))/(f - Sqrt[d^2*e^
2 - 2*c*d*e*f + (1 + c^2)*f^2]))])/((d*e - c*f)*Sqrt[d^2*e^2 - 2*c*d*e*f +
(1 + c^2)*f^2]) + (2*b^2*d*PolyLog[2, -((E^ArcCsch[c + d*x]*(d*e - c*f))/(f
 + Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]))])/((d*e - c*f)*Sqrt[d^2*e^2
- 2*c*d*e*f + (1 + c^2)*f^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
```

```
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5469

```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Csch[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6322

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csch[x]*Co
th[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx = -\left(d\,\text{Subst}\left(\int \frac{(a+bx)^2\coth(x)\text{csch}(x)}{(de-cf+f\text{csch}(x))^2}\,dx, x, \text{csch}^{-1}(c+dx)\right)\right)$$

$$= -\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\,\text{Subst}\left(\int \frac{a+bx}{de-cf+f\text{csch}(x)}\,dx, x, \text{csch}^{-1}(c+dx)\right)}{f}$$

$$= -\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\,\text{Subst}\left(\int \left(\frac{a+bx}{de-cf} + \frac{f(a+bx)}{(-de+cf)\left(f+de\left(1-\frac{cf}{de}\right)\sinh(x)\right)}\right)\right.}{f}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(de-cf)} - \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2bd)\,\text{Subst}\left(\int \frac{a+}{f+de\left(1-\frac{c}{d}\right.}\right.}{}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(de-cf)} - \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(4bd)\,\text{Subst}\left(\int \frac{}{2e^xf-de(1}\right.}{}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(de-cf)} - \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(4bd)\,\text{Subst}\left(\int \frac{}{2f+2dee^x(}\right.}{\sqrt{}}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(de-cf)} - \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{2bd\left(a+b\text{csch}^{-1}(c+d\right.}{(de-cf)\sqrt{d^2}}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(de-cf)} - \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{2bd\left(a+b\text{csch}^{-1}(c+d\right.}{(de-cf)\sqrt{d^2}}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(de-cf)} - \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{2bd\left(a+b\text{csch}^{-1}(c+d\right.}{(de-cf)\sqrt{d^2}}$$

Mathematica [C] time = 12.88, size = 2061, normalized size = 4.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] -(a^2/(f*(e + f*x))) - (2*a*b*(c + d*x)^2*(f + (d*e - c*f)/(c + d*x))^2*(Ar
cCsch[c + d*x]/(f + (d*e)/(c + d*x) - (c*f)/(c + d*x)) - (2*ArcTan[(d*e - c
*f - f*Tanh[ArcCsch[c + d*x]/2])/Sqrt[-(d^2*e^2) + 2*c*d*e*f - (1 + c^2)*f^
2]])/Sqrt[-(d^2*e^2) + 2*c*d*e*f - (1 + c^2)*f^2]))/(d*(-(d*e) + c*f)*(e +
f*x)^2) - (b^2*(c + d*x)^2*(f + (d*e - c*f)/(c + d*x))^2*(ArcCsch[c + d*x]^
2/((-(d*e) + c*f)*(f + (d*e)/(c + d*x) - (c*f)/(c + d*x))) + (2*(((-I)*Pi*A
rcTanh[(-(d*e) + c*f + f*Tanh[ArcCsch[c + d*x]/2])/Sqrt[f^2 + (d*e - c*f)^2
]])/Sqrt[f^2 + (d*e - c*f)^2] - (2*(Pi/2 - I*ArcCsch[c + d*x])*ArcTanh[((f
- I*(d*e - c*f))*Cot[(Pi/2 - I*ArcCsch[c + d*x])/2])/Sqrt[-(d^2*e^2) + 2*c*
```

```
d*e*f - f^2 - c^2*f^2]] - 2*ArcCos[((-I)*f)/(d*e - c*f)]*ArcTanh[((-f - I*(
d*e - c*f))*Tan[(Pi/2 - I*ArcCsch[c + d*x])/2])/Sqrt[-(d^2*e^2) + 2*c*d*e*f
 - f^2 - c^2*f^2]] + (ArcCos[((-I)*f)/(d*e - c*f)] - (2*I)*(ArcTanh[((f - I
*(d*e - c*f))*Cot[(Pi/2 - I*ArcCsch[c + d*x])/2])/Sqrt[-(d^2*e^2) + 2*c*d*e
*f - f^2 - c^2*f^2]] - ArcTanh[((-f - I*(d*e - c*f))*Tan[(Pi/2 - I*ArcCsch[
c + d*x])/2])/Sqrt[-(d^2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2]]))*Log[Sqrt[-(d^
2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2]/(Sqrt[2]*E^((I/2)*(Pi/2 - I*ArcCsch[c +
 d*x]))*Sqrt[(-I)*(d*e - c*f)]*Sqrt[f + (d*e - c*f)/(c + d*x)])] + (ArcCos[
((-I)*f)/(d*e - c*f)] + (2*I)*(ArcTanh[((f - I*(d*e - c*f))*Cot[(Pi/2 - I*A
rcCsch[c + d*x])/2])/Sqrt[-(d^2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2]] - ArcTan
h[((-f - I*(d*e - c*f))*Tan[(Pi/2 - I*ArcCsch[c + d*x])/2])/Sqrt[-(d^2*e^2)
 + 2*c*d*e*f - f^2 - c^2*f^2]]))*Log[(E^((I/2)*(Pi/2 - I*ArcCsch[c + d*x]))
*Sqrt[-(d^2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2])/(Sqrt[2]*Sqrt[(-I)*(d*e - c*
f)]*Sqrt[f + (d*e - c*f)/(c + d*x)])] - (ArcCos[((-I)*f)/(d*e - c*f)] + (2*
I)*ArcTanh[((-f - I*(d*e - c*f))*Tan[(Pi/2 - I*ArcCsch[c + d*x])/2])/Sqrt[-
(d^2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2]])*Log[1 - (I*(f - I*Sqrt[-(d^2*e^2)
+ 2*c*d*e*f - f^2 - c^2*f^2])*(f - I*(d*e - c*f) - Sqrt[-(d^2*e^2) + 2*c*d*
e*f - f^2 - c^2*f^2]*Tan[(Pi/2 - I*ArcCsch[c + d*x])/2]))/((d*e - c*f)*(f -
 I*(d*e - c*f) + Sqrt[-(d^2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2]*Tan[(Pi/2 - I
*ArcCsch[c + d*x])/2]))] + (-ArcCos[((-I)*f)/(d*e - c*f)] + (2*I)*ArcTanh[(
(-f - I*(d*e - c*f))*Tan[(Pi/2 - I*ArcCsch[c + d*x])/2])/Sqrt[-(d^2*e^2) +
2*c*d*e*f - f^2 - c^2*f^2]])*Log[1 - (I*(f + I*Sqrt[-(d^2*e^2) + 2*c*d*e*f
- f^2 - c^2*f^2])*(f - I*(d*e - c*f) - Sqrt[-(d^2*e^2) + 2*c*d*e*f - f^2 -
c^2*f^2]*Tan[(Pi/2 - I*ArcCsch[c + d*x])/2]))/((d*e - c*f)*(f - I*(d*e - c*
f) + Sqrt[-(d^2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2]*Tan[(Pi/2 - I*ArcCsch[c +
 d*x])/2]))] + I*(PolyLog[2, (I*(f - I*Sqrt[-(d^2*e^2) + 2*c*d*e*f - f^2 -
c^2*f^2])*(f - I*(d*e - c*f) - Sqrt[-(d^2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2]
*Tan[(Pi/2 - I*ArcCsch[c + d*x])/2]))/((d*e - c*f)*(f - I*(d*e - c*f) + Sqr
t[-(d^2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2]*Tan[(Pi/2 - I*ArcCsch[c + d*x])/2
]))] - PolyLog[2, (I*(f + I*Sqrt[-(d^2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2])*(
f - I*(d*e - c*f) - Sqrt[-(d^2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2]*Tan[(Pi/2
- I*ArcCsch[c + d*x])/2]))/((d*e - c*f)*(f - I*(d*e - c*f) + Sqrt[-(d^2*e^2
) + 2*c*d*e*f - f^2 - c^2*f^2]*Tan[(Pi/2 - I*ArcCsch[c + d*x])/2]))]))/Sqrt
[-(d^2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2]))/(d*e - c*f)))/(d*(e + f*x)^2)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^{2} \operatorname{arcsch}\left(dx+c\right)^{2}+2\,ab \operatorname{arcsch}\left(dx+c\right)+a^{2}}{f^{2}x^{2}+2\,efx+e^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccsch(d*x + c)^2 + 2*a*b*arccsch(d*x + c) + a^2)/(f^2*x^2 +
 2*e*f*x + e^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{arcsch}\left(dx+c\right)+a\right)^{2}}{\left(fx+e\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(d*x + c) + a)^2/(f*x + e)^2, x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \,\mathrm{arccsch}\left(dx+c\right)\right)^{2}}{\left(fx+e\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(d*x+c))^2/(f*x+e)^2,x)
```

```
[Out] int((a+b*arccsch(d*x+c))^2/(f*x+e)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2 \log\left(\sqrt{d^2x^2 + 2\,cdx + c^2 + 1} + 1\right)^2}{f^2x + ef} - \frac{a^2}{f^2x + ef} - \int -\frac{\left(b^2d^2fx^2 + 2\,b^2cdfx + \left(c^2f + f\right)b^2\right)\log\left(dx + c\right)^2 - 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -b^2*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)^2/(f^2*x + e*f) - a^2/(f^2*
x + e*f) - integrate(-((b^2*d^2*f*x^2 + 2*b^2*c*d*f*x + (c^2*f + f)*b^2)*lo
g(d*x + c)^2 - 2*(a*b*d^2*f*x^2 + 2*a*b*c*d*f*x + (c^2*f + f)*a*b)*log(d*x
+ c) + 2*(a*b*d^2*f*x^2 + 2*a*b*c*d*f*x + (c^2*f + f)*a*b - (b^2*d^2*f*x^2
+ 2*b^2*c*d*f*x + (c^2*f + f)*b^2)*log(d*x + c) + (b^2*c*d*e + (c^2*f + f)*
a*b + (a*b*d^2*f + b^2*d^2*f)*x^2 + (2*a*b*c*d*f + (d^2*e + c*d*f)*b^2)*x -
 (b^2*d^2*f*x^2 + 2*b^2*c*d*f*x + (c^2*f + f)*b^2)*log(d*x + c))*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1))*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)*((b^2*d^2*f*x^2 + 2*b^2*c*d*f*x + (c^2*f + f)*
b^2)*log(d*x + c)^2 - 2*(a*b*d^2*f*x^2 + 2*a*b*c*d*f*x + (c^2*f + f)*a*b)*l
og(d*x + c)))/(d^2*f^3*x^4 + c^2*e^2*f + 2*(d^2*e*f^2 + c*d*f^3)*x^3 + e^2*
f + (d^2*e^2*f + 4*c*d*e*f^2 + c^2*f^3 + f^3)*x^2 + 2*(c*d*e^2*f + c^2*e*f^
2 + e*f^2)*x + (d^2*f^3*x^4 + c^2*e^2*f + 2*(d^2*e*f^2 + c*d*f^3)*x^3 + e^2
*f + (d^2*e^2*f + 4*c*d*e*f^2 + c^2*f^3 + f^3)*x^2 + 2*(c*d*e^2*f + c^2*e*f
^2 + e*f^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c+d\,x}\right)\right)}^2}{{\left(e+f\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c + d*x)))^2/(e + f*x)^2,x)
```

```
[Out] int((a + b*asinh(1/(c + d*x)))^2/(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acsch}{\left(c + d x \right)}\right)^{2}}{\left(e + f x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(d*x+c))**2/(f*x+e)**2,x)
```

```
[Out] Integral((a + b*acsch(c + d*x))**2/(e + f*x)**2, x)
```

3.13 $\int \frac{\left(a+b\,\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^3}\,dx$

Optimal. Leaf size=1024

$$\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2 d^2}{2f(de-cf)^2}-\frac{bf\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)d^2}{(de-cf)\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)\left(f+\frac{de-cf}{c+dx}\right)}-\frac{2b\left(a+b\text{csch}^{-1}(c+dx)\right)\log\left(\cdots\right)}{(de-cf)^2\sqrt{d^2e^2-2cdf\cdots}}$$

```
[Out] 1/2*d^2*(a+b*arccsch(d*x+c))^2/f/(-c*f+d*e)^2-1/2*(a+b*arccsch(d*x+c))^2/f/
(f*x+e)^2+b^2*d^2*f*ln(f+(-c*f+d*e)/(d*x+c))/(-c*f+d*e)^2/(d^2*e^2-2*c*d*e*
f+(c^2+1)*f^2)+b*d^2*f^2*(a+b*arccsch(d*x+c))*ln(1+(1/(d*x+c)+(1+1/(d*x+c)^
2)^(1/2))*(-c*f+d*e)/(f-(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)^
2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(3/2)-b*d^2*f^2*(a+b*arccsch(d*x+c))*ln(1
+(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*c*d*e*f+(c^2+1)
*f^2)^(1/2)))/(-c*f+d*e)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(3/2)+b^2*d^2*f^
2*polylog(2,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f-(d^2*e^2-2*c*d
*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(3/2
)-b^2*d^2*f^2*polylog(2,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d
^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)^2/(d^2*e^2-2*c*d*e*f+(c^2+
1)*f^2)^(3/2)-2*b*d^2*(a+b*arccsch(d*x+c))*ln(1+(1/(d*x+c)+(1+1/(d*x+c)^2)^
(1/2))*(-c*f+d*e)/(f-(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)^2/(
d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)+2*b*d^2*(a+b*arccsch(d*x+c))*ln(1+(1/(
d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
^(1/2)))/(-c*f+d*e)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)-2*b^2*d^2*polyl
og(2,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f-(d^2*e^2-2*c*d*e*f+(c
^2+1)*f^2)^(1/2)))/(-c*f+d*e)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)+2*b^2
*d^2*polylog(2,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*
c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(
1/2)-b*d^2*f*(a+b*arccsch(d*x+c))*(1+1/(d*x+c)^2)^(1/2)/(-c*f+d*e)/(d^2*e^2
-2*c*d*e*f+(c^2+1)*f^2)/(f+(-c*f+d*e)/(d*x+c))
```

Rubi [A] time = 2.30, antiderivative size = 1024, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {6322, 5469, 4191, 3322, 2264, 2190, 2279, 2391, 3324, 2668, 31}

$$\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2 d^2}{2f(de-cf)^2}-\frac{bf\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)d^2}{(de-cf)\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)\left(f+\frac{de-cf}{c+dx}\right)}-\frac{2b\left(a+b\text{csch}^{-1}(c+dx)\right)\log\left(\cdots\right)}{(de-cf)^2\sqrt{d^2e^2-2cdf\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c + d*x])^2/(e + f*x)^3,x]
```

```
[Out] -((b*d^2*f*Sqrt[1 + (c + d*x)^(-2)]*(a + b*ArcCsch[c + d*x]))/((d*e - c*f)*
(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(f + (d*e - c*f)/(c + d*x)))) + (d^2*
(a + b*ArcCsch[c + d*x])^2)/(2*f*(d*e - c*f)^2) - (a + b*ArcCsch[c + d*x])^
2/(2*f*(e + f*x)^2) + (b*d^2*f^2*(a + b*ArcCsch[c + d*x])*Log[1 + (E^ArcCsc
h[c + d*x]*(d*e - c*f))/(f - Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])])/(
(d*e - c*f)^2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^(3/2)) - (2*b*d^2*(a +
b*ArcCsch[c + d*x])*Log[1 + (E^ArcCsch[c + d*x]*(d*e - c*f))/(f - Sqrt[d^2*
e^2 - 2*c*d*e*f + (1 + c^2)*f^2])])/((d*e - c*f)^2*Sqrt[d^2*e^2 - 2*c*d*e*f
 + (1 + c^2)*f^2]) - (b*d^2*f^2*(a + b*ArcCsch[c + d*x])*Log[1 + (E^ArcCsch
[c + d*x]*(d*e - c*f))/(f + Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])])/((
d*e - c*f)^2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^(3/2)) + (2*b*d^2*(a + b
```

```
*ArcCsch[c + d*x])*Log[1 + (E^ArcCsch[c + d*x]*(d*e - c*f))/(f + Sqrt[d^2*e
^2 - 2*c*d*e*f + (1 + c^2)*f^2])])/((d*e - c*f)^2*Sqrt[d^2*e^2 - 2*c*d*e*f
+ (1 + c^2)*f^2]) + (b^2*d^2*f*Log[f + (d*e - c*f)/(c + d*x)])/((d*e - c*f)
^2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) + (b^2*d^2*f^2*PolyLog[2, -((E^Ar
cCsch[c + d*x]*(d*e - c*f))/(f - Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])
)])/((d*e - c*f)^2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^(3/2)) - (2*b^2*d^
2*PolyLog[2, -((E^ArcCsch[c + d*x]*(d*e - c*f))/(f - Sqrt[d^2*e^2 - 2*c*d*e
*f + (1 + c^2)*f^2]))])/((d*e - c*f)^2*Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)
*f^2]) - (b^2*d^2*f^2*PolyLog[2, -((E^ArcCsch[c + d*x]*(d*e - c*f))/(f + Sq
rt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]))])/((d*e - c*f)^2*(d^2*e^2 - 2*c*d
*e*f + (1 + c^2)*f^2)^(3/2)) + (2*b^2*d^2*PolyLog[2, -((E^ArcCsch[c + d*x]*
(d*e - c*f))/(f + Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]))])/((d*e - c*f
)^2*Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5469

```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Csch[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6322

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Csch[x]*Co
th[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)^2}{(e+fx)^3}\,dx = -\left(d^2\,\mathrm{Subst}\left(\int \frac{(a+bx)^2\coth(x)\mathrm{csch}(x)}{(de-cf+f\,\mathrm{csch}(x))^3}\,dx, x, \mathrm{csch}^{-1}(c+dx)\right)\right)$$

$$= -\frac{\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)^2}{2f(e+fx)^2} + \frac{\left(bd^2\right)\mathrm{Subst}\left(\int \frac{a+bx}{(de-cf+f\,\mathrm{csch}(x))^2}\,dx, x, \mathrm{csch}^{-1}(c+d\right.}{f}$$

$$= -\frac{\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)^2}{2f(e+fx)^2} + \frac{\left(bd^2\right)\mathrm{Subst}\left(\int\left(\frac{a+bx}{(de-cf)^2} + \frac{2f(a+bx)}{(de-cf)^2\left(-f-de\left(1-\frac{cf}{de}\right)\sinh(x\right.}\right.\right.}{}$$

$$= \frac{d^2\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)^2}{2f(de-cf)^2} - \frac{\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)^2}{2f(e+fx)^2} + \frac{\left(2bd^2\right)\mathrm{Subst}\left(\int \frac{}{-f-de(}\right.}{}$$

$$= -\frac{bd^2f\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)}{(de-cf)\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)\left(f+\frac{de-cf}{c+dx}\right)} + \frac{d^2\left(a+b\,\mathrm{csch}^{-1}(c+dx\right.}{2f(de-cf)^2}$$

$$= -\frac{bd^2f\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)}{(de-cf)\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)\left(f+\frac{de-cf}{c+dx}\right)} + \frac{d^2\left(a+b\,\mathrm{csch}^{-1}(c+dx\right.}{2f(de-cf)^2}$$

$$= -\frac{bd^2f\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)}{(de-cf)\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)\left(f+\frac{de-cf}{c+dx}\right)} + \frac{d^2\left(a+b\,\mathrm{csch}^{-1}(c+dx\right.}{2f(de-cf)^2}$$

$$= -\frac{bd^2f\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)}{(de-cf)\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)\left(f+\frac{de-cf}{c+dx}\right)} + \frac{d^2\left(a+b\,\mathrm{csch}^{-1}(c+dx\right.}{2f(de-cf)^2}$$

$$= -\frac{bd^2f\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)}{(de-cf)\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)\left(f+\frac{de-cf}{c+dx}\right)} + \frac{d^2\left(a+b\,\mathrm{csch}^{-1}(c+dx\right.}{2f(de-cf)^2}$$

$$= -\frac{bd^2f\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\,\mathrm{csch}^{-1}(c+dx)\right)}{(de-cf)\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)\left(f+\frac{de-cf}{c+dx}\right)} + \frac{d^2\left(a+b\,\mathrm{csch}^{-1}(c+dx\right.}{2f(de-cf)^2}$$

Mathematica [C] time = 14.29, size = 8350, normalized size = 8.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c + d*x])^2/(e + f*x)^3,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{arcsch}(dx+c)^2 + 2\,ab \operatorname{arcsch}(dx+c) + a^2}{f^3x^3 + 3\,ef^2x^2 + 3\,e^2fx + e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(d*x+c))^2/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccsch(d*x + c)^2 + 2*a*b*arccsch(d*x + c) + a^2)/(f^3*x^3 +
 3*e*f^2*x^2 + 3*e^2*f*x + e^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(dx+c) + a)^2}{(fx+e)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(d*x+c))^2/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(d*x + c) + a)^2/(f*x + e)^3, x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsch}(dx+c))^2}{(fx+e)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(d*x+c))^2/(f*x+e)^3,x)
```

```
[Out] int((a+b*arccsch(d*x+c))^2/(f*x+e)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2 \log\left(\sqrt{d^2x^2 + 2\,cdx + c^2 + 1} + 1\right)^2}{2\left(f^3x^2 + 2\,ef^2x + e^2f\right)} - \frac{a^2}{2\left(f^3x^2 + 2\,ef^2x + e^2f\right)} - \int -\frac{\left(b^2d^2fx^2 + 2\,b^2cdfx + \left(c^2f + f\right)b^2\right)\log(c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(d*x+c))^2/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)^2/(f^3*x^2 + 2*e*f^2*x
+ e^2*f) - 1/2*a^2/(f^3*x^2 + 2*e*f^2*x + e^2*f) - integrate(-((b^2*d^2*f*x
^2 + 2*b^2*c*d*f*x + (c^2*f + f)*b^2)*log(d*x + c)^2 - 2*(a*b*d^2*f*x^2 + 2
*a*b*c*d*f*x + (c^2*f + f)*a*b)*log(d*x + c) + (2*a*b*d^2*f*x^2 + 4*a*b*c*d
*f*x + 2*(c^2*f + f)*a*b - 2*(b^2*d^2*f*x^2 + 2*b^2*c*d*f*x + (c^2*f + f)*b
^2)*log(d*x + c) + (b^2*c*d*e + 2*(c^2*f + f)*a*b + (2*a*b*d^2*f + b^2*d^2*
f)*x^2 + (4*a*b*c*d*f + (d^2*e + c*d*f)*b^2)*x - 2*(b^2*d^2*f*x^2 + 2*b^2*c
*d*f*x + (c^2*f + f)*b^2)*log(d*x + c))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*
log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1)*((b^2*d^2*f*x^2 + 2*b^2*c*d*f*x + (c^2*f + f)*b^2)*log(d*x + c)^2 - 2*(
a*b*d^2*f*x^2 + 2*a*b*c*d*f*x + (c^2*f + f)*a*b)*log(d*x + c)))/(d^2*f^4*x^
5 + c^2*e^3*f + (3*d^2*e*f^3 + 2*c*d*f^4)*x^4 + e^3*f + (3*d^2*e^2*f^2 + 6*
c*d*e*f^3 + c^2*f^4 + f^4)*x^3 + (d^2*e^3*f + 6*c*d*e^2*f^2 + 3*c^2*e*f^3 +
 3*e*f^3)*x^2 + (2*c*d*e^3*f + 3*c^2*e^2*f^2 + 3*e^2*f^2)*x + (d^2*f^4*x^5
+ c^2*e^3*f + (3*d^2*e*f^3 + 2*c*d*f^4)*x^4 + e^3*f + (3*d^2*e^2*f^2 + 6*c*
```

```
d*e*f^3 + c^2*f^4 + f^4)*x^3 + (d^2*e^3*f + 6*c*d*e^2*f^2 + 3*c^2*e*f^3 + 3
*e*f^3)*x^2 + (2*c*d*e^3*f + 3*c^2*e^2*f^2 + 3*e^2*f^2)*x)*sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c+d\,x}\right)\right)^2}{\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c + d*x)))^2/(e + f*x)^3,x)
[Out] int((a + b*asinh(1/(c + d*x)))^2/(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{acsch}\,(c+dx)\right)^2}{\left(e+fx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(d*x+c))**2/(f*x+e)**3,x)
[Out] Integral((a + b*acsch(c + d*x))**2/(e + f*x)**3, x)
```

3.14 $\int x^3 \mathrm{csch}^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=114

$$\frac{1}{4}x^4\mathrm{csch}^{-1}\left(\sqrt{x}\right)-\frac{(-x-1)^{7/2}\sqrt{x}}{28\sqrt{-x}}-\frac{3(-x-1)^{5/2}\sqrt{x}}{20\sqrt{-x}}-\frac{(-x-1)^{3/2}\sqrt{x}}{4\sqrt{-x}}-\frac{\sqrt{-x-1}\sqrt{x}}{4\sqrt{-x}}$$

```
[Out] 1/4*x^4*arccsch(x^(1/2))-1/4*(-1-x)^(3/2)*x^(1/2)/(-x)^(1/2)-3/20*(-1-x)^(5
/2)*x^(1/2)/(-x)^(1/2)-1/28*(-1-x)^(7/2)*x^(1/2)/(-x)^(1/2)-1/4*(-1-x)^(1/2
)*x^(1/2)/(-x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6346, 12, 43}

$$\frac{1}{4}x^4\mathrm{csch}^{-1}\left(\sqrt{x}\right)-\frac{(-x-1)^{7/2}\sqrt{x}}{28\sqrt{-x}}-\frac{3(-x-1)^{5/2}\sqrt{x}}{20\sqrt{-x}}-\frac{(-x-1)^{3/2}\sqrt{x}}{4\sqrt{-x}}-\frac{\sqrt{-x-1}\sqrt{x}}{4\sqrt{-x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCsch[Sqrt[x]],x]
```

```
[Out] -(Sqrt[-1 - x]*Sqrt[x])/(4*Sqrt[-x]) - ((-1 - x)^(3/2)*Sqrt[x])/(4*Sqrt[-x]
) - (3*(-1 - x)^(5/2)*Sqrt[x])/(20*Sqrt[-x]) - ((-1 - x)^(7/2)*Sqrt[x])/(28
*Sqrt[-x]) + (x^4*ArcCsch[Sqrt[x]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6346

```
Int[((a_.) + ArcCsch[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCsch[u]))/(d*(m + 1)), x] - Dist[(b*u)/(d*(
m + 1)*Sqrt[-u^2]), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(u*Sq
rt[-1 - u^2]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inv
erseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Func
tionOfExponentialQ[u, x]
```

Rubi steps

$$\int x^3\text{csch}^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{4}x^4\text{csch}^{-1}\left(\sqrt{x}\right) - \frac{\sqrt{x}\int\frac{x^3}{2\sqrt{-1-x}}\,dx}{4\sqrt{-x}}$$
$$= \frac{1}{4}x^4\text{csch}^{-1}\left(\sqrt{x}\right) - \frac{\sqrt{x}\int\frac{x^3}{\sqrt{-1-x}}\,dx}{8\sqrt{-x}}$$
$$= \frac{1}{4}x^4\text{csch}^{-1}\left(\sqrt{x}\right) - \frac{\sqrt{x}\int\left(-\frac{1}{\sqrt{-1-x}} - 3\sqrt{-1-x} - 3(-1-x)^{3/2} - (-1-x)^{5/2}\right)dx}{8\sqrt{-x}}$$
$$= -\frac{\sqrt{-1-x}\sqrt{x}}{4\sqrt{-x}} - \frac{(-1-x)^{3/2}\sqrt{x}}{4\sqrt{-x}} - \frac{3(-1-x)^{5/2}\sqrt{x}}{20\sqrt{-x}} - \frac{(-1-x)^{7/2}\sqrt{x}}{28\sqrt{-x}} + \frac{1}{4}x^4\text{csch}^{-1}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.41

$$\frac{1}{4}x^4\text{csch}^{-1}\left(\sqrt{x}\right) + \frac{1}{140}\sqrt{\frac{1}{x}+1}\left(5x^3 - 6x^2 + 8x - 16\right)\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCsch[Sqrt[x]],x]
```

```
[Out] (Sqrt[1 + x^(-1)]*Sqrt[x]*(-16 + 8*x - 6*x^2 + 5*x^3))/140 + (x^4*ArcCsch[S
qrt[x]])/4
```

fricas [A] time = 0.58, size = 55, normalized size = 0.48

$$\frac{1}{4}\,x^4\log\left(\frac{x\sqrt{\frac{x+1}{x}} + \sqrt{x}}{x}\right) + \frac{1}{140}\left(5\,x^3 - 6\,x^2 + 8\,x - 16\right)\sqrt{x}\sqrt{\frac{x+1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsch(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log((x*sqrt((x + 1)/x) + sqrt(x))/x) + 1/140*(5*x^3 - 6*x^2 + 8*x -
 16)*sqrt(x)*sqrt((x + 1)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,\text{arcsch}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsch(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccsch(sqrt(x)), x)
```

maple [A] time = 0.05, size = 43, normalized size = 0.38

$$\frac{x^4\text{arccsch}\left(\sqrt{x}\right)}{4} + \frac{(1+x)\left(5x^3 - 6x^2 + 8x - 16\right)}{140\sqrt{\frac{1+x}{x}}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccsch(x^(1/2)),x)
```

```
[Out] 1/4*x^4*arccsch(x^(1/2))+1/140*(1+x)*(5*x^3-6*x^2+8*x-16)/((1+x)/x)^(1/2)/x
^(1/2)
```

maxima [A] time = 0.31, size = 58, normalized size = 0.51

$$\frac{1}{28}\,x^{\frac{7}{2}}\left(\frac{1}{x}+1\right)^{\frac{7}{2}} - \frac{3}{20}\,x^{\frac{5}{2}}\left(\frac{1}{x}+1\right)^{\frac{5}{2}} + \frac{1}{4}\,x^4\operatorname{arcsch}\left(\sqrt{x}\right) + \frac{1}{4}\,x^{\frac{3}{2}}\left(\frac{1}{x}+1\right)^{\frac{3}{2}} - \frac{1}{4}\,\sqrt{x}\sqrt{\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsch(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/28*x^(7/2)*(1/x + 1)^(7/2) - 3/20*x^(5/2)*(1/x + 1)^(5/2) + 1/4*x^4*arccs
ch(sqrt(x)) + 1/4*x^(3/2)*(1/x + 1)^(3/2) - 1/4*sqrt(x)*sqrt(1/x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\operatorname{asinh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(1/x^(1/2)),x)
```

```
[Out] int(x^3*asinh(1/x^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\operatorname{acsch}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acsch(x**(1/2)),x)
```

```
[Out] Integral(x**3*acsch(sqrt(x)), x)
```

3.15 $\int x^2 \text{csch}^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=89

$$\frac{1}{3}x^3\text{csch}^{-1}\left(\sqrt{x}\right)+\frac{(-x-1)^{5/2}\sqrt{x}}{15\sqrt{-x}}+\frac{2(-x-1)^{3/2}\sqrt{x}}{9\sqrt{-x}}+\frac{\sqrt{-x-1}\sqrt{x}}{3\sqrt{-x}}$$

```
[Out] 1/3*x^3*arccsch(x^(1/2))+2/9*(-1-x)^(3/2)*x^(1/2)/(-x)^(1/2)+1/15*(-1-x)^(5
/2)*x^(1/2)/(-x)^(1/2)+1/3*(-1-x)^(1/2)*x^(1/2)/(-x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6346, 12, 43}

$$\frac{1}{3}x^3\text{csch}^{-1}\left(\sqrt{x}\right)+\frac{(-x-1)^{5/2}\sqrt{x}}{15\sqrt{-x}}+\frac{2(-x-1)^{3/2}\sqrt{x}}{9\sqrt{-x}}+\frac{\sqrt{-x-1}\sqrt{x}}{3\sqrt{-x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCsch[Sqrt[x]],x]
```

```
[Out] (Sqrt[-1 - x]*Sqrt[x])/(3*Sqrt[-x]) + (2*(-1 - x)^(3/2)*Sqrt[x])/(9*Sqrt[-x
]) + ((-1 - x)^(5/2)*Sqrt[x])/(15*Sqrt[-x]) + (x^3*ArcCsch[Sqrt[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6346

```
Int[((a_.) + ArcCsch[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCsch[u]))/(d*(m + 1)), x] - Dist[(b*u)/(d*(
m + 1)*Sqrt[-u^2]), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(u*Sq
rt[-1 - u^2]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inv
erseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Func
tionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\text{csch}^{-1}\left(\sqrt{x}\right)dx &= \frac{1}{3}x^3\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int\frac{x^2}{2\sqrt{-1-x}}\,dx}{3\sqrt{-x}}\\
&= \frac{1}{3}x^3\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int\frac{x^2}{\sqrt{-1-x}}\,dx}{6\sqrt{-x}}\\
&= \frac{1}{3}x^3\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int\left(\frac{1}{\sqrt{-1-x}}+2\sqrt{-1-x}+(-1-x)^{3/2}\right)dx}{6\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}\sqrt{x}}{3\sqrt{-x}}+\frac{2(-1-x)^{3/2}\sqrt{x}}{9\sqrt{-x}}+\frac{(-1-x)^{5/2}\sqrt{x}}{15\sqrt{-x}}+\frac{1}{3}x^3\text{csch}^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.47

$$\frac{1}{3}x^3\text{csch}^{-1}\left(\sqrt{x}\right)+\frac{1}{45}\sqrt{\frac{1}{x}+1}\left(3x^2-4x+8\right)\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCsch[Sqrt[x]],x]
```

```
[Out] (Sqrt[1 + x^(-1)]*Sqrt[x]*(8 - 4*x + 3*x^2))/45 + (x^3*ArcCsch[Sqrt[x]])/3
```

fricas [A] time = 0.59, size = 50, normalized size = 0.56

$$\frac{1}{3}x^3\log\left(\frac{x\sqrt{\frac{x+1}{x}}+\sqrt{x}}{x}\right)+\frac{1}{45}\left(3x^2-4x+8\right)\sqrt{x}\sqrt{\frac{x+1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccsch(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log((x*sqrt((x + 1)/x) + sqrt(x))/x) + 1/45*(3*x^2 - 4*x + 8)*sqrt(
x)*sqrt((x + 1)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{arcsch}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccsch(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccsch(sqrt(x)), x)
```

maple [A] time = 0.05, size = 38, normalized size = 0.43

$$\frac{x^3\operatorname{arccsch}\left(\sqrt{x}\right)}{3}+\frac{(1+x)\left(3x^2-4x+8\right)}{45\sqrt{\frac{1+x}{x}}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccsch(x^(1/2)),x)
```

```
[Out] 1/3*x^3*arccsch(x^(1/2))+1/45*(1+x)*(3*x^2-4*x+8)/((1+x)/x)^(1/2)/x^(1/2)
```

maxima [A] time = 0.32, size = 46, normalized size = 0.52

$$\frac{1}{15}x^{\frac{5}{2}}\left(\frac{1}{x}+1\right)^{\frac{5}{2}}+\frac{1}{3}x^3\operatorname{arcsch}\left(\sqrt{x}\right)-\frac{2}{9}x^{\frac{3}{2}}\left(\frac{1}{x}+1\right)^{\frac{3}{2}}+\frac{1}{3}\sqrt{x}\sqrt{\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccsch(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/15*x^(5/2)*(1/x + 1)^(5/2) + 1/3*x^3*arccsch(sqrt(x)) - 2/9*x^(3/2)*(1/x
+ 1)^(3/2) + 1/3*sqrt(x)*sqrt(1/x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{asinh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(1/x^(1/2)),x)
```

```
[Out] int(x^2*asinh(1/x^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acsch}\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acsch(x**(1/2)),x)
```

```
[Out] Integral(x**2*acsch(sqrt(x)), x)
```

3.16 $\int x\text{csch}^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=64

$$\frac{1}{2}x^2\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{(-x-1)^{3/2}\sqrt{x}}{6\sqrt{-x}}-\frac{\sqrt{-x-1}\sqrt{x}}{2\sqrt{-x}}$$

```
[Out] 1/2*x^2*arccsch(x^(1/2))-1/6*(-1-x)^(3/2)*x^(1/2)/(-x)^(1/2)-1/2*(-1-x)^(1/
2)*x^(1/2)/(-x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6346, 12, 43}

$$\frac{1}{2}x^2\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{(-x-1)^{3/2}\sqrt{x}}{6\sqrt{-x}}-\frac{\sqrt{-x-1}\sqrt{x}}{2\sqrt{-x}}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCsch[Sqrt[x]],x]
```

```
[Out] -(Sqrt[-1 - x]*Sqrt[x])/(2*Sqrt[-x]) - ((-1 - x)^(3/2)*Sqrt[x])/(6*Sqrt[-x]
) + (x^2*ArcCsch[Sqrt[x]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6346

```
Int[((a_.) + ArcCsch[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCsch[u]))/(d*(m + 1)), x] - Dist[(b*u)/(d*(
m + 1)*Sqrt[-u^2]), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(u*Sq
rt[-1 - u^2]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inv
erseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Func
tionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int x\text{csch}^{-1}\left(\sqrt{x}\right) dx &= \frac{1}{2}x^2\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \frac{x}{2\sqrt{-1-x}} dx}{2\sqrt{-x}}\\ &= \frac{1}{2}x^2\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \frac{x}{\sqrt{-1-x}} dx}{4\sqrt{-x}}\\ &= \frac{1}{2}x^2\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \left(-\frac{1}{\sqrt{-1-x}}-\sqrt{-1-x}\right) dx}{4\sqrt{-x}}\\ &= -\frac{\sqrt{-1-x}\sqrt{x}}{2\sqrt{-x}}-\frac{(-1-x)^{3/2}\sqrt{x}}{6\sqrt{-x}}+\frac{1}{2}x^2\text{csch}^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.55

$$\frac{1}{2}x^2\text{csch}^{-1}\left(\sqrt{x}\right)+\frac{1}{6}\sqrt{\frac{1}{x}+1}\,(x-2)\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCsch[Sqrt[x]],x]
[Out] (Sqrt[1 + x^(-1)]*(-2 + x)*Sqrt[x])/6 + (x^2*ArcCsch[Sqrt[x]])/2
```

fricas [A] time = 0.76, size = 43, normalized size = 0.67

$$\frac{1}{2}\,x^2\log\left(\frac{x\sqrt{\frac{x+1}{x}}+\sqrt{x}}{x}\right)+\frac{1}{6}\,(x-2)\sqrt{x}\,\sqrt{\frac{x+1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccsch(x^(1/2)),x, algorithm="fricas")
[Out] 1/2*x^2*log((x*sqrt((x + 1)/x) + sqrt(x))/x) + 1/6*(x - 2)*sqrt(x)*sqrt((x
+ 1)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\text{arcsch}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccsch(x^(1/2)),x, algorithm="giac")
[Out] integrate(x*arccsch(sqrt(x)), x)
```

maple [A] time = 0.04, size = 31, normalized size = 0.48

$$\frac{x^2\text{arccsch}\left(\sqrt{x}\right)}{2}+\frac{(1+x)\,(x-2)}{6\sqrt{\frac{1+x}{x}}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccsch(x^(1/2)),x)
[Out] 1/2*x^2*arccsch(x^(1/2))+1/6*(1+x)*(x-2)/((1+x)/x)^(1/2)/x^(1/2)
```

maxima [A] time = 0.32, size = 34, normalized size = 0.53

$$\frac{1}{6}\,x^{\frac{3}{2}}\left(\frac{1}{x}+1\right)^{\frac{3}{2}}+\frac{1}{2}\,x^2\,\text{arcsch}\left(\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x}\,\sqrt{\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccsch(x^(1/2)),x, algorithm="maxima")
[Out] 1/6*x^(3/2)*(1/x + 1)^(3/2) + 1/2*x^2*arccsch(sqrt(x)) - 1/2*sqrt(x)*sqrt(1
/x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\text{asinh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(1/x^(1/2)),x)
```

```
[Out] int(x*asinh(1/x^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acsch}\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acsch(x**(1/2)),x)
```

```
[Out] Integral(x*acsch(sqrt(x)), x)
```

3.17 $\int \operatorname{csch}^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=31

$$\frac{\sqrt{-x-1}\sqrt{x}}{\sqrt{-x}}+x\operatorname{csch}^{-1}\left(\sqrt{x}\right)$$

```
[Out] x*arccsch(x^(1/2))+(-1-x)^(1/2)*x^(1/2)/(-x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6344, 12, 32}

$$\frac{\sqrt{-x-1}\sqrt{x}}{\sqrt{-x}}+x\operatorname{csch}^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsch[Sqrt[x]],x]
```

```
[Out] (Sqrt[-1 - x]*Sqrt[x])/Sqrt[-x] + x*ArcCsch[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6344

```
Int[ArcCsch[u_], x_Symbol] :> Simp[x*ArcCsch[u], x] - Dist[u/Sqrt[-u^2], In
t[SimplifyIntegrand[(x*D[u, x])/(u*Sqrt[-1 - u^2]), x], x], x] /; InverseFu
nctionFreeQ[u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^{-1}\left(\sqrt{x}\right) dx &= x\operatorname{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \frac{1}{2\sqrt{-1-x}}\,dx}{\sqrt{-x}}\\
&= x\operatorname{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \frac{1}{\sqrt{-1-x}}\,dx}{2\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}\sqrt{x}}{\sqrt{-x}}+x\operatorname{csch}^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.77

$$\sqrt{\frac{1}{x}+1}\sqrt{x}+x\operatorname{csch}^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsch[Sqrt[x]],x]
```

```
[Out] Sqrt[1 + x^(-1)]*Sqrt[x] + x*ArcCsch[Sqrt[x]]
```

fricas [A] time = 0.52, size = 36, normalized size = 1.16

$$x \log\left(\frac{x\sqrt{\frac{x+1}{x}} + \sqrt{x}}{x}\right) + \sqrt{x}\sqrt{\frac{x+1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2)),x, algorithm="fricas")
[Out] x*log((x*sqrt((x + 1)/x) + sqrt(x))/x) + sqrt(x)*sqrt((x + 1)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arcsch}\left(\sqrt{x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2)),x, algorithm="giac")
[Out] integrate(arccsch(sqrt(x)), x)
```

maple [A] time = 0.05, size = 24, normalized size = 0.77

$$x \operatorname{arccsch}\left(\sqrt{x}\right) + \frac{1+x}{\sqrt{\frac{1+x}{x}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsch(x^(1/2)),x)
[Out] x*arccsch(x^(1/2))+1/((1+x)/x)^(1/2)/x^(1/2)*(1+x)
```

maxima [A] time = 0.32, size = 18, normalized size = 0.58

$$x \operatorname{arcsch}\left(\sqrt{x}\right) + \sqrt{x}\sqrt{\frac{1}{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2)),x, algorithm="maxima")
[Out] x*arccsch(sqrt(x)) + sqrt(x)*sqrt(1/x + 1)
```

mupad [B] time = 2.57, size = 18, normalized size = 0.58

$$x \operatorname{asinh}\left(\frac{1}{\sqrt{x}}\right) + \sqrt{x}\sqrt{\frac{1}{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(1/x^(1/2)),x)
[Out] x*asinh(1/x^(1/2)) + x^(1/2)*(1/x + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acsch}\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsch(x**(1/2)),x)
[Out] Integral(acsch(sqrt(x)), x)
```

3.18 $\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}\,dx$

Optimal. Leaf size=46

$$-\text{Li}_2\left(e^{2\operatorname{csch}^{-1}\left(\sqrt{x}\right)}\right)+\operatorname{csch}^{-1}\left(\sqrt{x}\right)^2-2\operatorname{csch}^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\operatorname{csch}^{-1}\left(\sqrt{x}\right)}\right)$$

```
[Out] arccsch(x^(1/2))^2-2*arccsch(x^(1/2))*ln(1-(1/x^(1/2)+(1/x+1)^(1/2))^2)-pol
ylog(2,(1/x^(1/2)+(1/x+1)^(1/2))^2)
```

Rubi [A] time = 0.10, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6282, 5659, 3716, 2190, 2279, 2391}

$$-\text{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}\left(\sqrt{x}\right)}\right)+\operatorname{csch}^{-1}\left(\sqrt{x}\right)^2-2\operatorname{csch}^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\operatorname{csch}^{-1}\left(\sqrt{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsch[Sqrt[x]]/x,x]
```

```
[Out] ArcCsch[Sqrt[x]]^2 - 2*ArcCsch[Sqrt[x]]*Log[1 - E^(2*ArcCsch[Sqrt[x]])] - P
olyLog[2, E^(2*ArcCsch[Sqrt[x]])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6282

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\operatorname{csch}^{-1}(x)}{x}\,dx, x, \sqrt{x}\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= -\left(2\,\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)\right)\right)$$
$$= \sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 + 4\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)\right)$$
$$= \sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) + 2\,\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \sin$$
$$= \sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) + \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-}}$$
$$= \sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) - \text{Li}_2\left(e^{2\sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right)$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.98

$$\text{Li}_2\left(e^{-2\operatorname{csch}^{-1}\left(\sqrt{x}\right)}\right) - \operatorname{csch}^{-1}\left(\sqrt{x}\right)\left(\operatorname{csch}^{-1}\left(\sqrt{x}\right) + 2\log\left(1-e^{-2\operatorname{csch}^{-1}\left(\sqrt{x}\right)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCsch[Sqrt[x]]/x,x]
[Out] -(ArcCsch[Sqrt[x]]*(ArcCsch[Sqrt[x]] + 2*Log[1 - E^(-2*ArcCsch[Sqrt[x]])]))
 + PolyLog[2, E^(-2*ArcCsch[Sqrt[x]])]
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2))/x,x, algorithm="fricas")
[Out] integral(arccsch(sqrt(x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2))/x,x, algorithm="giac")
[Out] integrate(arccsch(sqrt(x))/x, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsch(x^(1/2))/x,x)
[Out] int(arccsch(x^(1/2))/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2))/x,x, algorithm="maxima")
[Out] integrate(arccsch(sqrt(x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(\frac{1}{\sqrt{x}}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(1/x^(1/2))/x,x)
[Out] int(asinh(1/x^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acsch}\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsch(x**(1/2))/x,x)
[Out] Integral(acsch(sqrt(x))/x, x)
```

3.19 $\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^2}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{-x-1}}{2\sqrt{-x}\sqrt{x}}-\frac{\sqrt{x}\tan^{-1}\left(\sqrt{-x-1}\right)}{2\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}$$

```
[Out] -arccsch(x^(1/2))/x+1/2*(-1-x)^(1/2)/(-x)^(1/2)/x^(1/2)-1/2*arctan((-1-x)^(
1/2))*x^(1/2)/(-x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6346, 12, 51, 63, 204}

$$\frac{\sqrt{-x-1}}{2\sqrt{-x}\sqrt{x}}-\frac{\sqrt{x}\tan^{-1}\left(\sqrt{-x-1}\right)}{2\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsch[Sqrt[x]]/x^2,x]
```

```
[Out] Sqrt[-1 - x]/(2*Sqrt[-x]*Sqrt[x]) - ArcCsch[Sqrt[x]]/x - (Sqrt[x]*ArcTan[Sq
rt[-1 - x]])/(2*Sqrt[-x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 6346

```
Int[((a_.) + ArcCsch[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCsch[u]))/(d*(m + 1)), x] - Dist[(b*u)/(d*(
m + 1)*Sqrt[-u^2]), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(u*Sq
rt[-1 - u^2]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inv
erseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Func
```

```
tionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{x}\int \frac{1}{2\sqrt{-1-x}\,x^2}\,dx}{\sqrt{-x}}\\
&= -\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{x}\int \frac{1}{\sqrt{-1-x}\,x^2}\,dx}{2\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}}{2\sqrt{-x}\sqrt{x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}-\frac{\sqrt{x}\int \frac{1}{\sqrt{-1-x}\,x}\,dx}{4\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}}{2\sqrt{-x}\sqrt{x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{x}\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{-1-x}\right)}{2\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}}{2\sqrt{-x}\sqrt{x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}-\frac{\sqrt{x}\tan^{-1}\left(\sqrt{-1-x}\right)}{2\sqrt{-x}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.67

$$\frac{\sqrt{\frac{x+1}{x}}}{2\sqrt{x}}-\frac{1}{2}\sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsch[Sqrt[x]]/x^2,x]
[Out] Sqrt[(1 + x)/x]/(2*Sqrt[x]) - ArcCsch[Sqrt[x]]/x - ArcSinh[1/Sqrt[x]]/2
```

fricas [A] time = 0.63, size = 44, normalized size = 0.70

$$-\frac{(x+2)\log\left(\frac{x\sqrt{\frac{x+1}{x}}+\sqrt{x}}{x}\right)-\sqrt{x}\sqrt{\frac{x+1}{x}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2))/x^2,x, algorithm="fricas")
[Out] -1/2*((x + 2)*log((x*sqrt((x + 1)/x) + sqrt(x))/x) - sqrt(x)*sqrt((x + 1)/x
))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2))/x^2,x, algorithm="giac")
[Out] integrate(arccsch(sqrt(x))/x^2, x)
```

maple [A] time = 0.06, size = 45, normalized size = 0.71

$$-\frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{1+x}\,\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{1+x}}\right)x+\sqrt{1+x}\right)}{2\sqrt{\frac{1+x}{x}}\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsch(x^(1/2))/x^2,x)
```

```
[Out] -arccsch(x^(1/2))/x+1/2*(1+x)^(1/2)*(-arctanh(1/(1+x)^(1/2))*x+(1+x)^(1/2))
/((1+x)/x)^(1/2)/x^(3/2)
```

maxima [A] time = 0.32, size = 65, normalized size = 1.03

$$\frac{\sqrt{x}\sqrt{\frac{1}{x}+1}}{2\left(x\left(\frac{1}{x}+1\right)-1\right)}-\frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{x}-\frac{1}{4}\log\left(\sqrt{x}\sqrt{\frac{1}{x}+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x}\sqrt{\frac{1}{x}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x)*sqrt(1/x + 1)/(x*(1/x + 1) - 1) - arccsch(sqrt(x))/x - 1/4*log(
sqrt(x)*sqrt(1/x + 1) + 1) + 1/4*log(sqrt(x)*sqrt(1/x + 1) - 1)
```

mupad [B] time = 2.22, size = 33, normalized size = 0.52

$$\frac{\sqrt{\frac{1}{x}+1}}{2\sqrt{x}}-\frac{2\operatorname{asinh}\left(\frac{1}{\sqrt{x}}\right)\left(\frac{1}{2\sqrt{x}}+\frac{\sqrt{x}}{4}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(1/x^(1/2))/x^2,x)
```

```
[Out] (1/x + 1)^(1/2)/(2*x^(1/2)) - (2*asinh(1/x^(1/2))*(1/(2*x^(1/2)) + x^(1/2)/
4))/x^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acsch}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsch(x**(1/2))/x**2,x)
```

```
[Out] Integral(acsch(sqrt(x))/x**2, x)
```

3.20 $\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^3} \, dx$

Optimal. Leaf size=90

$$\frac{\sqrt{-x-1}}{8\sqrt{-x}\,x^{3/2}}-\frac{\operatorname{csch}^{-1}(\sqrt{x})}{2x^2}-\frac{3\sqrt{-x-1}}{16\sqrt{-x}\,\sqrt{x}}+\frac{3\sqrt{x}\,\tan^{-1}(\sqrt{-x-1})}{16\sqrt{-x}}$$

```
[Out] -1/2*arccsch(x^(1/2))/x^2+1/8*(-1-x)^(1/2)/x^(3/2)/(-x)^(1/2)-3/16*(-1-x)^(
1/2)/(-x)^(1/2)/x^(1/2)+3/16*arctan((-1-x)^(1/2))*x^(1/2)/(-x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6346, 12, 51, 63, 204}

$$\frac{\sqrt{-x-1}}{8\sqrt{-x}\,x^{3/2}}-\frac{\operatorname{csch}^{-1}(\sqrt{x})}{2x^2}-\frac{3\sqrt{-x-1}}{16\sqrt{-x}\,\sqrt{x}}+\frac{3\sqrt{x}\,\tan^{-1}(\sqrt{-x-1})}{16\sqrt{-x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsch[Sqrt[x]]/x^3,x]
```

```
[Out] Sqrt[-1 - x]/(8*Sqrt[-x]*x^(3/2)) - (3*Sqrt[-1 - x])/(16*Sqrt[-x]*Sqrt[x])
- ArcCsch[Sqrt[x]]/(2*x^2) + (3*Sqrt[x]*ArcTan[Sqrt[-1 - x]])/(16*Sqrt[-x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 6346

```
Int[((a_.) + ArcCsch[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCsch[u]))/(d*(m + 1)), x] - Dist[(b*u)/(d*(
m + 1)*Sqrt[-u^2]), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(u*Sq
rt[-1 - u^2]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inv
erseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Func
```

```
tionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{x}\int \frac{1}{2\sqrt{-1-x}\,x^3}\,dx}{2\sqrt{-x}}\\
&= -\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{x}\int \frac{1}{\sqrt{-1-x}\,x^3}\,dx}{4\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}}{8\sqrt{-x}\,x^{3/2}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{\left(3\sqrt{x}\right)\int \frac{1}{\sqrt{-1-x}\,x^2}\,dx}{16\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}}{8\sqrt{-x}\,x^{3/2}}-\frac{3\sqrt{-1-x}}{16\sqrt{-x}\,\sqrt{x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{\left(3\sqrt{x}\right)\int \frac{1}{\sqrt{-1-x}\,x}\,dx}{32\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}}{8\sqrt{-x}\,x^{3/2}}-\frac{3\sqrt{-1-x}}{16\sqrt{-x}\,\sqrt{x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{\left(3\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{-1-x}\right)}{16\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}}{8\sqrt{-x}\,x^{3/2}}-\frac{3\sqrt{-1-x}}{16\sqrt{-x}\,\sqrt{x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3\sqrt{x}\,\tan^{-1}\left(\sqrt{-1-x}\right)}{16\sqrt{-x}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.52

$$\frac{3x^2\sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)+\sqrt{\frac{1}{x}+1}\,(2-3x)\sqrt{x}-8\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{16x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsch[Sqrt[x]]/x^3,x]
```

```
[Out] (Sqrt[1 + x^(-1)]*(2 - 3*x)*Sqrt[x] - 8*ArcCsch[Sqrt[x]] + 3*x^2*ArcSinh[1/
Sqrt[x]])/(16*x^2)
```

fricas [A] time = 0.70, size = 53, normalized size = 0.59

$$-\frac{(3\,x-2)\sqrt{x}\,\sqrt{\frac{x+1}{x}}-\left(3\,x^2-8\right)\log\left(\frac{x\sqrt{\frac{x+1}{x}}+\sqrt{x}}{x}\right)}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*((3*x - 2)*sqrt(x)*sqrt((x + 1)/x) - (3*x^2 - 8)*log((x*sqrt((x + 1)/
x) + sqrt(x))/x))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccsch(sqrt(x))/x^3, x)
```

maple [A] time = 0.05, size = 57, normalized size = 0.63

$$-\frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\sqrt{1+x}\,\left(-3\operatorname{arctanh}\left(\frac{1}{\sqrt{1+x}}\right)x^{2}+3x\sqrt{1+x}\,-2\sqrt{1+x}\right)}{16\sqrt{\frac{1+x}{x}}\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsch(x^(1/2))/x^3,x)
```

```
[Out] -1/2*arccsch(x^(1/2))/x^2-1/16*(1+x)^(1/2)*(-3*arctanh(1/(1+x)^(1/2))*x^2+3
*x*(1+x)^(1/2)-2*(1+x)^(1/2))/((1+x)/x)^(1/2)/x^(5/2)
```

maxima [A] time = 0.32, size = 92, normalized size = 1.02

$$-\frac{3\,x^{\frac{3}{2}}\left(\frac{1}{x}+1\right)^{\frac{3}{2}}-5\,\sqrt{x}\,\sqrt{\frac{1}{x}+1}}{16\left(x^{2}\left(\frac{1}{x}+1\right)^{2}-2\,x\left(\frac{1}{x}+1\right)+1\right)}-\frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{2\,x^{2}}+\frac{3}{32}\,\log\left(\sqrt{x}\,\sqrt{\frac{1}{x}+1}\,+1\right)-\frac{3}{32}\,\log\left(\sqrt{x}\,\sqrt{\frac{1}{x}+1}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*x^(3/2)*(1/x + 1)^(3/2) - 5*sqrt(x)*sqrt(1/x + 1))/(x^2*(1/x + 1)^
2 - 2*x*(1/x + 1) + 1) - 1/2*arccsch(sqrt(x))/x^2 + 3/32*log(sqrt(x)*sqrt(1
/x + 1) + 1) - 3/32*log(sqrt(x)*sqrt(1/x + 1) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}\left(\frac{1}{\sqrt{x}}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(1/x^(1/2))/x^3,x)
```

```
[Out] int(asinh(1/x^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acsch}\left(\sqrt{x}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsch(x**(1/2))/x**3,x)
```

```
[Out] Integral(acsch(sqrt(x))/x**3, x)
```

3.21 $\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x^4} \, dx$

Optimal. Leaf size=115

$$-\frac{5\sqrt{-x-1}}{72\sqrt{-x}\,x^{3/2}}+\frac{\sqrt{-x-1}}{18\sqrt{-x}\,x^{5/2}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5\sqrt{-x-1}}{48\sqrt{-x}\,\sqrt{x}}-\frac{5\sqrt{x}\tan^{-1}\left(\sqrt{-x-1}\right)}{48\sqrt{-x}}$$

```
[Out] -1/3*arccsch(x^(1/2))/x^3+1/18*(-1-x)^(1/2)/x^(5/2)/(-x)^(1/2)-5/72*(-1-x)^
(1/2)/x^(3/2)/(-x)^(1/2)+5/48*(-1-x)^(1/2)/(-x)^(1/2)/x^(1/2)-5/48*arctan((
-1-x)^(1/2))*x^(1/2)/(-x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6346, 12, 51, 63, 204}

$$-\frac{5\sqrt{-x-1}}{72\sqrt{-x}\,x^{3/2}}+\frac{\sqrt{-x-1}}{18\sqrt{-x}\,x^{5/2}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5\sqrt{-x-1}}{48\sqrt{-x}\,\sqrt{x}}-\frac{5\sqrt{x}\tan^{-1}\left(\sqrt{-x-1}\right)}{48\sqrt{-x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsch[Sqrt[x]]/x^4,x]
```

```
[Out] Sqrt[-1 - x]/(18*Sqrt[-x]*x^(5/2)) - (5*Sqrt[-1 - x])/(72*Sqrt[-x]*x^(3/2))
 + (5*Sqrt[-1 - x])/(48*Sqrt[-x]*Sqrt[x]) - ArcCsch[Sqrt[x]]/(3*x^3) - (5*S
qrt[x]*ArcTan[Sqrt[-1 - x]])/(48*Sqrt[-x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 6346

```
Int[((a_.) + ArcCsch[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCsch[u]))/(d*(m + 1)), x] - Dist[(b*u)/(d*(
m + 1)*Sqrt[-u^2]), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(u*Sq
```

```
rt[-1 - u^2]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inv
erseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Func
tionOfExponentialQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x^4}\,dx &= -\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{\sqrt{x}\int \frac{1}{2\sqrt{-1-x}\,x^4}\,dx}{3\sqrt{-x}}\\
&= -\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{\sqrt{x}\int \frac{1}{\sqrt{-1-x}\,x^4}\,dx}{6\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}}{18\sqrt{-x}\,x^{5/2}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{\left(5\sqrt{x}\right)\int \frac{1}{\sqrt{-1-x}\,x^3}\,dx}{36\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}}{18\sqrt{-x}\,x^{5/2}}-\frac{5\sqrt{-1-x}}{72\sqrt{-x}\,x^{3/2}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{\left(5\sqrt{x}\right)\int \frac{1}{\sqrt{-1-x}\,x^2}\,dx}{48\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}}{18\sqrt{-x}\,x^{5/2}}-\frac{5\sqrt{-1-x}}{72\sqrt{-x}\,x^{3/2}}+\frac{5\sqrt{-1-x}}{48\sqrt{-x}\,\sqrt{x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{\left(5\sqrt{x}\right)\int \frac{1}{\sqrt{-1-x}\,x}\,dx}{96\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}}{18\sqrt{-x}\,x^{5/2}}-\frac{5\sqrt{-1-x}}{72\sqrt{-x}\,x^{3/2}}+\frac{5\sqrt{-1-x}}{48\sqrt{-x}\,\sqrt{x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{\left(5\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\right.}{48\sqrt{-x}}\\
&= \frac{\sqrt{-1-x}}{18\sqrt{-x}\,x^{5/2}}-\frac{5\sqrt{-1-x}}{72\sqrt{-x}\,x^{3/2}}+\frac{5\sqrt{-1-x}}{48\sqrt{-x}\,\sqrt{x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{5\sqrt{x}\tan^{-1}\left(\sqrt{-1-x}\right)}{48\sqrt{-x}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.45

$$\frac{-15x^3\sinh^{-1}\left(\frac{1}{\sqrt{x}}\right)+\sqrt{\frac{1}{x}+1}\left(15x^2-10x+8\right)\sqrt{x}-48\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{144x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsch[Sqrt[x]]/x^4,x]
```

```
[Out] (Sqrt[1 + x^(-1)]*Sqrt[x]*(8 - 10*x + 15*x^2) - 48*ArcCsch[Sqrt[x]] - 15*x^
3*ArcSinh[1/Sqrt[x]])/(144*x^3)
```

fricas [A] time = 0.65, size = 58, normalized size = 0.50

$$\frac{\left(15\,x^2-10\,x+8\right)\sqrt{x}\sqrt{\frac{x+1}{x}}-3\left(5\,x^3+16\right)\log\left(\frac{x\sqrt{\frac{x+1}{x}}+\sqrt{x}}{x}\right)}{144\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/144*((15*x^2 - 10*x + 8)*sqrt(x)*sqrt((x + 1)/x) - 3*(5*x^3 + 16)*log((x*
sqrt((x + 1)/x) + sqrt(x))/x))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccsch(sqrt(x))/x^4, x)
```

maple [A] time = 0.06, size = 67, normalized size = 0.58

$$-\frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{3x^3}+\frac{\sqrt{1+x}\,\left(-15\operatorname{arctanh}\left(\frac{1}{\sqrt{1+x}}\right)x^3+15x^2\sqrt{1+x}-10x\sqrt{1+x}+8\sqrt{1+x}\right)}{144\sqrt{\frac{1+x}{x}}\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsch(x^(1/2))/x^4,x)
```

```
[Out] -1/3*arccsch(x^(1/2))/x^3+1/144*(1+x)^(1/2)*(-15*arctanh(1/(1+x)^(1/2))*x^3
+15*x^2*(1+x)^(1/2)-10*x*(1+x)^(1/2)+8*(1+x)^(1/2))/((1+x)/x)^(1/2)/x^(7/2)
```

maxima [A] time = 0.32, size = 116, normalized size = 1.01

$$\frac{15\,x^{\frac{5}{2}}\left(\frac{1}{x}+1\right)^{\frac{5}{2}}-40\,x^{\frac{3}{2}}\left(\frac{1}{x}+1\right)^{\frac{3}{2}}+33\,\sqrt{x}\sqrt{\frac{1}{x}+1}}{144\left(x^3\left(\frac{1}{x}+1\right)^3-3\,x^2\left(\frac{1}{x}+1\right)^2+3\,x\left(\frac{1}{x}+1\right)-1\right)}-\frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{3\,x^3}-\frac{5}{96}\log\left(\sqrt{x}\sqrt{\frac{1}{x}+1}+1\right)+\frac{5}{96}\log\left(\sqrt{x}\sqrt{\frac{1}{x}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(x^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] 1/144*(15*x^(5/2)*(1/x + 1)^(5/2) - 40*x^(3/2)*(1/x + 1)^(3/2) + 33*sqrt(x)
*sqrt(1/x + 1))/(x^3*(1/x + 1)^3 - 3*x^2*(1/x + 1)^2 + 3*x*(1/x + 1) - 1) -
 1/3*arccsch(sqrt(x))/x^3 - 5/96*log(sqrt(x)*sqrt(1/x + 1) + 1) + 5/96*log(
sqrt(x)*sqrt(1/x + 1) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}\left(\frac{1}{\sqrt{x}}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(1/x^(1/2))/x^4,x)
```

```
[Out] int(asinh(1/x^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acsch}\left(\sqrt{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsch(x**(1/2))/x**4,x)
```

```
[Out] Integral(acsch(sqrt(x))/x**4, x)
```

3.22 $\int \operatorname{csch}^{-1}\left(\frac{1}{x}\right) dx$

Optimal. Leaf size=16

$$x \sinh^{-1}(x) - \sqrt{x^2+1}$$

```
[Out] x*arcsinh(x)-(x^2+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6328, 5653, 261}

$$x \sinh^{-1}(x) - \sqrt{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsch[x^(-1)],x]
```

```
[Out] -Sqrt[1 + x^2] + x*ArcSinh[x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 6328

```
Int[ArcCsch[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcSinh[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^{-1}\left(\frac{1}{x}\right) dx &= \int \sinh^{-1}(x)\, dx \\
&= x \sinh^{-1}(x) - \int \frac{x}{\sqrt{1+x^2}}\, dx \\
&= -\sqrt{1+x^2} + x \sinh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.12

$$x \operatorname{csch}^{-1}\left(\frac{1}{x}\right) - \sqrt{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsch[x^(-1)],x]
```

```
[Out] -Sqrt[1 + x^2] + x*ArcCsch[x^(-1)]
```

fricas [A] time = 0.73, size = 22, normalized size = 1.38

$$x \log\left(x + \sqrt{x^2+1}\right) - \sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(1/x),x, algorithm="fricas")
```

```
[Out] x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arcsch}\left(\frac{1}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(1/x),x, algorithm="giac")
```

```
[Out] integrate(arccsch(1/x), x)
```

maple [A] time = 0.05, size = 29, normalized size = 1.81

$$x \operatorname{arccsch}\left(\frac{1}{x}\right) - \frac{x^2\left(1+\frac{1}{x^2}\right)}{\sqrt{\left(1+\frac{1}{x^2}\right)x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsch(1/x),x)
```

```
[Out] x*arccsch(1/x)-1/((1+1/x^2)*x^2)^(1/2)*x^2*(1+1/x^2)
```

maxima [A] time = 0.31, size = 16, normalized size = 1.00

$$x \operatorname{arcsch}\left(\frac{1}{x}\right) - \sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(1/x),x, algorithm="maxima")
```

```
[Out] x*arccsch(1/x) - sqrt(x^2 + 1)
```

mupad [B] time = 0.07, size = 14, normalized size = 0.88

$$x \operatorname{asinh}(x) - \sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x),x)
```

```
[Out] x*asinh(x) - (x^2 + 1)^(1/2)
```

sympy [A] time = 0.12, size = 14, normalized size = 0.88

$$x \operatorname{acsch}\left(\frac{1}{x}\right) - \sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsch(1/x),x)
```

```
[Out] x*acsch(1/x) - sqrt(x**2 + 1)
```

3.23 $\int \frac{\operatorname{csch}^{-1}(ax^n)}{x} \, dx$

Optimal. Leaf size=61

$$-\frac{\operatorname{Li}_2\left(e^{2\operatorname{csch}^{-1}(ax^n)}\right)}{2n}+\frac{\operatorname{csch}^{-1}(ax^n)^2}{2n}-\frac{\operatorname{csch}^{-1}(ax^n)\log\left(1-e^{2\operatorname{csch}^{-1}(ax^n)}\right)}{n}$$

```
[Out] 1/2*arccsch(a*x^n)^2/n-arccsch(a*x^n)*ln(1-(1/a/(x^n)+(1+1/a^2/(x^n)^2)^(1/
2))^2)/n-1/2*polylog(2,(1/a/(x^n)+(1+1/a^2/(x^n)^2)^(1/2))^2)/n
```

Rubi [A] time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6282, 5659, 3716, 2190, 2279, 2391}

$$-\frac{\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(ax^n)}\right)}{2n}+\frac{\operatorname{csch}^{-1}(ax^n)^2}{2n}-\frac{\operatorname{csch}^{-1}(ax^n)\log\left(1-e^{2\operatorname{csch}^{-1}(ax^n)}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsch[a*x^n]/x,x]
```

```
[Out] ArcCsch[a*x^n]^2/(2*n) - (ArcCsch[a*x^n]*Log[1 - E^(2*ArcCsch[a*x^n])])/n -
 PolyLog[2, E^(2*ArcCsch[a*x^n])]/(2*n)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6282

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^{-1}(ax^n)}{x}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\operatorname{csch}^{-1}(ax)}{x}\,dx, x, x^n\right)}{n}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)}{x}\,dx, x, x^{-n}\right)}{n}$$
$$= -\frac{\operatorname{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n}$$
$$= \frac{\sinh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} + \frac{2\operatorname{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n}$$
$$= \frac{\sinh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\sinh^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} + \frac{\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}\right.}{n}$$
$$= \frac{\sinh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\sinh^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} + \frac{\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{2n}$$
$$= \frac{\sinh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\sinh^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} - \frac{\operatorname{Li}_2\left(e^{2\sinh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{2n}$$

Mathematica [C] time = 0.10, size = 64, normalized size = 1.05

$$\log(x)\left(\operatorname{csch}^{-1}(ax^n) - \sinh^{-1}\left(\frac{x^{-n}}{a}\right)\right) - \frac{x^{-n}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{x^{-2n}}{a^2}\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsch[a*x^n]/x,x]
```

```
[Out] -(HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, -(1/(a^2*x^(2*n)))]/(a*n*x
^n)) + (ArcCsch[a*x^n] - ArcSinh[1/(a*x^n)])*Log[x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsch}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arccsch(a*x^n)/x, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccsch}\left(a\,x^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsch(a*x^n)/x,x)
```

```
[Out] int(arccsch(a*x^n)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}n\int \frac{x^{2\,n}\log(x)}{a^{2}xx^{2\,n}+\left(a^{2}xx^{2\,n}+x\right)\sqrt{a^{2}x^{2\,n}+1}+x}\,dx+n\int \frac{\log(x)}{a^{2}xx^{2\,n}+x}\,dx-\log(a)\log(x)-\log(x)\log\left(x^{n}\right)+\log(x)\,l$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] a^2*n*integrate(x^(2*n)*log(x)/(a^2*x*x^(2*n) + (a^2*x*x^(2*n) + x)*sqrt(a^
2*x^(2*n) + 1) + x), x) + n*integrate(log(x)/(a^2*x*x^(2*n) + x), x) - log(
a)*log(x) - log(x)*log(x^n) + log(x)*log(sqrt(a^2*x^(2*n) + 1) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(\frac{1}{a\,x^{n}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(1/(a*x^n))/x,x)
```

```
[Out] int(asinh(1/(a*x^n))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acsch}\left(ax^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsch(a*x**n)/x,x)
```

```
[Out] Integral(acsch(a*x**n)/x, x)
```

3.24 $\int \frac{\operatorname{csch}^{-1}(ax^5)}{x}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{10}\operatorname{Li}_2\left(e^{2\operatorname{csch}^{-1}(ax^5)}\right)+\frac{1}{10}\operatorname{csch}^{-1}\left(ax^5\right)^2-\frac{1}{5}\operatorname{csch}^{-1}\left(ax^5\right)\log\left(1-e^{2\operatorname{csch}^{-1}(ax^5)}\right)$$

```
[Out] 1/10*arccsch(a*x^5)^2-1/5*arccsch(a*x^5)*ln(1-(1/a/x^5+(1+1/a^2/x^10)^(1/2)
)^2)-1/10*polylog(2,(1/a/x^5+(1+1/a^2/x^10)^(1/2))^2)
```

Rubi [A] time = 0.10, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6282, 5659, 3716, 2190, 2279, 2391}

$$-\frac{1}{10}\text{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(ax^5)}\right)+\frac{1}{10}\operatorname{csch}^{-1}\left(ax^5\right)^2-\frac{1}{5}\operatorname{csch}^{-1}\left(ax^5\right)\log\left(1-e^{2\operatorname{csch}^{-1}(ax^5)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsch[a*x^5]/x,x]
```

```
[Out] ArcCsch[a*x^5]^2/10 - (ArcCsch[a*x^5]*Log[1 - E^(2*ArcCsch[a*x^5])])/5 - Po
lyLog[2, E^(2*ArcCsch[a*x^5])]/10
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6282

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^{-1}\left(ax^5\right)}{x}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{\operatorname{csch}^{-1}(ax)}{x}\,dx, x, x^5\right)\\
&= -\left(\frac{1}{5}\operatorname{Subst}\left(\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)}{x}\,dx, x, \frac{1}{x^5}\right)\right)\\
&= -\left(\frac{1}{5}\operatorname{Subst}\left(\int x\coth(x)\,dx, x, \operatorname{csch}^{-1}\left(ax^5\right)\right)\right)\\
&= \frac{1}{10}\operatorname{csch}^{-1}\left(ax^5\right)^2 + \frac{2}{5}\operatorname{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \operatorname{csch}^{-1}\left(ax^5\right)\right)\\
&= \frac{1}{10}\operatorname{csch}^{-1}\left(ax^5\right)^2 - \frac{1}{5}\operatorname{csch}^{-1}\left(ax^5\right)\log\left(1-e^{2\operatorname{csch}^{-1}\left(ax^5\right)}\right) + \frac{1}{5}\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x\right.\\
&= \frac{1}{10}\operatorname{csch}^{-1}\left(ax^5\right)^2 - \frac{1}{5}\operatorname{csch}^{-1}\left(ax^5\right)\log\left(1-e^{2\operatorname{csch}^{-1}\left(ax^5\right)}\right) + \frac{1}{10}\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x,\right.\\
&= \frac{1}{10}\operatorname{csch}^{-1}\left(ax^5\right)^2 - \frac{1}{5}\operatorname{csch}^{-1}\left(ax^5\right)\log\left(1-e^{2\operatorname{csch}^{-1}\left(ax^5\right)}\right) - \frac{1}{10}\operatorname{Li}_2\left(e^{2\operatorname{csch}^{-1}\left(ax^5\right)}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.91

$$\frac{1}{10}\left(\operatorname{Li}_2\left(e^{-2\operatorname{csch}^{-1}\left(ax^5\right)}\right) - \operatorname{csch}^{-1}\left(ax^5\right)\left(\operatorname{csch}^{-1}\left(ax^5\right) + 2\log\left(1-e^{-2\operatorname{csch}^{-1}\left(ax^5\right)}\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCsch[a*x^5]/x,x]
[Out] (-(ArcCsch[a*x^5]*(ArcCsch[a*x^5] + 2*Log[1 - E^(-2*ArcCsch[a*x^5])])) + Po
lyLog[2, E^(-2*ArcCsch[a*x^5])])/10
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{arcsch}\left(ax^5\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(a*x^5)/x,x, algorithm="fricas")
[Out] integral(arccsch(a*x^5)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsch}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(a*x^5)/x,x, algorithm="giac")
[Out] integrate(arccsch(a*x^5)/x, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccsch}\left(a\,x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsch(a*x^5)/x,x)
```

```
[Out] int(arccsch(a*x^5)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$5\,a^2\int\frac{x^9\log(x)}{a^2x^{10}+\left(a^2x^{10}+1\right)^{\frac{3}{2}}+1}\,dx-\frac{1}{2}\,\log\left(a^2x^{10}+1\right)\log(x)-\log(a)\log(x)-\frac{5}{2}\,\log(x)^2+\log(x)\log\left(\sqrt{a^2x^{10}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] 5*a^2*integrate(x^9*log(x)/(a^2*x^10 + (a^2*x^10 + 1)^(3/2) + 1), x) - 1/2*
log(a^2*x^10 + 1)*log(x) - log(a)*log(x) - 5/2*log(x)^2 + log(x)*log(sqrt(a
^2*x^10 + 1) + 1) - 1/20*dilog(-a^2*x^10)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\mathrm{asinh}\left(\frac{1}{a\,x^5}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(1/(a*x^5))/x,x)
```

```
[Out] int(asinh(1/(a*x^5))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{acsch}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsch(a*x**5)/x,x)
```

```
[Out] Integral(acsch(a*x**5)/x, x)
```

3.25 $\int \operatorname{csch}^{-1}\left(ce^{a+bx}\right) dx$

Optimal. Leaf size=77

$$-\frac{\operatorname{Li}_2\left(e^{2\operatorname{csch}^{-1}\left(ce^{a+bx}\right)}\right)}{2b}+\frac{\operatorname{csch}^{-1}\left(ce^{a+bx}\right)^2}{2b}-\frac{\operatorname{csch}^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\operatorname{csch}^{-1}\left(ce^{a+bx}\right)}\right)}{b}$$

```
[Out] 1/2*arccsch(c*exp(b*x+a))^2/b-arccsch(c*exp(b*x+a))*ln(1-(1/c/exp(b*x+a)+(1
+1/c^2/exp(b*x+a)^2)^(1/2))^2)/b-1/2*polylog(2,(1/c/exp(b*x+a)+(1+1/c^2/exp
(b*x+a)^2)^(1/2))^2)/b
```

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2282, 6282, 5659, 3716, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}\left(ce^{a+bx}\right)}\right)}{2b}+\frac{\operatorname{csch}^{-1}\left(ce^{a+bx}\right)^2}{2b}-\frac{\operatorname{csch}^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\operatorname{csch}^{-1}\left(ce^{a+bx}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsch[c*E^(a + b*x)],x]
```

```
[Out] ArcCsch[c*E^(a + b*x)]^2/(2*b) - (ArcCsch[c*E^(a + b*x)]*Log[1 - E^(2*ArcCs
ch[c*E^(a + b*x)])])/b - PolyLog[2, E^(2*ArcCsch[c*E^(a + b*x)])]/(2*b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6282

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^{-1}\left(ce^{a+bx}\right)dx &= \frac{\text{Subst}\left(\int \frac{\operatorname{csch}^{-1}(cx)}{x}\,dx, x, e^{a+bx}\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \frac{\sinh^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, e^{-a-bx}\right)}{b} \\
&= -\frac{\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\right)}{b} \\
&= \frac{\sinh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\right)}{b} \\
&= \frac{\sinh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\sinh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{\text{Subst}\left(\int \log\left(1-e^{2x}\right)dx, x\right.}{b} \\
&= \frac{\sinh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\sinh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sin}\right.}{2b} \\
&= \frac{\sinh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\sinh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} - \frac{\text{Li}_2\left(e^{2\sinh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{2b}
\end{aligned}$$

Mathematica [B] time = 0.60, size = 236, normalized size = 3.06

$$\frac{e^{-a-bx}\sqrt{c^2e^{2(a+bx)}+1}\left(-4\text{Li}_2\left(\frac{1}{2}\left(1-\sqrt{e^{2(a+bx)}c^2+1}\right)\right)+\log^2\left(-c^2e^{2(a+bx)}\right)+2\log^2\left(\frac{1}{2}\left(\sqrt{c^2e^{2(a+bx)}+1}+1\right)\right)-4\right.}{8bc\sqrt{\frac{e^{-2(a+bx)}}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsch[c*E^(a + b*x)],x]
```

```
[Out] x*ArcCsch[c*E^(a + b*x)] + (E^(-a - b*x)*Sqrt[1 + c^2*E^(2*(a + b*x))]*(Log
[-(c^2*E^(2*(a + b*x)))]^2 + ArcTanh[Sqrt[1 + c^2*E^(2*(a + b*x))]]*(-8*b*x
 + 4*Log[-(c^2*E^(2*(a + b*x)))]) - 4*Log[-(c^2*E^(2*(a + b*x)))]*Log[(1 +
Sqrt[1 + c^2*E^(2*(a + b*x))])/2] + 2*Log[(1 + Sqrt[1 + c^2*E^(2*(a + b*x))
])/2]^2 - 4*PolyLog[2, (1 - Sqrt[1 + c^2*E^(2*(a + b*x))])/2]))/(8*b*c*Sqrt
[1 + 1/(c^2*E^(2*(a + b*x)))])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(c*exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arcsch}\left(ce^{(bx+a)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(c*exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccsch(c*e^(b*x + a)), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \operatorname{arccsch}\left(c\,\mathrm{e}^{bx+a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsch(c*exp(b*x+a)),x)
```

```
[Out] int(arccsch(c*exp(b*x+a)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$bc^2 \int \frac{xe^{(2\,bx+2\,a)}}{c^2e^{(2\,bx+2\,a)} + \left(c^2e^{(2\,bx+2\,a)} + 1\right)^{\frac{3}{2}} + 1}\,dx - \frac{1}{2}\,bx^2 - \left(a + \log(c)\right)x + x\log\left(\sqrt{c^2e^{(2\,bx+2\,a)} + 1} + 1\right) - \frac{2\,bx\log\left(c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(c*exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] b*c^2*integrate(x*e^(2*b*x + 2*a)/(c^2*e^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2*
a) + 1)^(3/2) + 1), x) - 1/2*b*x^2 - (a + log(c))*x + x*log(sqrt(c^2*e^(2*b
*x + 2*a) + 1) + 1) - 1/4*(2*b*x*log(c^2*e^(2*b*x + 2*a) + 1) + dilog(-c^2*
e^(2*b*x + 2*a)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}\left(\frac{\mathrm{e}^{-a-b\,x}}{c}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(exp(- a - b*x)/c),x)
```

```
[Out] int(asinh(exp(- a - b*x)/c), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acsch}\left(ce^{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsch(c*exp(b*x+a)),x)
```

```
[Out] Integral(acsch(c*exp(a + b*x)), x)
```

3.26 $\int e^{\operatorname{csch}^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=52

$$\frac{x^{m+1}\, {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}(-m-1); \frac{1-m}{2}; -\frac{1}{a^2x^2}\right)}{m+1} + \frac{x^m}{am}$$

```
[Out] x^m/a/m+x^(1+m)*hypergeom([-1/2, -1/2-1/2*m],[1/2-1/2*m],-1/a^2/x^2)/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6336, 30, 339, 364}

$$\frac{x^{m+1}\, {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}(-m-1); \frac{1-m}{2}; -\frac{1}{a^2x^2}\right)}{m+1} + \frac{x^m}{am}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x]*x^m,x]
```

```
[Out] x^m/(a*m) + (x^(1 + m)*Hypergeometric2F1[-1/2, (-1 - m)/2, (1 - m)/2, -(1/(
a^2*x^2))])/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\begin{aligned}
\int e^{\operatorname{csch}^{-1}(ax)} x^m \, dx &= \frac{\int x^{-1+m} \, dx}{a} + \int \sqrt{1+\frac{1}{a^2x^2}}\, x^m \, dx \\
&= \frac{x^m}{am} - \left(\left(\frac{1}{x}\right)^m x^m\right) \operatorname{Subst}\left(\int x^{-2-m}\sqrt{1+\frac{x^2}{a^2}} \, dx, x, \frac{1}{x}\right) \\
&= \frac{x^m}{am} + \frac{x^{1+m}\, {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}(-1-m); \frac{1-m}{2}; -\frac{1}{a^2x^2}\right)}{1+m}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 1.04

$$\frac{x^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-m-1);\frac{1}{2}(-m-1)+1;-\frac{1}{a^2x^2}\right)}{m+1}+\frac{x^m}{am}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCsch[a*x]*x^m,x]
```

```
[Out] x^m/(a*m) + (x^(1 + m)*Hypergeometric2F1[-1/2, (-1 - m)/2, 1 + (-1 - m)/2,
-(1/(a^2*x^2))])/(1 + m)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{axx^m\sqrt{\frac{a^2x^2+1}{a^2x^2}}+x^m}{ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x*x^m*sqrt((a^2*x^2 + 1)/(a^2*x^2)) + x^m)/(a*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\left(\frac{1}{ax}+\sqrt{1+\frac{1}{x^2a^2}}\right)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))*x^m,x)
```

```
[Out] int((1/a/x+(1+1/x^2/a^2)^(1/2))*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{x^m\Gamma\left(\frac{1}{2}m\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{2}m\\ \frac{1}{2}m+1\end{matrix};-a^2x^2\right)}{2\Gamma\left(\frac{1}{2}m+1\right)}}{a}+\frac{x^m}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + 1)*x^m/x, x)/a + x^m/(a*m)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \left(\sqrt{\frac{1}{a^2\,x^2}+1}+\frac{1}{a\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] int(x^m*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x)), x)
```

sympy [A] time = 5.99, size = 51, normalized size = 0.98

$$-\frac{x^m\Gamma\left(-\frac{m}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\middle| a^2x^2e^{i\pi}\right)}{2a\Gamma\left(1-\frac{m}{2}\right)}+\frac{\begin{cases}\frac{x^m}{m} & \text{for}\ m\neq 0\\ \log\left(x\right) & \text{otherwise}\end{cases}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))*x**m,x)
```

```
[Out] -x**m*gamma(-m/2)*hyper((-1/2, m/2), (m/2 + 1,), a**2*x**2*exp_polar(I*pi))
/(2*a*gamma(1 - m/2)) + Piecewise((x**m/m, Ne(m, 0)), (log(x), True))/a
```

3.27 $\int e^{\operatorname{csch}^{-1}(ax)} x^4\, dx$

Optimal. Leaf size=54

$$\frac{1}{5}x^5\left(\frac{1}{a^2x^2}+1\right)^{3/2}-\frac{2x^3\left(\frac{1}{a^2x^2}+1\right)^{3/2}}{15a^2}+\frac{x^4}{4a}$$

```
[Out] -2/15*(1+1/a^2/x^2)^(3/2)*x^3/a^2+1/4*x^4/a+1/5*(1+1/a^2/x^2)^(3/2)*x^5
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = $\{6336, 30, 271, 264\}$

$$\frac{1}{5}x^5\left(\frac{1}{a^2x^2}+1\right)^{3/2}-\frac{2x^3\left(\frac{1}{a^2x^2}+1\right)^{3/2}}{15a^2}+\frac{x^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x]*x^4,x]
```

```
[Out] (-2*(1 + 1/(a^2*x^2))^(3/2)*x^3)/(15*a^2) + x^4/(4*a) + ((1 + 1/(a^2*x^2))^
(3/2)*x^5)/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\int e^{\operatorname{csch}^{-1}(ax)}x^4\,dx = \frac{\int x^3\,dx}{a} + \int \sqrt{1+\frac{1}{a^2x^2}}\,x^4\,dx$$
$$= \frac{x^4}{4a} + \frac{1}{5}\left(1+\frac{1}{a^2x^2}\right)^{3/2}x^5 - \frac{2\int \sqrt{1+\frac{1}{a^2x^2}}\,x^2\,dx}{5a^2}$$
$$= -\frac{2\left(1+\frac{1}{a^2x^2}\right)^{3/2}x^3}{15a^2} + \frac{x^4}{4a} + \frac{1}{5}\left(1+\frac{1}{a^2x^2}\right)^{3/2}x^5$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.91

$$\frac{x\sqrt{\frac{1}{a^2x^2}+1}\left(3a^4x^4+a^2x^2-2\right)}{15a^4} + \frac{x^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCsch[a*x]*x^4,x]
```

```
[Out] x^4/(4*a) + (Sqrt[1 + 1/(a^2*x^2)]*x*(-2 + a^2*x^2 + 3*a^4*x^4))/(15*a^4)
```

fricas [A] time = 0.40, size = 53, normalized size = 0.98

$$\frac{15\,a^3x^4 + 4\left(3\,a^4x^5 + a^2x^3 - 2\,x\right)\sqrt{\frac{a^2x^2+1}{a^2x^2}}}{60\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^4,x, algorithm="fricas")
```

```
[Out] 1/60*(15*a^3*x^4 + 4*(3*a^4*x^5 + a^2*x^3 - 2*x)*sqrt((a^2*x^2 + 1)/(a^2*x^
2)))/a^4
```

giac [A] time = 0.14, size = 78, normalized size = 1.44

$$-\frac{a^2x^2+1}{2\,a^5} + \frac{2\,|a|\mathrm{sgn}(x)}{15\,a^6} + \frac{12\left(a^2x^2+1\right)^{\frac{5}{2}}|a|\mathrm{sgn}(x) - 20\left(a^2x^2+1\right)^{\frac{3}{2}}|a|\mathrm{sgn}(x) + 15\left(a^2x^2+1\right)^2a}{60\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^4,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*x^2 + 1)/a^5 + 2/15*abs(a)*sgn(x)/a^6 + 1/60*(12*(a^2*x^2 + 1)^(5
/2)*abs(a)*sgn(x) - 20*(a^2*x^2 + 1)^(3/2)*abs(a)*sgn(x) + 15*(a^2*x^2 + 1)
^2*a)/a^6
```

maple [A] time = 0.09, size = 53, normalized size = 0.98

$$\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,x\left(a^2x^2+1\right)\left(3a^2x^2-2\right)}{15a^4} + \frac{x^4}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))*x^4,x)
```

```
[Out] 1/15*((a^2*x^2+1)/a^2/x^2)^(1/2)*x/a^4*(a^2*x^2+1)*(3*a^2*x^2-2)+1/4*x^4/a
```

maxima [A] time = 0.33, size = 50, normalized size = 0.93

$$\frac{x^4}{4\,a}+\frac{3\,a^2x^5\left(\frac{1}{a^2x^2}+1\right)^{\frac{5}{2}}-5\,x^3\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^4,x, algorithm="maxima")
```

```
[Out] 1/4*x^4/a + 1/15*(3*a^2*x^5*(1/(a^2*x^2) + 1)^(5/2) - 5*x^3*(1/(a^2*x^2) +
1)^(3/2))/a^2
```

mupad [B] time = 2.18, size = 41, normalized size = 0.76

$$\sqrt{\frac{1}{a^2\,x^2}+1}\,\left(\frac{x^5}{5}-\frac{2\,x}{15\,a^4}+\frac{x^3}{15\,a^2}\right)+\frac{x^4}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] (1/(a^2*x^2) + 1)^(1/2)*(x^5/5 - (2*x)/(15*a^4) + x^3/(15*a^2)) + x^4/(4*a)
```

sympy [A] time = 3.01, size = 63, normalized size = 1.17

$$\frac{x^4\sqrt{a^2x^2+1}}{5a}+\frac{x^4}{4a}+\frac{x^2\sqrt{a^2x^2+1}}{15a^3}-\frac{2\sqrt{a^2x^2+1}}{15a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))*x**4,x)
```

```
[Out] x**4*sqrt(a**2*x**2 + 1)/(5*a) + x**4/(4*a) + x**2*sqrt(a**2*x**2 + 1)/(15*
a**3) - 2*sqrt(a**2*x**2 + 1)/(15*a**5)
```

3.28 $\int e^{\text{csch}^{-1}(ax)} x^3\,dx$

Optimal. Leaf size=75

$$\frac{x^2\sqrt{\frac{1}{a^2x^2}+1}}{8a^2}+\frac{1}{4}x^4\sqrt{\frac{1}{a^2x^2}+1}-\frac{\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{8a^4}+\frac{x^3}{3a}$$

```
[Out] 1/3*x^3/a-1/8*arctanh((1+1/a^2/x^2)^(1/2))/a^4+1/8*x^2*(1+1/a^2/x^2)^(1/2)/
a^2+1/4*x^4*(1+1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6336, 30, 266, 47, 51, 63, 208}

$$\frac{1}{4}x^4\sqrt{\frac{1}{a^2x^2}+1}+\frac{x^2\sqrt{\frac{1}{a^2x^2}+1}}{8a^2}-\frac{\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{8a^4}+\frac{x^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x]*x^3,x]
```

```
[Out] (Sqrt[1 + 1/(a^2*x^2)]*x^2)/(8*a^2) + x^3/(3*a) + (Sqrt[1 + 1/(a^2*x^2)]*x^
4)/4 - ArcTanh[Sqrt[1 + 1/(a^2*x^2)]]/(8*a^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$
\begin{aligned}
\int e^{\operatorname{csch}^{-1}(ax)}x^3\,dx &= \frac{\int x^2\,dx}{a}+\int\sqrt{1+\frac{1}{a^2x^2}}\,x^3\,dx\\
&= \frac{x^3}{3a}-\frac{1}{2}\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a^2}}}{x^3}\,dx,x,\frac{1}{x^2}\right)\\
&= \frac{x^3}{3a}+\frac{1}{4}\sqrt{1+\frac{1}{a^2x^2}}\,x^4-\frac{\operatorname{Subst}\left(\int\frac{1}{x^2\sqrt{1+\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{8a^2}\\
&= \frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^2}{8a^2}+\frac{x^3}{3a}+\frac{1}{4}\sqrt{1+\frac{1}{a^2x^2}}\,x^4+\frac{\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{16a^4}\\
&= \frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^2}{8a^2}+\frac{x^3}{3a}+\frac{1}{4}\sqrt{1+\frac{1}{a^2x^2}}\,x^4+\frac{\operatorname{Subst}\left(\int\frac{1}{-a^2+a^2x^2}\,dx,x,\sqrt{1+\frac{1}{a^2x^2}}\right)}{8a^2}\\
&= \frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^2}{8a^2}+\frac{x^3}{3a}+\frac{1}{4}\sqrt{1+\frac{1}{a^2x^2}}\,x^4-\frac{\tanh^{-1}\left(\sqrt{1+\frac{1}{a^2x^2}}\right)}{8a^4}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 76, normalized size = 1.01

$$\frac{a^2x^2\left(6a^2x^2\sqrt{\frac{1}{a^2x^2}+1}+3\sqrt{\frac{1}{a^2x^2}+1}+8ax\right)-3\log\left(x\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)\right)}{24a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x]*x^3,x]
```

```
[Out] (a^2*x^2*(3*Sqrt[1 + 1/(a^2*x^2)] + 8*a*x + 6*a^2*Sqrt[1 + 1/(a^2*x^2)]*x^2
) - 3*Log[(1 + Sqrt[1 + 1/(a^2*x^2)])*x])/(24*a^4)
```

fricas [A] time = 0.40, size = 79, normalized size = 1.05

$$\frac{8\,a^3x^3+3\left(2\,a^4x^4+a^2x^2\right)\sqrt{\frac{a^2x^2+1}{a^2x^2}}+3\,\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(8*a^3*x^3 + 3*(2*a^4*x^4 + a^2*x^2)*sqrt((a^2*x^2 + 1)/(a^2*x^2)) + 3
*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x))/a^4
```

giac [A] time = 0.15, size = 69, normalized size = 0.92

$$\frac{1}{8}\sqrt{a^2x^2+1}\left(\frac{2x^2|a|\mathrm{sgn}(x)}{a^2}+\frac{|a|\mathrm{sgn}(x)}{a^4}\right)x+\frac{x^3}{3a}+\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)\mathrm{sgn}(x)}{8a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^3,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a^2*x^2 + 1)*(2*x^2*abs(a)*sgn(x)/a^2 + abs(a)*sgn(x)/a^4)*x + 1/3
*x^3/a + 1/8*log(-x*abs(a) + sqrt(a^2*x^2 + 1))*sgn(x)/a^4
```

maple [A] time = 0.05, size = 109, normalized size = 1.45

$$-\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,x\left(-2x\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}a^4+x\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+\ln\left(x+\sqrt{\frac{a^2x^2+1}{a^2}}\right)\right)}{8\sqrt{\frac{a^2x^2+1}{a^2}}\,a^4}+\frac{x^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))*x^3,x)
```

```
[Out] -1/8*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(-2*x*((a^2*x^2+1)/a^2)^(3/2)*a^4+x*((a^
2*x^2+1)/a^2)^(1/2)*a^2+ln(x+((a^2*x^2+1)/a^2)^(1/2)))/((a^2*x^2+1)/a^2)^(1
/2)/a^4+1/3*x^3/a
```

maxima [A] time = 0.32, size = 107, normalized size = 1.43

$$\frac{x^3}{3a}+\frac{\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}+\sqrt{\frac{1}{a^2x^2}+1}}{8\left(a^4\left(\frac{1}{a^2x^2}+1\right)^2-2a^4\left(\frac{1}{a^2x^2}+1\right)+a^4\right)}-\frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)}{16a^4}+\frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^3,x, algorithm="maxima")
```

```
[Out] 1/3*x^3/a + 1/8*((1/(a^2*x^2) + 1)^(3/2) + sqrt(1/(a^2*x^2) + 1))/(a^4*(1/(
a^2*x^2) + 1)^2 - 2*a^4*(1/(a^2*x^2) + 1) + a^4) - 1/16*log(sqrt(1/(a^2*x^2
) + 1) + 1)/a^4 + 1/16*log(sqrt(1/(a^2*x^2) + 1) - 1)/a^4
```

mupad [B] time = 2.41, size = 61, normalized size = 0.81

$$\frac{x^4\sqrt{\frac{1}{a^2x^2}+1}}{4}-\frac{\mathrm{atanh}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{8a^4}+\frac{x^3}{3a}+\frac{x^2\sqrt{\frac{1}{a^2x^2}+1}}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] (x^4*(1/(a^2*x^2) + 1)^(1/2))/4 - atanh((1/(a^2*x^2) + 1)^(1/2))/(8*a^4) +
x^3/(3*a) + (x^2*(1/(a^2*x^2) + 1)^(1/2))/(8*a^2)
```

sympy [A] time = 4.60, size = 73, normalized size = 0.97

$$\frac{ax^5}{4\sqrt{a^2x^2+1}}+\frac{x^3}{3a}+\frac{3x^3}{8a\sqrt{a^2x^2+1}}+\frac{x}{8a^3\sqrt{a^2x^2+1}}-\frac{\mathrm{asinh}\,(ax)}{8a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))*x**3,x)
```

```
[Out] a*x**5/(4*sqrt(a**2*x**2 + 1)) + x**3/(3*a) + 3*x**3/(8*a*sqrt(a**2*x**2 +
1)) + x/(8*a**3*sqrt(a**2*x**2 + 1)) - asinh(a*x)/(8*a**4)
```

3.29 $\int e^{\text{csch}^{-1}(ax)} x^2\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}x^3\left(\frac{1}{a^2x^2}+1\right)^{3/2}+\frac{x^2}{2a}$$

```
[Out] 1/2*x^2/a+1/3*(1+1/a^2/x^2)^(3/2)*x^3
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6336, 30, 264}

$$\frac{1}{3}x^3\left(\frac{1}{a^2x^2}+1\right)^{3/2}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x]*x^2,x]
```

```
[Out] x^2/(2*a) + ((1 + 1/(a^2*x^2))^(3/2)*x^3)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\begin{aligned}\int e^{\text{csch}^{-1}(ax)}x^2\,dx &= \frac{\int x\,dx}{a}+\int \sqrt{1+\frac{1}{a^2x^2}}\,x^2\,dx\\ &= \frac{x^2}{2a}+\frac{1}{3}\left(1+\frac{1}{a^2x^2}\right)^{3/2}x^3\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 1.23

$$\frac{2\sqrt{\frac{1}{a^2x^2}+1}\left(a^2x^3+x\right)+3ax^2}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCsch[a*x]*x^2,x]
```

```
[Out] (3*a*x^2 + 2*Sqrt[1 + 1/(a^2*x^2)]*(x + a^2*x^3))/(6*a^2)
```

fricas [A] time = 0.56, size = 41, normalized size = 1.32

$$\frac{3\,ax^2+2\left(a^2x^3+x\right)\sqrt{\frac{a^2x^2+1}{a^2x^2}}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^2,x, algorithm="fricas")
[Out] 1/6*(3*a*x^2 + 2*(a^2*x^3 + x)*sqrt((a^2*x^2 + 1)/(a^2*x^2)))/a^2
```

giac [A] time = 0.14, size = 44, normalized size = 1.42

$$\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}|a|\mathrm{sgn}(x)}{3\,a^4}+\frac{a^2x^2+1}{2\,a^3}-\frac{|a|\mathrm{sgn}(x)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^2,x, algorithm="giac")
[Out] 1/3*(a^2*x^2 + 1)^(3/2)*abs(a)*sgn(x)/a^4 + 1/2*(a^2*x^2 + 1)/a^3 - 1/3*abs
(a)*sgn(x)/a^4
```

maple [A] time = 0.04, size = 43, normalized size = 1.39

$$\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,x\left(a^2x^2+1\right)}{3a^2}+\frac{x^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))*x^2,x)
[Out] 1/3*((a^2*x^2+1)/a^2/x^2)^(1/2)*x/a^2*(a^2*x^2+1)+1/2*x^2/a
```

maxima [A] time = 0.31, size = 25, normalized size = 0.81

$$\frac{1}{3}\,x^3\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}+\frac{x^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^2,x, algorithm="maxima")
[Out] 1/3*x^3*(1/(a^2*x^2) + 1)^(3/2) + 1/2*x^2/a
```

mupad [B] time = 2.17, size = 33, normalized size = 1.06

$$\left(\frac{x}{3\,a^2}+\frac{x^3}{3}\right)\sqrt{\frac{1}{a^2\,x^2}+1}+\frac{x^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x)),x)
[Out] (x/(3*a^2) + x^3/3)*(1/(a^2*x^2) + 1)^(1/2) + x^2/(2*a)
```

sympy [A] time = 2.71, size = 41, normalized size = 1.32

$$\frac{x^2\sqrt{a^2x^2+1}}{3a}+\frac{x^2}{2a}+\frac{\sqrt{a^2x^2+1}}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))*x**2,x)
[Out] x**2*sqrt(a**2*x**2 + 1)/(3*a) + x**2/(2*a) + sqrt(a**2*x**2 + 1)/(3*a**3)
```

3.30 $\int e^{\text{csch}^{-1}(ax)} x\,dx$

Optimal. Leaf size=47

$$\frac{1}{2}x^2\sqrt{\frac{1}{a^2x^2}+1}+\frac{\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{2a^2}+\frac{x}{a}$$

```
[Out] x/a+1/2*arctanh((1+1/a^2/x^2)^(1/2))/a^2+1/2*x^2*(1+1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6336, 8, 266, 47, 63, 208}

$$\frac{1}{2}x^2\sqrt{\frac{1}{a^2x^2}+1}+\frac{\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{2a^2}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x]*x,x]
```

```
[Out] x/a + (Sqrt[1 + 1/(a^2*x^2)]*x^2)/2 + ArcTanh[Sqrt[1 + 1/(a^2*x^2)]]/(2*a^2
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
```

]

Rubi steps

$$
\begin{aligned}
\int e^{\operatorname{csch}^{-1}(ax)}x\,dx &= \frac{\int 1\,dx}{a}+\int\sqrt{1+\frac{1}{a^2x^2}}\,x\,dx\\
&= \frac{x}{a}-\frac{1}{2}\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a^2}}}{x^2}\,dx,x,\frac{1}{x^2}\right)\\
&= \frac{x}{a}+\frac{1}{2}\sqrt{1+\frac{1}{a^2x^2}}\,x^2-\frac{\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{4a^2}\\
&= \frac{x}{a}+\frac{1}{2}\sqrt{1+\frac{1}{a^2x^2}}\,x^2-\frac{1}{2}\operatorname{Subst}\left(\int\frac{1}{-a^2+a^2x^2}\,dx,x,\sqrt{1+\frac{1}{a^2x^2}}\right)\\
&= \frac{x}{a}+\frac{1}{2}\sqrt{1+\frac{1}{a^2x^2}}\,x^2+\frac{\tanh^{-1}\left(\sqrt{1+\frac{1}{a^2x^2}}\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.00

$$
\frac{ax\left(ax\sqrt{\frac{1}{a^2x^2}+1}+2\right)+\log\left(x\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)\right)}{2a^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x]*x,x]
```

```
[Out] (a*x*(2 + a*Sqrt[1 + 1/(a^2*x^2)]*x) + Log[(1 + Sqrt[1 + 1/(a^2*x^2)])*x])/
(2*a^2)
```

fricas [A] time = 0.77, size = 64, normalized size = 1.36

$$
\frac{a^2x^2\sqrt{\frac{a^2x^2+1}{a^2x^2}}+2\,ax-\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax\right)}{2\,a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2*sqrt((a^2*x^2 + 1)/(a^2*x^2)) + 2*a*x - log(a*x*sqrt((a^2*x^2
+ 1)/(a^2*x^2)) - a*x))/a^2
```

giac [A] time = 0.15, size = 52, normalized size = 1.11

$$
\frac{\sqrt{a^2x^2+1}\,x|a|\operatorname{sgn}(x)}{2\,a^2}+\frac{x}{a}-\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)\operatorname{sgn}(x)}{2\,a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a^2*x^2 + 1)*x*abs(a)*sgn(x)/a^2 + x/a - 1/2*log(-x*abs(a) + sqrt(
a^2*x^2 + 1))*sgn(x)/a^2
```

maple [**B**] time = 0.05, size = 85, normalized size = 1.81

$$\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\, x\left(x\sqrt{\frac{a^2x^2+1}{a^2}}\, a^2+\ln\left(x+\sqrt{\frac{a^2x^2+1}{a^2}}\right)\right)}{2\sqrt{\frac{a^2x^2+1}{a^2}}\, a^2}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))*x,x)
```

```
[Out] 1/2*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(x*((a^2*x^2+1)/a^2)^(1/2)*a^2+ln(x+((a^2
*x^2+1)/a^2)^(1/2)))/((a^2*x^2+1)/a^2)^(1/2)/a^2+x/a
```

maxima [**A**] time = 0.31, size = 78, normalized size = 1.66

$$\frac{x}{a}+\frac{\sqrt{\frac{1}{a^2x^2}+1}}{2\left(a^2\left(\frac{1}{a^2x^2}+1\right)-a^2\right)}+\frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)}{4\,a^2}-\frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x,x, algorithm="maxima")
```

```
[Out] x/a + 1/2*sqrt(1/(a^2*x^2) + 1)/(a^2*(1/(a^2*x^2) + 1) - a^2) + 1/4*log(sqr
t(1/(a^2*x^2) + 1) + 1)/a^2 - 1/4*log(sqrt(1/(a^2*x^2) + 1) - 1)/a^2
```

mupad [**B**] time = 2.21, size = 39, normalized size = 0.83

$$\frac{\operatorname{atanh}\left(\sqrt{\frac{1}{a^2\,x^2}+1}\right)}{2\,a^2}+\frac{x}{a}+\frac{x^2\,\sqrt{\frac{1}{a^2\,x^2}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] atanh((1/(a^2*x^2) + 1)^(1/2))/(2*a^2) + x/a + (x^2*(1/(a^2*x^2) + 1)^(1/2)
)/2
```

sympy [**A**] time = 3.25, size = 29, normalized size = 0.62

$$\frac{x\sqrt{a^2x^2+1}}{2a}+\frac{x}{a}+\frac{\operatorname{asinh}(ax)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))*x,x)
```

```
[Out] x*sqrt(a**2*x**2 + 1)/(2*a) + x/a + asinh(a*x)/(2*a**2)
```

3.31 $\int e^{\text{csch}^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\frac{\log(x)}{a}+xe^{\text{csch}^{-1}(ax)}-\frac{\text{csch}^{-1}(ax)}{a}$$

```
[Out] (1/a/x+(1+1/a^2/x^2)^(1/2))*x-arccsch(a*x)/a+ln(x)/a
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6331, 29, 242, 277, 215}

$$x\sqrt{\frac{1}{a^2x^2}+1}+\frac{\log(x)}{a}-\frac{\text{csch}^{-1}(ax)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcCsch[a*x],x]
```

```
[Out] Sqrt[1 + 1/(a^2*x^2)]*x - ArcCsch[a*x]/a + Log[x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6331

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)], x_Symbol] :> Dist[1/a, Int[1/x^p, x], x] +
 Int[Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, p}, x]
```

Rubi steps

$$\begin{aligned}\int e^{\operatorname{csch}^{-1}(ax)}\,dx &= \frac{\int \frac{1}{x}\,dx}{a} + \int \sqrt{1+\frac{1}{a^2x^2}}\,dx\\ &= \frac{\log(x)}{a} - \operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{1+\frac{1}{a^2x^2}}\,x + \frac{\log(x)}{a} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2}\\ &= \sqrt{1+\frac{1}{a^2x^2}}\,x - \frac{\operatorname{csch}^{-1}(ax)}{a} + \frac{\log(x)}{a}\end{aligned}$$

Mathematica [**A**] time = 0.02, size = 35, normalized size = 1.46

$$\frac{ax\sqrt{\frac{1}{a^2x^2}+1} + \log(ax) - \sinh^{-1}\left(\frac{1}{ax}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x],x]
[Out] (a*Sqrt[1 + 1/(a^2*x^2)]*x - ArcSinh[1/(a*x)] + Log[a*x])/a
```

fricas [**B**] time = 0.45, size = 86, normalized size = 3.58

$$\frac{ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - \log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - ax + 1\right) + \log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - ax - 1\right) + \log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x+(1+1/a^2/x^2)^(1/2),x, algorithm="fricas")
[Out] (a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2))
 - a*x + 1) + log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x - 1) + log(x))/a
```

giac [**A**] time = 0.13, size = 66, normalized size = 2.75

$$-\frac{\left(\log\left(\sqrt{a^2x^2+1}+1\right)\operatorname{sgn}(x) - \log\left(\sqrt{a^2x^2+1}-1\right)\operatorname{sgn}(x) - 2\sqrt{a^2x^2+1}\operatorname{sgn}(x)\right)|a|}{2a^2} + \frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x+(1+1/a^2/x^2)^(1/2),x, algorithm="giac")
[Out] -1/2*(log(sqrt(a^2*x^2 + 1) + 1)*sgn(x) - log(sqrt(a^2*x^2 + 1) - 1)*sgn(x)
 - 2*sqrt(a^2*x^2 + 1)*sgn(x))*abs(a)/a^2 + log(abs(x))/a
```

maple [**B**] time = 0.05, size = 113, normalized size = 4.71

$$-\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,x\left(-\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2 + \ln\left(\frac{2\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+2}{a^2x}\right)\right)}{\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2} + \frac{\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a/x+(1+1/x^2/a^2)^(1/2),x)
```

```
[Out] -((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(-(1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*a^2+
ln(2*((1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*a^2+1)/a^2/x))/(1/a^2)^(1/2)/((
a^2*x^2+1)/a^2)^(1/2)/a^2+ln(x)/a
```

maxima [A] time = 0.31, size = 64, normalized size = 2.67

$$x\sqrt{\frac{1}{a^2x^2}+1}-\frac{\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}+1\right)}{2\,a}+\frac{\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{2\,a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x+(1+1/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x*sqrt(1/(a^2*x^2) + 1) - 1/2*log(a*x*sqrt(1/(a^2*x^2) + 1) + 1)/a + 1/2*lo
g(a*x*sqrt(1/(a^2*x^2) + 1) - 1)/a + log(x)/a
```

mupad [B] time = 2.25, size = 36, normalized size = 1.50

$$\frac{\ln(x)}{a}+x\sqrt{\frac{1}{a^2\,x^2}+1}+\frac{\mathrm{asin}\left(\frac{1\mathrm{i}}{a\,x}\right)\,1\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x),x)
```

```
[Out] log(x)/a + (asin(1i/(a*x))*1i)/a + x*(1/(a^2*x^2) + 1)^(1/2)
```

sympy [A] time = 1.14, size = 48, normalized size = 2.00

$$\frac{x}{\sqrt{1+\frac{1}{a^2x^2}}}+\frac{\log\left(x\right)}{a}-\frac{\mathrm{asinh}\left(\frac{1}{ax}\right)}{a}+\frac{1}{a^2x\sqrt{1+\frac{1}{a^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x+(1+1/a**2/x**2)**(1/2),x)
```

```
[Out] x/sqrt(1 + 1/(a**2*x**2)) + log(x)/a - asinh(1/(a*x))/a + 1/(a**2*x*sqrt(1
+ 1/(a**2*x**2)))
```

3.32 $\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=38

$$-\sqrt{\frac{1}{a^2x^2}+1}+\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)-\frac{1}{ax}$$

```
[Out] -1/a/x+arctanh((1+1/a^2/x^2)^(1/2))-(1+1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6336, 30, 266, 50, 63, 208}

$$-\sqrt{\frac{1}{a^2x^2}+1}+\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x]/x,x]
```

```
[Out] -Sqrt[1 + 1/(a^2*x^2)] - 1/(a*x) + ArcTanh[Sqrt[1 + 1/(a^2*x^2)]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
```

]

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x}\,dx &= \frac{\int \frac{1}{x^2}\,dx}{a} + \int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x}\,dx\\
&= -\frac{1}{ax} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a^2}}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= -\sqrt{1+\frac{1}{a^2x^2}} - \frac{1}{ax} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)\\
&= -\sqrt{1+\frac{1}{a^2x^2}} - \frac{1}{ax} - a^2\operatorname{Subst}\left(\int \frac{1}{-a^2+a^2x^2}\,dx, x, \sqrt{1+\frac{1}{a^2x^2}}\right)\\
&= -\sqrt{1+\frac{1}{a^2x^2}} - \frac{1}{ax} + \tanh^{-1}\left(\sqrt{1+\frac{1}{a^2x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 1.11

$$-\sqrt{\frac{1}{a^2x^2}+1} + \log\left(x\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)\right) - \frac{1}{ax}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x]/x,x]
[Out] -Sqrt[1 + 1/(a^2*x^2)] - 1/(a*x) + Log[(1 + Sqrt[1 + 1/(a^2*x^2)])*x]
```

fricas [A] time = 0.59, size = 64, normalized size = 1.68

$$-\frac{ax\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - ax\right) + ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} + ax + 1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x,x, algorithm="fricas")
[Out] -(a*x*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x) + a*x*sqrt((a^2*x^2 + 1)
/(a^2*x^2)) + a*x + 1)/(a*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.04, size = 107, normalized size = 2.82

$$\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\left(-a^2\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}+\sqrt{\frac{a^2x^2+1}{a^2}}\,x^2a^2+\ln\left(x+\sqrt{\frac{a^2x^2+1}{a^2}}\right)x\right)}{\sqrt{\frac{a^2x^2+1}{a^2}}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))/x,x)
```

```
[Out] ((a^2*x^2+1)/a^2/x^2)^(1/2)*(-a^2*((a^2*x^2+1)/a^2)^(3/2)+((a^2*x^2+1)/a^2)
^(1/2)*x^2*a^2+ln(x+((a^2*x^2+1)/a^2)^(1/2))*x)/((a^2*x^2+1)/a^2)^(1/2)-1/a
/x
```

maxima [A] time = 0.31, size = 54, normalized size = 1.42

$$-\sqrt{\frac{1}{a^2x^2}+1}-\frac{1}{ax}+\frac{1}{2}\log\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{1}{a^2x^2}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] -sqrt(1/(a^2*x^2) + 1) - 1/(a*x) + 1/2*log(sqrt(1/(a^2*x^2) + 1) + 1) - 1/2
*log(sqrt(1/(a^2*x^2) + 1) - 1)
```

mupad [B] time = 2.46, size = 34, normalized size = 0.89

$$\operatorname{atanh}\left(\sqrt{\frac{1}{a^2\,x^2}+1}\right)-\sqrt{\frac{1}{a^2\,x^2}+1}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))/x,x)
```

```
[Out] atanh((1/(a^2*x^2) + 1)^(1/2)) - (1/(a^2*x^2) + 1)^(1/2) - 1/(a*x)
```

sympy [A] time = 4.81, size = 41, normalized size = 1.08

$$-\frac{ax}{\sqrt{a^2x^2+1}}+\operatorname{asinh}(ax)-\frac{1}{ax}-\frac{1}{ax\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))/x,x)
```

```
[Out] -a*x/sqrt(a**2*x**2 + 1) + asinh(a*x) - 1/(a*x) - 1/(a*x*sqrt(a**2*x**2 + 1
))
```

3.33 $\int \frac{e^{\text{csch}^{-1}(ax)}}{x^2} \, dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{2x}-\frac{1}{2ax^2}-\frac{1}{2}a\text{csch}^{-1}(ax)$$

```
[Out] -1/2/a/x^2-1/2*a*arccsch(a*x)-1/2*(1+1/a^2/x^2)^(1/2)/x
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6336, 30, 335, 195, 215}

$$-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{2x}-\frac{1}{2ax^2}-\frac{1}{2}a\text{csch}^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x]/x^2,x]
```

```
[Out] -1/(2*a*x^2) - Sqrt[1 + 1/(a^2*x^2)]/(2*x) - (a*ArcCsch[a*x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^2}\,dx = \frac{\int \frac{1}{x^3}\,dx}{a} + \int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^2}\,dx$$
$$= -\frac{1}{2ax^2} - \operatorname{Subst}\left(\int \sqrt{1+\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{2ax^2} - \frac{\sqrt{1+\frac{1}{a^2x^2}}}{2x} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{2ax^2} - \frac{\sqrt{1+\frac{1}{a^2x^2}}}{2x} - \frac{1}{2}\operatorname{acsch}^{-1}(ax)$$

Mathematica [A] time = 0.03, size = 43, normalized size = 1.08

$$-\frac{ax\sqrt{\frac{1}{a^2x^2}+1} + a^2x^2\sinh^{-1}\left(\frac{1}{ax}\right)+1}{2ax^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x]/x^2,x]
```

```
[Out] -1/2*(1 + a*Sqrt[1 + 1/(a^2*x^2)]*x + a^2*x^2*ArcSinh[1/(a*x)])/(a*x^2)
```

fricas [B] time = 0.46, size = 102, normalized size = 2.55

$$-\frac{a^2x^2\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - ax + 1\right) - a^2x^2\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - ax - 1\right) + ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} + 1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*x^2*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x + 1) - a^2*x^2*lo
g(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x - 1) + a*x*sqrt((a^2*x^2 + 1)/(a^
2*x^2)) + 1)/(a*x^2)
```

giac [B] time = 0.14, size = 82, normalized size = 2.05

$$-\frac{a^4|a|\log\left(\sqrt{a^2x^2+1}+1\right)\operatorname{sgn}(x) - a^4|a|\log\left(\sqrt{a^2x^2+1}-1\right)\operatorname{sgn}(x) + \frac{2\left(\sqrt{a^2x^2+1}\,a^4|a|\operatorname{sgn}(x)+a^5\right)}{a^2x^2}}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(a^4*abs(a)*log(sqrt(a^2*x^2 + 1) + 1)*sgn(x) - a^4*abs(a)*log(sqrt(a^
2*x^2 + 1) - 1)*sgn(x) + 2*(sqrt(a^2*x^2 + 1)*a^4*abs(a)*sgn(x) + a^5)/(a^2
*x^2))/a^4
```

maple [B] time = 0.05, size = 145, normalized size = 3.62

$$-\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\left(a^2\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}\sqrt{\frac{1}{a^2}} - \sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}\,x^2a^2 + \ln\left(\frac{2\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+2}{a^2x}\right)x^2\right)}{2x\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}} - \frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))/x^2,x)
```

```
[Out] -1/2*((a^2*x^2+1)/a^2/x^2)^(1/2)/x*(a^2*((a^2*x^2+1)/a^2)^(3/2)*(1/a^2)^(1/
2)-(1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*x^2*a^2+ln(2*((1/a^2)^(1/2)*((a^2*
x^2+1)/a^2)^(1/2)*a^2+1)/a^2/x)*x^2)/(1/a^2)^(1/2)/((a^2*x^2+1)/a^2)^(1/2)-
1/2/a/x^2
```

maxima [B] time = 0.32, size = 86, normalized size = 2.15

$$-\frac{a^2 x \sqrt{\frac{1}{a^2 x^2}+1}}{2\left(a^2 x^2\left(\frac{1}{a^2 x^2}+1\right)-1\right)}-\frac{1}{4}\, a \log\left(a x \sqrt{\frac{1}{a^2 x^2}+1}+1\right)+\frac{1}{4}\, a \log\left(a x \sqrt{\frac{1}{a^2 x^2}+1}-1\right)-\frac{1}{2\, a x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*x*sqrt(1/(a^2*x^2) + 1)/(a^2*x^2*(1/(a^2*x^2) + 1) - 1) - 1/4*a*lo
g(a*x*sqrt(1/(a^2*x^2) + 1) + 1) + 1/4*a*log(a*x*sqrt(1/(a^2*x^2) + 1) - 1)
 - 1/2/(a*x^2)
```

mupad [B] time = 2.56, size = 42, normalized size = 1.05

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{\frac{1}{a^2}}}{x}\right)}{2\sqrt{\frac{1}{a^2}}}-\frac{\sqrt{\frac{1}{a^2\, x^2}+1}}{2\, x}-\frac{1}{2\, a\, x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))/x^2,x)
```

```
[Out] - asinh((1/a^2)^(1/2)/x)/(2*(1/a^2)^(1/2)) - (1/(a^2*x^2) + 1)^(1/2)/(2*x)
- 1/(2*a*x^2)
```

sympy [A] time = 3.23, size = 36, normalized size = 0.90

$$-\frac{a \operatorname{asinh}\left(\frac{1}{a x}\right)}{2}-\frac{\sqrt{1+\frac{1}{a^2 x^2}}}{2x}-\frac{1}{2 a x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))/x**2,x)
```

```
[Out] -a*asinh(1/(a*x))/2 - sqrt(1 + 1/(a**2*x**2))/(2*x) - 1/(2*a*x**2)
```

3.34 $\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{3}a^2\left(\frac{1}{a^2x^2}+1\right)^{3/2}-\frac{1}{3ax^3}$$

```
[Out] -1/3*a^2*(1+1/a^2/x^2)^(3/2)-1/3/a/x^3
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6336, 30, 261}

$$-\frac{1}{3}a^2\left(\frac{1}{a^2x^2}+1\right)^{3/2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x]/x^3,x]
```

```
[Out] -(a^2*(1 + 1/(a^2*x^2))^(3/2))/3 - 1/(3*a*x^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^3}\,dx &= \frac{\int \frac{1}{x^4}\,dx}{a}+\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^3}\,dx\\ &= -\frac{1}{3}a^2\left(1+\frac{1}{a^2x^2}\right)^{3/2}-\frac{1}{3ax^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 37, normalized size = 1.19

$$-\frac{ax\sqrt{\frac{1}{a^2x^2}+1}\,\left(a^2x^2+1\right)+1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCsch[a*x]/x^3,x]
```

```
[Out] -1/3*(1 + a*Sqrt[1 + 1/(a^2*x^2)]*x*(1 + a^2*x^2))/(a*x^3)
```

fricas [A] time = 0.70, size = 47, normalized size = 1.52

$$-\frac{a^3x^3 + \left(a^3x^3 + ax\right)\sqrt{\frac{a^2x^2+1}{a^2x^2}} + 1}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/3*(a^3*x^3 + (a^3*x^3 + a*x)*sqrt((a^2*x^2 + 1)/(a^2*x^2)) + 1)/(a*x^3)
```

giac [B] time = 0.17, size = 69, normalized size = 2.23

$$\frac{2\left(3\left(x|a| - \sqrt{a^2x^2+1}\right)^4 a^2\mathrm{sgn}(x) + a^2\mathrm{sgn}(x)\right)}{3\left(\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)^3} - \frac{1}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*(x*abs(a) - sqrt(a^2*x^2 + 1))^4*a^2*sgn(x) + a^2*sgn(x))/((x*abs(a)
 - sqrt(a^2*x^2 + 1))^2 - 1)^3 - 1/3/(a*x^3)
```

maple [A] time = 0.06, size = 42, normalized size = 1.35

$$-\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,\left(a^2x^2+1\right)}{3x^2} - \frac{1}{3x^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))/x^3,x)
```

```
[Out] -1/3*((a^2*x^2+1)/a^2/x^2)^(1/2)/x^2*(a^2*x^2+1)-1/3/x^3/a
```

maxima [A] time = 0.31, size = 25, normalized size = 0.81

$$-\frac{1}{3}\,a^2\left(\frac{1}{a^2x^2} + 1\right)^{\frac{3}{2}} - \frac{1}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/3*a^2*(1/(a^2*x^2) + 1)^(3/2) - 1/3/(a*x^3)
```

mupad [B] time = 2.16, size = 42, normalized size = 1.35

$$-\frac{\frac{x\sqrt{\frac{1}{a^2\,x^2}+1}}{3} + \frac{1}{3\,a}}{x^3} - \frac{a^2\,\sqrt{\frac{1}{a^2\,x^2}+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))/x^3,x)
```

```
[Out] - ((x*(1/(a^2*x^2) + 1)^(1/2))/3 + 1/(3*a))/x^3 - (a^2*(1/(a^2*x^2) + 1)^(1
/2))/3
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))/x**3,x)
[Out] Exception raised: TypeError
```

3.35 $\int \frac{e^{\text{csch}^{-1}(ax)}}{x^4} \, dx$

Optimal. Leaf size=65

$$\frac{1}{8}a^3\text{csch}^{-1}(ax) - \frac{a^2\sqrt{\frac{1}{a^2x^2}+1}}{8x} - \frac{\sqrt{\frac{1}{a^2x^2}+1}}{4x^3} - \frac{1}{4ax^4}$$

```
[Out] -1/4/a/x^4+1/8*a^3*arccsch(a*x)-1/4*(1+1/a^2/x^2)^(1/2)/x^3-1/8*a^2*(1+1/a^
2/x^2)^(1/2)/x
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6336, 30, 335, 279, 321, 215}

$$-\frac{a^2\sqrt{\frac{1}{a^2x^2}+1}}{8x} - \frac{\sqrt{\frac{1}{a^2x^2}+1}}{4x^3} + \frac{1}{8}a^3\text{csch}^{-1}(ax) - \frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x]/x^4,x]
```

```
[Out] -1/(4*a*x^4) - Sqrt[1 + 1/(a^2*x^2)]/(4*x^3) - (a^2*Sqrt[1 + 1/(a^2*x^2)])/
(8*x) + (a^3*ArcCsch[a*x])/8
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^4}\,dx &= \frac{\int \frac{1}{x^5}\,dx}{a} + \int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^4}\,dx\\
&= -\frac{1}{4ax^4} - \operatorname{Subst}\left(\int x^2\sqrt{1+\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{1}{4ax^4} - \frac{\sqrt{1+\frac{1}{a^2x^2}}}{4x^3} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{1}{4ax^4} - \frac{\sqrt{1+\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1+\frac{1}{a^2x^2}}}{8x} + \frac{1}{8}a^2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{1}{4ax^4} - \frac{\sqrt{1+\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1+\frac{1}{a^2x^2}}}{8x} + \frac{1}{8}a^3\operatorname{csch}^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 0.82

$$\frac{a^4x^4\sinh^{-1}\left(\frac{1}{ax}\right) - ax\sqrt{\frac{1}{a^2x^2}+1}\,\left(a^2x^2+2\right) - 2}{8ax^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x]/x^4,x]
[Out] (-2 - a*Sqrt[1 + 1/(a^2*x^2)]*x*(2 + a^2*x^2) + a^4*x^4*ArcSinh[1/(a*x)])/(
8*a*x^4)
```

fricas [B] time = 0.68, size = 113, normalized size = 1.74

$$\frac{a^4x^4\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - ax + 1\right) - a^4x^4\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - ax - 1\right) - \left(a^3x^3 + 2\,ax\right)\sqrt{\frac{a^2x^2+1}{a^2x^2}} - 2}{8\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^4,x, algorithm="fricas")
[Out] 1/8*(a^4*x^4*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x + 1) - a^4*x^4*log
(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x - 1) - (a^3*x^3 + 2*a*x)*sqrt((a^2
*x^2 + 1)/(a^2*x^2)) - 2)/(a*x^4)
```

giac [A] time = 0.16, size = 103, normalized size = 1.58

$$\frac{a^6|a|\log\left(\sqrt{a^2x^2+1}+1\right)\operatorname{sgn}(x) - a^6|a|\log\left(\sqrt{a^2x^2+1}-1\right)\operatorname{sgn}(x) - \frac{2\left(\left(a^2x^2+1\right)^{\frac{3}{2}}a^6|a|\operatorname{sgn}(x) + \sqrt{a^2x^2+1}\,a^6|a|\operatorname{sgn}(x) + 2\,a^7\right)}{a^4x^4}}{16\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] 1/16*(a^6*abs(a)*log(sqrt(a^2*x^2 + 1) + 1)*sgn(x) - a^6*abs(a)*log(sqrt(a^
2*x^2 + 1) - 1)*sgn(x) - 2*((a^2*x^2 + 1)^(3/2)*a^6*abs(a)*sgn(x) + sqrt(a^
2*x^2 + 1)*a^6*abs(a)*sgn(x) + 2*a^7)/(a^4*x^4))/a^4
```

maple [B] time = 0.06, size = 173, normalized size = 2.66

$$\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,a^2\left(\sqrt{\frac{1}{a^2}}\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}x^2a^2-\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}\,x^4a^2+\ln\left(\frac{2\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+2}{a^2x}\right)x^4-2\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}\sqrt{\frac{1}{a^2}}\right)}{8x^3\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}}-\frac{1}{4x^4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))/x^4,x)
```

```
[Out] 1/8*((a^2*x^2+1)/a^2/x^2)^(1/2)/x^3*a^2*((1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(3
/2)*x^2*a^2-(1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*x^4*a^2+ln(2*((1/a^2)^(1/
2)*((a^2*x^2+1)/a^2)^(1/2)*a^2+1)/a^2/x)*x^4-2*((a^2*x^2+1)/a^2)^(3/2)*(1/a
^2)^(1/2))/(1/a^2)^(1/2)/((a^2*x^2+1)/a^2)^(1/2)-1/4/x^4/a
```

maxima [B] time = 0.32, size = 129, normalized size = 1.98

$$\frac{1}{16}\,a^3\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}+1\right)-\frac{1}{16}\,a^3\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}-1\right)-\frac{a^6x^3\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}+a^4x\sqrt{\frac{1}{a^2x^2}+1}}{8\left(a^4x^4\left(\frac{1}{a^2x^2}+1\right)^2-2\,a^2x^2\left(\frac{1}{a^2x^2}+1\right)+1\right)}-\frac{1}{4\,a x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] 1/16*a^3*log(a*x*sqrt(1/(a^2*x^2) + 1) + 1) - 1/16*a^3*log(a*x*sqrt(1/(a^2*
x^2) + 1) - 1) - 1/8*(a^6*x^3*(1/(a^2*x^2) + 1)^(3/2) + a^4*x*sqrt(1/(a^2*x
^2) + 1))/(a^4*x^4*(1/(a^2*x^2) + 1)^2 - 2*a^2*x^2*(1/(a^2*x^2) + 1) + 1) -
 1/4/(a*x^4)
```

mupad [B] time = 2.54, size = 61, normalized size = 0.94

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{\frac{1}{a^2}}}{x}\right)}{8\left(\frac{1}{a^2}\right)^{3/2}}-\frac{\sqrt{\frac{1}{a^2\,x^2}+1}}{4\,x^3}-\frac{1}{4\,a\,x^4}-\frac{a^2\,\sqrt{\frac{1}{a^2\,x^2}+1}}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))/x^4,x)
```

```
[Out] asinh((1/a^2)^(1/2)/x)/(8*(1/a^2)^(3/2)) - (1/(a^2*x^2) + 1)^(1/2)/(4*x^3)
- 1/(4*a*x^4) - (a^2*(1/(a^2*x^2) + 1)^(1/2))/(8*x)
```

sympy [A] time = 4.25, size = 83, normalized size = 1.28

$$\frac{a^3\operatorname{asinh}\left(\frac{1}{ax}\right)}{8}-\frac{a^2}{8x\sqrt{1+\frac{1}{a^2x^2}}}-\frac{3}{8x^3\sqrt{1+\frac{1}{a^2x^2}}}-\frac{1}{4ax^4}-\frac{1}{4a^2x^5\sqrt{1+\frac{1}{a^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))/x**4,x)
```

```
[Out] a**3*asinh(1/(a*x))/8 - a**2/(8*x*sqrt(1 + 1/(a**2*x**2))) - 3/(8*x**3*sqrt
(1 + 1/(a**2*x**2))) - 1/(4*a*x**4) - 1/(4*a**2*x**5*sqrt(1 + 1/(a**2*x**2)
))
```

3.36 $\int \frac{e^{\text{csch}^{-1}(ax)}}{x^5} \, dx$

Optimal. Leaf size=51

$$-\frac{1}{5}a^4\left(\frac{1}{a^2x^2}+1\right)^{5/2}+\frac{1}{3}a^4\left(\frac{1}{a^2x^2}+1\right)^{3/2}-\frac{1}{5ax^5}$$

```
[Out] 1/3*a^4*(1+1/a^2/x^2)^(3/2)-1/5*a^4*(1+1/a^2/x^2)^(5/2)-1/5/a/x^5
```

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6336, 30, 266, 43}

$$-\frac{1}{5}a^4\left(\frac{1}{a^2x^2}+1\right)^{5/2}+\frac{1}{3}a^4\left(\frac{1}{a^2x^2}+1\right)^{3/2}-\frac{1}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x]/x^5,x]
```

```
[Out] (a^4*(1 + 1/(a^2*x^2))^(3/2))/3 - (a^4*(1 + 1/(a^2*x^2))^(5/2))/5 - 1/(5*a*
x^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^5}\,dx &= \frac{\int \frac{1}{x^6}\,dx}{a} + \int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^5}\,dx \\ &= -\frac{1}{5ax^5} - \frac{1}{2}\operatorname{Subst}\left(\int x\sqrt{1+\frac{x}{a^2}}\,dx, x, \frac{1}{x^2}\right) \\ &= -\frac{1}{5ax^5} - \frac{1}{2}\operatorname{Subst}\left(\int \left(-a^2\sqrt{1+\frac{x}{a^2}} + a^2\left(1+\frac{x}{a^2}\right)^{3/2}\right)dx, x, \frac{1}{x^2}\right) \\ &= \frac{1}{3}a^4\left(1+\frac{1}{a^2x^2}\right)^{3/2} - \frac{1}{5}a^4\left(1+\frac{1}{a^2x^2}\right)^{5/2} - \frac{1}{5ax^5}\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 0.90

$$\frac{ax\sqrt{\frac{1}{a^2x^2}+1}\left(2a^4x^4 - a^2x^2 - 3\right) - 3}{15ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCsch[a*x]/x^5,x]
```

```
[Out] (-3 + a*Sqrt[1 + 1/(a^2*x^2)]*x*(-3 - a^2*x^2 + 2*a^4*x^4))/(15*a*x^5)
```

fricas [A] time = 1.02, size = 58, normalized size = 1.14

$$\frac{2\,a^5x^5 + \left(2\,a^5x^5 - a^3x^3 - 3\,ax\right)\sqrt{\frac{a^2x^2+1}{a^2x^2}} - 3}{15\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] 1/15*(2*a^5*x^5 + (2*a^5*x^5 - a^3*x^3 - 3*a*x)*sqrt((a^2*x^2 + 1)/(a^2*x^2
)) - 3)/(a*x^5)
```

giac [B] time = 0.20, size = 124, normalized size = 2.43

$$\frac{4\left(15\left(x|a| - \sqrt{a^2x^2+1}\right)^6 a^4\operatorname{sgn}(x) + 5\left(x|a| - \sqrt{a^2x^2+1}\right)^4 a^4\operatorname{sgn}(x) + 5\left(x|a| - \sqrt{a^2x^2+1}\right)^2 a^4\operatorname{sgn}(x) - a^4\operatorname{sgn}(x)\right.}{15\left(\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] 4/15*(15*(x*abs(a) - sqrt(a^2*x^2 + 1))^6*a^4*sgn(x) + 5*(x*abs(a) - sqrt(a
^2*x^2 + 1))^4*a^4*sgn(x) + 5*(x*abs(a) - sqrt(a^2*x^2 + 1))^2*a^4*sgn(x) -
 a^4*sgn(x))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^5 - 1/5/(a*x^5)
```

maple [A] time = 0.05, size = 52, normalized size = 1.02

$$\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\left(a^2x^2+1\right)\left(2a^2x^2-3\right)}{15x^4} - \frac{1}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))/x^5,x)
```

```
[Out] 1/15*((a^2*x^2+1)/a^2/x^2)^(1/2)/x^4*(a^2*x^2+1)*(2*a^2*x^2-3)-1/5/a/x^5
```

maxima [A] time = 0.31, size = 41, normalized size = 0.80

$$-\frac{1}{5}\,a^4\left(\frac{1}{a^2x^2}+1\right)^{\frac{5}{2}}+\frac{1}{3}\,a^4\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}-\frac{1}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] -1/5*a^4*(1/(a^2*x^2) + 1)^(5/2) + 1/3*a^4*(1/(a^2*x^2) + 1)^(3/2) - 1/5/(a
*x^5)
```

mupad [B] time = 2.20, size = 61, normalized size = 1.20

$$\frac{2\,a^4\,\sqrt{\frac{1}{a^2\,x^2}+1}}{15}-\frac{\frac{x\,\sqrt{\frac{1}{a^2\,x^2}+1}}{5}+\frac{1}{5\,a}}{x^5}-\frac{a^2\,\sqrt{\frac{1}{a^2\,x^2}+1}}{15\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))/x^5,x)
```

```
[Out] (2*a^4*(1/(a^2*x^2) + 1)^(1/2))/15 - ((x*(1/(a^2*x^2) + 1)^(1/2))/5 + 1/(5*
a))/x^5 - (a^2*(1/(a^2*x^2) + 1)^(1/2))/(15*x^2)
```

sympy [A] time = 2.70, size = 65, normalized size = 1.27

$$\frac{2a^3\sqrt{a^2x^2+1}}{15x}-\frac{a\sqrt{a^2x^2+1}}{15x^3}-\frac{\sqrt{a^2x^2+1}}{5ax^5}-\frac{1}{5ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))/x**5,x)
```

```
[Out] 2*a**3*sqrt(a**2*x**2 + 1)/(15*x) - a*sqrt(a**2*x**2 + 1)/(15*x**3) - sqrt(
a**2*x**2 + 1)/(5*a*x**5) - 1/(5*a*x**5)
```

3.37 $\int e^{\text{csch}^{-1}\left(ax^2\right)}x^m\,dx$

Optimal. Leaf size=59

$$\frac{x^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}(-m-1);\frac{3-m}{4};-\frac{1}{a^2x^4}\right)}{m+1}-\frac{x^{m-1}}{a(1-m)}$$

```
[Out] -x^(-1+m)/a/(1-m)+x^(1+m)*hypergeom([-1/2, -1/4-1/4*m],[3/4-1/4*m],-1/a^2/x
^4)/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6336, 30, 339, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}(-m-1);\frac{3-m}{4};-\frac{1}{a^2x^4}\right)}{m+1}-\frac{x^{m-1}}{a(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x^2]*x^m,x]
```

```
[Out] -(x^(-1 + m)/(a*(1 - m))) + (x^(1 + m)*Hypergeometric2F1[-1/2, (-1 - m)/4,
(3 - m)/4, -(1/(a^2*x^4))])/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\int e^{\text{csch}^{-1}(ax^2)}x^m\,dx = \frac{\int x^{-2+m}\,dx}{a} + \int \sqrt{1+\frac{1}{a^2x^4}}\,x^m\,dx$$
$$= -\frac{x^{-1+m}}{a(1-m)} - \left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int x^{-2-m}\sqrt{1+\frac{x^4}{a^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{x^{-1+m}}{a(1-m)} + \frac{x^{1+m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4}(-1-m);\frac{3-m}{4};-\frac{1}{a^2x^4}\right)}{1+m}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.93

$$x^{m-1}\left(\frac{x^2\,{}_2F_1\left(-\frac{1}{2},-\frac{m}{4}-\frac{1}{4};\frac{3}{4}-\frac{m}{4};-\frac{1}{a^2x^4}\right)}{m+1}+\frac{1}{a(m-1)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCsch[a*x^2]*x^m,x]
```

```
[Out] x^(-1 + m)*(1/(a*(-1 + m)) + (x^2*Hypergeometric2F1[-1/2, -1/4 - m/4, 3/4 -
 m/4, -(1/(a^2*x^4))])/(1 + m))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax^2x^m\sqrt{\frac{a^2x^4+1}{a^2x^4}}+x^m}{ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*x^m*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + x^m)/(a*x^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{a\,x^2}+\sqrt{1+\frac{1}{a^2x^4}}\right)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^m,x)
```

```
[Out] int((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^m,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is m-2 equal to -1?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \left(\sqrt{\frac{1}{a^2 x^4}+1}+\frac{1}{a\,x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] int(x^m*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)), x)
```

sympy [A] time = 6.03, size = 66, normalized size = 1.12

$$-\frac{x x^m \Gamma\left(-\frac{m}{4}-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{m}{4}-\frac{1}{4} \\ \frac{3}{4}-\frac{m}{4} \end{matrix}\middle| \frac{e^{i\pi}}{a^2 x^4}\right)}{4\Gamma\left(\frac{3}{4}-\frac{m}{4}\right)}+\frac{\begin{cases} \frac{x^m}{mx-x} & \text{for } m \neq 1 \\ \log(x) & \text{otherwise} \end{cases}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))*x**m,x)
```

```
[Out] -x*x**m*gamma(-m/4 - 1/4)*hyper((-1/2, -m/4 - 1/4), (3/4 - m/4,), exp_polar
(I*pi)/(a**2*x**4))/(4*gamma(3/4 - m/4)) + Piecewise((x**m/(m*x - x), Ne(m,
 1)), (log(x), True))/a
```

3.38 $\int e^{\text{csch}^{-1}\left(ax^2\right)}x^4\,dx$

Optimal. Leaf size=202

$$\frac{2x\sqrt{\frac{1}{a^2x^4}+1}}{5a^2}+\frac{1}{5}x^5\sqrt{\frac{1}{a^2x^4}+1}-\frac{2\sqrt{\frac{1}{a^2x^4}+1}}{5a^2x\left(a+\frac{1}{x^2}\right)}-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)F\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{5a^{7/2}\sqrt{\frac{1}{a^2x^4}+1}}+\frac{2\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)}{5a^{7/2}\sqrt{}}$$

```
[Out] 1/3*x^3/a-2/5*(1+1/a^2/x^4)^(1/2)/a^2/(a+1/x^2)/x+2/5*x*(1+1/a^2/x^4)^(1/2)
/a^2+1/5*x^5*(1+1/a^2/x^4)^(1/2)+2/5*(a+1/x^2)*(cos(2*arccot(x*a^(1/2)))^2)
^(1/2)/cos(2*arccot(x*a^(1/2)))*EllipticE(sin(2*arccot(x*a^(1/2))),1/2*2^(1
/2))*((a^2+1/x^4)/(a+1/x^2)^2)^(1/2)/a^(7/2)/(1+1/a^2/x^4)^(1/2)-1/5*(a+1/x
^2)*(cos(2*arccot(x*a^(1/2)))^2)^(1/2)/cos(2*arccot(x*a^(1/2)))*EllipticF(s
in(2*arccot(x*a^(1/2))),1/2*2^(1/2))*((a^2+1/x^4)/(a+1/x^2)^2)^(1/2)/a^(7/2
)/(1+1/a^2/x^4)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6336, 30, 335, 277, 325, 305, 220, 1196}

$$\frac{1}{5}x^5\sqrt{\frac{1}{a^2x^4}+1}+\frac{2x\sqrt{\frac{1}{a^2x^4}+1}}{5a^2}-\frac{2\sqrt{\frac{1}{a^2x^4}+1}}{5a^2x\left(a+\frac{1}{x^2}\right)}-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)F\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{5a^{7/2}\sqrt{\frac{1}{a^2x^4}+1}}+\frac{2\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)}{5a^{7/2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x^2]*x^4,x]
```

```
[Out] (-2*Sqrt[1 + 1/(a^2*x^4)])/(5*a^2*(a + x^(-2))*x) + (2*Sqrt[1 + 1/(a^2*x^4)
]*x)/(5*a^2) + x^3/(3*a) + (Sqrt[1 + 1/(a^2*x^4)]*x^5)/5 + (2*Sqrt[(a^2 + x
^(-4))/(a + x^(-2))^2]*(a + x^(-2))*EllipticE[2*ArcCot[Sqrt[a]*x], 1/2])/(5
*a^(7/2)*Sqrt[1 + 1/(a^2*x^4)]) - (Sqrt[(a^2 + x^(-4))/(a + x^(-2))^2]*(a +
 x^(-2))*EllipticF[2*ArcCot[Sqrt[a]*x], 1/2])/(5*a^(7/2)*Sqrt[1 + 1/(a^2*x^
4)])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\int e^{\text{csch}^{-1}\left(ax^2\right)}x^4\,dx=\frac{\int x^2\,dx}{a}+\int\sqrt{1+\frac{1}{a^2x^4}}\,x^4\,dx$$

$$=\frac{x^3}{3a}-\text{Subst}\left(\int\frac{\sqrt{1+\frac{x^4}{a^2}}}{x^6}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{x^3}{3a}+\frac{1}{5}\sqrt{1+\frac{1}{a^2x^4}}\,x^5-\frac{2\,\text{Subst}\left(\int\frac{1}{x^2\sqrt{1+\frac{x^4}{a^2}}}\,dx,x,\frac{1}{x}\right)}{5a^2}$$

$$=\frac{2\sqrt{1+\frac{1}{a^2x^4}}\,x}{5a^2}+\frac{x^3}{3a}+\frac{1}{5}\sqrt{1+\frac{1}{a^2x^4}}\,x^5-\frac{2\,\text{Subst}\left(\int\frac{x^2}{\sqrt{1+\frac{x^4}{a^2}}}\,dx,x,\frac{1}{x}\right)}{5a^4}$$

$$=\frac{2\sqrt{1+\frac{1}{a^2x^4}}\,x}{5a^2}+\frac{x^3}{3a}+\frac{1}{5}\sqrt{1+\frac{1}{a^2x^4}}\,x^5-\frac{2\,\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^4}{a^2}}}\,dx,x,\frac{1}{x}\right)}{5a^3}+\frac{2\,\text{Subst}\left(\int\frac{1-\frac{x^2}{a}}{\sqrt{1+\frac{x^4}{a^2}}}\right.}{5a^3}$$

$$=-\frac{2\sqrt{1+\frac{1}{a^2x^4}}}{5a^2\left(a+\frac{1}{x^2}\right)x}+\frac{2\sqrt{1+\frac{1}{a^2x^4}}\,x}{5a^2}+\frac{x^3}{3a}+\frac{1}{5}\sqrt{1+\frac{1}{a^2x^4}}\,x^5+\frac{2\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)E\left(2\cot^{-}\right.}{5a^{7/2}\sqrt{1+\frac{1}{a^2x^4}}}$$

Mathematica [C] time = 0.25, size = 112, normalized size = 0.55

$$\frac{4\sqrt{2}\,x^5e^{-\text{csch}^{-1}\left(ax^2\right)}\left(\frac{e^{\text{csch}^{-1}\left(ax^2\right)}}{e^{2\text{csch}^{-1}\left(ax^2\right)}-1}\right)^{5/2}\left(4\left(1-e^{2\text{csch}^{-1}\left(ax^2\right)}\right)^{5/2}{}_2F_1\left(\frac{3}{4},\frac{7}{2};\frac{7}{4};e^{2\text{csch}^{-1}\left(ax^2\right)}\right)+7e^{2\text{csch}^{-1}\left(ax^2\right)}-4\right)}{21\left(ax^2\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x^2]*x^4,x]
```

```
[Out] (4*Sqrt[2]*(E^ArcCsch[a*x^2]/(-1 + E^(2*ArcCsch[a*x^2])))^(5/2)*x^5*(-4 + 7
*E^(2*ArcCsch[a*x^2]) + 4*(1 - E^(2*ArcCsch[a*x^2]))^(5/2)*Hypergeometric2F
1[3/4, 7/2, 7/4, E^(2*ArcCsch[a*x^2])]))/(21*E^ArcCsch[a*x^2]*(a*x^2)^(5/2)
)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax^4\sqrt{\frac{a^2x^4+1}{a^2x^4}}+x^2}{a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^4,x, algorithm="fricas")
```

```
[Out] integral((a*x^4*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + x^2)/a, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^4,x, algorithm="giac")
```

```
[Out] integrate(x^4*(sqrt(1/(a^2*x^4) + 1) + 1/(a*x^2)), x)
```

maple [C] time = 0.07, size = 150, normalized size = 0.74

$$\frac{\sqrt{\frac{a^2x^4+1}{a^2x^4}}\, x^2 \left(\sqrt{ia}\, x^7 a^3 + x^3 a\sqrt{ia} + 2i\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\text{EllipticF}\left(x\sqrt{ia}\,, i\right) - 2i\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\text{Elli}\right.}{5a\left(a^2x^4+1\right)\sqrt{ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^4,x)
```

```
[Out] 1/5*((a^2*x^4+1)/a^2/x^4)^(1/2)*x^2*((I*a)^(1/2)*x^7*a^3+x^3*a*(I*a)^(1/2)+
2*I*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)*EllipticF(x*(I*a)^(1/2),I)-2*I*(1-I
*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)*EllipticE(x*(I*a)^(1/2),I))/a/(a^2*x^4+1)/(
I*a)^(1/2)+1/3*x^3/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^3}{3\,a} + \frac{\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}; -a^2x^4\right)}{4\,\Gamma\left(\frac{7}{4}\right)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^4,x, algorithm="maxima")
```

```
[Out] 1/3*x^3/a + integrate(sqrt(a^2*x^4 + 1)*x^2, x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(\sqrt{\frac{1}{a^2\,x^4}+1}+\frac{1}{a\,x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] int(x^4*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)), x)
```

sympy [C] time = 2.46, size = 48, normalized size = 0.24

$$-\frac{x^5\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| \frac{e^{i\pi}}{a^2x^4}\right)}{4\Gamma\left(-\frac{1}{4}\right)} + \frac{x^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))*x**4,x)
```

```
[Out] -x**5*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), exp_polar(I*pi)/(a**2*x**4))
/(4*gamma(-1/4)) + x**3/(3*a)
```

3.39 $\int e^{\text{csch}^{-1}(ax^2)}x^3\,dx$

Optimal. Leaf size=52

$$\frac{1}{4}x^4\sqrt{\frac{1}{a^2x^4}+1}+\frac{\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^4}+1}\right)}{4a^2}+\frac{x^2}{2a}$$

```
[Out] 1/2*x^2/a+1/4*arctanh((1+1/a^2/x^4)^(1/2))/a^2+1/4*x^4*(1+1/a^2/x^4)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6336, 30, 266, 47, 63, 208}

$$\frac{1}{4}x^4\sqrt{\frac{1}{a^2x^4}+1}+\frac{\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^4}+1}\right)}{4a^2}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x^2]*x^3,x]
```

```
[Out] x^2/(2*a) + (Sqrt[1 + 1/(a^2*x^4)]*x^4)/4 + ArcTanh[Sqrt[1 + 1/(a^2*x^4)]]/
(4*a^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\begin{aligned}
\int e^{\operatorname{csch}^{-1}\left(ax^2\right)}x^3\,dx &= \frac{\int x\,dx}{a}+\int\sqrt{1+\frac{1}{a^2x^4}}\,x^3\,dx\\
&= \frac{x^2}{2a}-\frac{1}{4}\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a^2}}}{x^2}\,dx,x,\frac{1}{x^4}\right)\\
&= \frac{x^2}{2a}+\frac{1}{4}\sqrt{1+\frac{1}{a^2x^4}}\,x^4-\frac{\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a^2}}}\,dx,x,\frac{1}{x^4}\right)}{8a^2}\\
&= \frac{x^2}{2a}+\frac{1}{4}\sqrt{1+\frac{1}{a^2x^4}}\,x^4-\frac{1}{4}\operatorname{Subst}\left(\int\frac{1}{-a^2+a^2x^2}\,dx,x,\sqrt{1+\frac{1}{a^2x^4}}\right)\\
&= \frac{x^2}{2a}+\frac{1}{4}\sqrt{1+\frac{1}{a^2x^4}}\,x^4+\frac{\tanh^{-1}\left(\sqrt{1+\frac{1}{a^2x^4}}\right)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 1.02

$$\frac{ax^2\left(ax^2\sqrt{\frac{1}{a^2x^4}+1}+2\right)+\log\left(x^2\left(\sqrt{\frac{1}{a^2x^4}+1}+1\right)\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x^2]*x^3,x]
```

```
[Out] (a*x^2*(2 + a*Sqrt[1 + 1/(a^2*x^4)]*x^2) + Log[(1 + Sqrt[1 + 1/(a^2*x^4)])*
x^2])/(4*a^2)
```

fricas [A] time = 0.65, size = 70, normalized size = 1.35

$$\frac{a^2x^4\sqrt{\frac{a^2x^4+1}{a^2x^4}}+2\,ax^2-\log\left(ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}-ax^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*x^4*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + 2*a*x^2 - log(a*x^2*sqrt((a^2*
x^4 + 1)/(a^2*x^4)) - a*x^2))/a^2
```

giac [A] time = 0.15, size = 57, normalized size = 1.10

$$\frac{2\,ax^2+\left(\sqrt{a^2x^4+1}\,x^2-\frac{\log\left(-x^2|a|+\sqrt{a^2x^4+1}\right)}{|a|}\right)|a|}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a*x^2 + (sqrt(a^2*x^4 + 1)*x^2 - log(-x^2*abs(a) + sqrt(a^2*x^4 + 1)
)/abs(a))*abs(a))/a^2
```

maple [B] time = 0.17, size = 94, normalized size = 1.81

$$\frac{\sqrt{\frac{a^2x^4+1}{a^2x^4}}\, x^2 \left(x^2\sqrt{\frac{a^2x^4+1}{a^2}}\, a^2 + \ln\left(x^2 + \sqrt{\frac{a^2x^4+1}{a^2}}\right)\right)}{4\sqrt{\frac{a^2x^4+1}{a^2}}\, a^2} + \frac{x^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^3,x)
```

```
[Out] 1/4*((a^2*x^4+1)/a^2/x^4)^(1/2)*x^2*(x^2*((a^2*x^4+1)/a^2)^(1/2)*a^2+ln(x^2
+((a^2*x^4+1)/a^2)^(1/2)))/((a^2*x^4+1)/a^2)^(1/2)/a^2+1/2*x^2/a
```

maxima [A] time = 0.33, size = 81, normalized size = 1.56

$$\frac{x^2}{2\,a} + \frac{\sqrt{\frac{1}{a^2x^4}+1}}{4\left(a^2\left(\frac{1}{a^2x^4}+1\right)-a^2\right)} + \frac{\log\left(\sqrt{\frac{1}{a^2x^4}+1}+1\right)}{8\,a^2} - \frac{\log\left(\sqrt{\frac{1}{a^2x^4}+1}-1\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2/a + 1/4*sqrt(1/(a^2*x^4) + 1)/(a^2*(1/(a^2*x^4) + 1) - a^2) + 1/8*l
og(sqrt(1/(a^2*x^4) + 1) + 1)/a^2 - 1/8*log(sqrt(1/(a^2*x^4) + 1) - 1)/a^2
```

mupad [B] time = 2.61, size = 42, normalized size = 0.81

$$\frac{\operatorname{atanh}\left(\sqrt{\frac{1}{a^2\,x^4}+1}\right)}{4\,a^2} + \frac{x^4\,\sqrt{\frac{1}{a^2\,x^4}+1}}{4} + \frac{x^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] atanh((1/(a^2*x^4) + 1)^(1/2))/(4*a^2) + (x^4*(1/(a^2*x^4) + 1)^(1/2))/4 +
x^2/(2*a)
```

sympy [A] time = 3.68, size = 36, normalized size = 0.69

$$\frac{x^2\sqrt{a^2x^4+1}}{4a} + \frac{x^2}{2a} + \frac{\operatorname{asinh}\left(ax^2\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))*x**3,x)
```

```
[Out] x**2*sqrt(a**2*x**4 + 1)/(4*a) + x**2/(2*a) + asinh(a*x**2)/(4*a**2)
```

3.40 $\int e^{\operatorname{csch}^{-1}(ax^2)}x^2\,dx$

Optimal. Leaf size=86

$$\frac{1}{3}x^3\sqrt{\frac{1}{a^2x^4}+1}-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)F\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{3a^{5/2}\sqrt{\frac{1}{a^2x^4}+1}}+\frac{x}{a}$$

```
[Out] x/a+1/3*x^3*(1+1/a^2/x^4)^(1/2)-1/3*(a+1/x^2)*(cos(2*arccot(x*a^(1/2)))^2)^
(1/2)/cos(2*arccot(x*a^(1/2)))*EllipticF(sin(2*arccot(x*a^(1/2))),1/2*2^(1/
2))*((a^2+1/x^4)/(a+1/x^2)^2)^(1/2)/a^(5/2)/(1+1/a^2/x^4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6336, 8, 335, 277, 220}

$$\frac{1}{3}x^3\sqrt{\frac{1}{a^2x^4}+1}-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)F\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{3a^{5/2}\sqrt{\frac{1}{a^2x^4}+1}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x^2]*x^2,x]
```

```
[Out] x/a + (Sqrt[1 + 1/(a^2*x^4)]*x^3)/3 - (Sqrt[(a^2 + x^(-4))/(a + x^(-2))^2]*
(a + x^(-2))*EllipticF[2*ArcCot[Sqrt[a]*x], 1/2])/(3*a^(5/2)*Sqrt[1 + 1/(a^
2*x^4)])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
```

]

Rubi steps

$$
\begin{aligned}
\int e^{\text{csch}^{-1}\left(ax^2\right)}x^2\,dx &= \frac{\int 1\,dx}{a}+\int \sqrt{1+\frac{1}{a^2x^4}}\,x^2\,dx \\
&= \frac{x}{a}-\text{Subst}\left(\int \frac{\sqrt{1+\frac{x^4}{a^2}}}{x^4}\,dx,x,\frac{1}{x}\right) \\
&= \frac{x}{a}+\frac{1}{3}\sqrt{1+\frac{1}{a^2x^4}}\,x^3-\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{a^2}}}\,dx,x,\frac{1}{x}\right)}{3a^2} \\
&= \frac{x}{a}+\frac{1}{3}\sqrt{1+\frac{1}{a^2x^4}}\,x^3-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)F\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{3a^{5/2}\sqrt{1+\frac{1}{a^2x^4}}}
\end{aligned}
$$

Mathematica [C] time = 0.23, size = 113, normalized size = 1.31

$$
-\frac{2\sqrt{2}\,xe^{-\text{csch}^{-1}\left(ax^2\right)}\left(\frac{e^{\text{csch}^{-1}\left(ax^2\right)}}{e^{2\text{csch}^{-1}\left(ax^2\right)}-1}\right)^{3/2}\left(\left(1-e^{2\text{csch}^{-1}\left(ax^2\right)}\right)^{3/2}\left(-{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};e^{2\text{csch}^{-1}\left(ax^2\right)}\right)\right)-2e^{2\text{csch}^{-1}\left(ax^2\right)}+1\right)}{3a\sqrt{ax^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x^2]*x^2,x]
[Out] (-2*Sqrt[2]*(E^ArcCsch[a*x^2]/(-1 + E^(2*ArcCsch[a*x^2])))^(3/2)*x*(1 - 2*E
^(2*ArcCsch[a*x^2]) - (1 - E^(2*ArcCsch[a*x^2]))^(3/2)*Hypergeometric2F1[1/
4, 1/2, 5/4, E^(2*ArcCsch[a*x^2])]))/(3*a*E^ArcCsch[a*x^2]*Sqrt[a*x^2])
```

fricas [F] time = 2.60, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}+1}{a},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^2,x, algorithm="fricas")
[Out] integral((a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + 1)/a, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int x^2\left(\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}\right)dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^2,x, algorithm="giac")
[Out] integrate(x^2*(sqrt(1/(a^2*x^4) + 1) + 1/(a*x^2)), x)
```

maple [C] time = 0.05, size = 104, normalized size = 1.21

$$\frac{\sqrt{\frac{a^2x^4+1}{a^2x^4}}\, x^2 \left(\sqrt{ia}\, x^5a^2 + 2\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{ia}\,, i\right) + x\sqrt{ia}\right)}{3\left(a^2x^4+1\right)\sqrt{ia}} + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^2,x)
```

```
[Out] 1/3*((a^2*x^4+1)/a^2/x^4)^(1/2)*x^2*((I*a)^(1/2)*x^5*a^2+2*(1-I*a*x^2)^(1/2
)*(1+I*a*x^2)^(1/2)*EllipticF(x*(I*a)^(1/2),I)+x*(I*a)^(1/2))/(a^2*x^4+1)/(
I*a)^(1/2)+x/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{a} + \frac{\frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix};-a^2x^4\right)}{4\,\Gamma\left(\frac{5}{4}\right)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^2,x, algorithm="maxima")
```

```
[Out] x/a + integrate(sqrt(a^2*x^4 + 1), x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(\sqrt{\frac{1}{a^2\,x^4}+1} + \frac{1}{a\,x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] int(x^2*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)), x)
```

sympy [C] time = 2.16, size = 41, normalized size = 0.48

$$-\frac{x^3\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{e^{i\pi}}{a^2x^4}\right)}{4\Gamma\left(\frac{1}{4}\right)} + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))*x**2,x)
```

```
[Out] -x**3*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), exp_polar(I*pi)/(a**2*x**4))/
(4*gamma(1/4)) + x/a
```

3.41 $\int e^{\text{csch}^{-1}(ax^2)} x \, dx$

Optimal. Leaf size=40

$$\frac{1}{2}x^2\sqrt{\frac{1}{a^2x^4}+1}-\frac{\text{csch}^{-1}\left(ax^2\right)}{2a}+\frac{\log(x)}{a}$$

```
[Out] -1/2*arccsch(a*x^2)/a+ln(x)/a+1/2*x^2*(1+1/a^2/x^4)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6336, 29, 335, 275, 277, 215}

$$\frac{1}{2}x^2\sqrt{\frac{1}{a^2x^4}+1}-\frac{\text{csch}^{-1}\left(ax^2\right)}{2a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x^2]*x,x]
```

```
[Out] (Sqrt[1 + 1/(a^2*x^4)]*x^2)/2 - ArcCsch[a*x^2]/(2*a) + Log[x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$
\begin{aligned}
\int e^{\operatorname{csch}^{-1}(ax^2)}x\,dx &= \frac{\int \frac{1}{x}\,dx}{a} + \int \sqrt{1+\frac{1}{a^2x^4}}\,x\,dx \\
&= \frac{\log(x)}{a} - \text{Subst}\left(\int \frac{\sqrt{1+\frac{x^4}{a^2}}}{x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\log(x)}{a} - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right) \\
&= \frac{1}{2}\sqrt{1+\frac{1}{a^2x^4}}\,x^2 + \frac{\log(x)}{a} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a^2} \\
&= \frac{1}{2}\sqrt{1+\frac{1}{a^2x^4}}\,x^2 - \frac{\operatorname{csch}^{-1}\left(ax^2\right)}{2a} + \frac{\log(x)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 42, normalized size = 1.05

$$
\frac{ax^2\sqrt{\frac{1}{a^2x^4}+1} + \log\left(ax^2\right) - \sinh^{-1}\left(\frac{1}{ax^2}\right)}{2a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x^2]*x,x]
[Out] (a*Sqrt[1 + 1/(a^2*x^4)]*x^2 - ArcSinh[1/(a*x^2)] + Log[a*x^2])/(2*a)
```

fricas [B] time = 0.61, size = 88, normalized size = 2.20

$$
\frac{2\,ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}} - \log\left(ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}+1\right) + \log\left(ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}-1\right) + 4\,\log(x)}{4\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x,x, algorithm="fricas")
[Out] 1/4*(2*a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) - log(a*x^2*sqrt((a^2*x^4 + 1)/(
a^2*x^4)) + 1) + log(a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) - 1) + 4*log(x))/a
```

giac [A] time = 0.15, size = 61, normalized size = 1.52

$$
\frac{(a-|a|)\log\left(\sqrt{a^2x^4+1}+1\right) + (a+|a|)\log\left(\sqrt{a^2x^4+1}-1\right) + 2\sqrt{a^2x^4+1}\,|a|}{4\,a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x,x, algorithm="giac")
[Out] 1/4*((a - abs(a))*log(sqrt(a^2*x^4 + 1) + 1) + (a + abs(a))*log(sqrt(a^2*x^
4 + 1) - 1) + 2*sqrt(a^2*x^4 + 1)*abs(a))/a^2
```

maple [B] time = 0.15, size = 116, normalized size = 2.90

$$
\frac{\sqrt{\frac{a^2x^4+1}{a^2x^4}}\,x^2\left(\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{a^2x^4+1}{a^2}}\,a^2 - \ln\left(\frac{2\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{a^2x^4+1}{a^2}}\,a^2+2}{a^2x^2}\right)\right)}{2\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{a^2x^4+1}{a^2}}\,a^2} + \frac{\ln(x)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x,x)
```

```
[Out] 1/2*((a^2*x^4+1)/a^2/x^4)^(1/2)*x^2*((1/a^2)^(1/2)*((a^2*x^4+1)/a^2)^(1/2)*
a^2-ln(2*((1/a^2)^(1/2)*((a^2*x^4+1)/a^2)^(1/2)*a^2+1)/a^2/x^2))/(1/a^2)^(1
/2)/((a^2*x^4+1)/a^2)^(1/2)/a^2+ln(x)/a
```

maxima [B] time = 0.37, size = 71, normalized size = 1.78

$$\frac{1}{2}\,x^2\sqrt{\frac{1}{a^2x^4}+1}-\frac{\log\left(ax^2\sqrt{\frac{1}{a^2x^4}+1}+1\right)}{4\,a}+\frac{\log\left(ax^2\sqrt{\frac{1}{a^2x^4}+1}-1\right)}{4\,a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sqrt(1/(a^2*x^4) + 1) - 1/4*log(a*x^2*sqrt(1/(a^2*x^4) + 1) + 1)/a
+ 1/4*log(a*x^2*sqrt(1/(a^2*x^4) + 1) - 1)/a + log(x)/a
```

mupad [B] time = 2.97, size = 43, normalized size = 1.08

$$\frac{x^2\,\sqrt{\frac{1}{a^2\,x^4}+1}}{2}-\frac{\ln\left(\frac{1}{x^2}\right)}{2\,a}-\frac{\mathrm{asinh}\left(\frac{\sqrt{\frac{1}{a^2}}}{x^2}\right)\,\sqrt{\frac{1}{a^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] (x^2*(1/(a^2*x^4) + 1)^(1/2))/2 - log(1/x^2)/(2*a) - (asinh((1/a^2)^(1/2)/x
^2)*(1/a^2)^(1/2))/2
```

sympy [A] time = 6.79, size = 58, normalized size = 1.45

$$\frac{x^2}{2\sqrt{1+\frac{1}{a^2x^4}}}+\frac{\log{(x)}}{a}-\frac{\operatorname{asinh}{\left(\frac{1}{ax^2}\right)}}{2a}+\frac{1}{2a^2x^2\sqrt{1+\frac{1}{a^2x^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))*x,x)
```

```
[Out] x**2/(2*sqrt(1 + 1/(a**2*x**4))) + log(x)/a - asinh(1/(a*x**2))/(2*a) + 1/(
2*a**2*x**2*sqrt(1 + 1/(a**2*x**4)))
```

3.42 $\int e^{\operatorname{csch}^{-1}\left(ax^2\right)}\,dx$

Optimal. Leaf size=165

$$x\sqrt{\frac{1}{a^2x^4}+1}-\frac{2\sqrt{\frac{1}{a^2x^4}+1}}{x\left(a+\frac{1}{x^2}\right)}-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)F\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{a^{3/2}\sqrt{\frac{1}{a^2x^4}+1}}+\frac{2\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)E\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{a^{3/2}\sqrt{\frac{1}{a^2x^4}+1}}-$$

```
[Out] -1/a/x-2*(1+1/a^2/x^4)^(1/2)/(a+1/x^2)/x+x*(1+1/a^2/x^4)^(1/2)+2*(a+1/x^2)*
(cos(2*arccot(x*a^(1/2)))^2)^(1/2)/cos(2*arccot(x*a^(1/2)))*EllipticE(sin(2
*arccot(x*a^(1/2))),1/2*2^(1/2))*((a^2+1/x^4)/(a+1/x^2)^2)^(1/2)/a^(3/2)/(1
+1/a^2/x^4)^(1/2)-(a+1/x^2)*(cos(2*arccot(x*a^(1/2)))^2)^(1/2)/cos(2*arccot
(x*a^(1/2)))*EllipticF(sin(2*arccot(x*a^(1/2))),1/2*2^(1/2))*((a^2+1/x^4)/(
a+1/x^2)^2)^(1/2)/a^(3/2)/(1+1/a^2/x^4)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6331, 30, 242, 277, 305, 220, 1196}

$$x\sqrt{\frac{1}{a^2x^4}+1}-\frac{2\sqrt{\frac{1}{a^2x^4}+1}}{x\left(a+\frac{1}{x^2}\right)}-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)F\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{a^{3/2}\sqrt{\frac{1}{a^2x^4}+1}}+\frac{2\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)E\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{a^{3/2}\sqrt{\frac{1}{a^2x^4}+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x^2],x]
```

```
[Out] -(1/(a*x)) - (2*Sqrt[1 + 1/(a^2*x^4)])/((a + x^(-2))*x) + Sqrt[1 + 1/(a^2*x
^4)]*x + (2*Sqrt[(a^2 + x^(-4))/(a + x^(-2))^2]*(a + x^(-2))*EllipticE[2*Ar
cCot[Sqrt[a]*x], 1/2])/(a^(3/2)*Sqrt[1 + 1/(a^2*x^4)]) - (Sqrt[(a^2 + x^(-4
))/(a + x^(-2))^2]*(a + x^(-2))*EllipticF[2*ArcCot[Sqrt[a]*x], 1/2])/(a^(3/
2)*Sqrt[1 + 1/(a^2*x^4)])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 6331

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)], x_Symbol] :> Dist[1/a, Int[1/x^p, x], x] +
 Int[Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int e^{\operatorname{csch}^{-1}(ax^2)}\,dx &= \frac{\int \frac{1}{x^2}\,dx}{a} + \int \sqrt{1+\frac{1}{a^2x^4}}\,dx \\
&= -\frac{1}{ax} - \operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x^4}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{1}{ax} + \sqrt{1+\frac{1}{a^2x^4}}\,x - \frac{2\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} \\
&= -\frac{1}{ax} + \sqrt{1+\frac{1}{a^2x^4}}\,x - \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} + \frac{2\operatorname{Subst}\left(\int \frac{1-\frac{x^2}{a}}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= -\frac{1}{ax} - \frac{2\sqrt{1+\frac{1}{a^2x^4}}}{\left(a+\frac{1}{x^2}\right)x} + \sqrt{1+\frac{1}{a^2x^4}}\,x + \frac{2\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)E\left(2\cot^{-1}\left(\sqrt{a}\,x\right)\middle|\frac{1}{2}\right)}{a^{3/2}\sqrt{1+\frac{1}{a^2x^4}}} - \sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)}}
\end{aligned}
$$

Mathematica [C] time = 0.15, size = 96, normalized size = 0.58

$$
\frac{\sqrt{2}\,x e^{\operatorname{csch}^{-1}(ax^2)}\sqrt{\frac{e^{\operatorname{csch}^{-1}(ax^2)}}{e^{2\operatorname{csch}^{-1}(ax^2)}-1}}\left(4\sqrt{1-e^{2\operatorname{csch}^{-1}(ax^2)}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};e^{2\operatorname{csch}^{-1}(ax^2)}\right)-3\right)}{3\sqrt{ax^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x^2],x]
```

```
[Out] (Sqrt[2]*E^ArcCsch[a*x^2]*Sqrt[E^ArcCsch[a*x^2]/(-1 + E^(2*ArcCsch[a*x^2]))
]*x*(-3 + 4*Sqrt[1 - E^(2*ArcCsch[a*x^2])]*Hypergeometric2F1[3/4, 3/2, 7/4,
 E^(2*ArcCsch[a*x^2])]))/(3*Sqrt[a*x^2])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}+1}{ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x^2+(1+1/a^2/x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + 1)/(a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x^2+(1+1/a^2/x^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(1/(a^2*x^4) + 1) + 1/(a*x^2), x)
```

maple [C] time = 0.06, size = 146, normalized size = 0.88

$$\frac{\sqrt{\frac{a^2x^4+1}{a^2x^4}}\,x\left(-\sqrt{ia}\,x^4a^2+2i\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,x\,\mathrm{EllipticF}\left(x\sqrt{ia}\,,i\right)a-2i\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,x\,\mathrm{EllipticE}\right.}{\left(a^2x^4+1\right)\sqrt{ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a/x^2+(1+1/a^2/x^4)^(1/2),x)
```

```
[Out] ((a^2*x^4+1)/a^2/x^4)^(1/2)*x*(-(I*a)^(1/2)*x^4*a^2+2*I*(1-I*a*x^2)^(1/2)*(
1+I*a*x^2)^(1/2)*x*EllipticF(x*(I*a)^(1/2),I)*a-2*I*(1-I*a*x^2)^(1/2)*(1+I*
a*x^2)^(1/2)*x*EllipticE(x*(I*a)^(1/2),I)*a-(I*a)^(1/2))/(a^2*x^4+1)/(I*a)^
(1/2)-1/a/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix};-a^2x^4\right)}{4\,x\Gamma\left(\frac{3}{4}\right)}}{a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x^2+(1+1/a^2/x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^4 + 1)/x^2, x)/a - 1/(a*x)
```

mupad [B] time = 2.33, size = 24, normalized size = 0.15

$$x\,{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{4};\,\frac{3}{4};\,-\frac{1}{a^2\,x^4}\right)-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2),x)
```

```
[Out] x*hypergeom([-1/2, -1/4], 3/4, -1/(a^2*x^4)) - 1/(a*x)
```

sympy **[C]** time = 0.78, size = 42, normalized size = 0.25

$$-\frac{x\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{e^{i\pi}}{a^2x^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x**2+(1+1/a**2/x**4)**(1/2),x)
```

```
[Out] -x*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), exp_polar(I*pi)/(a**2*x**4))/(4*
gamma(3/4)) - 1/(a*x)
```

3.43 $\int \frac{e^{\mathrm{csch}^{-1}(ax^2)}}{x} dx$

Optimal. Leaf size=46

$$-\frac{1}{2}\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^4}+1}\right)-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2+1/2*arctanh((1+1/a^2/x^4)^(1/2))-1/2*(1+1/a^2/x^4)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6336, 30, 266, 50, 63, 208}

$$-\frac{1}{2}\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^4}+1}\right)-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x^2]/x,x]
```

```
[Out] -Sqrt[1 + 1/(a^2*x^4)]/2 - 1/(2*a*x^2) + ArcTanh[Sqrt[1 + 1/(a^2*x^4)]]/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
```

]

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x}\,dx &= \frac{\int \frac{1}{x^3}\,dx}{a} + \int \frac{\sqrt{1+\frac{1}{a^2x^4}}}{x}\,dx \\
&= -\frac{1}{2ax^2} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a^2}}}{x}\,dx, x, \frac{1}{x^4}\right) \\
&= -\frac{1}{2}\sqrt{1+\frac{1}{a^2x^4}} - \frac{1}{2ax^2} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a^2}}}\,dx, x, \frac{1}{x^4}\right) \\
&= -\frac{1}{2}\sqrt{1+\frac{1}{a^2x^4}} - \frac{1}{2ax^2} - \frac{1}{2}a^2\operatorname{Subst}\left(\int \frac{1}{-a^2+a^2x^2}\,dx, x, \sqrt{1+\frac{1}{a^2x^4}}\right) \\
&= -\frac{1}{2}\sqrt{1+\frac{1}{a^2x^4}} - \frac{1}{2ax^2} + \frac{1}{2}\tanh^{-1}\left(\sqrt{1+\frac{1}{a^2x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 22, normalized size = 0.48

$$\tanh^{-1}\left(e^{\operatorname{csch}^{-1}(ax^2)}\right) - \frac{1}{2}e^{\operatorname{csch}^{-1}(ax^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x^2]/x,x]
[Out] -1/2*E^ArcCsch[a*x^2] + ArcTanh[E^ArcCsch[a*x^2]]
```

fricas [B] time = 0.87, size = 74, normalized size = 1.61

$$-\frac{ax^2\log\left(ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}} - ax^2\right) + ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}} + ax^2 + 1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x,x, algorithm="fricas")
[Out] -1/2*(a*x^2*log(a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) - a*x^2) + a*x^2*sqrt((
a^2*x^4 + 1)/(a^2*x^4)) + a*x^2 + 1)/(a*x^2)
```

giac [A] time = 0.16, size = 66, normalized size = 1.43

$$-\frac{a^2\log\left(-x^2|a| + \sqrt{a^2x^4+1}\right) - \frac{2\,a^2}{\left(x^2|a| - \sqrt{a^2x^4+1}\right)^2 - 1} + \frac{a}{x^2}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x,x, algorithm="giac")
[Out] -1/2*(a^2*log(-x^2*abs(a) + sqrt(a^2*x^4 + 1)) - 2*a^2/((x^2*abs(a) - sqrt(
a^2*x^4 + 1))^2 - 1) + a/x^2)/a^2
```

maple [B] time = 0.16, size = 86, normalized size = 1.87

$$-\frac{\sqrt{\frac{a^2x^4+1}{a^2x^4}}\left(-\ln\left(x^2+\sqrt{\frac{a^2x^4+1}{a^2}}\right)x^2+\sqrt{\frac{a^2x^4+1}{a^2}}\right)}{2\sqrt{\frac{a^2x^4+1}{a^2}}}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x,x)
```

```
[Out] -1/2*((a^2*x^4+1)/a^2/x^4)^(1/2)*(-ln(x^2+((a^2*x^4+1)/a^2)^(1/2))*x^2+((a^
2*x^4+1)/a^2)^(1/2))/((a^2*x^4+1)/a^2)^(1/2)-1/2/a/x^2
```

maxima [A] time = 0.31, size = 54, normalized size = 1.17

$$-\frac{1}{2}\sqrt{\frac{1}{a^2x^4}+1}-\frac{1}{2\,ax^2}+\frac{1}{4}\log\left(\sqrt{\frac{1}{a^2x^4}+1}+1\right)-\frac{1}{4}\log\left(\sqrt{\frac{1}{a^2x^4}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(1/(a^2*x^4) + 1) - 1/2/(a*x^2) + 1/4*log(sqrt(1/(a^2*x^4) + 1) +
1) - 1/4*log(sqrt(1/(a^2*x^4) + 1) - 1)
```

mupad [B] time = 2.38, size = 36, normalized size = 0.78

$$\frac{\operatorname{atanh}\left(\sqrt{\frac{1}{a^2\,x^4}+1}\right)}{2}-\frac{\sqrt{\frac{1}{a^2\,x^4}+1}}{2}-\frac{1}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2))/x,x)
```

```
[Out] atanh((1/(a^2*x^4) + 1)^(1/2))/2 - (1/(a^2*x^4) + 1)^(1/2)/2 - 1/(2*a*x^2)
```

sympy [A] time = 10.69, size = 54, normalized size = 1.17

$$-\frac{ax^2}{2\sqrt{a^2x^4+1}}+\frac{\operatorname{asinh}\left(ax^2\right)}{2}-\frac{1}{2ax^2}-\frac{1}{2ax^2\sqrt{a^2x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))/x,x)
```

```
[Out] -a*x**2/(2*sqrt(a**2*x**4 + 1)) + asinh(a*x**2)/2 - 1/(2*a*x**2) - 1/(2*a*x
**2*sqrt(a**2*x**4 + 1))
```

3.44 $\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^2}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{\frac{1}{a^2x^4}+1}}{3x}-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)F\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{3\sqrt{a}\sqrt{\frac{1}{a^2x^4}+1}}-\frac{1}{3ax^3}$$

```
[Out] -1/3/a/x^3-1/3*(1+1/a^2/x^4)^(1/2)/x-1/3*(a+1/x^2)*(cos(2*arccot(x*a^(1/2))
)^2)^(1/2)/cos(2*arccot(x*a^(1/2)))*EllipticF(sin(2*arccot(x*a^(1/2))),1/2*
2^(1/2))*((a^2+1/x^4)/(a+1/x^2)^2)^(1/2)/a^(1/2)/(1+1/a^2/x^4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6336, 30, 335, 195, 220}

$$-\frac{\sqrt{\frac{1}{a^2x^4}+1}}{3x}-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)F\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{3\sqrt{a}\sqrt{\frac{1}{a^2x^4}+1}}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x^2]/x^2,x]
```

```
[Out] -1/(3*a*x^3) - Sqrt[1 + 1/(a^2*x^4)]/(3*x) - (Sqrt[(a^2 + x^(-4))/(a + x^(-
2))^2]*(a + x^(-2))*EllipticF[2*ArcCot[Sqrt[a]*x], 1/2])/(3*Sqrt[a]*Sqrt[1
+ 1/(a^2*x^4)])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^2}\,dx &= \frac{\int \frac{1}{x^4}\,dx}{a}+\int \frac{\sqrt{1+\frac{1}{a^2x^4}}}{x^2}\,dx\\
&= -\frac{1}{3ax^3}-\operatorname{Subst}\left(\int \sqrt{1+\frac{x^4}{a^2}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{1}{3ax^3}-\frac{\sqrt{1+\frac{1}{a^2x^4}}}{3x}-\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{1}{3ax^3}-\frac{\sqrt{1+\frac{1}{a^2x^4}}}{3x}-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)F\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{3\sqrt{a}\sqrt{1+\frac{1}{a^2x^4}}}
\end{aligned}$$

Mathematica [C] time = 0.16, size = 96, normalized size = 1.05

$$-\frac{ax\sqrt{\frac{e^{\operatorname{csch}^{-1}(ax^2)}}{2e^{2\operatorname{csch}^{-1}(ax^2)}-2}}\left(4\sqrt{1-e^{2\operatorname{csch}^{-1}(ax^2)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};e^{2\operatorname{csch}^{-1}(ax^2)}\right)+e^{2\operatorname{csch}^{-1}(ax^2)}-1\right)}{3\sqrt{ax^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x^2]/x^2,x]
```

```
[Out] -1/3*(a*Sqrt[E^ArcCsch[a*x^2]/(-2 + 2*E^(2*ArcCsch[a*x^2]))]*x*(-1 + E^(2*A
rcCsch[a*x^2]) + 4*Sqrt[1 - E^(2*ArcCsch[a*x^2])]*Hypergeometric2F1[1/4, 1/
2, 5/4, E^(2*ArcCsch[a*x^2])]))/Sqrt[a*x^2]
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}+1}{ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + 1)/(a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a^2*x^4) + 1) + 1/(a*x^2))/x^2, x)
```

maple [C] time = 0.05, size = 111, normalized size = 1.22

$$-\frac{\sqrt{\frac{a^{2}x^{4}+1}{a^{2}x^{4}}}\,\left(-2\sqrt{-ia\,x^{2}+1}\,\sqrt{ia\,x^{2}+1}\,\operatorname{EllipticF}\left(x\sqrt{ia}\,,i\right)x^{3}a^{2}+\sqrt{ia}\,x^{4}a^{2}+\sqrt{ia}\right)}{3x\left(a^{2}x^{4}+1\right)\sqrt{ia}}-\frac{1}{3x^{3}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^2,x)
```

```
[Out] -1/3*((a^2*x^4+1)/a^2/x^4)^(1/2)*(-2*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)*El
lipticF(x*(I*a)^(1/2),I)*x^3*a^2+(I*a)^(1/2)*x^4*a^2+(I*a)^(1/2))/x/(a^2*x^
4+1)/(I*a)^(1/2)-1/3/x^3/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix};-a^{2}x^{4}\right)}{4\,x^{3}\Gamma\left(\frac{1}{4}\right)}}{a}-\frac{1}{3\,ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^4 + 1)/x^4, x)/a - 1/3/(a*x^3)
```

mupad [B] time = 2.37, size = 27, normalized size = 0.30

$$-\frac{{}_{2}\mathrm{F}_{1}\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{1}{a^{2}\,x^{4}}\right)}{x}-\frac{1}{3\,a\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2))/x^2,x)
```

```
[Out] - hypergeom([-1/2, 1/4], 5/4, -1/(a^2*x^4))/x - 1/(3*a*x^3)
```

sympy [C] time = 2.28, size = 42, normalized size = 0.46

$$-\frac{\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^{i\pi}}{a^{2}x^{4}}\right)}{4x\Gamma\left(\frac{5}{4}\right)}-\frac{1}{3ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))/x**2,x)
```

```
[Out] -gamma(1/4)*hyper((-1/2, 1/4), (5/4,), exp_polar(I*pi)/(a**2*x**4))/(4*x*ga
mma(5/4)) - 1/(3*a*x**3)
```

3.45 $\int \frac{e^{\text{csch}^{-1}(ax^2)}}{x^3}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{\frac{1}{a^2x^4}+1}}{4x^2}-\frac{1}{4ax^4}-\frac{1}{4}a\text{csch}^{-1}\left(ax^2\right)$$

[Out] -1/4/a/x^4-1/4*a*arccsch(a*x^2)-1/4*(1+1/a^2/x^4)^(1/2)/x^2

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6336, 30, 335, 275, 195, 215}

$$-\frac{\sqrt{\frac{1}{a^2x^4}+1}}{4x^2}-\frac{1}{4ax^4}-\frac{1}{4}a\text{csch}^{-1}\left(ax^2\right)$$

Antiderivative was successfully verified.

[In] Int[E^ArcCsch[a*x^2]/x^3,x]

[Out] -1/(4*a*x^4) - Sqrt[1 + 1/(a^2*x^4)]/(4*x^2) - (a*ArcCsch[a*x^2])/4

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^3}\,dx &= \frac{\int \frac{1}{x^5}\,dx}{a} + \int \frac{\sqrt{1+\frac{1}{a^2x^4}}}{x^3}\,dx\\ &= -\frac{1}{4ax^4} - \operatorname{Subst}\left(\int x\sqrt{1+\frac{x^4}{a^2}}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{1}{4ax^4} - \frac{1}{2}\operatorname{Subst}\left(\int \sqrt{1+\frac{x^2}{a^2}}\,dx, x, \frac{1}{x^2}\right)\\ &= -\frac{1}{4ax^4} - \frac{\sqrt{1+\frac{1}{a^2x^4}}}{4x^2} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x^2}\right)\\ &= -\frac{1}{4ax^4} - \frac{\sqrt{1+\frac{1}{a^2x^4}}}{4x^2} - \frac{1}{4}a\operatorname{csch}^{-1}\left(ax^2\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 24, normalized size = 0.57

$$-\frac{1}{8}a\left(2\operatorname{csch}^{-1}\left(ax^2\right)+e^{2\operatorname{csch}^{-1}\left(ax^2\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x^2]/x^3,x]
```

```
[Out] -1/8*(a*(E^(2*ArcCsch[a*x^2]) + 2*ArcCsch[a*x^2]))
```

fricas [B] time = 0.72, size = 101, normalized size = 2.40

$$-\frac{a^2x^4\log\left(ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}+1\right)-a^2x^4\log\left(ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}-1\right)+2\,ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}+2}{8\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(a^2*x^4*log(a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + 1) - a^2*x^4*log(a*
x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) - 1) + 2*a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^
4)) + 2)/(a*x^4)
```

giac [B] time = 0.14, size = 76, normalized size = 1.81

$$-\frac{a^4|a|\log\left(\sqrt{a^2x^4+1}+1\right)-a^4|a|\log\left(\sqrt{a^2x^4+1}-1\right)+\frac{2\left(\sqrt{a^2x^4+1}\,a^4|a|+a^5\right)}{a^2x^4}}{8\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(a^4*abs(a)*log(sqrt(a^2*x^4 + 1) + 1) - a^4*abs(a)*log(sqrt(a^2*x^4 +
 1) - 1) + 2*(sqrt(a^2*x^4 + 1)*a^4*abs(a) + a^5)/(a^2*x^4))/a^4
```

maple [B] time = 0.15, size = 114, normalized size = 2.71

$$-\frac{\sqrt{\frac{a^2x^4+1}{a^2x^4}}\left(\ln\left(\frac{2\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^4+1}{a^2}}\,a^2+2}{a^2x^2}\right)x^4+\sqrt{\frac{a^2x^4+1}{a^2}}\sqrt{\frac{1}{a^2}}\right)}{4x^2\sqrt{\frac{a^2x^4+1}{a^2}}\sqrt{\frac{1}{a^2}}}-\frac{1}{4x^4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^3,x)
```

```
[Out] -1/4*((a^2*x^4+1)/a^2/x^4)^(1/2)/x^2*(ln(2*((1/a^2)^(1/2)*((a^2*x^4+1)/a^2)
^(1/2)*a^2+1)/a^2/x^2)*x^4+((a^2*x^4+1)/a^2)^(1/2)*(1/a^2)^(1/2))/((a^2*x^4
+1)/a^2)^(1/2)/(1/a^2)^(1/2)-1/4/x^4/a
```

maxima [B] time = 0.31, size = 92, normalized size = 2.19

$$-\frac{a^{2}x^{2}\sqrt{\frac{1}{a^{2}x^{4}}+1}}{4\left(a^{2}x^{4}\left(\frac{1}{a^{2}x^{4}}+1\right)-1\right)}-\frac{1}{8}\,a\log\left(ax^{2}\sqrt{\frac{1}{a^{2}x^{4}}+1}+1\right)+\frac{1}{8}\,a\log\left(ax^{2}\sqrt{\frac{1}{a^{2}x^{4}}+1}-1\right)-\frac{1}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*x^2*sqrt(1/(a^2*x^4) + 1)/(a^2*x^4*(1/(a^2*x^4) + 1) - 1) - 1/8*a*
log(a*x^2*sqrt(1/(a^2*x^4) + 1) + 1) + 1/8*a*log(a*x^2*sqrt(1/(a^2*x^4) + 1
) - 1) - 1/4/(a*x^4)
```

mupad [B] time = 2.83, size = 42, normalized size = 1.00

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{\frac{1}{a^2}}}{x^2}\right)}{4\sqrt{\frac{1}{a^2}}}-\frac{\sqrt{\frac{1}{a^2\,x^4}+1}}{4\,x^2}-\frac{1}{4\,a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2))/x^3,x)
```

```
[Out] - asinh((1/a^2)^(1/2)/x^2)/(4*(1/a^2)^(1/2)) - (1/(a^2*x^4) + 1)^(1/2)/(4*x
^2) - 1/(4*a*x^4)
```

sympy [A] time = 3.97, size = 39, normalized size = 0.93

$$-\frac{a\operatorname{asinh}\left(\frac{1}{ax^{2}}\right)}{4}-\frac{\sqrt{1+\frac{1}{a^{2}x^{4}}}}{4x^{2}}-\frac{1}{4ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))/x**3,x)
```

```
[Out] -a*asinh(1/(a*x**2))/4 - sqrt(1 + 1/(a**2*x**4))/(4*x**2) - 1/(4*a*x**4)
```

3.46 $\int \frac{e^{\text{csch}^{-1}\left(ax^2\right)}}{x^4}\,dx$

Optimal. Leaf size=181

$$-\frac{\sqrt{\frac{1}{a^2x^4}+1}}{5x^3}-\frac{2a^2\sqrt{\frac{1}{a^2x^4}+1}}{5x\left(a+\frac{1}{x^2}\right)}-\frac{\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)F\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{5\sqrt{\frac{1}{a^2x^4}+1}}+\frac{2\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)E\left(2\cot^{-1}\right.}{5\sqrt{\frac{1}{a^2x^4}+1}}$$

```
[Out] -1/5/a/x^5-1/5*(1+1/a^2/x^4)^(1/2)/x^3-2/5*a^2*(1+1/a^2/x^4)^(1/2)/(a+1/x^2
)/x+2/5*(a+1/x^2)*(cos(2*arccot(x*a^(1/2)))^2)^(1/2)/cos(2*arccot(x*a^(1/2)
))*EllipticE(sin(2*arccot(x*a^(1/2))),1/2*2^(1/2))*a^(1/2)*((a^2+1/x^4)/(a+
1/x^2)^2)^(1/2)/(1+1/a^2/x^4)^(1/2)-1/5*(a+1/x^2)*(cos(2*arccot(x*a^(1/2)))
^2)^(1/2)/cos(2*arccot(x*a^(1/2)))*EllipticF(sin(2*arccot(x*a^(1/2))),1/2*2
^(1/2))*a^(1/2)*((a^2+1/x^4)/(a+1/x^2)^2)^(1/2)/(1+1/a^2/x^4)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6336, 30, 335, 279, 305, 220, 1196}

$$-\frac{2a^2\sqrt{\frac{1}{a^2x^4}+1}}{5x\left(a+\frac{1}{x^2}\right)}-\frac{\sqrt{\frac{1}{a^2x^4}+1}}{5x^3}-\frac{\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)F\left(2\cot^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{5\sqrt{\frac{1}{a^2x^4}+1}}+\frac{2\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)E\left(2\cot^{-1}\right.}{5\sqrt{\frac{1}{a^2x^4}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x^2]/x^4,x]
```

```
[Out] -1/(5*a*x^5) - Sqrt[1 + 1/(a^2*x^4)]/(5*x^3) - (2*a^2*Sqrt[1 + 1/(a^2*x^4)]
)/(5*(a + x^(-2))*x) + (2*Sqrt[a]*Sqrt[(a^2 + x^(-4))/(a + x^(-2))^2]*(a +
x^(-2))*EllipticE[2*ArcCot[Sqrt[a]*x], 1/2])/(5*Sqrt[1 + 1/(a^2*x^4)]) - (S
qrt[a]*Sqrt[(a^2 + x^(-4))/(a + x^(-2))^2]*(a + x^(-2))*EllipticF[2*ArcCot[
Sqrt[a]*x], 1/2])/(5*Sqrt[1 + 1/(a^2*x^4)])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^4}\,dx &= \frac{\int \frac{1}{x^6}\,dx}{a} + \int \frac{\sqrt{1+\frac{1}{a^2x^4}}}{x^4}\,dx \\
&= -\frac{1}{5ax^5} - \operatorname{Subst}\left(\int x^2\sqrt{1+\frac{x^4}{a^2}}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{1}{5ax^5} - \frac{\sqrt{1+\frac{1}{a^2x^4}}}{5x^3} - \frac{2}{5}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{1}{5ax^5} - \frac{\sqrt{1+\frac{1}{a^2x^4}}}{5x^3} - \frac{1}{5}(2a)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \frac{1}{x}\right) + \frac{1}{5}(2a)\operatorname{Subst}\left(\int \frac{1-\frac{x^2}{a}}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \right. \\
&= -\frac{1}{5ax^5} - \frac{\sqrt{1+\frac{1}{a^2x^4}}}{5x^3} - \frac{2a^2\sqrt{1+\frac{1}{a^2x^4}}}{5\left(a+\frac{1}{x^2}\right)x} + \frac{2\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)E\left(2\cot^{-1}\left(\sqrt{a}\,x\right)\middle|\frac{1}{2}\right)}{5\sqrt{1+\frac{1}{a^2x^4}}} - \frac{\sqrt{a}\ \cdots}{\cdots}
\end{aligned}$$

Mathematica [C] time = 0.21, size = 114, normalized size = 0.63

$$\frac{\left(ax^2\right)^{3/2}\left(4e^{2\operatorname{csch}^{-1}(ax^2)}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2\operatorname{csch}^{-1}(ax^2)}\right)+3\left(1-e^{2\operatorname{csch}^{-1}(ax^2)}\right)^{3/2}\right)}{6x^3\sqrt{2-2e^{2\operatorname{csch}^{-1}(ax^2)}}\sqrt{\frac{e^{\operatorname{csch}^{-1}(ax^2)}}{e^{2\operatorname{csch}^{-1}(ax^2)}-1}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[a*x^2]/x^4,x]
```

```
[Out] ((a*x^2)^(3/2)*(3*(1 - E^(2*ArcCsch[a*x^2]))^(3/2) + 4*E^(2*ArcCsch[a*x^2])
*Hypergeometric2F1[-1/2, 3/4, 7/4, E^(2*ArcCsch[a*x^2])]))/(6*Sqrt[2 - 2*E^
(2*ArcCsch[a*x^2])]*Sqrt[E^ArcCsch[a*x^2]/(-1 + E^(2*ArcCsch[a*x^2]))]*x^3)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}+1}{ax^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^4,x, algorithm="fricas")
[Out] integral((a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + 1)/(a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^4,x, algorithm="giac")
[Out] integrate((sqrt(1/(a^2*x^4) + 1) + 1/(a*x^2))/x^4, x)
```

maple [C] time = 0.06, size = 171, normalized size = 0.94

$$\frac{\sqrt{\frac{a^2x^4+1}{a^2x^4}}\left(-2\sqrt{ia}\,x^8a^4+2ia^3\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,x^5\,\text{EllipticF}\left(x\sqrt{ia},i\right)-2ia^3\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,x^5\,\text{E}\right.}{5x^3\left(a^2x^4+1\right)\sqrt{ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^4,x)
[Out] 1/5*((a^2*x^4+1)/a^2/x^4)^(1/2)*(-2*(I*a)^(1/2)*x^8*a^4+2*I*a^3*(1-I*a*x^2)
^(1/2)*(1+I*a*x^2)^(1/2)*x^5*EllipticF(x*(I*a)^(1/2),I)-2*I*a^3*(1-I*a*x^2)
^(1/2)*(1+I*a*x^2)^(1/2)*x^5*EllipticE(x*(I*a)^(1/2),I)-3*(I*a)^(1/2)*x^4*a
^2-(I*a)^(1/2))/x^3/(a^2*x^4+1)/(I*a)^(1/2)-1/5/a/x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix};-a^2x^4\right)}{4\,x^5\Gamma\left(-\frac{1}{4}\right)}}{a}-\frac{1}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(a^2*x^4 + 1)/x^6, x)/a - 1/5/(a*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{a^2\,x^4}+1}+\frac{1}{a\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2))/x^4,x)
```

```
[Out] int(((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2))/x^4, x)
```

sympy [C] time = 2.56, size = 44, normalized size = 0.24

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^{i\pi}}{a^2x^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)}-\frac{1}{5ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))/x**4,x)
```

```
[Out] -gamma(3/4)*hyper((-1/2, 3/4), (7/4,), exp_polar(I*pi)/(a**2*x**4))/(4*x**3
*gamma(7/4)) - 1/(5*a*x**5)
```

3.47 $\int \frac{e^{\mathrm{csch}^{-1}(ax^2)}}{x^5}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{6}a^2\left(\frac{1}{a^2x^4}+1\right)^{3/2}-\frac{1}{6ax^6}$$

```
[Out] -1/6*a^2*(1+1/a^2/x^4)^(3/2)-1/6/a/x^6
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6336, 30, 261}

$$-\frac{1}{6}a^2\left(\frac{1}{a^2x^4}+1\right)^{3/2}-\frac{1}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[a*x^2]/x^5,x]
```

```
[Out] -(a^2*(1 + 1/(a^2*x^4))^(3/2))/6 - 1/(6*a*x^6)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6336

```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Dist[1/a, Int[x^(m
 - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p}, x
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\mathrm{csch}^{-1}(ax^2)}}{x^5}\,dx &= \frac{\int \frac{1}{x^7}\,dx}{a}+\int \frac{\sqrt{1+\frac{1}{a^2x^4}}}{x^5}\,dx\\ &= -\frac{1}{6}a^2\left(1+\frac{1}{a^2x^4}\right)^{3/2}-\frac{1}{6ax^6}\end{aligned}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 1.26

$$-\frac{ax^2\sqrt{\frac{1}{a^2x^4}+1}\,\left(a^2x^4+1\right)+1}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCsch[a*x^2]/x^5,x]
```

```
[Out] -1/6*(1 + a*Sqrt[1 + 1/(a^2*x^4)]*x^2*(1 + a^2*x^4))/(a*x^6)
```

fricas [A] time = 0.56, size = 49, normalized size = 1.58

$$-\frac{a^3x^6 + \left(a^3x^6 + ax^2\right)\sqrt{\frac{a^2x^4+1}{a^2x^4}} + 1}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^5,x, algorithm="fricas")
[Out] -1/6*(a^3*x^6 + (a^3*x^6 + a*x^2)*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + 1)/(a*x^6
)
```

giac [B] time = 0.14, size = 71, normalized size = 2.29

$$\frac{\frac{2\left(3\left(x^2|a|-\sqrt{a^2x^4+1}\right)^4a^4+a^4\right)}{\left(\left(x^2|a|-\sqrt{a^2x^4+1}\right)^2-1\right)^3}-\frac{a}{x^6}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^5,x, algorithm="giac")
[Out] 1/6*(2*(3*(x^2*abs(a) - sqrt(a^2*x^4 + 1))^4*a^4 + a^4)/((x^2*abs(a) - sqrt
(a^2*x^4 + 1))^2 - 1)^3 - a/x^6)/a^2
```

maple [A] time = 0.14, size = 42, normalized size = 1.35

$$-\frac{\sqrt{\frac{a^2x^4+1}{a^2x^4}}\left(a^2x^4+1\right)}{6x^4}-\frac{1}{6x^6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^5,x)
[Out] -1/6*((a^2*x^4+1)/a^2/x^4)^(1/2)/x^4*(a^2*x^4+1)-1/6/x^6/a
```

maxima [A] time = 0.32, size = 25, normalized size = 0.81

$$-\frac{1}{6}\,a^2\left(\frac{1}{a^2x^4}+1\right)^{\frac{3}{2}}-\frac{1}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^5,x, algorithm="maxima")
[Out] -1/6*a^2*(1/(a^2*x^4) + 1)^(3/2) - 1/6/(a*x^6)
```

mupad [B] time = 2.16, size = 44, normalized size = 1.42

$$-\frac{\frac{1}{6\,a}+\frac{x^2\sqrt{\frac{1}{a^2\,x^4}+1}}{6}}{x^6}-\frac{a^2\sqrt{\frac{1}{a^2\,x^4}+1}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2))/x^5,x)
[Out] - (1/(6*a) + (x^2*(1/(a^2*x^4) + 1)^(1/2))/6)/x^6 - (a^2*(1/(a^2*x^4) + 1)^
(1/2))/6
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))/x**5,x)
```

```
[Out] Exception raised: TypeError
```

3.48 $\int e^{2\text{csch}^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=64

$$\frac{2x^m \,_2F_1\left(-\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};-\frac{1}{a^2x^2}\right)}{am}-\frac{2x^{m-1}}{a^2(1-m)}+\frac{x^{m+1}}{m+1}$$

```
[Out] -2*x^(-1+m)/a^2/(1-m)+x^(1+m)/(1+m)+2*x^m*hypergeom([-1/2, -1/2*m],[1-1/2*m
],-1/a^2/x^2)/a/m
```

Rubi [A] time = 0.32, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6338, 6742, 339, 364}

$$\frac{2x^m \,_2F_1\left(-\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};-\frac{1}{a^2x^2}\right)}{am}-\frac{2x^{m-1}}{a^2(1-m)}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCsch[a*x])*x^m,x]
```

```
[Out] (-2*x^(-1 + m))/(a^2*(1 - m)) + x^(1 + m)/(1 + m) + (2*x^m*Hypergeometric2F
1[-1/2, -m/2, 1 - m/2, -(1/(a^2*x^2))])/(a*m)
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6338

```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\operatorname{csch}^{-1}(ax)}x^m\,dx &= \int \left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)^2 x^m\,dx \\
&= \int \left(\frac{2x^{-2+m}}{a^2}+\frac{2\sqrt{1+\frac{1}{a^2x^2}}\,x^{-1+m}}{a}+x^m\right)dx \\
&= -\frac{2x^{-1+m}}{a^2(1-m)}+\frac{x^{1+m}}{1+m}+\frac{2\int \sqrt{1+\frac{1}{a^2x^2}}\,x^{-1+m}\,dx}{a} \\
&= -\frac{2x^{-1+m}}{a^2(1-m)}+\frac{x^{1+m}}{1+m}-\frac{\left(2\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int x^{-1-m}\sqrt{1+\frac{x^2}{a^2}}\,dx,x,\frac{1}{x}\right)}{a} \\
&= -\frac{2x^{-1+m}}{a^2(1-m)}+\frac{x^{1+m}}{1+m}+\frac{2x^m\,{}_2F_1\left(-\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};-\frac{1}{a^2x^2}\right)}{am}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.89

$$
x^m\left(\frac{2\,{}_2F_1\left(-\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};-\frac{1}{a^2x^2}\right)}{am}+\frac{2}{a^2(m-1)x}+\frac{x}{m+1}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCsch[a*x])*x^m,x]
```

```
[Out] x^m*(2/(a^2*(-1 + m)*x) + x/(1 + m) + (2*Hypergeometric2F1[-1/2, -1/2*m, 1
- m/2, -(1/(a^2*x^2))])/(a*m))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\frac{2\,axx^m\sqrt{\frac{a^2x^2+1}{a^2x^2}}+\left(a^2x^2+2\right)x^m}{a^2x^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^m,x, algorithm="fricas")
```

```
[Out] integral((2*a*x*x^m*sqrt((a^2*x^2 + 1)/(a^2*x^2)) + (a^2*x^2 + 2)*x^m)/(a^2
*x^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$
\int \left(\frac{1}{ax}+\sqrt{1+\frac{1}{x^2a^2}}\right)^2 x^m\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))^2*x^m,x)
```

```
[Out] int((1/a/x+(1+1/x^2/a^2)^(1/2))^2*x^m,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is m-2 equal to -1?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \left(\sqrt{\frac{1}{a^2\,x^2}+1}+\frac{1}{a\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] int(x^m*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2, x)
```

sympy [A] time = 7.15, size = 71, normalized size = 1.11

$$\begin{cases} \frac{x^{m+1}}{m+1} & \text{for}\: m \neq -1 \\ \log{\left(x \right)} & \text{otherwise} \end{cases} - \frac{x^{m} \Gamma\left(- \frac{m}{2}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{1}{2}, - \frac{m}{2} \\ 1 - \frac{m}{2} \end{matrix}\middle| {\frac{e^{i \pi}}{a^{2} x^{2}}} \right)}}{a \Gamma\left(1 - \frac{m}{2}\right)} + \frac{2 \left(\begin{cases} \frac{x^{m}}{m x - x} & \text{for}\: m \neq 1 \\ \log{\left(x \right)} & \text{otherwise} \end{cases}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2*x**m,x)
```

```
[Out] Piecewise((x**(m + 1)/(m + 1), Ne(m, -1)), (log(x), True)) - x**m*gamma(-m/
2)*hyper((-1/2, -m/2), (1 - m/2,), exp_polar(I*pi)/(a**2*x**2))/(a*gamma(1
- m/2)) + 2*Piecewise((x**m/(m*x - x), Ne(m, 1)), (log(x), True))/a**2
```

3.49 $\int e^{2\text{csch}^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=85

$$\frac{2x^3}{3a^2}+\frac{x^4\sqrt{\frac{1}{a^2x^2}+1}}{2a}-\frac{\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{4a^5}+\frac{x^2\sqrt{\frac{1}{a^2x^2}+1}}{4a^3}+\frac{x^5}{5}$$

```
[Out] 2/3*x^3/a^2+1/5*x^5-1/4*arctanh((1+1/a^2/x^2)^(1/2))/a^5+1/4*x^2*(1+1/a^2/x
^2)^(1/2)/a^3+1/2*x^4*(1+1/a^2/x^2)^(1/2)/a
```

Rubi [A] time = 0.25, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6338, 6742, 266, 47, 51, 63, 208}

$$\frac{x^4\sqrt{\frac{1}{a^2x^2}+1}}{2a}+\frac{2x^3}{3a^2}+\frac{x^2\sqrt{\frac{1}{a^2x^2}+1}}{4a^3}-\frac{\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{4a^5}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCsch[a*x])*x^4,x]
```

```
[Out] (Sqrt[1 + 1/(a^2*x^2)]*x^2)/(4*a^3) + (2*x^3)/(3*a^2) + (Sqrt[1 + 1/(a^2*x^
2)]*x^4)/(2*a) + x^5/5 - ArcTanh[Sqrt[1 + 1/(a^2*x^2)]]/(4*a^5)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6338

```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\text{csch}^{-1}(ax)}x^4\,dx &= \int\left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)^2 x^4\,dx\\
&= \int\left(\frac{2x^2}{a^2}+\frac{2\sqrt{1+\frac{1}{a^2x^2}}\,x^3}{a}+x^4\right)dx\\
&= \frac{2x^3}{3a^2}+\frac{x^5}{5}+\frac{2\int\sqrt{1+\frac{1}{a^2x^2}}\,x^3\,dx}{a}\\
&= \frac{2x^3}{3a^2}+\frac{x^5}{5}-\frac{\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a^2}}}{x^3}\,dx,x,\frac{1}{x^2}\right)}{a}\\
&= \frac{2x^3}{3a^2}+\frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^4}{2a}+\frac{x^5}{5}-\frac{\text{Subst}\left(\int\frac{1}{x^2\sqrt{1+\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{4a^3}\\
&= \frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^2}{4a^3}+\frac{2x^3}{3a^2}+\frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^4}{2a}+\frac{x^5}{5}+\frac{\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{8a^5}\\
&= \frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^2}{4a^3}+\frac{2x^3}{3a^2}+\frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^4}{2a}+\frac{x^5}{5}+\frac{\text{Subst}\left(\int\frac{1}{-a^2+a^2x^2}\,dx,x,\sqrt{1+\frac{1}{a^2x^2}}\right)}{4a^3}\\
&= \frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^2}{4a^3}+\frac{2x^3}{3a^2}+\frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^4}{2a}+\frac{x^5}{5}-\frac{\tanh^{-1}\left(\sqrt{1+\frac{1}{a^2x^2}}\right)}{4a^5}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 84, normalized size = 0.99

$$
\frac{a^2x^2\left(12a^3x^3+30a^2x^2\sqrt{\frac{1}{a^2x^2}+1}+15\sqrt{\frac{1}{a^2x^2}+1}+40ax\right)-15\log\left(x\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)\right)}{60a^5}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCsch[a*x])*x^4,x]
```

```
[Out] (a^2*x^2*(15*Sqrt[1 + 1/(a^2*x^2)] + 40*a*x + 30*a^2*Sqrt[1 + 1/(a^2*x^2)]*
x^2 + 12*a^3*x^3) - 15*Log[(1 + Sqrt[1 + 1/(a^2*x^2)])*x])/(60*a^5)
```

fricas [A] time = 0.67, size = 87, normalized size = 1.02

$$
\frac{12\,a^5x^5+40\,a^3x^3+15\left(2\,a^4x^4+a^2x^2\right)\sqrt{\frac{a^2x^2+1}{a^2x^2}}+15\,\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax\right)}{60\,a^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^4,x, algorithm="fricas")
```

```
[Out] 1/60*(12*a^5*x^5 + 40*a^3*x^3 + 15*(2*a^4*x^4 + a^2*x^2)*sqrt((a^2*x^2 + 1)
/(a^2*x^2)) + 15*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x))/a^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

maple [A] time = 0.04, size = 114, normalized size = 1.34

$$\frac{x^5}{5}+\frac{2x^3}{3a^2}-\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\, x\left(-2x\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}} a^4+x\sqrt{\frac{a^2x^2+1}{a^2}}\, a^2+\ln\left(x+\sqrt{\frac{a^2x^2+1}{a^2}}\right)\right)}{4a^5\sqrt{\frac{a^2x^2+1}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))^2*x^4,x)
```

```
[Out] 1/5*x^5+2/3*x^3/a^2-1/4/a^5*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(-2*x*((a^2*x^2+1
)/a^2)^(3/2)*a^4+x*((a^2*x^2+1)/a^2)^(1/2)*a^2+ln(x+((a^2*x^2+1)/a^2)^(1/2)
))/((a^2*x^2+1)/a^2)^(1/2)
```

maxima [A] time = 0.31, size = 117, normalized size = 1.38

$$\frac{1}{5}x^5+\frac{2\,x^3}{3\,a^2}+\frac{\frac{2\left(\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}+\sqrt{\frac{1}{a^2x^2}+1}\right)}{a^4\left(\frac{1}{a^2x^2}+1\right)^2-2\,a^4\left(\frac{1}{a^2x^2}+1\right)+a^4}-\frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)}{a^4}+\frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{a^4}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^4,x, algorithm="maxima")
```

```
[Out] 1/5*x^5 + 2/3*x^3/a^2 + 1/8*(2*((1/(a^2*x^2) + 1)^(3/2) + sqrt(1/(a^2*x^2)
+ 1))/(a^4*(1/(a^2*x^2) + 1)^2 - 2*a^4*(1/(a^2*x^2) + 1) + a^4) - log(sqrt(
1/(a^2*x^2) + 1) + 1)/a^4 + log(sqrt(1/(a^2*x^2) + 1) - 1)/a^4)/a
```

mupad [B] time = 2.17, size = 73, normalized size = 0.86

$$\frac{x^5}{5}+\frac{2\,x^3}{3\,a^2}+\frac{x^4\,\sqrt{\frac{1}{a^2\,x^2}+1}}{2\,a}+\frac{x^2\,\sqrt{\frac{1}{a^2\,x^2}+1}}{4\,a^3}+\frac{\mathrm{atan}\left(\sqrt{\frac{1}{a^2\,x^2}+1}\;1{}\mathrm{i}\right)\,1{}\mathrm{i}}{4\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] (atan((1/(a^2*x^2) + 1)^(1/2)*1i)*1i)/(4*a^5) + x^5/5 + (2*x^3)/(3*a^2) + (
x^4*(1/(a^2*x^2) + 1)^(1/2))/(2*a) + (x^2*(1/(a^2*x^2) + 1)^(1/2))/(4*a^3)
```

sympy [**A**] time = 4.72, size = 82, normalized size = 0.96

$$\frac{x^5}{5}+\frac{x^5}{2\sqrt{a^2x^2+1}}+\frac{2x^3}{3a^2}+\frac{3x^3}{4a^2\sqrt{a^2x^2+1}}+\frac{x}{4a^4\sqrt{a^2x^2+1}}-\frac{\operatorname{asinh}(ax)}{4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2*x**4,x)
```

```
[Out] x**5/5 + x**5/(2*sqrt(a**2*x**2 + 1)) + 2*x**3/(3*a**2) + 3*x**3/(4*a**2*sq
rt(a**2*x**2 + 1)) + x/(4*a**4*sqrt(a**2*x**2 + 1)) - asinh(a*x)/(4*a**5)
```

3.50 $\int e^{2\text{csch}^{-1}(ax)} x^3\,dx$

Optimal. Leaf size=38

$$\frac{x^2}{a^2}+\frac{2x^3\left(\frac{1}{a^2x^2}+1\right)^{3/2}}{3a}+\frac{x^4}{4}$$

```
[Out] x^2/a^2+2/3*(1+1/a^2/x^2)^(3/2)*x^3/a+1/4*x^4
```

Rubi [A] time = 0.22, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6338, 6742, 264}

$$\frac{2x^3\left(\frac{1}{a^2x^2}+1\right)^{3/2}}{3a}+\frac{x^2}{a^2}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCsch[a*x])*x^3,x]
```

```
[Out] x^2/a^2 + (2*(1 + 1/(a^2*x^2))^(3/2)*x^3)/(3*a) + x^4/4
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6338

```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int e^{2\text{csch}^{-1}(ax)}x^3\,dx &= \int \left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)^2 x^3\,dx\\
&= \int \left(\frac{2x}{a^2}+\frac{2\sqrt{1+\frac{1}{a^2x^2}}\,x^2}{a}+x^3\right)dx\\
&= \frac{x^2}{a^2}+\frac{x^4}{4}+\frac{2\int \sqrt{1+\frac{1}{a^2x^2}}\,x^2\,dx}{a}\\
&= \frac{x^2}{a^2}+\frac{2\left(1+\frac{1}{a^2x^2}\right)^{3/2}x^3}{3a}+\frac{x^4}{4}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 1.16

$$\frac{x^2}{a^2}+\frac{2\sqrt{\frac{1}{a^2x^2}+1}\left(a^2x^3+x\right)}{3a^3}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCsch[a*x])*x^3,x]
```

```
[Out] x^2/a^2 + x^4/4 + (2*Sqrt[1 + 1/(a^2*x^2)]*(x + a^2*x^3))/(3*a^3)
```

fricas [A] time = 0.53, size = 49, normalized size = 1.29

$$\frac{3\,a^3x^4+12\,ax^2+8\,\left(a^2x^3+x\right)\sqrt{\frac{a^2x^2+1}{a^2x^2}}}{12\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^3*x^4 + 12*a*x^2 + 8*(a^2*x^3 + x)*sqrt((a^2*x^2 + 1)/(a^2*x^2)))
/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

maple [A] time = 0.05, size = 61, normalized size = 1.61

$$\frac{\frac{1}{4}a^2x^4+\frac{1}{2}x^2}{a^2}+\frac{2\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,x\left(a^2x^2+1\right)}{3a^3}+\frac{x^2}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))^2*x^3,x)
```

```
[Out] 1/a^2*(1/4*a^2*x^4+1/2*x^2)+2/3/a^3*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(a^2*x^2+
1)+1/2*x^2/a^2
```

maxima [A] time = 0.31, size = 32, normalized size = 0.84

$$\frac{1}{4}\,x^4+\frac{2\,x^3\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}}{3\,a}+\frac{x^2}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 2/3*x^3*(1/(a^2*x^2) + 1)^(3/2)/a + x^2/a^2
```

mupad [B] time = 2.14, size = 40, normalized size = 1.05

$$\sqrt{\frac{1}{a^2\,x^2}+1}\,\left(\frac{2\,x}{3\,a^3}+\frac{2\,x^3}{3\,a}\right)+\frac{x^4}{4}+\frac{x^2}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] (1/(a^2*x^2) + 1)^(1/2)*((2*x)/(3*a^3) + (2*x^3)/(3*a)) + x^4/4 + x^2/a^2
```

sympy [A] time = 2.77, size = 51, normalized size = 1.34

$$\frac{x^4}{4} + \frac{2x^2\sqrt{a^2x^2+1}}{3a^2} + \frac{x^2}{a^2} + \frac{2\sqrt{a^2x^2+1}}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2*x**3,x)
```

```
[Out] x**4/4 + 2*x**2*sqrt(a**2*x**2 + 1)/(3*a**2) + x**2/a**2 + 2*sqrt(a**2*x**2
 + 1)/(3*a**4)
```

3.51 $\int e^{2\text{csch}^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=52

$$\frac{x^2\sqrt{\frac{1}{a^2x^2}+1}}{a}+\frac{2x}{a^2}+\frac{\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{a^3}+\frac{x^3}{3}$$

[Out] 2*x/a^2+1/3*x^3+arctanh((1+1/a^2/x^2)^(1/2))/a^3+x^2*(1+1/a^2/x^2)^(1/2)/a

Rubi [A] time = 0.24, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6338, 6742, 266, 47, 63, 208}

$$\frac{x^2\sqrt{\frac{1}{a^2x^2}+1}}{a}+\frac{\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{a^3}+\frac{2x}{a^2}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCsch[a*x])*x^2,x]

[Out] (2*x)/a^2 + (Sqrt[1 + 1/(a^2*x^2)]*x^2)/a + x^3/3 + ArcTanh[Sqrt[1 + 1/(a^2*x^2)]]/a^3

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6338

```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int e^{2\operatorname{csch}^{-1}(ax)}x^2\,dx &= \int \left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)^2 x^2\,dx\\
&= \int \left(\frac{2}{a^2}+\frac{2\sqrt{1+\frac{1}{a^2x^2}}\,x}{a}+x^2\right)dx\\
&= \frac{2x}{a^2}+\frac{x^3}{3}+\frac{2\int \sqrt{1+\frac{1}{a^2x^2}}\,x\,dx}{a}\\
&= \frac{2x}{a^2}+\frac{x^3}{3}-\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a^2}}}{x^2}\,dx,x,\frac{1}{x^2}\right)}{a}\\
&= \frac{2x}{a^2}+\frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^2}{a}+\frac{x^3}{3}-\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{2a^3}\\
&= \frac{2x}{a^2}+\frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^2}{a}+\frac{x^3}{3}-\frac{\operatorname{Subst}\left(\int \frac{1}{-a^2+a^2x^2}\,dx,x,\sqrt{1+\frac{1}{a^2x^2}}\right)}{a}\\
&= \frac{2x}{a^2}+\frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^2}{a}+\frac{x^3}{3}+\frac{\tanh^{-1}\left(\sqrt{1+\frac{1}{a^2x^2}}\right)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 1.10

$$\frac{ax\left(a^2x^2+3ax\sqrt{\frac{1}{a^2x^2}+1}+6\right)+3\log\left(x\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)\right)}{3a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCsch[a*x])*x^2,x]
[Out] (a*x*(6 + 3*a*Sqrt[1 + 1/(a^2*x^2)]*x + a^2*x^2) + 3*Log[(1 + Sqrt[1 + 1/(a
^2*x^2)])*x])/(3*a^3)
```

fricas [A] time = 0.77, size = 72, normalized size = 1.38

$$\frac{a^3x^3+3\,a^2x^2\sqrt{\frac{a^2x^2+1}{a^2x^2}}+6\,ax-3\,\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^2,x, algorithm="fricas")
[Out] 1/3*(a^3*x^3 + 3*a^2*x^2*sqrt((a^2*x^2 + 1)/(a^2*x^2)) + 6*a*x - 3*log(a*x*
sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

maple [A] time = 0.05, size = 90, normalized size = 1.73

$$\frac{x^3}{3}+\frac{2x}{a^2}+\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,x\left(x\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+\ln\left(x+\sqrt{\frac{a^2x^2+1}{a^2}}\right)\right)}{a^3\sqrt{\frac{a^2x^2+1}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))^2*x^2,x)
```

```
[Out] 1/3*x^3+2*x/a^2+1/a^3*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(x*((a^2*x^2+1)/a^2)^(1
/2)*a^2+ln(x+((a^2*x^2+1)/a^2)^(1/2)))/((a^2*x^2+1)/a^2)^(1/2)
```

maxima [A] time = 0.32, size = 89, normalized size = 1.71

$$\frac{1}{3}\,x^3+\frac{\frac{2\sqrt{\frac{1}{a^2x^2}+1}}{a^2\left(\frac{1}{a^2x^2}+1\right)-a^2}+\frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)}{a^2}-\frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{a^2}}{2\,a}+\frac{2\,x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*(2*sqrt(1/(a^2*x^2) + 1)/(a^2*(1/(a^2*x^2) + 1) - a^2) + log(
sqrt(1/(a^2*x^2) + 1) + 1)/a^2 - log(sqrt(1/(a^2*x^2) + 1) - 1)/a^2)/a + 2*
x/a^2
```

mupad [B] time = 2.13, size = 51, normalized size = 0.98

$$\frac{2\,x}{a^2}+\frac{x^3}{3}+\frac{x^2\,\sqrt{\frac{1}{a^2\,x^2}+1}}{a}-\frac{\mathrm{atan}\left(\sqrt{\frac{1}{a^2\,x^2}+1}\;1{}\mathrm{i}\right)\,1{}\mathrm{i}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] (2*x)/a^2 - (atan((1/(a^2*x^2) + 1)^(1/2)*1i)*1i)/a^3 + x^3/3 + (x^2*(1/(a^
2*x^2) + 1)^(1/2))/a
```

sympy [A] time = 3.33, size = 36, normalized size = 0.69

$$\frac{x^3}{3}+\frac{x\sqrt{a^2x^2+1}}{a^2}+\frac{2x}{a^2}+\frac{\operatorname{asinh}{(ax)}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2*x**2,x)
```

```
[Out] x**3/3 + x*sqrt(a**2*x**2 + 1)/a**2 + 2*x/a**2 + asinh(a*x)/a**3
```

3.52 $\int e^{2\text{csch}^{-1}(ax)}x\,dx$

Optimal. Leaf size=43

$$\frac{2x\sqrt{\frac{1}{a^2x^2}+1}}{a}+\frac{2\log(x)}{a^2}-\frac{2\text{csch}^{-1}(ax)}{a^2}+\frac{x^2}{2}$$

[Out] 1/2*x^2-2*arccsch(a*x)/a^2+2*ln(x)/a^2+2*x*(1+1/a^2/x^2)^(1/2)/a

Rubi [A] time = 0.16, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6338, 6742, 242, 277, 215}

$$\frac{2x\sqrt{\frac{1}{a^2x^2}+1}}{a}+\frac{2\log(x)}{a^2}-\frac{2\text{csch}^{-1}(ax)}{a^2}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCsch[a*x])*x,x]

[Out] (2*Sqrt[1 + 1/(a^2*x^2)]*x)/a + x^2/2 - (2*ArcCsch[a*x])/a^2 + (2*Log[x])/a^2

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6338

```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 + 1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int e^{2\operatorname{csch}^{-1}(ax)}x\,dx &= \int \left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)^2 x\,dx \\
&= \int \left(\frac{2\sqrt{1+\frac{1}{a^2x^2}}}{a}+\frac{2}{a^2x}+x\right)dx \\
&= \frac{x^2}{2}+\frac{2\log(x)}{a^2}+\frac{2\int \sqrt{1+\frac{1}{a^2x^2}}\,dx}{a} \\
&= \frac{x^2}{2}+\frac{2\log(x)}{a^2}-\frac{2\,\text{Subst}\left(\int \frac{\sqrt{1+\frac{x^2}{a^2}}}{x^2}\,dx,x,\frac{1}{x}\right)}{a} \\
&= \frac{2\sqrt{1+\frac{1}{a^2x^2}}\,x}{a}+\frac{x^2}{2}+\frac{2\log(x)}{a^2}-\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^3} \\
&= \frac{2\sqrt{1+\frac{1}{a^2x^2}}\,x}{a}+\frac{x^2}{2}-\frac{2\operatorname{csch}^{-1}(ax)}{a^2}+\frac{2\log(x)}{a^2}
\end{aligned}$$

Mathematica [**A**] time = 0.05, size = 44, normalized size = 1.02

$$\frac{ax\left(4\sqrt{\frac{1}{a^2x^2}+1}+ax\right)-4\sinh^{-1}\left(\frac{1}{ax}\right)+4\log(x)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCsch[a*x])*x,x]
```

```
[Out] (a*x*(4*Sqrt[1 + 1/(a^2*x^2)] + a*x) - 4*ArcSinh[1/(a*x)] + 4*Log[x])/(2*a^
2)
```

fricas [**B**] time = 0.61, size = 99, normalized size = 2.30

$$\frac{a^2x^2+4\,ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-4\,\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax+1\right)+4\,\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax-1\right)+4\,\log(x)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2 + 4*a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - 4*log(a*x*sqrt((a^2*x^
2 + 1)/(a^2*x^2)) - a*x + 1) + 4*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*
x - 1) + 4*log(x))/a^2
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Warning, integration of abs or sign assumes
```

```
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

maple [B] time = 0.06, size = 120, normalized size = 2.79

$$\frac{x^{2}}{2}+\frac{2\ln(x)}{a^{2}}+\frac{2\sqrt{\frac{a^{2}x^{2}+1}{a^{2}x^{2}}}\,x\left(\sqrt{\frac{1}{a^{2}}}\,\sqrt{\frac{a^{2}x^{2}+1}{a^{2}}}\,a^{2}-\ln\left(\frac{2\sqrt{\frac{1}{a^{2}}}\,\sqrt{\frac{a^{2}x^{2}+1}{a^{2}}}\,a^{2}+2}{a^{2}x}\right)\right)}{a^{3}\sqrt{\frac{a^{2}x^{2}+1}{a^{2}}}\,\sqrt{\frac{1}{a^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))^2*x,x)
```

```
[Out] 1/2*x^2+2*ln(x)/a^2+2/a^3*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*((1/a^2)^(1/2)*((a^
2*x^2+1)/a^2)^(1/2)*a^2-ln(2*((1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*a^2+1)/
a^2/x))/((a^2*x^2+1)/a^2)^(1/2)/(1/a^2)^(1/2)
```

maxima [A] time = 0.31, size = 75, normalized size = 1.74

$$\frac{1}{2}x^{2}+\frac{2x\sqrt{\frac{1}{a^{2}x^{2}}+1}-\frac{\log\left(ax\sqrt{\frac{1}{a^{2}x^{2}}+1}+1\right)}{a}+\frac{\log\left(ax\sqrt{\frac{1}{a^{2}x^{2}}+1}-1\right)}{a}}{a}+\frac{2\log(x)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + (2*x*sqrt(1/(a^2*x^2) + 1) - log(a*x*sqrt(1/(a^2*x^2) + 1) + 1)/a
 + log(a*x*sqrt(1/(a^2*x^2) + 1) - 1)/a)/a + 2*log(x)/a^2
```

mupad [B] time = 2.20, size = 52, normalized size = 1.21

$$\frac{x^{2}}{2}-\frac{2\ln\left(\frac{1}{x}\right)}{a^{2}}+\frac{2x\sqrt{\frac{1}{a^{2}x^{2}}+1}}{a}-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{\frac{1}{a^{2}}}}{x}\right)}{a^{3}\sqrt{\frac{1}{a^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] x^2/2 - (2*log(1/x))/a^2 + (2*x*(1/(a^2*x^2) + 1)^(1/2))/a - (2*asinh((1/a^
2)^(1/2)/x))/(a^3*(1/a^2)^(1/2))
```

sympy [A] time = 3.79, size = 63, normalized size = 1.47

$$\frac{x^{2}}{2}+\frac{2x}{a\sqrt{1+\frac{1}{a^{2}x^{2}}}}+\frac{2\log(x)}{a^{2}}-\frac{2\,\mathrm{asinh}\left(\frac{1}{ax}\right)}{a^{2}}+\frac{2}{a^{3}x\sqrt{1+\frac{1}{a^{2}x^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2*x,x)
```

```
[Out] x**2/2 + 2*x/(a*sqrt(1 + 1/(a**2*x**2))) + 2*log(x)/a**2 - 2*asinh(1/(a*x))
/a**2 + 2/(a**3*x*sqrt(1 + 1/(a**2*x**2)))
```

3.53 $\int e^{2\text{csch}^{-1}(ax)}\,dx$

Optimal. Leaf size=47

$$-\frac{2\sqrt{\frac{1}{a^2x^2}+1}}{a}+\frac{2\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{a}-\frac{2}{a^2x}+x$$

```
[Out] -2/a^2/x+x+2*arctanh((1+1/a^2/x^2)^(1/2))/a-2*(1+1/a^2/x^2)^(1/2)/a
```

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6333, 6742, 266, 50, 63, 208}

$$-\frac{2\sqrt{\frac{1}{a^2x^2}+1}}{a}+\frac{2\tanh^{-1}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{a}-\frac{2}{a^2x}+x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCsch[a*x]),x]
```

```
[Out] (-2*Sqrt[1 + 1/(a^2*x^2)])/a - 2/(a^2*x) + x + (2*ArcTanh[Sqrt[1 + 1/(a^2*x
^2)]])/a
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6333

```
Int[E^(ArcCsch[u_]*(n_.)), x_Symbol] :> Int[(1/u + Sqrt[1 + 1/u^2])^n, x] /
; IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int e^{2\operatorname{csch}^{-1}(ax)}\,dx &= \int \left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)^2 dx\\
&= \int \left(1+\frac{2}{a^2x^2}+\frac{2\sqrt{1+\frac{1}{a^2x^2}}}{ax}\right) dx\\
&= -\frac{2}{a^2x}+x+\frac{2\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x}\,dx}{a}\\
&= -\frac{2}{a^2x}+x-\frac{\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a^2}}}{x}\,dx,x,\frac{1}{x^2}\right)}{a}\\
&= -\frac{2\sqrt{1+\frac{1}{a^2x^2}}}{a}-\frac{2}{a^2x}+x-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{a}\\
&= -\frac{2\sqrt{1+\frac{1}{a^2x^2}}}{a}-\frac{2}{a^2x}+x-(2a)\,\text{Subst}\left(\int \frac{1}{-a^2+a^2x^2}\,dx,x,\sqrt{1+\frac{1}{a^2x^2}}\right)\\
&= -\frac{2\sqrt{1+\frac{1}{a^2x^2}}}{a}-\frac{2}{a^2x}+x+\frac{2\tanh^{-1}\left(\sqrt{1+\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 1.11

$$-\frac{2\sqrt{\frac{1}{a^2x^2}+1}}{a}+\frac{2\log\left(ax\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)\right)}{a}-\frac{2}{a^2x}+x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCsch[a*x]),x]
```

```
[Out] (-2*Sqrt[1 + 1/(a^2*x^2)])/a - 2/(a^2*x) + x + (2*Log[a*(1 + Sqrt[1 + 1/(a^
2*x^2)])*x])/a
```

fricas [A] time = 0.94, size = 73, normalized size = 1.55

$$\frac{a^2x^2-2\,ax\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax\right)-2\,ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-2\,ax-2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - 2*a*x*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x) - 2*a*x*sqrt(
(a^2*x^2 + 1)/(a^2*x^2)) - 2*a*x - 2)/(a^2*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.05, size = 112, normalized size = 2.38

$$x-\frac{2}{a^{2}x}+\frac{2\sqrt{\frac{a^{2}x^{2}+1}{a^{2}x^{2}}}\left(-a^{2}\left(\frac{a^{2}x^{2}+1}{a^{2}}\right)^{\frac{3}{2}}+\sqrt{\frac{a^{2}x^{2}+1}{a^{2}}}\,x^{2}a^{2}+\ln\left(x+\sqrt{\frac{a^{2}x^{2}+1}{a^{2}}}\right)x\right)}{a\sqrt{\frac{a^{2}x^{2}+1}{a^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))^2,x)
```

```
[Out] x-2/a^2/x+2/a*((a^2*x^2+1)/a^2/x^2)^(1/2)*(-a^2*((a^2*x^2+1)/a^2)^(3/2)+((a
^2*x^2+1)/a^2)^(1/2)*x^2*a^2+ln(x+((a^2*x^2+1)/a^2)^(1/2))*x)/((a^2*x^2+1)/
a^2)^(1/2)
```

maxima [A] time = 0.31, size = 59, normalized size = 1.26

$$x-\frac{2\sqrt{\frac{1}{a^{2}x^{2}}+1}-\log\left(\sqrt{\frac{1}{a^{2}x^{2}}+1}+1\right)+\log\left(\sqrt{\frac{1}{a^{2}x^{2}}+1}-1\right)}{a}-\frac{2}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] x - (2*sqrt(1/(a^2*x^2) + 1) - log(sqrt(1/(a^2*x^2) + 1) + 1) + log(sqrt(1/
(a^2*x^2) + 1) - 1))/a - 2/(a^2*x)
```

mupad [B] time = 2.26, size = 47, normalized size = 1.00

$$x-\frac{2\sqrt{\frac{1}{a^{2}\,x^{2}}+1}}{a}-\frac{2}{a^{2}\,x}-\frac{\mathrm{atan}\left(\sqrt{\frac{1}{a^{2}\,x^{2}}+1}\,1\mathrm{i}\right)\,2\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] x - (atan((1/(a^2*x^2) + 1)^(1/2)*1i)*2i)/a - (2*(1/(a^2*x^2) + 1)^(1/2))/a
 - 2/(a^2*x)
```

sympy [A] time = 3.63, size = 49, normalized size = 1.04

$$x-\frac{2x}{\sqrt{a^{2}x^{2}+1}}+\frac{2\operatorname{asinh}(ax)}{a}-\frac{2}{a^{2}x}-\frac{2}{a^{2}x\sqrt{a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2,x)
```

```
[Out] x - 2*x/sqrt(a**2*x**2 + 1) + 2*asinh(a*x)/a - 2/(a**2*x) - 2/(a**2*x*sqrt(
a**2*x**2 + 1))
```

3.54 $\int \frac{e^{2\text{csch}^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=38

$$-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{ax}-\frac{1}{a^2x^2}-\text{csch}^{-1}(ax)+\log(x)$$

```
[Out] -1/a^2/x^2-arccsch(a*x)+ln(x)-(1+1/a^2/x^2)^(1/2)/a/x
```

Rubi [A] time = 0.21, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6338, 6742, 335, 195, 215}

$$-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{ax}-\frac{1}{a^2x^2}-\text{csch}^{-1}(ax)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCsch[a*x])/x,x]
```

```
[Out] -(1/(a^2*x^2)) - Sqrt[1 + 1/(a^2*x^2)]/(a*x) - ArcCsch[a*x] + Log[x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6338

```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x}\,dx &= \int \frac{\left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)^2}{x}\,dx \\
&= \int \left(\frac{2}{a^2x^3}+\frac{2\sqrt{1+\frac{1}{a^2x^2}}}{ax^2}+\frac{1}{x}\right)dx \\
&= -\frac{1}{a^2x^2}+\log(x)+\frac{2\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^2}\,dx}{a} \\
&= -\frac{1}{a^2x^2}+\log(x)-\frac{2\,\text{Subst}\left(\int \sqrt{1+\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= -\frac{1}{a^2x^2}-\frac{\sqrt{1+\frac{1}{a^2x^2}}}{ax}+\log(x)-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= -\frac{1}{a^2x^2}-\frac{\sqrt{1+\frac{1}{a^2x^2}}}{ax}-\operatorname{csch}^{-1}(ax)+\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 39, normalized size = 1.03

$$
-\frac{ax\sqrt{\frac{1}{a^2x^2}+1}+1}{a^2x^2}-\sinh^{-1}\left(\frac{1}{ax}\right)+\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCsch[a*x])/x,x]
```

```
[Out] -((1 + a*Sqrt[1 + 1/(a^2*x^2)]*x)/(a^2*x^2)) - ArcSinh[1/(a*x)] + Log[x]
```

fricas [B] time = 0.60, size = 112, normalized size = 2.95

$$
-\frac{a^2x^2\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax+1\right)-a^2x^2\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax-1\right)-a^2x^2\log(x)+ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}+1}{a^2x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] -(a^2*x^2*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x + 1) - a^2*x^2*log(a*
x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x - 1) - a^2*x^2*log(x) + a*x*sqrt((a^2
*x^2 + 1)/(a^2*x^2)) + 1)/(a^2*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
```

```
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

maple [B] time = 0.05, size = 150, normalized size = 3.95

$$\ln(x)-\frac{1}{x^2a^2}-\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\left(a^2\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}\sqrt{\frac{1}{a^2}}-\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}\,x^2a^2+\ln\left(\frac{2\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+2}{a^2x}\right)x^2\right)}{ax\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))^2/x,x)
```

```
[Out] ln(x)-1/x^2/a^2-1/a*((a^2*x^2+1)/a^2/x^2)^(1/2)/x*(a^2*((a^2*x^2+1)/a^2)^(3
/2)*(1/a^2)^(1/2)-(1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*x^2*a^2+ln(2*((1/a^
2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*a^2+1)/a^2/x)*x^2)/(1/a^2)^(1/2)/((a^2*x^2
+1)/a^2)^(1/2)
```

maxima [B] time = 0.32, size = 93, normalized size = 2.45

$$-\frac{\frac{2\,a^2x\sqrt{\frac{1}{a^2x^2}+1}}{a^2x^2\left(\frac{1}{a^2x^2}+1\right)-1}+a\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}+1\right)-a\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{2\,a}-\frac{1}{a^2x^2}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^2*x*sqrt(1/(a^2*x^2) + 1)/(a^2*x^2*(1/(a^2*x^2) + 1) - 1) + a*log
(a*x*sqrt(1/(a^2*x^2) + 1) + 1) - a*log(a*x*sqrt(1/(a^2*x^2) + 1) - 1))/a -
 1/(a^2*x^2) + log(x)
```

mupad [B] time = 2.25, size = 44, normalized size = 1.16

$$-\ln\left(\frac{1}{x}\right)-\mathrm{asinh}\left(\frac{1}{a\,x}\right)-\frac{1}{a^2\,x^2}-\frac{\sqrt{\frac{1}{a^2\,x^2}+1}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2/x,x)
```

```
[Out] - log(1/x) - asinh(1/(a*x)) - 1/(a^2*x^2) - (1/(a^2*x^2) + 1)^(1/2)/(a*x)
```

sympy [A] time = 4.08, size = 34, normalized size = 0.89

$$\log(x)-\mathrm{asinh}\left(\frac{1}{ax}\right)-\frac{\sqrt{1+\frac{1}{a^2x^2}}}{ax}-\frac{1}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2/x,x)
```

```
[Out] log(x) - asinh(1/(a*x)) - sqrt(1 + 1/(a**2*x**2))/(a*x) - 1/(a**2*x**2)
```

3.55 $\int \frac{e^{2\mathrm{csch}^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=34

$$-\frac{2}{3a^2x^3}-\frac{2}{3}a\left(\frac{1}{a^2x^2}+1\right)^{3/2}-\frac{1}{x}$$

[Out] -2/3*a*(1+1/a^2/x^2)^(3/2)-2/3/a^2/x^3-1/x

Rubi [A] time = 0.20, antiderivative size = 54, normalized size of antiderivative = 1.59, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6338, 6715, 2117}

$$-\frac{1}{6}a\left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)^3-\frac{1}{2}a\sqrt{\frac{1}{a^2x^2}+1}-\frac{1}{2x}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCsch[a*x])/x^2,x]

[Out] -(a*Sqrt[1 + 1/(a^2*x^2)])/2 - (a*(Sqrt[1 + 1/(a^2*x^2)] + 1/(a*x))^3)/6 - 1/(2*x)

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 6338

```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\mathrm{csch}^{-1}(ax)}}{x^2}\,dx &= \int \frac{\left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)^2}{x^2}\,dx\\
&= -\mathrm{Subst}\left(\int \left(\frac{x}{a}+\sqrt{1+\frac{x^2}{a^2}}\right)^2 dx, x, \frac{1}{x}\right)\\
&= -\left(\frac{1}{2}a\,\mathrm{Subst}\left(\int \left(1+x^2\right) dx, x, \sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)\right)\\
&= -\frac{1}{2}a\sqrt{1+\frac{1}{a^2x^2}}-\frac{1}{6}a\left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)^3-\frac{1}{2x}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.35

$$-\frac{3a^2x^2 + 2ax\sqrt{\frac{1}{a^2x^2}+1}\left(a^2x^2+1\right) + 2}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCsch[a*x])/x^2,x]
```

```
[Out] -1/3*(2 + 3*a^2*x^2 + 2*a*Sqrt[1 + 1/(a^2*x^2)]*x*(1 + a^2*x^2))/(a^2*x^3)
```

fricas [B] time = 0.59, size = 57, normalized size = 1.68

$$-\frac{2\,a^3x^3 + 3\,a^2x^2 + 2\left(a^3x^3 + ax\right)\sqrt{\frac{a^2x^2+1}{a^2x^2}} + 2}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*a^3*x^3 + 3*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt((a^2*x^2 + 1)/(a^2*x^2
)) + 2)/(a^2*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

maple [B] time = 0.05, size = 63, normalized size = 1.85

$$\frac{-\frac{a^2}{x} - \frac{1}{3x^3}}{a^2} - \frac{2\sqrt{\frac{a^2x^2+1}{a^2x^2}}\left(a^2x^2+1\right)}{3a\,x^2} - \frac{1}{3x^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))^2/x^2,x)
```

```
[Out] 1/a^2*(-a^2/x-1/3/x^3)-2/3/a*((a^2*x^2+1)/a^2/x^2)^(1/2)/x^2*(a^2*x^2+1)-1/
3/x^3/a^2
```

maxima [A] time = 0.31, size = 28, normalized size = 0.82

$$-\frac{2}{3}a\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}} - \frac{1}{x} - \frac{2}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^2,x, algorithm="maxima")
```

```
[Out] -2/3*a*(1/(a^2*x^2) + 1)^(3/2) - 1/x - 2/3/(a^2*x^3)
```

mupad [B] time = 2.21, size = 51, normalized size = 1.50

$$-\frac{\frac{2}{3\,a^2} + \frac{2\,x\,\sqrt{\frac{1}{a^2\,x^2}+1}}{3\,a}}{x^3} - \frac{\frac{2\,a\,x\,\sqrt{\frac{1}{a^2\,x^2}+1}}{3} + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2/x^2,x)
```

```
[Out] - (2/(3*a^2) + (2*x*(1/(a^2*x^2) + 1)^(1/2))/(3*a))/x^3 - ((2*a*x*(1/(a^2*x
^2) + 1)^(1/2))/3 + 1)/x
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2/x**2,x)
```

```
[Out] Exception raised: TypeError
```

3.56 $\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=73

$$-\frac{1}{2a^2x^4}-\frac{a\sqrt{\frac{1}{a^2x^2}+1}}{4x}-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{2ax^3}+\frac{1}{4}a^2\operatorname{csch}^{-1}(ax)-\frac{1}{2x^2}$$

```
[Out] -1/2/a^2/x^4-1/2/x^2+1/4*a^2*arccsch(a*x)-1/2*(1+1/a^2/x^2)^(1/2)/a/x^3-1/4
*a*(1+1/a^2/x^2)^(1/2)/x
```

Rubi [A] time = 0.23, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6338, 6742, 335, 279, 321, 215}

$$-\frac{a\sqrt{\frac{1}{a^2x^2}+1}}{4x}-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{2ax^3}-\frac{1}{2a^2x^4}+\frac{1}{4}a^2\operatorname{csch}^{-1}(ax)-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCsch[a*x])/x^3,x]
```

```
[Out] -1/(2*a^2*x^4) - Sqrt[1 + 1/(a^2*x^2)]/(2*a*x^3) - 1/(2*x^2) - (a*Sqrt[1 +
1/(a^2*x^2)])/(4*x) + (a^2*ArcCsch[a*x])/4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6338

```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^3}\,dx &= \int \frac{\left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)^2}{x^3}\,dx\\
&= \int \left(\frac{2}{a^2x^5}+\frac{2\sqrt{1+\frac{1}{a^2x^2}}}{ax^4}+\frac{1}{x^3}\right)dx\\
&= -\frac{1}{2a^2x^4}-\frac{1}{2x^2}+\frac{2\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^4}\,dx}{a}\\
&= -\frac{1}{2a^2x^4}-\frac{1}{2x^2}-\frac{2\,\text{Subst}\left(\int x^2\sqrt{1+\frac{x^2}{a^2}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= -\frac{1}{2a^2x^4}-\frac{\sqrt{1+\frac{1}{a^2x^2}}}{2ax^3}-\frac{1}{2x^2}-\frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= -\frac{1}{2a^2x^4}-\frac{\sqrt{1+\frac{1}{a^2x^2}}}{2ax^3}-\frac{1}{2x^2}-\frac{a\sqrt{1+\frac{1}{a^2x^2}}}{4x}+\frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{1}{2a^2x^4}-\frac{\sqrt{1+\frac{1}{a^2x^2}}}{2ax^3}-\frac{1}{2x^2}-\frac{a\sqrt{1+\frac{1}{a^2x^2}}}{4x}+\frac{1}{4}a^2\operatorname{csch}^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 73, normalized size = 1.00

$$-\frac{1}{2a^2x^4}+\left(-\frac{1}{2ax^3}-\frac{a}{4x}\right)\sqrt{\frac{a^2x^2+1}{a^2x^2}}+\frac{1}{4}a^2\sinh^{-1}\left(\frac{1}{ax}\right)-\frac{1}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCsch[a*x])/x^3,x]
```

```
[Out] -1/2*1/(a^2*x^4) - 1/(2*x^2) + (-1/2*1/(a*x^3) - a/(4*x))*Sqrt[(1 + a^2*x^2
)/(a^2*x^2)] + (a^2*ArcSinh[1/(a*x)])/4
```

fricas [B] time = 0.51, size = 121, normalized size = 1.66

$$\frac{a^4x^4\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax+1\right)-a^4x^4\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax-1\right)-2\,a^2x^2-\left(a^3x^3+2\,ax\right)\sqrt{\frac{a^2x^2+1}{a^2x^2}}-2}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a^4*x^4*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x + 1) - a^4*x^4*log
(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x - 1) - 2*a^2*x^2 - (a^3*x^3 + 2*a*
x)*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - 2)/(a^2*x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

maple [B] time = 0.05, size = 176, normalized size = 2.41

$$-\frac{1}{2x^2}-\frac{1}{2a^2x^4}+\frac{a\sqrt{\frac{a^2x^2+1}{a^2x^2}}\left(\sqrt{\frac{1}{a^2}}\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}x^2a^2-\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}\,x^4a^2+\ln\left(\frac{2\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+2}{a^2x}\right)x^4-2\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}\cdots\right)}{4x^3\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))^2/x^3,x)
```

```
[Out] -1/2/x^2-1/2/a^2/x^4+1/4*a*((a^2*x^2+1)/a^2/x^2)^(1/2)/x^3*((1/a^2)^(1/2)*(
(a^2*x^2+1)/a^2)^(3/2)*x^2*a^2-(1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*x^4*a^
2+ln(2*((1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*a^2+1)/a^2/x)*x^4-2*((a^2*x^2
+1)/a^2)^(3/2)*(1/a^2)^(1/2))/(1/a^2)^(1/2)/((a^2*x^2+1)/a^2)^(1/2)
```

maxima [B] time = 0.31, size = 139, normalized size = 1.90

$$\frac{a^3\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}+1\right)-a^3\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}-1\right)-\frac{2\left(a^6x^3\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}+a^4x\sqrt{\frac{1}{a^2x^2}+1}\right)}{a^4x^4\left(\frac{1}{a^2x^2}+1\right)^2-2\,a^2x^2\left(\frac{1}{a^2x^2}+1\right)+1}}{8\,a}-\frac{1}{2\,x^2}-\frac{1}{2\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(a^3*log(a*x*sqrt(1/(a^2*x^2) + 1) + 1) - a^3*log(a*x*sqrt(1/(a^2*x^2)
+ 1) - 1) - 2*(a^6*x^3*(1/(a^2*x^2) + 1)^(3/2) + a^4*x*sqrt(1/(a^2*x^2) + 1
))/(a^4*x^4*(1/(a^2*x^2) + 1)^2 - 2*a^2*x^2*(1/(a^2*x^2) + 1) + 1))/a - 1/2
/x^2 - 1/2/(a^2*x^4)
```

mupad [B] time = 2.31, size = 68, normalized size = 0.93

$$\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{\frac{1}{a^2}}}{x}\right)}{4\sqrt{\frac{1}{a^2}}}-\frac{1}{2\,a^2\,x^4}-\frac{a\,\sqrt{\frac{1}{a^2\,x^2}+1}}{4\,x}-\frac{1}{2\,x^2}-\frac{\sqrt{\frac{1}{a^2\,x^2}+1}}{2\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2/x^3,x)
```

```
[Out] (a*asinh((1/a^2)^(1/2)/x))/(4*(1/a^2)^(1/2)) - 1/(2*a^2*x^4) - (a*(1/(a^2*x
^2) + 1)^(1/2))/(4*x) - 1/(2*x^2) - (1/(a^2*x^2) + 1)^(1/2)/(2*a*x^3)
```

sympy [A] time = 4.61, size = 92, normalized size = 1.26

$$\frac{a^2\,\mathrm{asinh}\left(\frac{1}{ax}\right)}{4}-\frac{a}{4x\sqrt{1+\frac{1}{a^2x^2}}}-\frac{1}{2x^2}-\frac{3}{4ax^3\sqrt{1+\frac{1}{a^2x^2}}}-\frac{1}{2a^2x^4}-\frac{1}{2a^3x^5\sqrt{1+\frac{1}{a^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2/x**3,x)
```

```
[Out] a**2*asinh(1/(a*x))/4 - a/(4*x*sqrt(1 + 1/(a**2*x**2))) - 1/(2*x**2) - 3/(4
*a*x**3*sqrt(1 + 1/(a**2*x**2))) - 1/(2*a**2*x**4) - 1/(2*a**3*x**5*sqrt(1
+ 1/(a**2*x**2)))
```

3.57 $\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=58

$$-\frac{2}{5a^2x^5}-\frac{2}{5}a^3\left(\frac{1}{a^2x^2}+1\right)^{5/2}+\frac{2}{3}a^3\left(\frac{1}{a^2x^2}+1\right)^{3/2}-\frac{1}{3x^3}$$

```
[Out] 2/3*a^3*(1+1/a^2/x^2)^(3/2)-2/5*a^3*(1+1/a^2/x^2)^(5/2)-2/5/a^2/x^5-1/3/x^3
```

Rubi [A] time = 0.23, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6338, 6742, 266, 43}

$$-\frac{2}{5}a^3\left(\frac{1}{a^2x^2}+1\right)^{5/2}+\frac{2}{3}a^3\left(\frac{1}{a^2x^2}+1\right)^{3/2}-\frac{2}{5a^2x^5}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCsch[a*x])/x^4,x]
```

```
[Out] (2*a^3*(1 + 1/(a^2*x^2))^(3/2))/3 - (2*a^3*(1 + 1/(a^2*x^2))^(5/2))/5 - 2/(
5*a^2*x^5) - 1/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6338

```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)^2}{x^4}\,dx$$
$$= \int \left(\frac{2}{a^2x^6}+\frac{2\sqrt{1+\frac{1}{a^2x^2}}}{ax^5}+\frac{1}{x^4}\right)dx$$
$$= -\frac{2}{5a^2x^5}-\frac{1}{3x^3}+\frac{2\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^5}\,dx}{a}$$
$$= -\frac{2}{5a^2x^5}-\frac{1}{3x^3}-\frac{\text{Subst}\left(\int x\sqrt{1+\frac{x}{a^2}}\,dx, x, \frac{1}{x^2}\right)}{a}$$
$$= -\frac{2}{5a^2x^5}-\frac{1}{3x^3}-\frac{\text{Subst}\left(\int \left(-a^2\sqrt{1+\frac{x}{a^2}}+a^2\left(1+\frac{x}{a^2}\right)^{3/2}\right)dx, x, \frac{1}{x^2}\right)}{a}$$
$$= \frac{2}{3}a^3\left(1+\frac{1}{a^2x^2}\right)^{3/2}-\frac{2}{5}a^3\left(1+\frac{1}{a^2x^2}\right)^{5/2}-\frac{2}{5a^2x^5}-\frac{1}{3x^3}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 0.93

$$-\frac{5a^2x^2+2ax\sqrt{\frac{1}{a^2x^2}+1}\left(-2a^4x^4+a^2x^2+3\right)+6}{15a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCsch[a*x])/x^4,x]
```

```
[Out] -1/15*(6 + 5*a^2*x^2 + 2*a*Sqrt[1 + 1/(a^2*x^2)]*x*(3 + a^2*x^2 - 2*a^4*x^4
))/(a^2*x^5)
```

fricas [A] time = 0.73, size = 67, normalized size = 1.16

$$\frac{4\,a^5x^5-5\,a^2x^2+2\left(2\,a^5x^5-a^3x^3-3\,ax\right)\sqrt{\frac{a^2x^2+1}{a^2x^2}}-6}{15\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/15*(4*a^5*x^5 - 5*a^2*x^2 + 2*(2*a^5*x^5 - a^3*x^3 - 3*a*x)*sqrt((a^2*x^2
 + 1)/(a^2*x^2)) - 6)/(a^2*x^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

maple [A] time = 0.05, size = 73, normalized size = 1.26

$$\frac{-\frac{a^2}{3x^3}-\frac{1}{5x^5}}{a^2}+\frac{2\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,\left(a^2x^2+1\right)\left(2a^2x^2-3\right)}{15a\,x^4}-\frac{1}{5x^5a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))^2/x^4,x)
```

```
[Out] 1/a^2*(-1/3*a^2/x^3-1/5/x^5)+2/15/a*((a^2*x^2+1)/a^2/x^2)^(1/2)/x^4*(a^2*x^
2+1)*(2*a^2*x^2-3)-1/5/x^5/a^2
```

maxima [A] time = 0.30, size = 52, normalized size = 0.90

$$-\frac{2\left(3\,a^4\left(\frac{1}{a^2x^2}+1\right)^{\frac{5}{2}}-5\,a^4\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}\right)}{15\,a}-\frac{1}{3\,x^3}-\frac{2}{5\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^4,x, algorithm="maxima")
```

```
[Out] -2/15*(3*a^4*(1/(a^2*x^2) + 1)^(5/2) - 5*a^4*(1/(a^2*x^2) + 1)^(3/2))/a - 1
/3/x^3 - 2/5/(a^2*x^5)
```

mupad [B] time = 2.26, size = 67, normalized size = 1.16

$$\frac{4\,a^3\,\sqrt{\frac{1}{a^2\,x^2}+1}}{15}-\frac{\frac{2\,a\,x\,\sqrt{\frac{1}{a^2\,x^2}+1}}{15}+\frac{1}{3}}{x^3}-\frac{\frac{2}{5\,a^2}+\frac{2\,x\,\sqrt{\frac{1}{a^2\,x^2}+1}}{5\,a}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2/x^4,x)
```

```
[Out] (4*a^3*(1/(a^2*x^2) + 1)^(1/2))/15 - ((2*a*x*(1/(a^2*x^2) + 1)^(1/2))/15 +
1/3)/x^3 - (2/(5*a^2) + (2*x*(1/(a^2*x^2) + 1)^(1/2))/(5*a))/x^5
```

sympy [A] time = 3.07, size = 76, normalized size = 1.31

$$\frac{4a^2\sqrt{a^2x^2+1}}{15x}-\frac{2\sqrt{a^2x^2+1}}{15x^3}-\frac{1}{3x^3}-\frac{2\sqrt{a^2x^2+1}}{5a^2x^5}-\frac{2}{5a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2/x**4,x)
```

```
[Out] 4*a**2*sqrt(a**2*x**2 + 1)/(15*x) - 2*sqrt(a**2*x**2 + 1)/(15*x**3) - 1/(3*
x**3) - 2*sqrt(a**2*x**2 + 1)/(5*a**2*x**5) - 2/(5*a**2*x**5)
```

3.58 $\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=96

$$-\frac{1}{8}a^4\text{csch}^{-1}(ax)-\frac{1}{3a^2x^6}-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{3ax^5}-\frac{a\sqrt{\frac{1}{a^2x^2}+1}}{12x^3}+\frac{a^3\sqrt{\frac{1}{a^2x^2}+1}}{8x}-\frac{1}{4x^4}$$

```
[Out] -1/3/a^2/x^6-1/4/x^4-1/8*a^4*arccsch(a*x)-1/3*(1+1/a^2/x^2)^(1/2)/a/x^5-1/1
2*a*(1+1/a^2/x^2)^(1/2)/x^3+1/8*a^3*(1+1/a^2/x^2)^(1/2)/x
```

Rubi [A] time = 0.25, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6338, 6742, 335, 279, 321, 215}

$$\frac{a^3\sqrt{\frac{1}{a^2x^2}+1}}{8x}-\frac{a\sqrt{\frac{1}{a^2x^2}+1}}{12x^3}-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{3ax^5}-\frac{1}{3a^2x^6}-\frac{1}{8}a^4\text{csch}^{-1}(ax)-\frac{1}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCsch[a*x])/x^5,x]
```

```
[Out] -1/(3*a^2*x^6) - Sqrt[1 + 1/(a^2*x^2)]/(3*a*x^5) - 1/(4*x^4) - (a*Sqrt[1 +
1/(a^2*x^2)])/(12*x^3) + (a^3*Sqrt[1 + 1/(a^2*x^2)])/(8*x) - (a^4*ArcCsch[a
*x])/8
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6338

```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^5}\,dx &= \int \frac{\left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)^2}{x^5}\,dx\\
&= \int \left(\frac{2}{a^2x^7}+\frac{2\sqrt{1+\frac{1}{a^2x^2}}}{ax^6}+\frac{1}{x^5}\right)dx\\
&= -\frac{1}{3a^2x^6}-\frac{1}{4x^4}+\frac{2\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^6}\,dx}{a}\\
&= -\frac{1}{3a^2x^6}-\frac{1}{4x^4}-\frac{2\,\text{Subst}\left(\int x^4\sqrt{1+\frac{x^2}{a^2}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= -\frac{1}{3a^2x^6}-\frac{\sqrt{1+\frac{1}{a^2x^2}}}{3ax^5}-\frac{1}{4x^4}-\frac{\text{Subst}\left(\int \frac{x^4}{\sqrt{1+\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{3a}\\
&= -\frac{1}{3a^2x^6}-\frac{\sqrt{1+\frac{1}{a^2x^2}}}{3ax^5}-\frac{1}{4x^4}-\frac{a\sqrt{1+\frac{1}{a^2x^2}}}{12x^3}+\frac{1}{4}a\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{1}{3a^2x^6}-\frac{\sqrt{1+\frac{1}{a^2x^2}}}{3ax^5}-\frac{1}{4x^4}-\frac{a\sqrt{1+\frac{1}{a^2x^2}}}{12x^3}+\frac{a^3\sqrt{1+\frac{1}{a^2x^2}}}{8x}-\frac{1}{8}a^3\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^2}}}\,dx,x\right.\\
&= -\frac{1}{3a^2x^6}-\frac{\sqrt{1+\frac{1}{a^2x^2}}}{3ax^5}-\frac{1}{4x^4}-\frac{a\sqrt{1+\frac{1}{a^2x^2}}}{12x^3}+\frac{a^3\sqrt{1+\frac{1}{a^2x^2}}}{8x}-\frac{1}{8}a^4\text{csch}^{-1}(ax)
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 74, normalized size = 0.77

$$
\frac{\frac{(3a^2x^2+4)\left(-2ax\sqrt{\frac{1}{a^2x^2}+1}+a^3x^3\sqrt{\frac{1}{a^2x^2}+1}-2\right)}{x^6}-3a^6\sinh^{-1}\left(\frac{1}{ax}\right)}{24a^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCsch[a*x])/x^5,x]
```

```
[Out] (((4 + 3*a^2*x^2)*(-2 - 2*a*Sqrt[1 + 1/(a^2*x^2)]*x + a^3*Sqrt[1 + 1/(a^2*x
^2)]*x^3))/x^6 - 3*a^6*ArcSinh[1/(a*x)])/(24*a^2)
```

fricas [A] time = 2.31, size = 131, normalized size = 1.36

$$
-\frac{3\,a^6x^6\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax+1\right)-3\,a^6x^6\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax-1\right)+6\,a^2x^2-\left(3\,a^5x^5-2\,a^3x^3-8\,ax\right)\sqrt{\frac{a^2x}{a^2}}}{24\,a^2x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(3*a^6*x^6*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x + 1) - 3*a^6*x
^6*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x - 1) + 6*a^2*x^2 - (3*a^5*x^
5 - 2*a^3*x^3 - 8*a*x)*sqrt((a^2*x^2 + 1)/(a^2*x^2)) + 8)/(a^2*x^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

maple [B] time = 0.05, size = 209, normalized size = 2.18

$$-\frac{1}{3a^2x^6}-\frac{1}{4x^4}-\frac{a\sqrt{\frac{a^2x^2+1}{a^2x^2}}\left(3\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}\sqrt{\frac{1}{a^2}}\,x^4a^4-3\sqrt{\frac{a^2x^2+1}{a^2}}\,\sqrt{\frac{1}{a^2}}\,x^6a^4+3\ln\left(\frac{2\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+2}{a^2x}\right)x^6a^2-6\sqrt{\frac{1}{a^2}}\,\left(\right.}{24x^5\sqrt{\frac{a^2x^2+1}{a^2}}\,\sqrt{\frac{1}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1+1/x^2/a^2)^(1/2))^2/x^5,x)
```

```
[Out] -1/3/a^2/x^6-1/4/x^4-1/24*a*((a^2*x^2+1)/a^2/x^2)^(1/2)/x^5*(3*((a^2*x^2+1)
/a^2)^(3/2)*(1/a^2)^(1/2)*x^4*a^4-3*((a^2*x^2+1)/a^2)^(1/2)*(1/a^2)^(1/2)*x
^6*a^4+3*ln(2*((1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*a^2+1)/a^2/x)*x^6*a^2-
6*(1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(3/2)*x^2*a^2+8*((a^2*x^2+1)/a^2)^(3/2)*(
1/a^2)^(1/2))/((a^2*x^2+1)/a^2)^(1/2)/(1/a^2)^(1/2)
```

maxima [B] time = 0.31, size = 180, normalized size = 1.88

$$-\frac{3\,a^5\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}+1\right)-3\,a^5\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}-1\right)-\frac{2\left(3\,a^{10}x^5\left(\frac{1}{a^2x^2}+1\right)^{\frac{5}{2}}-8\,a^8x^3\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}-3\,a^6x\sqrt{\frac{1}{a^2x^2}+1}\right)}{a^6x^6\left(\frac{1}{a^2x^2}+1\right)^3-3\,a^4x^4\left(\frac{1}{a^2x^2}+1\right)^2+3\,a^2x^2\left(\frac{1}{a^2x^2}+1\right)-1}}{48\,a}-\frac{1}{4\,x^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/48*(3*a^5*log(a*x*sqrt(1/(a^2*x^2) + 1) + 1) - 3*a^5*log(a*x*sqrt(1/(a^2
*x^2) + 1) - 1) - 2*(3*a^10*x^5*(1/(a^2*x^2) + 1)^(5/2) - 8*a^8*x^3*(1/(a^2
*x^2) + 1)^(3/2) - 3*a^6*x*sqrt(1/(a^2*x^2) + 1))/(a^6*x^6*(1/(a^2*x^2) + 1
)^3 - 3*a^4*x^4*(1/(a^2*x^2) + 1)^2 + 3*a^2*x^2*(1/(a^2*x^2) + 1) - 1))/a -
 1/4/x^4 - 1/3/(a^2*x^6)
```

mupad [B] time = 2.37, size = 89, normalized size = 0.93

$$\frac{a^3\sqrt{\frac{1}{a^2\,x^2}+1}}{8\,x}-\frac{1}{3\,a^2\,x^6}-\frac{a\sqrt{\frac{1}{a^2\,x^2}+1}}{12\,x^3}-\frac{1}{4\,x^4}-\frac{\sqrt{\frac{1}{a^2\,x^2}+1}}{3\,a\,x^5}-\frac{a^3\,\mathrm{asinh}\left(\frac{\sqrt{\frac{1}{a^2}}}{x}\right)}{8\sqrt{\frac{1}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2/x^5,x)
```

```
[Out] (a^3*(1/(a^2*x^2) + 1)^(1/2))/(8*x) - 1/(3*a^2*x^6) - (a*(1/(a^2*x^2) + 1)^
(1/2))/(12*x^3) - 1/(4*x^4) - (1/(a^2*x^2) + 1)^(1/2)/(3*a*x^5) - (a^3*asin
h((1/a^2)^(1/2)/x))/(8*(1/a^2)^(1/2))
```

sympy [A] time = 6.33, size = 114, normalized size = 1.19

$$-\frac{a^4\operatorname{asinh}\left(\frac{1}{ax}\right)}{8}+\frac{a^3}{8x\sqrt{1+\frac{1}{a^2x^2}}}+\frac{a}{24x^3\sqrt{1+\frac{1}{a^2x^2}}}-\frac{1}{4x^4}-\frac{5}{12ax^5\sqrt{1+\frac{1}{a^2x^2}}}-\frac{1}{3a^2x^6}-\frac{1}{3a^3x^7\sqrt{1+\frac{1}{a^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2/x**5,x)
```

```
[Out] -a**4*asinh(1/(a*x))/8 + a**3/(8*x*sqrt(1 + 1/(a**2*x**2))) + a/(24*x**3*sq
rt(1 + 1/(a**2*x**2))) - 1/(4*x**4) - 5/(12*a*x**5*sqrt(1 + 1/(a**2*x**2)))
 - 1/(3*a**2*x**6) - 1/(3*a**3*x**7*sqrt(1 + 1/(a**2*x**2)))
```

3.59 $\int \frac{e^{\text{csch}^{-1}(cx)}(dx)^m}{1+c^2x^2} dx$

Optimal. Leaf size=85

$$\frac{(dx)^m {}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};-c^2x^2\right)}{cm} - \frac{d(dx)^{m-1} {}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};-\frac{1}{c^2x^2}\right)}{c^2(1-m)}$$

```
[Out] -d*(d*x)^(-1+m)*hypergeom([1/2, 1/2-1/2*m],[3/2-1/2*m],-1/c^2/x^2)/c^2/(1-m
)+(d*x)^m*hypergeom([1, 1/2*m],[1+1/2*m],-c^2*x^2)/c/m
```

Rubi [A] time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6342, 339, 364}

$$\frac{(dx)^m {}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};-c^2x^2\right)}{cm} - \frac{d(dx)^{m-1} {}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};-\frac{1}{c^2x^2}\right)}{c^2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCsch[c*x]*(d*x)^m)/(1 + c^2*x^2),x]
```

```
[Out] -((d*(d*x)^(-1 + m)*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, -(1/(c^2*x
^2))])/(c^2*(1 - m))) + ((d*x)^m*Hypergeometric2F1[1, m/2, (2 + m)/2, -(c^2
*x^2)])/(c*m)
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] && !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6342

```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d^2/(a*c^2), Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x] +
 Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m},
x] && EqQ[b - a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}(dx)^m}{1+c^2x^2}\,dx = \frac{d\int \frac{(dx)^{-1+m}}{1+c^2x^2}\,dx}{c} + \frac{d^2\int \frac{(dx)^{-2+m}}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{c^2}$$

$$= \frac{(dx)^m\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};-c^2x^2\right)}{cm} - \frac{\left(d\left(\frac{1}{x}\right)^{-1+m}(dx)^{-1+m}\right)\text{Subst}\left(\int \frac{x^{-m}}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$= -\frac{d(dx)^{-1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};-\frac{1}{c^2x^2}\right)}{c^2(1-m)} + \frac{(dx)^m\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};-c^2x^2\right)}{cm}$$

Mathematica [A] time = 0.25, size = 88, normalized size = 1.04

$$\frac{(dx)^m\left(\frac{x\sqrt{\frac{1}{c^2x^2}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m}{2}+1;-c^2x^2\right)}{\sqrt{c^2x^2+1}} + \frac{{}_2F_1\left(1,\frac{m}{2};\frac{m}{2}+1;-c^2x^2\right)}{c}\right)}{m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCsch[c*x]*(d*x)^m)/(1 + c^2*x^2),x]
```

```
[Out] ((d*x)^m*((Sqrt[1 + 1/(c^2*x^2)]*x*Hypergeometric2F1[1/2, m/2, 1 + m/2, -(c
^2*x^2)])/Sqrt[1 + c^2*x^2] + Hypergeometric2F1[1, m/2, 1 + m/2, -(c^2*x^2)
]/c))/m
```

fricas [F] time = 2.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m\,cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + (dx)^m}{c^3x^3+cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*(d*x)^m/(c^2*x^2+1),x, algorithm="fri
cas")
```

```
[Out] integral(((d*x)^m*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + (d*x)^m)/(c^3*x^3 + c
*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*(d*x)^m/(c^2*x^2+1),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}\right)(dx)^m}{c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(1+1/c^2/x^2)^(1/2))*(d*x)^m/(c^2*x^2+1),x)
```

```
[Out] int((1/c/x+(1+1/c^2/x^2)^(1/2))*(d*x)^m/(c^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m \left(\sqrt{\frac{1}{c^2x^2}+1} + \frac{1}{cx}\right)}{c^2x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*(d*x)^m/(c^2*x^2+1),x, algorithm="max
ima")
```

```
[Out] integrate((d*x)^m*(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\sqrt{\frac{1}{c^2\,x^2}+1} + \frac{1}{c\,x}\right)\,(d\,x)^m}{c^2\,x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x))*(d*x)^m)/(c^2*x^2 + 1),x)
```

```
[Out] int((((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x))*(d*x)^m)/(c^2*x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(dx)^m}{c^2x^3+x}\, dx + \int \frac{cx(dx)^m\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^3+x}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c**2/x**2)**(1/2))*(d*x)**m/(c**2*x**2+1),x)
```

```
[Out] (Integral((d*x)**m/(c**2*x**3 + x), x) + Integral(c*x*(d*x)**m*sqrt(1 + 1/(
c**2*x**2))/(c**2*x**3 + x), x))/c
```

3.60 $\int \frac{e^{\text{csch}^{-1}(cx)}x^5}{1+c^2x^2}\,dx$

Optimal. Leaf size=92

$$\frac{\tan^{-1}(cx)}{c^6}-\frac{x}{c^5}+\frac{x^3}{3c^3}+\frac{x^4\sqrt{\frac{1}{c^2x^2}+1}}{4c^2}+\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{8c^6}-\frac{3x^2\sqrt{\frac{1}{c^2x^2}+1}}{8c^4}$$

```
[Out] -x/c^5+1/3*x^3/c^3+arctan(c*x)/c^6+3/8*arctanh((1+1/c^2/x^2)^(1/2))/c^6-3/8
*x^2*(1+1/c^2/x^2)^(1/2)/c^4+1/4*x^4*(1+1/c^2/x^2)^(1/2)/c^2
```

Rubi [A] time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6342, 266, 51, 63, 208, 302, 203}

$$\frac{x^4\sqrt{\frac{1}{c^2x^2}+1}}{4c^2}+\frac{x^3}{3c^3}-\frac{3x^2\sqrt{\frac{1}{c^2x^2}+1}}{8c^4}+\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{8c^6}-\frac{x}{c^5}+\frac{\tan^{-1}(cx)}{c^6}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCsch[c*x]*x^5)/(1 + c^2*x^2),x]
```

```
[Out] -(x/c^5) - (3*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(8*c^4) + x^3/(3*c^3) + (Sqrt[1 +
1/(c^2*x^2)]*x^4)/(4*c^2) + ArcTan[c*x]/c^6 + (3*ArcTanh[Sqrt[1 + 1/(c^2*x^
2)]])/(8*c^6)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6342

```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d^2/(a*c^2), Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x] +
 Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m},
x] && EqQ[b - a*c^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\text{csch}^{-1}(cx)}x^5}{1+c^2x^2}\,dx &= \frac{\int \frac{x^3}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{c^2}+\frac{\int \frac{x^4}{1+c^2x^2}\,dx}{c}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{2c^2}+\frac{\int \left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4(1+c^2x^2)}\right)dx}{c}\\
&= -\frac{x}{c^5}+\frac{x^3}{3c^3}+\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{4c^2}+\frac{\int \frac{1}{1+c^2x^2}\,dx}{c^5}+\frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{8c^4}\\
&= -\frac{x}{c^5}-\frac{3\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{8c^4}+\frac{x^3}{3c^3}+\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{4c^2}+\frac{\tan^{-1}(cx)}{c^6}-\frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{16c^6}\\
&= -\frac{x}{c^5}-\frac{3\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{8c^4}+\frac{x^3}{3c^3}+\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{4c^2}+\frac{\tan^{-1}(cx)}{c^6}-\frac{3\,\text{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx,x,\sqrt{1+\frac{1}{c^2x^2}}\right)}{8c^4}\\
&= -\frac{x}{c^5}-\frac{3\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{8c^4}+\frac{x^3}{3c^3}+\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{4c^2}+\frac{\tan^{-1}(cx)}{c^6}+\frac{3\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{8c^6}
\end{aligned}
$$

Mathematica [A] time = 0.20, size = 85, normalized size = 0.92

$$
\frac{9\log\left(x\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)\right)+cx\left(8c^2x^2-9cx\sqrt{\frac{1}{c^2x^2}+1}+6c^3x^3\sqrt{\frac{1}{c^2x^2}+1}-24\right)+24\tan^{-1}(cx)}{24c^6}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCsch[c*x]*x^5)/(1 + c^2*x^2),x]
```

```
[Out] (c*x*(-24 - 9*c*Sqrt[1 + 1/(c^2*x^2)]*x + 8*c^2*x^2 + 6*c^3*Sqrt[1 + 1/(c^2
*x^2)]*x^3) + 24*ArcTan[c*x] + 9*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(24*c^
6)
```

fricas [A] time = 1.64, size = 90, normalized size = 0.98

$$
\frac{8\,c^3x^3-24\,cx+3\left(2\,c^4x^4-3\,c^2x^2\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}+24\,\arctan(cx)-9\,\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)}{24\,c^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^5/(c^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] 1/24*(8*c^3*x^3 - 24*c*x + 3*(2*c^4*x^4 - 3*c^2*x^2)*sqrt((c^2*x^2 + 1)/(c^
2*x^2)) + 24*arctan(c*x) - 9*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x))/
c^6
```

giac [A] time = 0.15, size = 89, normalized size = 0.97

$$\frac{1}{8}\sqrt{c^2x^2+1}\,x\left(\frac{2\,x^2|c|\mathrm{sgn}(x)}{c^4}-\frac{3\,|c|\mathrm{sgn}(x)}{c^6}\right)-\frac{3\,\log\left(-x|c|+\sqrt{c^2x^2+1}\right)\mathrm{sgn}(x)}{8\,c^6}+\frac{\arctan\left(cx\right)}{c^6}+\frac{c^6x^3-3\,c^4x}{3\,c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^5/(c^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c^2*x^2 + 1)*x*(2*x^2*abs(c)*sgn(x)/c^4 - 3*abs(c)*sgn(x)/c^6) - 3
/8*log(-x*abs(c) + sqrt(c^2*x^2 + 1))*sgn(x)/c^6 + arctan(c*x)/c^6 + 1/3*(c
^6*x^3 - 3*c^4*x)/c^9
```

maple [B] time = 0.12, size = 165, normalized size = 1.79

$$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\;x\left(2x\left(\frac{c^2x^2+1}{c^2}\right)^{\frac{3}{2}}c^4-5x\sqrt{\frac{c^2x^2+1}{c^2}}\;c^2-5\ln\left(x+\sqrt{\frac{c^2x^2+1}{c^2}}\right)+8\ln\left(x+\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)\right)}{8\sqrt{\frac{c^2x^2+1}{c^2}}\;c^6}+\frac{x^3}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(1+1/c^2/x^2)^(1/2))*x^5/(c^2*x^2+1),x)
```

```
[Out] 1/8*((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(2*x*((c^2*x^2+1)/c^2)^(3/2)*c^4-5*x*((c^
2*x^2+1)/c^2)^(1/2)*c^2-5*ln(x+((c^2*x^2+1)/c^2)^(1/2))+8*ln(x+(-(-c^2*x+(-
c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))/c^4)^(1/2)))/((c^2*x^2+1)/c^2)^(1/2)/c^6+1
/3*x^3/c^3-x/c^5+arctan(c*x)/c^6
```

maxima [B] time = 0.40, size = 162, normalized size = 1.76

$$\frac{c^2x^3-3\,x}{3\,c^5}-\frac{\frac{2\left(\frac{5\sqrt{\frac{c^2x^2+1}{x^2}}}{c}-\frac{3\left(\frac{c^2x^2+1}{x^2}\right)^{\frac{3}{2}}}{c^3}\right)}{\frac{2\left(c^2x^2+1\right)}{c^2x^2}-\frac{\left(c^2x^2+1\right)^2}{c^4x^4}-1}-3\,\log\left(\frac{\sqrt{\frac{c^2x^2+1}{x^2}}}{c}+1\right)+3\,\log\left(\frac{\sqrt{\frac{c^2x^2+1}{x^2}}}{c}-1\right)}{16\,c^6}+\frac{\arctan\left(cx\right)}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^5/(c^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] 1/3*(c^2*x^3 - 3*x)/c^5 - 1/16*(2*(5*sqrt((c^2*x^2 + 1)/x^2)/c - 3*((c^2*x^
2 + 1)/x^2)^(3/2)/c^3)/(2*(c^2*x^2 + 1)/(c^2*x^2) - (c^2*x^2 + 1)^2/(c^4*x^
4) - 1) - 3*log(sqrt((c^2*x^2 + 1)/x^2)/c + 1) + 3*log(sqrt((c^2*x^2 + 1)/x
^2)/c - 1))/c^6 + arctan(c*x)/c^6
```

mupad [B] time = 2.43, size = 79, normalized size = 0.86

$$\frac{3\,\mathrm{atanh}\left(\sqrt{\frac{1}{c^2\,x^2}+1}\right)}{8\,c^6}+\frac{3\,\mathrm{atan}\left(c\,x\right)-3\,c\,x+c^3\,x^3}{3\,c^6}+\frac{x^4\,\sqrt{\frac{1}{c^2\,x^2}+1}}{4\,c^2}-\frac{3\,x^2\,\sqrt{\frac{1}{c^2\,x^2}+1}}{8\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 + 1),x)
```

```
[Out] (3*atanh((1/(c^2*x^2) + 1)^(1/2)))/(8*c^6) + (3*atan(c*x) - 3*c*x + c^3*x^3
)/(3*c^6) + (x^4*(1/(c^2*x^2) + 1)^(1/2))/(4*c^2) - (3*x^2*(1/(c^2*x^2) + 1
)^(1/2))/(8*c^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4}{c^2x^2+1}\,dx + \int \frac{cx^5\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c**2/x**2)**(1/2))*x**5/(c**2*x**2+1),x)
```

```
[Out] (Integral(x**4/(c**2*x**2 + 1), x) + Integral(c*x**5*sqrt(1 + 1/(c**2*x**2)
)/(c**2*x**2 + 1), x))/c
```

3.61 $\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^4}{1+c^2x^2}\,dx$

Optimal. Leaf size=72

$$\frac{x^2}{2c^3}+\frac{x^3\sqrt{\frac{1}{c^2x^2}+1}}{3c^2}-\frac{\log\left(c^2x^2+1\right)}{2c^5}-\frac{2x\sqrt{\frac{1}{c^2x^2}+1}}{3c^4}$$

```
[Out] 1/2*x^2/c^3-1/2*ln(c^2*x^2+1)/c^5-2/3*x*(1+1/c^2/x^2)^(1/2)/c^4+1/3*x^3*(1+
1/c^2/x^2)^(1/2)/c^2
```

Rubi [A] time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6342, 271, 191, 266, 43}

$$\frac{x^3\sqrt{\frac{1}{c^2x^2}+1}}{3c^2}+\frac{x^2}{2c^3}-\frac{2x\sqrt{\frac{1}{c^2x^2}+1}}{3c^4}-\frac{\log\left(c^2x^2+1\right)}{2c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCsch[c*x]*x^4)/(1 + c^2*x^2),x]
```

```
[Out] (-2*Sqrt[1 + 1/(c^2*x^2)]*x)/(3*c^4) + x^2/(2*c^3) + (Sqrt[1 + 1/(c^2*x^2)]
*x^3)/(3*c^2) - Log[1 + c^2*x^2]/(2*c^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6342

```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d^2/(a*c^2), Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x] +
 Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m},
x] && EqQ[b - a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^4}{1+c^2x^2}\,dx = \frac{\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{c^2} + \frac{\int \frac{x^3}{1+c^2x^2}\,dx}{c}$$

$$= \frac{\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{3c^2} - \frac{2\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{3c^4} + \frac{\text{Subst}\left(\int \frac{x}{1+c^2x}\,dx, x, x^2\right)}{2c}$$

$$= -\frac{2\sqrt{1+\frac{1}{c^2x^2}}\,x}{3c^4} + \frac{\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{3c^2} + \frac{\text{Subst}\left(\int \left(\frac{1}{c^2} - \frac{1}{c^2(1+c^2x)}\right)dx, x, x^2\right)}{2c}$$

$$= -\frac{2\sqrt{1+\frac{1}{c^2x^2}}\,x}{3c^4} + \frac{x^2}{2c^3} + \frac{\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{3c^2} - \frac{\log(1+c^2x^2)}{2c^5}$$

Mathematica [A] time = 0.13, size = 64, normalized size = 0.89

$$\frac{cx\left(2c^2x^2\sqrt{\frac{1}{c^2x^2}+1} - 4\sqrt{\frac{1}{c^2x^2}+1} + 3cx\right) - 3\log\left(c^2x^2+1\right)}{6c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCsch[c*x]*x^4)/(1 + c^2*x^2),x]
```

```
[Out] (c*x*(-4*Sqrt[1 + 1/(c^2*x^2)] + 3*c*x + 2*c^2*Sqrt[1 + 1/(c^2*x^2)]*x^2) -
 3*Log[1 + c^2*x^2])/(6*c^5)
```

fricas [A] time = 0.56, size = 58, normalized size = 0.81

$$\frac{3\,c^2x^2 + 2\left(c^3x^3 - 2\,cx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}} - 3\,\log\left(c^2x^2+1\right)}{6\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^4/(c^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] 1/6*(3*c^2*x^2 + 2*(c^3*x^3 - 2*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 3*log(
c^2*x^2 + 1))/c^5
```

giac [A] time = 0.15, size = 85, normalized size = 1.18

$$-\frac{\log\left(c^2x^2+1\right)}{2\,c^5} + \frac{2\,|c|\text{sgn}(x)}{3\,c^6} + \frac{2\left(c^2x^2+1\right)^{\frac{3}{2}}c^{12}|c|\text{sgn}(x) - 6\,\sqrt{c^2x^2+1}\,c^{12}|c|\text{sgn}(x) + 3\left(c^2x^2+1\right)c^{13}}{6\,c^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^4/(c^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*log(c^2*x^2 + 1)/c^5 + 2/3*abs(c)*sgn(x)/c^6 + 1/6*(2*(c^2*x^2 + 1)^(3
/2)*c^12*abs(c)*sgn(x) - 6*sqrt(c^2*x^2 + 1)*c^12*abs(c)*sgn(x) + 3*(c^2*x^
2 + 1)*c^13)/c^18
```

maple [A] time = 0.06, size = 120, normalized size = 1.67

$$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x\left(\left(\frac{c^2x^2+1}{c^2}\right)^{\frac{3}{2}}c^2 - 3\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)}{3c^4\sqrt{\frac{c^2x^2+1}{c^2}}} + \frac{x^2}{2c^3} - \frac{\ln\left(c^2x^2+1\right)}{2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(1+1/c^2/x^2)^(1/2))*x^4/(c^2*x^2+1),x)
```

```
[Out] 1/3*((c^2*x^2+1)/c^2/x^2)^(1/2)*x/c^4*(((c^2*x^2+1)/c^2)^(3/2)*c^2-3*(-(-c^
2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))/c^4)^(1/2))/((c^2*x^2+1)/c^2)^(1/2)+
1/2*x^2/c^3-1/2*ln(c^2*x^2+1)/c^5
```

maxima [A] time = 0.32, size = 49, normalized size = 0.68

$$\frac{x^2}{2\,c^3} + \frac{\sqrt{c^2x^2+1}\left(c^2x^2-2\right)}{3\,c^5} - \frac{\log\left(c^2x^2+1\right)}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^4/(c^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] 1/2*x^2/c^3 + 1/3*sqrt(c^2*x^2 + 1)*(c^2*x^2 - 2)/c^5 - 1/2*log(c^2*x^2 + 1
)/c^5
```

mupad [B] time = 2.35, size = 61, normalized size = 0.85

$$\frac{x^3\,\sqrt{\frac{1}{c^2\,x^2}+1}}{3\,c^2} - \frac{2\,x\,\sqrt{\frac{1}{c^2\,x^2}+1}}{3\,c^4} - \frac{\ln\left(c^2\,x^2+1\right)-c^2\,x^2}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 + 1),x)
```

```
[Out] (x^3*(1/(c^2*x^2) + 1)^(1/2))/(3*c^2) - (2*x*(1/(c^2*x^2) + 1)^(1/2))/(3*c^
4) - (log(c^2*x^2 + 1) - c^2*x^2)/(2*c^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{c^2x^2+1}\,dx + \int \frac{cx^4\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c**2/x**2)**(1/2))*x**4/(c**2*x**2+1),x)
```

```
[Out] (Integral(x**3/(c**2*x**2 + 1), x) + Integral(c*x**4*sqrt(1 + 1/(c**2*x**2)
)/(c**2*x**2 + 1), x))/c
```

3.62 $\int \frac{e^{\text{csch}^{-1}(cx)}x^3}{1+c^2x^2}\,dx$

Optimal. Leaf size=59

$$-\frac{\tan^{-1}(cx)}{c^4}+\frac{x}{c^3}+\frac{x^2\sqrt{\frac{1}{c^2x^2}+1}}{2c^2}-\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{2c^4}$$

```
[Out] x/c^3-arctan(c*x)/c^4-1/2*arctanh((1+1/c^2/x^2)^(1/2))/c^4+1/2*x^2*(1+1/c^2
/x^2)^(1/2)/c^2
```

Rubi [A] time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6342, 266, 51, 63, 208, 321, 203}

$$\frac{x^2\sqrt{\frac{1}{c^2x^2}+1}}{2c^2}-\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{2c^4}+\frac{x}{c^3}-\frac{\tan^{-1}(cx)}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCsch[c*x]*x^3)/(1 + c^2*x^2),x]
```

```
[Out] x/c^3 + (Sqrt[1 + 1/(c^2*x^2)]*x^2)/(2*c^2) - ArcTan[c*x]/c^4 - ArcTanh[Sqr
t[1 + 1/(c^2*x^2)]]/(2*c^4)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6342

```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d^2/(a*c^2), Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x] +
 Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m},
x] && EqQ[b - a*c^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^3}{1+c^2x^2}\,dx &= \frac{\int \frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{c^2}+\frac{\int \frac{x^2}{1+c^2x^2}\,dx}{c}\\
&= \frac{x}{c^3}-\frac{\int \frac{1}{1+c^2x^2}\,dx}{c^3}-\frac{\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{2c^2}\\
&= \frac{x}{c^3}+\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{2c^2}-\frac{\tan^{-1}(cx)}{c^4}+\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{4c^4}\\
&= \frac{x}{c^3}+\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{2c^2}-\frac{\tan^{-1}(cx)}{c^4}+\frac{\operatorname{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx,x,\sqrt{1+\frac{1}{c^2x^2}}\right)}{2c^2}\\
&= \frac{x}{c^3}+\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{2c^2}-\frac{\tan^{-1}(cx)}{c^4}-\frac{\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{2c^4}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 54, normalized size = 0.92

$$
-\frac{-cx\left(cx\sqrt{\frac{1}{c^2x^2}+1}+2\right)+\log\left(x\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)\right)+2\tan^{-1}(cx)}{2c^4}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCsch[c*x]*x^3)/(1 + c^2*x^2),x]
```

```
[Out] -1/2*(-(c*x*(2 + c*Sqrt[1 + 1/(c^2*x^2)]*x)) + 2*ArcTan[c*x] + Log[(1 + Sqr
t[1 + 1/(c^2*x^2)])*x])/c^4
```

fricas [A] time = 0.89, size = 68, normalized size = 1.15

$$
\frac{c^2x^2\sqrt{\frac{c^2x^2+1}{c^2x^2}}+2\,cx-2\,\arctan(cx)+\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)}{2\,c^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^3/(c^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] 1/2*(c^2*x^2*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*c*x - 2*arctan(c*x) + log(c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x))/c^4
```

giac [A] time = 0.14, size = 61, normalized size = 1.03

$$\frac{\sqrt{c^2x^2+1}\,x|c|\mathrm{sgn}(x)}{2\,c^4}+\frac{x}{c^3}+\frac{\log\left(-x|c|+\sqrt{c^2x^2+1}\right)\mathrm{sgn}(x)}{2\,c^4}-\frac{\arctan\left(cx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^3/(c^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c^2*x^2 + 1)*x*abs(c)*sgn(x)/c^4 + x/c^3 + 1/2*log(-x*abs(c) + sqr
t(c^2*x^2 + 1))*sgn(x)/c^4 - arctan(c*x)/c^4
```

maple [B] time = 0.06, size = 133, normalized size = 2.25

$$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x\left(x\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2+\ln\left(x+\sqrt{\frac{c^2x^2+1}{c^2}}\right)-2\ln\left(x+\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)\right)}{2\sqrt{\frac{c^2x^2+1}{c^2}}\,c^4}+\frac{x}{c^3}-\frac{\arctan\left(cx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(1+1/c^2/x^2)^(1/2))*x^3/(c^2*x^2+1),x)
```

```
[Out] 1/2*((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(x*((c^2*x^2+1)/c^2)^(1/2)*c^2+ln(x+((c^2
*x^2+1)/c^2)^(1/2))-2*ln(x+(-(-c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))/c^4
)^(1/2)))/((c^2*x^2+1)/c^2)^(1/2)/c^4+x/c^3-arctan(c*x)/c^4
```

maxima [B] time = 0.49, size = 107, normalized size = 1.81

$$\frac{x}{c^3}+\frac{\frac{2\sqrt{\frac{c^2x^2+1}{x^2}}}{c\left(\frac{c^2x^2+1}{c^2x^2}-1\right)}-\log\left(\frac{\sqrt{\frac{c^2x^2+1}{x^2}}}{c}+1\right)+\log\left(\frac{\sqrt{\frac{c^2x^2+1}{x^2}}}{c}-1\right)}{4\,c^4}-\frac{\arctan\left(cx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^3/(c^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] x/c^3 + 1/4*(2*sqrt((c^2*x^2 + 1)/x^2)/(c*((c^2*x^2 + 1)/(c^2*x^2) - 1)) -
log(sqrt((c^2*x^2 + 1)/x^2)/c + 1) + log(sqrt((c^2*x^2 + 1)/x^2)/c - 1))/c^
4 - arctan(c*x)/c^4
```

mupad [B] time = 2.34, size = 51, normalized size = 0.86

$$\frac{x^2\sqrt{\frac{1}{c^2\,x^2}+1}}{2\,c^2}-\frac{\mathrm{atan}\left(c\,x\right)-c\,x}{c^4}-\frac{\mathrm{atanh}\left(\sqrt{\frac{1}{c^2\,x^2}+1}\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 + 1),x)
```

```
[Out] (x^2*(1/(c^2*x^2) + 1)^(1/2))/(2*c^2) - (atan(c*x) - c*x)/c^4 - atanh((1/(c
^2*x^2) + 1)^(1/2))/(2*c^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{x^2}{c^2x^2+1}\,dx+\int\frac{cx^3\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c**2/x**2)**(1/2))*x**3/(c**2*x**2+1),x)
```

```
[Out] (Integral(x**2/(c**2*x**2 + 1), x) + Integral(c*x**3*sqrt(1 + 1/(c**2*x**2)
)/(c**2*x**2 + 1), x))/c
```

3.63 $\int \frac{e^{\text{csch}^{-1}(cx)}x^2}{1+c^2x^2}\,dx$

Optimal. Leaf size=36

$$\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c^2}+\frac{\log\left(c^2x^2+1\right)}{2c^3}$$

[Out] 1/2*ln(c^2*x^2+1)/c^3+x*(1+1/c^2/x^2)^(1/2)/c^2

Rubi [A] time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6342, 191, 260}

$$\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c^2}+\frac{\log\left(c^2x^2+1\right)}{2c^3}$$

Antiderivative was successfully verified.

[In] Int[(E^ArcCsch[c*x]*x^2)/(1 + c^2*x^2),x]

[Out] (Sqrt[1 + 1/(c^2*x^2)]*x)/c^2 + Log[1 + c^2*x^2]/(2*c^3)

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6342

```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d^2/(a*c^2), Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x] +
 Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m},
x] && EqQ[b - a*c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\text{csch}^{-1}(cx)}x^2}{1+c^2x^2}\,dx &= \frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{c^2}+\frac{\int \frac{x}{1+c^2x^2}\,dx}{c}\\ &= \frac{\sqrt{1+\frac{1}{c^2x^2}}\,x}{c^2}+\frac{\log\left(1+c^2x^2\right)}{2c^3}\end{aligned}$$

Mathematica [A] time = 0.06, size = 35, normalized size = 0.97

$$\frac{2cx\sqrt{\frac{1}{c^2x^2}+1}+\log\left(c^2x^2+1\right)}{2c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(E^ArcCsch[c*x]*x^2)/(1 + c^2*x^2),x]

```
[Out] (2*c*Sqrt[1 + 1/(c^2*x^2)]*x + Log[1 + c^2*x^2])/(2*c^3)
```

fricas [A] time = 0.64, size = 38, normalized size = 1.06

$$\frac{2\,cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+\log\left(c^2x^2+1\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^2/(c^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + log(c^2*x^2 + 1))/c^3
```

giac [A] time = 0.14, size = 44, normalized size = 1.22

$$\frac{\sqrt{c^2x^2+1}\,|c|\mathrm{sgn}(x)}{c^4}+\frac{\log\left(c^2x^2+1\right)}{2\,c^3}-\frac{|c|\mathrm{sgn}(x)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^2/(c^2*x^2+1),x, algorithm="giac")
```

```
[Out] sqrt(c^2*x^2 + 1)*abs(c)*sgn(x)/c^4 + 1/2*log(c^2*x^2 + 1)/c^3 - abs(c)*sgn
(x)/c^4
```

maple [B] time = 0.06, size = 89, normalized size = 2.47

$$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\;x\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}}{\sqrt{\frac{c^2x^2+1}{c^2}}\;c^2}+\frac{\ln\left(c^2x^2+1\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(1+1/c^2/x^2)^(1/2))*x^2/(c^2*x^2+1),x)
```

```
[Out] ((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(-(-c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))/
c^4)^(1/2)/((c^2*x^2+1)/c^2)^(1/2)/c^2+1/2*ln(c^2*x^2+1)/c^3
```

maxima [A] time = 0.40, size = 31, normalized size = 0.86

$$\frac{\log\left(c^3x^2+c\right)}{2\,c^3}+\frac{\sqrt{c^2x^2+1}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^2/(c^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] 1/2*log(c^3*x^2 + c)/c^3 + sqrt(c^2*x^2 + 1)/c^3
```

mupad [B] time = 2.34, size = 31, normalized size = 0.86

$$\frac{\ln\left(c^2\,x^2+1\right)+2\,c\,x\,\sqrt{\frac{1}{c^2\,x^2}+1}}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 + 1),x)
```

```
[Out] (log(c^2*x^2 + 1) + 2*c*x*(1/(c^2*x^2) + 1)^(1/2))/(2*c^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{c^2x^2+1}\,dx + \int \frac{cx^2\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c**2/x**2)**(1/2))*x**2/(c**2*x**2+1),x)
```

```
[Out] (Integral(x/(c**2*x**2 + 1), x) + Integral(c*x**2*sqrt(1 + 1/(c**2*x**2))/(
c**2*x**2 + 1), x))/c
```

3.64 $\int \frac{e^{\text{csch}^{-1}(cx)}x}{1+c^2x^2}\,dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^2}+\frac{\tan^{-1}(cx)}{c^2}$$

[Out] arctan(c*x)/c^2+arctanh((1+1/c^2/x^2)^(1/2))/c^2

Rubi [A] time = 0.06, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6342, 266, 63, 208, 203}

$$\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^2}+\frac{\tan^{-1}(cx)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(E^ArcCsch[c*x]*x)/(1 + c^2*x^2),x]

[Out] ArcTan[c*x]/c^2 + ArcTanh[Sqrt[1 + 1/(c^2*x^2)]]/c^2

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6342

```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d^2/(a*c^2), Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x] +
 Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m},
x] && EqQ[b - a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\text{csch}^{-1}(cx)}x}{1+c^2x^2}\,dx = \frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x}\,dx}{c^2} + \frac{\int \frac{1}{1+c^2x^2}\,dx}{c}$$
$$= \frac{\tan^{-1}(cx)}{c^2} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2c^2}$$
$$= \frac{\tan^{-1}(cx)}{c^2} - \text{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx, x, \sqrt{1+\frac{1}{c^2x^2}}\right)$$
$$= \frac{\tan^{-1}(cx)}{c^2} + \frac{\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c^2}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 1.41

$$\frac{\log\left(x\left(\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1\right)\right)}{c^2} + \frac{\tan^{-1}(cx)}{c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCsch[c*x]*x)/(1 + c^2*x^2),x]
[Out] ArcTan[c*x]/c^2 + Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])]/c^2
```

fricas [A] time = 0.66, size = 38, normalized size = 1.41

$$\frac{\arctan(cx) - \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x/(c^2*x^2+1),x, algorithm="fricas")
[Out] (arctan(c*x) - log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x))/c^2
```

giac [A] time = 0.14, size = 34, normalized size = 1.26

$$-\frac{\log\left(-x|c| + \sqrt{c^2x^2+1}\right)\text{sgn}(x)}{c^2} + \frac{\arctan(cx)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x/(c^2*x^2+1),x, algorithm="giac")
[Out] -log(-x*abs(c) + sqrt(c^2*x^2 + 1))*sgn(x)/c^2 + arctan(c*x)/c^2
```

maple [B] time = 0.06, size = 85, normalized size = 3.15

$$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x\ln\left(x+\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)}{\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2} + \frac{\arctan(cx)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(1+1/c^2/x^2)^(1/2))*x/(c^2*x^2+1),x)
```

```
[Out] ((c^2*x^2+1)/c^2/x^2)^(1/2)*x*ln(x+(-(-c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1
/2))/c^4)^(1/2))/((c^2*x^2+1)/c^2)^(1/2)/c^2+arctan(c*x)/c^2
```

maxima [B] time = 0.41, size = 61, normalized size = 2.26

$$\frac{\log\left(\frac{\sqrt{\frac{c^2x^2+1}{x^2}}}{c}+1\right)-\log\left(\frac{\sqrt{\frac{c^2x^2+1}{x^2}}}{c}-1\right)}{2\,c^2}+\frac{\arctan\left(cx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x/(c^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*(log(sqrt((c^2*x^2 + 1)/x^2)/c + 1) - log(sqrt((c^2*x^2 + 1)/x^2)/c - 1
))/c^2 + arctan(c*x)/c^2
```

mupad [B] time = 2.36, size = 21, normalized size = 0.78

$$\frac{\mathrm{atanh}\left(\sqrt{\frac{1}{c^2\,x^2}+1}\right)+\mathrm{atan}\left(c\,x\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 + 1),x)
```

```
[Out] (atanh((1/(c^2*x^2) + 1)^(1/2)) + atan(c*x))/c^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{cx\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^2+1}\,dx+\int \frac{1}{c^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c**2/x**2)**(1/2))*x/(c**2*x**2+1),x)
```

```
[Out] (Integral(c*x*sqrt(1 + 1/(c**2*x**2))/(c**2*x**2 + 1), x) + Integral(1/(c**
2*x**2 + 1), x))/c
```

3.65 $\int \frac{e^{\mathrm{csch}^{-1}(cx)}}{1+c^2x^2}\,dx$

Optimal. Leaf size=33

$$-\frac{\log\left(c^2x^2+1\right)}{2c}+\frac{\log(x)}{c}-\frac{\mathrm{csch}^{-1}(cx)}{c}$$

```
[Out] -arccsch(c*x)/c+ln(x)/c-1/2*ln(c^2*x^2+1)/c
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6340, 335, 215, 266, 36, 29, 31}

$$-\frac{\log\left(c^2x^2+1\right)}{2c}+\frac{\log(x)}{c}-\frac{\mathrm{csch}^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[c*x]/(1 + c^2*x^2),x]
```

```
[Out] -(ArcCsch[c*x]/c) + Log[x]/c - Log[1 + c^2*x^2]/(2*c)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6340

```
Int[E^ArcCsch[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Dist[1/(a*c^2
), Int[1/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x] + Dist[1/c, Int[1/(x*(a + b*x^
2)), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b - a*c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{1+c^2x^2}\,dx &= \frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2}\,dx}{c^2}+\frac{\int \frac{1}{x(1+c^2x^2)}\,dx}{c}\\ &= -\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c^2}+\frac{\operatorname{Subst}\left(\int \frac{1}{x(1+c^2x)}\,dx, x, x^2\right)}{2c}\\ &= -\frac{\operatorname{csch}^{-1}(cx)}{c}+\frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2c}-\frac{1}{2}c\operatorname{Subst}\left(\int \frac{1}{1+c^2x}\,dx, x, x^2\right)\\ &= -\frac{\operatorname{csch}^{-1}(cx)}{c}+\frac{\log(x)}{c}-\frac{\log\left(1+c^2x^2\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 1.12

$$-\frac{\log\left(c^2x^2+1\right)}{2c}+\frac{\log(x)}{c}-\frac{\sinh^{-1}\left(\frac{1}{cx}\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[c*x]/(1 + c^2*x^2),x]
```

```
[Out] -(ArcSinh[1/(c*x)]/c) + Log[x]/c - Log[1 + c^2*x^2]/(2*c)
```

fricas [B] time = 0.67, size = 80, normalized size = 2.42

$$-\frac{\log\left(c^2x^2+1\right)+2\,\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-2\,\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)-2\,\log(x)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/(c^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*(log(c^2*x^2 + 1) + 2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1)
 - 2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - 2*log(x))/c
```

giac [B] time = 0.16, size = 70, normalized size = 2.12

$$-\frac{\log\left(c^2x^2+1\right)}{2\,c}-\frac{\left(|c|\operatorname{sgn}(x)-c\right)\log\left(\sqrt{c^2x^2+1}+1\right)}{2\,c^2}+\frac{\left(|c|\operatorname{sgn}(x)+c\right)\log\left(\sqrt{c^2x^2+1}-1\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/(c^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*log(c^2*x^2 + 1)/c - 1/2*(abs(c)*sgn(x) - c)*log(sqrt(c^2*x^2 + 1) + 1
)/c^2 + 1/2*(abs(c)*sgn(x) + c)*log(sqrt(c^2*x^2 + 1) - 1)/c^2
```

maple [B] time = 0.06, size = 172, normalized size = 5.21

$$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x\left(\sqrt{\frac{1}{c^2}}\,\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2-\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\,c^2\sqrt{\frac{1}{c^2}}-\ln\left(\frac{2\sqrt{\frac{1}{c^2}}\,\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2+2}{c^2x}\right)\right)}{\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2\sqrt{\frac{1}{c^2}}}+\frac{\ln(x)}{c}-\frac{\ln\left(c^2x^2+1\right.}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(1+1/c^2/x^2)^(1/2))/(c^2*x^2+1),x)
```

```
[Out] ((c^2*x^2+1)/c^2/x^2)^(1/2)*x*((1/c^2)^(1/2)*((c^2*x^2+1)/c^2)^(1/2)*c^2-(-
(-c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))/c^4)^(1/2)*c^2*(1/c^2)^(1/2)-ln(
2*((1/c^2)^(1/2)*((c^2*x^2+1)/c^2)^(1/2)*c^2+1)/x/c^2))/((c^2*x^2+1)/c^2)^(
1/2)/c^2/(1/c^2)^(1/2)+ln(x)/c-1/2*ln(c^2*x^2+1)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(c^{2}x^{2}+1\right)}{2\,c}+\frac{\log(x)}{c}+\int\frac{\sqrt{c^{2}x^{2}+1}}{c^{3}x^{3}+cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/(c^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*log(c^2*x^2 + 1)/c + log(x)/c + integrate(sqrt(c^2*x^2 + 1)/(c^3*x^3 +
 c*x), x)
```

mupad [B] time = 2.36, size = 38, normalized size = 1.15

$$-\mathrm{asinh}\left(\frac{\sqrt{\frac{1}{c^{2}}}}{x}\right)\sqrt{\frac{1}{c^{2}}}-\frac{\ln\left(c^{2}\,x^{2}+1\right)-2\,\ln(x)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x))/(c^2*x^2 + 1),x)
```

```
[Out] - asinh((1/c^2)^(1/2)/x)*(1/c^2)^(1/2) - (log(c^2*x^2 + 1) - 2*log(x))/(2*c
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{cx\sqrt{1+\frac{1}{c^{2}x^{2}}}}{c^{2}x^{3}+x}\,dx+\int\frac{1}{c^{2}x^{3}+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c**2/x**2)**(1/2))/(c**2*x**2+1),x)
```

```
[Out] (Integral(c*x*sqrt(1 + 1/(c**2*x**2))/(c**2*x**3 + x), x) + Integral(1/(c**
2*x**3 + x), x))/c
```

3.66 $\int \frac{e^{\mathbf{csch}^{-1}(cx)}}{x(1+c^2x^2)}\,dx$

Optimal. Leaf size=30

$$-\sqrt{\frac{1}{c^2x^2}+1}-\frac{1}{cx}-\tan^{-1}(cx)$$

[Out] -1/c/x-arctan(c*x)-(1+1/c^2/x^2)^(1/2)

Rubi [A] time = 0.07, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6342, 261, 325, 203}

$$-\sqrt{\frac{1}{c^2x^2}+1}-\frac{1}{cx}-\tan^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[E^ArcCsch[c*x]/(x*(1 + c^2*x^2)),x]

[Out] -Sqrt[1 + 1/(c^2*x^2)] - 1/(c*x) - ArcTan[c*x]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6342

```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d^2/(a*c^2), Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x] +
 Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m},
x] && EqQ[b - a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x\left(1+c^2x^2\right)}\,dx = \frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^3}\,dx}{c^2} + \frac{\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx}{c}$$
$$= -\sqrt{1+\frac{1}{c^2x^2}} - \frac{1}{cx} - c\int \frac{1}{1+c^2x^2}\,dx$$
$$= -\sqrt{1+\frac{1}{c^2x^2}} - \frac{1}{cx} - \tan^{-1}(cx)$$

Mathematica [A] time = 0.10, size = 30, normalized size = 1.00

$$-\sqrt{\frac{1}{c^2x^2}+1} - \frac{1}{cx} - \tan^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCsch[c*x]/(x*(1 + c^2*x^2)),x]
[Out] -Sqrt[1 + 1/(c^2*x^2)] - 1/(c*x) - ArcTan[c*x]
```

fricas [A] time = 0.54, size = 41, normalized size = 1.37

$$-\frac{cx\arctan(cx) + cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + cx + 1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x/(c^2*x^2+1),x, algorithm="fricas")
[Out] -(c*x*arctan(c*x) + c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + c*x + 1)/(c*x)
```

giac [A] time = 0.14, size = 43, normalized size = 1.43

$$\frac{2\,\mathrm{sgn}(x)}{\left(x|c| - \sqrt{c^2x^2+1}\right)^2 - 1} - \frac{1}{cx} - \arctan(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x/(c^2*x^2+1),x, algorithm="giac")
[Out] 2*sgn(x)/((x*abs(c) - sqrt(c^2*x^2 + 1))^2 - 1) - 1/(c*x) - arctan(c*x)
```

maple [B] time = 0.07, size = 154, normalized size = 5.13

$$-\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\left(\left(\frac{c^2x^2+1}{c^2}\right)^{\frac{3}{2}}c^2 - \sqrt{\frac{c^2x^2+1}{c^2}}\,x^2c^2 + \ln\left(x+\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)x - \ln\left(x+\sqrt{\frac{c^2x^2+1}{c^2}}\right)x\right)}{\sqrt{\frac{c^2x^2+1}{c^2}}} - \frac{1}{cx} - \text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(1+1/c^2/x^2)^(1/2))/x/(c^2*x^2+1),x)
[Out] -((c^2*x^2+1)/c^2/x^2)^(1/2)*(((c^2*x^2+1)/c^2)^(3/2)*c^2-((c^2*x^2+1)/c^2)
^(1/2)*x^2*c^2+ln(x+(-(-c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))/c^4)^(1/2)
```

```
)*x-ln(x+((c^2*x^2+1)/c^2)^(1/2))*x)/((c^2*x^2+1)/c^2)^(1/2)-1/c/x-arctan(c
*x)
```

maxima [A] time = 0.43, size = 34, normalized size = 1.13

$$-\frac{\sqrt{c^2x^2+1}}{cx}-\frac{1}{cx}-\arctan(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x/(c^2*x^2+1),x, algorithm="maxima")
[Out] -sqrt(c^2*x^2 + 1)/(c*x) - 1/(c*x) - arctan(c*x)
```

mupad [B] time = 2.15, size = 29, normalized size = 0.97

$$-\mathrm{atan}(c\,x)-\frac{x\,\sqrt{\frac{1}{c^2\,x^2}+1}+\frac{1}{c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x))/(x*(c^2*x^2 + 1)),x)
[Out] - atan(c*x) - (x*(1/(c^2*x^2) + 1)^(1/2) + 1/c)/x
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c**2/x**2)**(1/2))/x/(c**2*x**2+1),x)
[Out] Exception raised: TypeError
```

3.67 $\int \frac{e^{\text{csch}^{-1}(cx)}}{x^2(1+c^2x^2)}\,dx$

Optimal. Leaf size=60

$$-\frac{\sqrt{\frac{1}{c^2x^2}+1}}{2x}+\frac{1}{2}c\log\left(c^2x^2+1\right)-\frac{1}{2cx^2}-c\log(x)+\frac{1}{2}c\,\text{csch}^{-1}(cx)$$

```
[Out] -1/2/c/x^2+1/2*c*arccsch(c*x)-c*ln(x)+1/2*c*ln(c^2*x^2+1)-1/2*(1+1/c^2/x^2)
^(1/2)/x
```

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6342, 335, 321, 215, 266, 44}

$$-\frac{\sqrt{\frac{1}{c^2x^2}+1}}{2x}+\frac{1}{2}c\log\left(c^2x^2+1\right)-\frac{1}{2cx^2}-c\log(x)+\frac{1}{2}c\,\text{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[c*x]/(x^2*(1 + c^2*x^2)),x]
```

```
[Out] -1/(2*c*x^2) - Sqrt[1 + 1/(c^2*x^2)]/(2*x) + (c*ArcCsch[c*x])/2 - c*Log[x]
+ (c*Log[1 + c^2*x^2])/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6342

```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d^2/(a*c^2), Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x] +
 Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m},
x] && EqQ[b - a*c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^2(1+c^2x^2)}\,dx &= \frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^4}\,dx}{c^2}+\frac{\int \frac{1}{x^3(1+c^2x^2)}\,dx}{c}\\ &= -\frac{\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c^2}+\frac{\operatorname{Subst}\left(\int \frac{1}{x^2(1+c^2x)}\,dx,x,x^2\right)}{2c}\\ &= -\frac{\sqrt{1+\frac{1}{c^2x^2}}}{2x}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)+\frac{\operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)dx,x,x^2\right)}{2c}\\ &= -\frac{1}{2cx^2}-\frac{\sqrt{1+\frac{1}{c^2x^2}}}{2x}+\frac{1}{2}c\operatorname{csch}^{-1}(cx)-c\log(x)+\frac{1}{2}c\log\left(1+c^2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.11, size = 58, normalized size = 0.97

$$\frac{1}{2}\left(-\frac{\sqrt{\frac{1}{c^2x^2}+1}}{x}+c\log\left(c^2x^2+1\right)-\frac{1}{cx^2}-2c\log(x)+c\sinh^{-1}\left(\frac{1}{cx}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCsch[c*x]/(x^2*(1 + c^2*x^2)),x]
```

```
[Out] (-(1/(c*x^2)) - Sqrt[1 + 1/(c^2*x^2)]/x + c*ArcSinh[1/(c*x)] - 2*c*Log[x] +
 c*Log[1 + c^2*x^2])/2
```

fricas [B] time = 1.34, size = 130, normalized size = 2.17

$$\frac{c^2x^2\log\left(c^2x^2+1\right)+c^2x^2\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-c^2x^2\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)-2\,c^2x^2\log(x)-cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x^2/(c^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] 1/2*(c^2*x^2*log(c^2*x^2 + 1) + c^2*x^2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2
)) - c*x + 1) - c^2*x^2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) -
2*c^2*x^2*log(x) - c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 1)/(c*x^2)
```

giac [B] time = 0.15, size = 114, normalized size = 1.90

$$\frac{1}{2}c\log\left(c^2x^2+1\right)+\frac{1}{4}\left(|c|\operatorname{sgn}(x)-2c\right)\log\left(\sqrt{c^2x^2+1}+1\right)-\frac{1}{4}\left(|c|\operatorname{sgn}(x)+2c\right)\log\left(\sqrt{c^2x^2+1}-1\right)-\frac{\sqrt{c^2}}{2\left(\sqrt{c^2x^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x^2/(c^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*c*log(c^2*x^2 + 1) + 1/4*(abs(c)*sgn(x) - 2*c)*log(sqrt(c^2*x^2 + 1) +
1) - 1/4*(abs(c)*sgn(x) + 2*c)*log(sqrt(c^2*x^2 + 1) - 1) - 1/2*(sqrt(c^2*x
^2 + 1)*abs(c)*sgn(x) + c)/((sqrt(c^2*x^2 + 1) + 1)*(sqrt(c^2*x^2 + 1) - 1)
)
```

maple [B] time = 0.07, size = 210, normalized size = 3.50

$$-\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\left(c^2\left(\frac{c^2x^2+1}{c^2}\right)^{\frac{3}{2}}\sqrt{\frac{1}{c^2}}+\sqrt{\frac{c^2x^2+1}{c^2}}\sqrt{\frac{1}{c^2}}\,x^2c^2-2\sqrt{\frac{1}{c^2}}\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\,x^2c^2-\ln\left(\frac{2\sqrt{\frac{1}{c^2}}\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2+2}{c^2x}\right.\right.}{2x\sqrt{\frac{c^2x^2+1}{c^2}}\sqrt{\frac{1}{c^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(1+1/c^2/x^2)^(1/2))/x^2/(c^2*x^2+1),x)
```

```
[Out] -1/2*((c^2*x^2+1)/c^2/x^2)^(1/2)/x*(c^2*((c^2*x^2+1)/c^2)^(3/2)*(1/c^2)^(1/
2)+((c^2*x^2+1)/c^2)^(1/2)*(1/c^2)^(1/2)*x^2*c^2-2*(1/c^2)^(1/2)*(-(-c^2*x+
(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))/c^4)^(1/2)*x^2*c^2-ln(2*((1/c^2)^(1/2)*(
(c^2*x^2+1)/c^2)^(1/2)*c^2+1)/x/c^2)*x^2)/((c^2*x^2+1)/c^2)^(1/2)/(1/c^2)^(
1/2)-1/2/c/x^2-c*ln(x)+1/2*c*ln(c^2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,c\log\left(c^2x^2+1\right)-c\log(x)-\frac{1}{2\,cx^2}+\int\frac{\sqrt{c^2x^2+1}}{c^3x^5+cx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x^2/(c^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] 1/2*c*log(c^2*x^2 + 1) - c*log(x) - 1/2/(c*x^2) + integrate(sqrt(c^2*x^2 +
1)/(c^3*x^5 + c*x^3), x)
```

mupad [B] time = 2.42, size = 61, normalized size = 1.02

$$\frac{\mathrm{asinh}\left(\frac{\sqrt{\frac{1}{c^2}}}{x}\right)}{2\sqrt{\frac{1}{c^2}}}+\frac{c\,\ln\left(-c^2\,x^2-1\right)}{2}-c\,\ln(x)-\frac{\sqrt{\frac{1}{c^2\,x^2}+1}}{2\,x}-\frac{1}{2\,c\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x))/(x^2*(c^2*x^2 + 1)),x)
```

```
[Out] asinh((1/c^2)^(1/2)/x)/(2*(1/c^2)^(1/2)) + (c*log(- c^2*x^2 - 1))/2 - c*log
(x) - (1/(c^2*x^2) + 1)^(1/2)/(2*x) - 1/(2*c*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{cx\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^5+x^3}\,dx+\int\frac{1}{c^2x^5+x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c**2/x**2)**(1/2))/x**2/(c**2*x**2+1),x)
```

```
[Out] (Integral(c*x*sqrt(1 + 1/(c**2*x**2))/(c**2*x**5 + x**3), x) + Integral(1/(
c**2*x**5 + x**3), x))/c
```

3.68 $\int \frac{e^{\mathbf{csch}^{-1}(cx)}}{x^3(1+c^2x^2)} dx$

Optimal. Leaf size=61

$$-\frac{1}{3}c^2\left(\frac{1}{c^2x^2}+1\right)^{3/2}+c^2\sqrt{\frac{1}{c^2x^2}+1}+c^2\tan^{-1}(cx)-\frac{1}{3cx^3}+\frac{c}{x}$$

```
[Out] -1/3*c^2*(1+1/c^2/x^2)^(3/2)-1/3/c/x^3+c/x+c^2*arctan(c*x)+c^2*(1+1/c^2/x^2
)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6342, 266, 43, 325, 203}

$$-\frac{1}{3}c^2\left(\frac{1}{c^2x^2}+1\right)^{3/2}+c^2\sqrt{\frac{1}{c^2x^2}+1}+c^2\tan^{-1}(cx)-\frac{1}{3cx^3}+\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCsch[c*x]/(x^3*(1 + c^2*x^2)),x]
```

```
[Out] c^2*Sqrt[1 + 1/(c^2*x^2)] - (c^2*(1 + 1/(c^2*x^2))^(3/2))/3 - 1/(3*c*x^3) +
 c/x + c^2*ArcTan[c*x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6342

```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d^2/(a*c^2), Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x] +
 Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m},
x] && EqQ[b - a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\text{csch}^{-1}(cx)}}{x^3\left(1+c^2x^2\right)}\,dx = \frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^5}\,dx}{c^2} + \frac{\int \frac{1}{x^4\left(1+c^2x^2\right)}\,dx}{c}$$

$$= -\frac{1}{3cx^3} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2c^2} - c\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx$$

$$= -\frac{1}{3cx^3} + \frac{c}{x} - \frac{\text{Subst}\left(\int \left(-\frac{c^2}{\sqrt{1+\frac{x}{c^2}}} + c^2\sqrt{1+\frac{x}{c^2}}\right)dx, x, \frac{1}{x^2}\right)}{2c^2} + c^3\int \frac{1}{1+c^2x^2}\,dx$$

$$= c^2\sqrt{1+\frac{1}{c^2x^2}} - \frac{1}{3}c^2\left(1+\frac{1}{c^2x^2}\right)^{3/2} - \frac{1}{3cx^3} + \frac{c}{x} + c^2\tan^{-1}(cx)$$

Mathematica [A] time = 0.14, size = 54, normalized size = 0.89

$$\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(2c^2x^2-1\right)}{3x^2} + c^2\tan^{-1}(cx) - \frac{1}{3cx^3} + \frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCsch[c*x]/(x^3*(1 + c^2*x^2)),x]
```

```
[Out] -1/3*1/(c*x^3) + c/x + (Sqrt[1 + 1/(c^2*x^2)]*(-1 + 2*c^2*x^2))/(3*x^2) + c
^2*ArcTan[c*x]
```

fricas [A] time = 0.55, size = 70, normalized size = 1.15

$$\frac{3\,c^3x^3\arctan(cx) + 2\,c^3x^3 + 3\,c^2x^2 + \left(2\,c^3x^3 - cx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}} - 1}{3\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x^3/(c^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] 1/3*(3*c^3*x^3*arctan(c*x) + 2*c^3*x^3 + 3*c^2*x^2 + (2*c^3*x^3 - c*x)*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) - 1)/(c*x^3)
```

giac [A] time = 0.16, size = 82, normalized size = 1.34

$$c^2\arctan(cx) + \frac{4\left(3\left(x|c| - \sqrt{c^2x^2+1}\right)^2 - 1\right)c^2\text{sgn}(x)}{3\left(\left(x|c| - \sqrt{c^2x^2+1}\right)^2 - 1\right)^3} + \frac{3\,c^2x^2 - 1}{3\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x^3/(c^2*x^2+1),x, algorithm="giac")
```

```
[Out] c^2*arctan(c*x) + 4/3*(3*(x*abs(c) - sqrt(c^2*x^2 + 1))^2 - 1)*c^2*sgn(x)/(
(x*abs(c) - sqrt(c^2*x^2 + 1))^2 - 1)^3 + 1/3*(3*c^2*x^2 - 1)/(c*x^3)
```

maple [B] time = 0.07, size = 193, normalized size = 3.16

$$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\, c^2 \left(3\left(\frac{c^2x^2+1}{c^2}\right)^{\frac{3}{2}} x^2c^2 - 3\sqrt{\frac{c^2x^2+1}{c^2}}\, x^4c^2 - 3\ln\left(x+\sqrt{\frac{c^2x^2+1}{c^2}}\right)x^3 + 3\ln\left(x+\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)x\right.}{3x^2\sqrt{\frac{c^2x^2+1}{c^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(1+1/c^2/x^2)^(1/2))/x^3/(c^2*x^2+1),x)
```

```
[Out] 1/3*((c^2*x^2+1)/c^2/x^2)^(1/2)/x^2*c^2*(3*((c^2*x^2+1)/c^2)^(3/2)*x^2*c^2-
3*((c^2*x^2+1)/c^2)^(1/2)*x^4*c^2-3*ln(x+((c^2*x^2+1)/c^2)^(1/2))*x^3+3*ln(
x+(-(-c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))/c^4)^(1/2))*x^3-((c^2*x^2+1)
/c^2)^(3/2))/((c^2*x^2+1)/c^2)^(1/2)-1/3/c/x^3+c/x+c^2*arctan(c*x)
```

maxima [A] time = 0.43, size = 56, normalized size = 0.92

$$c^2\arctan(cx) + \frac{\left(2\,c^2x^2-1\right)\sqrt{c^2x^2+1}}{3\,cx^3} + \frac{3\,c^2x^2-1}{3\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x^3/(c^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] c^2*arctan(c*x) + 1/3*(2*c^2*x^2 - 1)*sqrt(c^2*x^2 + 1)/(c*x^3) + 1/3*(3*c^
2*x^2 - 1)/(c*x^3)
```

mupad [B] time = 2.22, size = 57, normalized size = 0.93

$$\frac{c+\frac{2\,c^2\,x\,\sqrt{\frac{1}{c^2\,x^2}+1}}{3}}{x} - \frac{\frac{x\,\sqrt{\frac{1}{c^2\,x^2}+1}}{3}+\frac{1}{3\,c}}{x^3} + c^2\,\mathrm{atan}\left(c\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x))/(x^3*(c^2*x^2 + 1)),x)
```

```
[Out] (c + (2*c^2*x*(1/(c^2*x^2) + 1)^(1/2))/3)/x - ((x*(1/(c^2*x^2) + 1)^(1/2))/
3 + 1/(3*c))/x^3 + c^2*atan(c*x)
```

sympy [A] time = 5.23, size = 75, normalized size = 1.23

$$-2c^5\left(\frac{\left(1+\frac{1}{c^2x^2}\right)^{\frac{3}{2}}}{6c^3} - \frac{\sqrt{1+\frac{1}{c^2x^2}}}{2c^3}\right) - \frac{c^3\,\mathrm{atan}\left(\frac{1}{x\sqrt{c^2}}\right)}{\sqrt{c^2}} + \frac{c}{x} - \frac{1}{3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(1+1/c**2/x**2)**(1/2))/x**3/(c**2*x**2+1),x)
```

```
[Out] -2*c**5*((1 + 1/(c**2*x**2))**(3/2)/(6*c**3) - sqrt(1 + 1/(c**2*x**2))/(2*c
**3)) - c**3*atan(1/(x*sqrt(c**2)))/sqrt(c**2) + c/x - 1/(3*c*x**3)
```

3.69 $\int \frac{\operatorname{csch}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=61

$$-\frac{\operatorname{Li}_2\left(e^{2\operatorname{csch}^{-1}(a+bx)}\right)}{2d}+\frac{\operatorname{csch}^{-1}(a+bx)^2}{2d}-\frac{\operatorname{csch}^{-1}(a+bx)\log\left(1-e^{2\operatorname{csch}^{-1}(a+bx)}\right)}{d}$$

```
[Out] 1/2*arccsch(b*x+a)^2/d-arccsch(b*x+a)*ln(1-(1/(b*x+a)+(1+1/(b*x+a)^2)^(1/2)
)^2)/d-1/2*polylog(2,(1/(b*x+a)+(1+1/(b*x+a)^2)^(1/2))^2)/d
```

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6320, 12, 6282, 5659, 3716, 2190, 2279, 2391}

$$-\frac{\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(a+bx)}\right)}{2d}+\frac{\operatorname{csch}^{-1}(a+bx)^2}{2d}-\frac{\operatorname{csch}^{-1}(a+bx)\log\left(1-e^{2\operatorname{csch}^{-1}(a+bx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsch[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ArcCsch[a + b*x]^2/(2*d) - (ArcCsch[a + b*x]*Log[1 - E^(2*ArcCsch[a + b*x])
])/d - PolyLog[2, E^(2*ArcCsch[a + b*x])]/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6282

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rule 6320

```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcCsch[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{b\operatorname{csch}^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{\operatorname{csch}^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sinh^{-1}(x)}{x}\,dx, x, \frac{1}{a+bx}\right)}{d}\\
&= -\frac{\operatorname{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{1}{a+bx}\right)\right)}{d}\\
&= \frac{\sinh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} + \frac{2\operatorname{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{1}{a+bx}\right)\right)}{d}\\
&= \frac{\sinh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\sinh^{-1}\left(\frac{1}{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} + \frac{\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x\right.}{d}\\
&= \frac{\sinh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\sinh^{-1}\left(\frac{1}{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} + \frac{\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sin}\right.}{2d}\\
&= \frac{\sinh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\sinh^{-1}\left(\frac{1}{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} - \frac{\operatorname{Li}_2\left(e^{2\sinh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 52, normalized size = 0.85

$$\frac{\operatorname{Li}_2\left(e^{-2\operatorname{csch}^{-1}(a+bx)}\right) - \operatorname{csch}^{-1}(a+bx)\left(\operatorname{csch}^{-1}(a+bx) + 2\log\left(1-e^{-2\operatorname{csch}^{-1}(a+bx)}\right)\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCsch[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] (-(ArcCsch[a + b*x]*(ArcCsch[a + b*x] + 2*Log[1 - E^(-2*ArcCsch[a + b*x])])
) + PolyLog[2, E^(-2*ArcCsch[a + b*x])])/(2*d)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcsch}\left(bx+a\right)}{bdx+ad},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arccsch(b*x + a)/(b*d*x + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcsch}\left(bx+a\right)}{dx+\frac{ad}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arccsch(b*x + a)/(d*x + a*d/b), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\text{arcsch}\left(bx+a\right)}{\frac{ad}{b}+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsch(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] int(arccsch(b*x+a)/(a*d/b+d*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,\log\left(b^2x^2+2\,abx+a^2+1\right)\log\left(bx+a\right)+\text{Li}_2\left(-b^2x^2-2\,abx-a^2\right)}{4\,d}-\frac{\log\left(bx+a\right)^2-2\,\log\left(bx+a\right)\log\left(\sqrt{b^2x\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsch(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] -1/4*(2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a) + dilog(-b^2*x^2 - 2*
a*b*x - a^2))/d - 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1) + 1))/d + integrate((b^2*x + a*b)*log(b*x + a)/(b^2*d*x^2
 + 2*a*b*d*x + a^2*d + (b^2*d*x^2 + 2*a*b*d*x + a^2*d + d)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1) + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\text{asinh}\left(\frac{1}{a+b\,x}\right)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(1/(a + b*x))/(d*x + (a*d)/b),x)
```

```
[Out] int(asinh(1/(a + b*x))/(d*x + (a*d)/b), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int \frac{\operatorname{acsch}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsch(b*x+a)/(a*d/b+d*x),x)
[Out] b*Integral(acsch(a + b*x)/(a + b*x), x)/d
```

3.70 $\int x^3 \text{csch}^{-1}\left(a+bx^4\right)\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\sqrt{\frac{1}{\left(a+bx^4\right)^2}+1}\right)}{4b}+\frac{\left(a+bx^4\right)\text{csch}^{-1}\left(a+bx^4\right)}{4b}$$

```
[Out] 1/4*(b*x^4+a)*arccsch(b*x^4+a)/b+1/4*arctanh((1+1/(b*x^4+a)^2)^(1/2))/b
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6715, 6314, 372, 266, 63, 207}

$$\frac{\tanh^{-1}\left(\sqrt{\frac{1}{\left(a+bx^4\right)^2}+1}\right)}{4b}+\frac{\left(a+bx^4\right)\text{csch}^{-1}\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCsch[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcCsch[a + b*x^4])/(4*b) + ArcTanh[Sqrt[1 + (a + b*x^4)^(-2)]
]/(4*b)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 6314

```
Int[ArcCsch[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcCsch[c + d*
x])/d, x] + Int[1/((c + d*x)*Sqrt[1 + 1/(c + d*x)^2]), x] /; FreeQ[{c, d},
x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int x^3\text{csch}^{-1}\left(a+bx^4\right)\,dx &= \frac{1}{4}\text{Subst}\left(\int \text{csch}^{-1}(a+bx)\,dx,x,x^4\right)\\
&= \frac{\left(a+bx^4\right)\text{csch}^{-1}\left(a+bx^4\right)}{4b}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}\,dx,x,x^4\right)\\
&= \frac{\left(a+bx^4\right)\text{csch}^{-1}\left(a+bx^4\right)}{4b}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^2}}\,x}\,dx,x,a+bx^4\right)}{4b}\\
&= \frac{\left(a+bx^4\right)\text{csch}^{-1}\left(a+bx^4\right)}{4b}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,\frac{1}{\left(a+bx^4\right)^2}\right)}{8b}\\
&= \frac{\left(a+bx^4\right)\text{csch}^{-1}\left(a+bx^4\right)}{4b}-\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+\frac{1}{\left(a+bx^4\right)^2}}\right)}{4b}\\
&= \frac{\left(a+bx^4\right)\text{csch}^{-1}\left(a+bx^4\right)}{4b}+\frac{\tanh^{-1}\left(\sqrt{1+\frac{1}{\left(a+bx^4\right)^2}}\right)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 74, normalized size = 1.61

$$\frac{\frac{\sqrt{\left(a+bx^4\right)^2+1}\,\sinh^{-1}\left(a+bx^4\right)}{\sqrt{\frac{1}{\left(a+bx^4\right)^2}+1}}+\left(a+bx^4\right)^2\text{csch}^{-1}\left(a+bx^4\right)}{4b\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCsch[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)^2*ArcCsch[a + b*x^4] + (Sqrt[1 + (a + b*x^4)^2]*ArcSinh[a + b*
x^4])/Sqrt[1 + (a + b*x^4)^(-2)])/(4*b*(a + b*x^4))
```

fricas [B] time = 0.61, size = 266, normalized size = 5.78

$$\frac{bx^4\log\left(\frac{\left(bx^4+a\right)\sqrt{\frac{b^2x^8+2\,abx^4+a^2+1}{b^2x^8+2\,abx^4+a^2}}+1}{bx^4+a}\right)+a\log\left(-bx^4+\left(bx^4+a\right)\sqrt{\frac{b^2x^8+2\,abx^4+a^2+1}{b^2x^8+2\,abx^4+a^2}}-a+1\right)-a\log\left(-bx^4+\left(bx^4+\right.\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsch(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b*x^4*log(((b*x^4 + a)*sqrt((b^2*x^8 + 2*a*b*x^4 + a^2 + 1)/(b^2*x^8 +
 2*a*b*x^4 + a^2)) + 1)/(b*x^4 + a)) + a*log(-b*x^4 + (b*x^4 + a)*sqrt((b^2
*x^8 + 2*a*b*x^4 + a^2 + 1)/(b^2*x^8 + 2*a*b*x^4 + a^2)) - a + 1) - a*log(-
```

```
b*x^4 + (b*x^4 + a)*sqrt((b^2*x^8 + 2*a*b*x^4 + a^2 + 1)/(b^2*x^8 + 2*a*b*x
^4 + a^2)) - a - 1) - log(-b*x^4 + (b*x^4 + a)*sqrt((b^2*x^8 + 2*a*b*x^4 +
a^2 + 1)/(b^2*x^8 + 2*a*b*x^4 + a^2)) - a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arcsch}\left(bx^4 + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsch(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccsch(b*x^4 + a), x)
```

maple [A] time = 0.08, size = 63, normalized size = 1.37

$$\frac{\operatorname{arccsch}\left(b\,x^4+a\right)x^4}{4}+\frac{\operatorname{arccsch}\left(b\,x^4+a\right)a}{4b}+\frac{\ln\left(b\,x^4+a+\left(b\,x^4+a\right)\sqrt{1+\frac{1}{\left(b\,x^4+a\right)^2}}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccsch(b*x^4+a),x)
```

```
[Out] 1/4*arccsch(b*x^4+a)*x^4+1/4/b*arccsch(b*x^4+a)*a+1/4/b*ln(b*x^4+a+(b*x^4+a
)*(1+1/(b*x^4+a)^2)^(1/2))
```

maxima [A] time = 0.31, size = 57, normalized size = 1.24

$$\frac{2\left(bx^4+a\right)\operatorname{arcsch}\left(bx^4+a\right)+\log\left(\sqrt{\frac{1}{\left(bx^4+a\right)^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{\left(bx^4+a\right)^2}+1}-1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsch(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arccsch(b*x^4 + a) + log(sqrt(1/(b*x^4 + a)^2 + 1) + 1)
- log(sqrt(1/(b*x^4 + a)^2 + 1) - 1))/b
```

mupad [B] time = 2.71, size = 42, normalized size = 0.91

$$\frac{\operatorname{atanh}\left(\sqrt{\frac{1}{\left(b\,x^4+a\right)^2}+1}\right)}{4\,b}+\frac{\operatorname{asinh}\left(\frac{1}{b\,x^4+a}\right)\left(b\,x^4+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(1/(a + b*x^4)),x)
```

```
[Out] atanh((1/(a + b*x^4)^2 + 1)^(1/2))/(4*b) + (asinh(1/(a + b*x^4))*(a + b*x^4
))/(4*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acsch(b*x**4+a),x)
```

```
[Out] Timed out
```

3.71 $\int x^{-1+n}\operatorname{csch}^{-1}(a+bx^n)\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\sqrt{\frac{1}{(a+bx^n)^2}+1}\right)}{bn}+\frac{(a+bx^n)\operatorname{csch}^{-1}(a+bx^n)}{bn}$$

```
[Out] (a+b*x^n)*arccsch(a+b*x^n)/b/n+arctanh((1+1/(a+b*x^n)^2)^(1/2))/b/n
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6715, 6314, 372, 266, 63, 207}

$$\frac{\tanh^{-1}\left(\sqrt{\frac{1}{(a+bx^n)^2}+1}\right)}{bn}+\frac{(a+bx^n)\operatorname{csch}^{-1}(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcCsch[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcCsch[a + b*x^n])/(b*n) + ArcTanh[Sqrt[1 + (a + b*x^n)^(-2)]
]/(b*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 6314

```
Int[ArcCsch[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcCsch[c + d*
x])/d, x] + Int[1/((c + d*x)*Sqrt[1 + 1/(c + d*x)^2]), x] /; FreeQ[{c, d},
x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
```

```
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x^{-1+n}\operatorname{csch}^{-1}(a+bx^n)\,dx = \frac{\operatorname{Subst}\left(\int \operatorname{csch}^{-1}(a+bx)\,dx, x, x^n\right)}{n}$$

$$= \frac{(a+bx^n)\operatorname{csch}^{-1}(a+bx^n)}{bn} + \frac{\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}\,dx, x, x^n\right)}{n}$$

$$= \frac{(a+bx^n)\operatorname{csch}^{-1}(a+bx^n)}{bn} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^2}}\,x}\,dx, x, a+bx^n\right)}{bn}$$

$$= \frac{(a+bx^n)\operatorname{csch}^{-1}(a+bx^n)}{bn} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{(a+bx^n)^2}\right)}{2bn}$$

$$= \frac{(a+bx^n)\operatorname{csch}^{-1}(a+bx^n)}{bn} - \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{(a+bx^n)^2}}\right)}{bn}$$

$$= \frac{(a+bx^n)\operatorname{csch}^{-1}(a+bx^n)}{bn} + \frac{\tanh^{-1}\left(\sqrt{1+\frac{1}{(a+bx^n)^2}}\right)}{bn}$$

Mathematica [A] time = 0.18, size = 74, normalized size = 1.61

$$\frac{\frac{\sqrt{(a+bx^n)^2+1}\,\sinh^{-1}(a+bx^n)}{\sqrt{\frac{1}{(a+bx^n)^2}+1}} + (a+bx^n)^2\operatorname{csch}^{-1}(a+bx^n)}{bn(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcCsch[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)^2*ArcCsch[a + b*x^n] + (Sqrt[1 + (a + b*x^n)^2]*ArcSinh[a + b*
x^n])/Sqrt[1 + (a + b*x^n)^(-2)])/(b*n*(a + b*x^n))
```

fricas [B] time = 0.65, size = 334, normalized size = 7.26

$$a\log\left(-b\cosh\left(n\log(x)\right) - b\sinh\left(n\log(x)\right) - a + \sqrt{\frac{2\,ab+\left(a^2+b^2+1\right)\cosh\left(n\log(x)\right)-\left(a^2-b^2+1\right)\sinh\left(n\log(x)\right)}{\cosh\left(n\log(x)\right)-\sinh\left(n\log(x)\right)}} + 1\right) - a\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccsch(a+b*x^n),x, algorithm="fricas")
```

```
[Out] (a*log(-b*cosh(n*log(x)) - b*sinh(n*log(x)) - a + sqrt((2*a*b + (a^2 + b^2
+ 1)*cosh(n*log(x)) - (a^2 - b^2 + 1)*sinh(n*log(x)))/(cosh(n*log(x)) - sin
h(n*log(x)))) + 1) - a*log(-b*cosh(n*log(x)) - b*sinh(n*log(x)) - a + sqrt(
(2*a*b + (a^2 + b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1)*sinh(n*log(x)))/(
cosh(n*log(x)) - sinh(n*log(x)))) - 1) + (b*cosh(n*log(x)) + b*sinh(n*log(x
)))*log((sqrt((2*a*b + (a^2 + b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1)*sin
```

```
h(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x)))) + 1)/(b*cosh(n*log(x)) + b*
sinh(n*log(x)) + a)) - log(-b*cosh(n*log(x)) - b*sinh(n*log(x)) - a + sqrt(
(2*a*b + (a^2 + b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1)*sinh(n*log(x)))/(
cosh(n*log(x)) - sinh(n*log(x))))))/(b*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{n-1} \operatorname{arcsch}(bx^n + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccsch(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 1)*arccsch(b*x^n + a), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int x^{-1+n} \operatorname{arccsch}(a + b x^n) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*arccsch(a+b*x^n),x)
```

```
[Out] int(x^(-1+n)*arccsch(a+b*x^n),x)
```

maxima [A] time = 0.31, size = 60, normalized size = 1.30

$$\frac{2\,(bx^n + a)\operatorname{arcsch}(bx^n + a) + \log\left(\sqrt{\frac{1}{(bx^n+a)^2} + 1} + 1\right) - \log\left(\sqrt{\frac{1}{(bx^n+a)^2} + 1} - 1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccsch(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x^n + a)*arccsch(b*x^n + a) + log(sqrt(1/(b*x^n + a)^2 + 1) + 1)
- log(sqrt(1/(b*x^n + a)^2 + 1) - 1))/(b*n)
```

mupad [B] time = 2.21, size = 40, normalized size = 0.87

$$\frac{\operatorname{atanh}\left(\sqrt{\frac{1}{(a+b\,x^n)^2} + 1}\right) + \operatorname{asinh}\left(\frac{1}{a+b\,x^n}\right)\,(a + b\,x^n)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*asinh(1/(a + b*x^n)),x)
```

```
[Out] (atanh((1/(a + b*x^n)^2 + 1)^(1/2)) + asinh(1/(a + b*x^n))*(a + b*x^n))/(b*
n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*acsch(a+b*x**n),x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```