Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.5-Inverse-hyperbolic-secant/7.5.2-Inverse-hyperbolic-secant-functions

Nasser M. Abbasi

July 28, 2021 Compiled on July 28, 2021 at 9:35am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [100]. This is test number [201].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (100)	% 0.00 (0)
Mathematica	% 98.00 (98)	% 2.00 (2)
Maple	% 74.00 (74)	% 26.00 (26)
Maxima	% 21.00 (21)	% 79.00 (79)
Fricas	% 69.00 (69)	% 31.00 (31)
Sympy	% 1.00 (1)	% 99.00 (99)
Giac	% 6.00 (6)	% 94.00 (94)
Mupad	% 56.00 (56)	% 44.00 (44)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

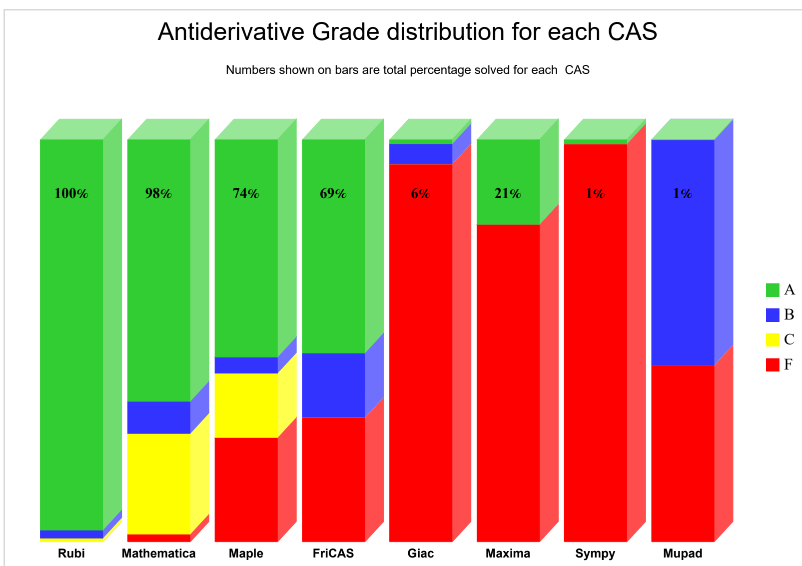
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

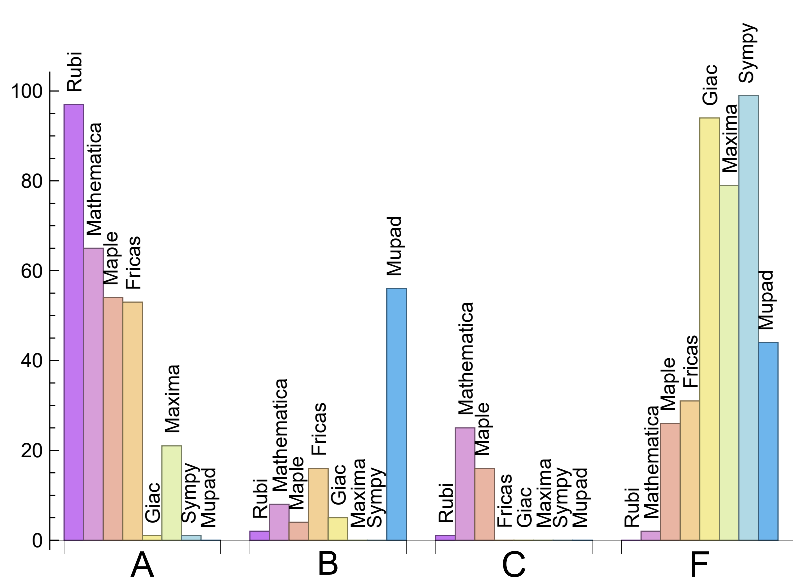
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.00	2.00	1.00	0.00
Mathematica	65.00	8.00	25.00	2.00
Maple	54.00	4.00	16.00	26.00
Maxima	21.00	0.00	0.00	79.00
Fricas	53.00	16.00	0.00	31.00
Sympy	1.00	0.00	0.00	99.00
Giac	1.00	5.00	0.00	94.00
Mupad	0.00	56.00	0.00	44.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	26	65.38 %	0.00 %	34.62 %
Maxima	79	93.67 %	0.00 %	6.33 %
Fricas	31	80.65 %	0.00 %	19.35 %
Sympy	99	85.86 %	14.14 %	0.00 %
Giac	94	87.23 %	0.00 %	12.77 %
Mupad	44	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

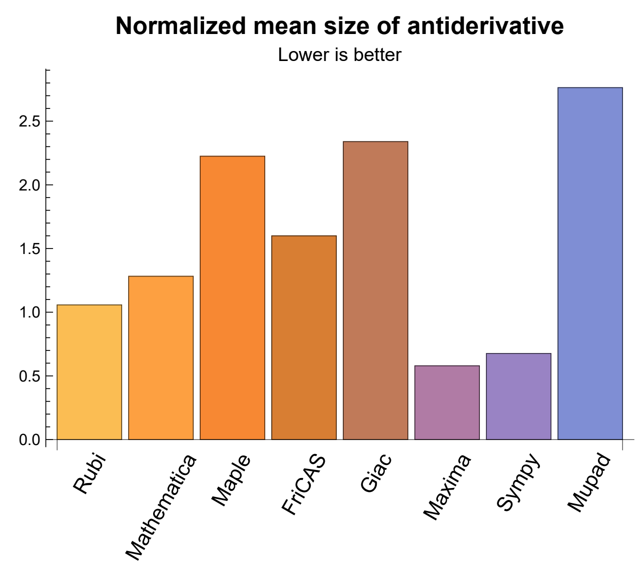
1.3 Performance

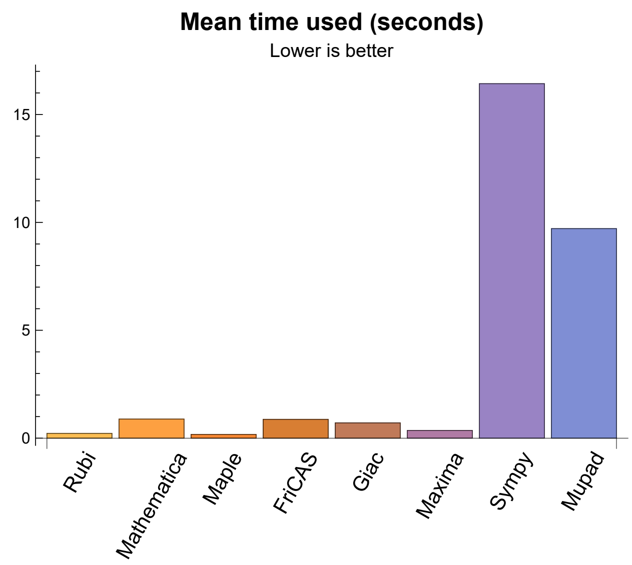
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.22	135.47	1.06	107.00	1.00
Mathematica	0.89	155.88	1.28	102.50	1.11
Maple	0.17	224.27	2.22	102.50	1.23
Maxima	0.36	49.76	0.58	46.00	0.67
Fricas	0.87	139.99	1.60	89.00	1.05
Sympy	16.43	48.00	0.68	48.00	0.68
Giac	0.71	168.50	2.34	161.00	2.12
Mupad	9.71	329.71	2.76	88.50	1.74

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {32, 34, 36, 37, 38, 39, 47, 53, 63, 97}

Mathematica {4, 5, 9, 10, 11, 12, 15, 16, 17, 18, 24, 30, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 92, 93, 94, 95, 96, 97, 98}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```python
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

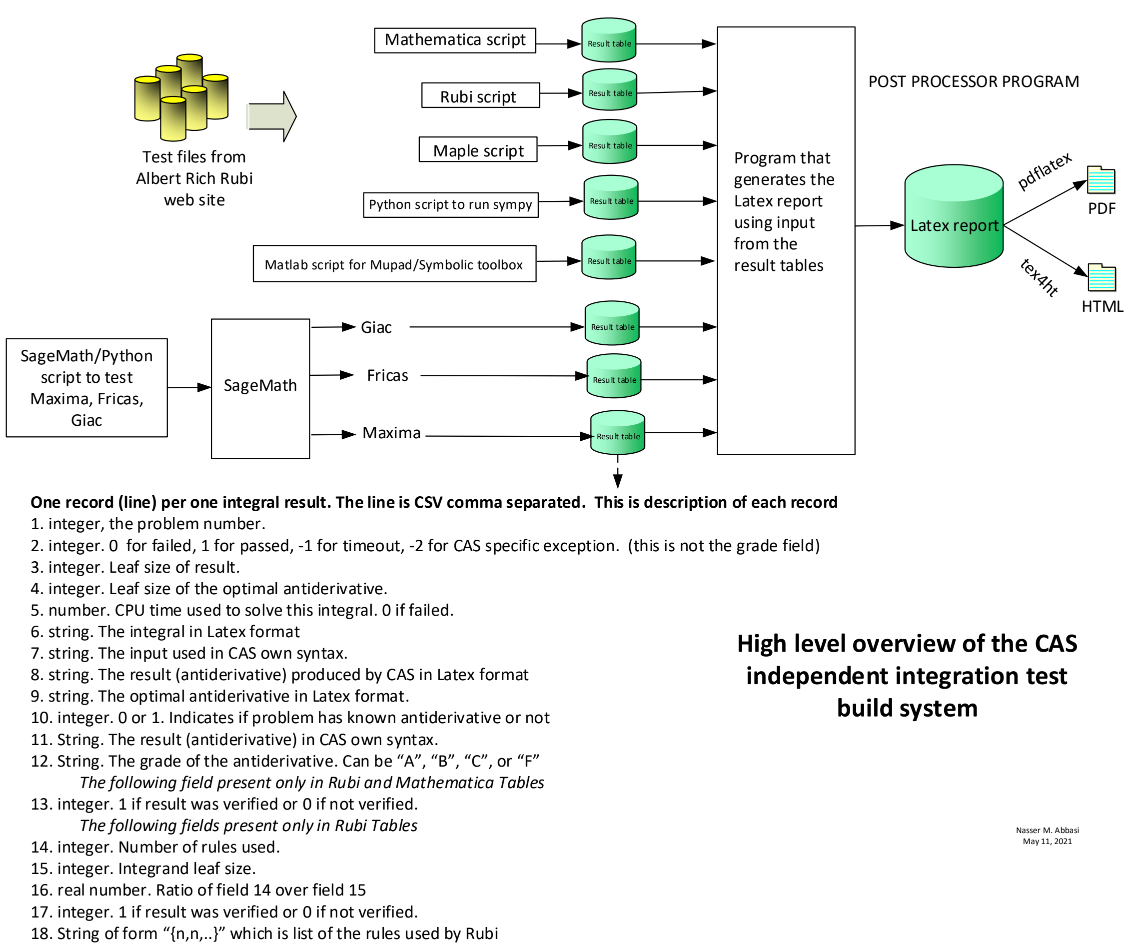
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100 }

B grade: { 38, 97 }

C grade: { 39 }

F grade: { }

2.1.2 Mathematica

A grade: { 8, 9, 10, 11, 12, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 32, 34, 39, 40, 41, 42, 43, 44, 47, 51, 53, 55, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 80, 82, 83, 84, 85, 86, 87, 89, 91, 93, 94, 95, 96, 98, 99, 100 }

B grade: { 4, 6, 7, 29, 31, 36, 38, 97 }

C grade: { 1, 2, 3, 5, 13, 14, 18, 33, 35, 37, 45, 46, 48, 49, 50, 52, 54, 63, 65, 67, 77, 79, 81, 90, 92 }

F grade: { 19, 88 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 9, 10, 11, 13, 14, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 66, 68, 70, 71, 72, 73, 74, 75, 89, 91, 93, 95, 97, 98, 99 }

B grade: { 6, 7, 8, 80 }

C grade: { 5, 33, 35, 37, 51, 55, 63, 65, 67, 69, 76, 78, 90, 92, 94, 96 }

F grade: { 12, 15, 16, 17, 18, 19, 30, 56, 57, 58, 59, 60, 61, 62, 64, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 100 }

2.1.4 Maxima

A grade: { 4, 20, 21, 22, 23, 25, 26, 27, 28, 32, 34, 39, 41, 43, 47, 66, 71, 73, 75, 99, 100 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24, 29, 30, 31, 33, 35, 36, 37, 38, 40, 42, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 72, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

2.1.5 FriCAS

A grade: { 1, 20, 21, 22, 23, 25, 26, 27, 28, 32, 33, 34, 35, 37, 39, 40, 41, 42, 43, 44, 45, 47, 49, 55, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 91, 94, 96, 97 }

B grade: { 2, 3, 4, 6, 7, 8, 36, 38, 51, 53, 90, 92, 93, 95, 99, 100 }

C grade: { }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24, 29, 30, 31, 46, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 64, 88, 98 }

2.1.6 Sympy

A grade: { 93 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100 }

2.1.7 Giac

A grade: { 82 }

B grade: { 45, 47, 49, 53, 71 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 48, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 }

2.1.8 Mupad

A grade: { }

B grade: { 4, 25, 28, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 49, 51, 53, 55, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100 }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 29, 30, 31, 46, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 88, 98 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	225	250	0	345	0	0	-1
normalized size	1	1.00	1.11	1.23	0.00	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.542	0.102	0.000	0.483	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	200	189	0	327	0	0	-1
normalized size	1	1.00	1.31	1.24	0.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.284	0.070	0.000	1.921	0.000	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	176	111	0	308	0	0	-1
normalized size	1	1.00	1.64	1.04	0.00	2.88	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.168	0.070	0.000	1.393	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	125	50	31	253	0	0	43
normalized size	1	1.00	2.84	1.14	0.70	5.75	0.00	0.00	0.98
time (sec)	N/A	0.056	0.314	0.047	0.303	0.762	0.000	0.000	2.161

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	332	886	0	0	0	0	-1
normalized size	1	1.00	1.95	5.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.204	0.955	0.000	0.666	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	244	542	0	651	0	0	-1
normalized size	1	1.00	3.49	7.74	0.00	9.30	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.257	0.084	0.000	0.707	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	315	1233	0	865	0	0	-1
normalized size	1	1.00	2.37	9.27	0.00	6.50	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.974	0.090	0.000	0.689	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	368	1946	0	987	0	0	-1
normalized size	1	1.00	1.87	9.88	0.00	5.01	0.00	0.00	-0.01
time (sec)	N/A	0.316	0.389	0.099	0.000	0.695	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	305	655	0	0	0	0	-1
normalized size	1	1.00	1.09	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	1.995	1.678	0.000	1.288	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	172	396	0	0	0	0	-1
normalized size	1	1.00	1.15	2.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.430	1.310	0.000	0.621	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	105	209	0	0	0	0	-1
normalized size	1	1.00	1.31	2.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.227	0.339	0.000	0.786	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	280	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.463	0.305	0.801	0.000	0.966	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	678	367	0	0	0	0	-1
normalized size	1	1.00	3.03	1.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.389	2.690	0.751	0.000	0.559	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	537	537	1439	1026	0	0	0	0	-1
normalized size	1	1.00	2.68	1.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.750	7.576	1.292	0.000	0.513	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	254	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	0.542	1.885	0.000	0.606	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	153	0	0	0	0	0	-1
normalized size	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.141	0.610	0.000	0.568	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	384	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.511	0.324	0.810	0.000	0.577	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	1849	0	0	0	0	0	-1
normalized size	1	1.00	5.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.575	46.916	0.896	0.000	1.036	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	965	965	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.338	9.954	1.667	0.000	0.558	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	84	54	58	57	0	0	-1
normalized size	1	1.00	0.51	0.33	0.35	0.35	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.044	0.074	0.324	0.636	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	72	49	46	52	0	0	-1
normalized size	1	1.00	0.57	0.39	0.37	0.41	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.031	0.069	0.338	0.519	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	56	42	34	45	0	0	-1
normalized size	1	1.00	0.64	0.48	0.39	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.026	0.062	0.376	0.630	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	67	36	19	39	0	0	-1
normalized size	1	1.00	1.56	0.84	0.44	0.91	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.096	0.059	0.388	0.562	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	65	0	0	0	0	-1
normalized size	1	1.00	0.98	1.41	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.095	0.039	0.101	0.000	0.570	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	111	64	65	45	0	0	40
normalized size	1	1.00	1.13	0.65	0.66	0.46	0.00	0.00	0.41
time (sec)	N/A	0.023	0.077	0.072	0.300	0.608	0.000	0.000	2.150

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	125	79	92	54	0	0	-1
normalized size	1	1.00	0.92	0.58	0.68	0.40	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.116	0.073	0.342	1.086	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	140	91	116	60	0	0	-1
normalized size	1	1.00	0.81	0.53	0.67	0.35	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.133	0.072	0.315	1.385	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	29	16	22	0	0	17
normalized size	1	1.00	1.19	1.38	0.76	1.05	0.00	0.00	0.81
time (sec)	N/A	0.007	0.025	0.061	0.318	0.635	0.000	0.000	1.349

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	219	116	0	0	0	0	-1
normalized size	1	1.00	3.59	1.90	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.105	0.983	0.150	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.107	0.047	0.180	0.000	0.696	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	249	140	0	0	0	0	-1
normalized size	1	1.00	3.23	1.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	1.173	0.148	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	83	65	64	47	65	0	0	75
normalized size	1	1.30	1.02	1.00	0.73	1.02	0.00	0.00	1.17
time (sec)	N/A	0.038	0.094	0.057	0.379	0.552	0.000	0.000	1.469

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	97	118	0	95	0	0	521
normalized size	1	1.00	1.15	1.40	0.00	1.13	0.00	0.00	6.20
time (sec)	N/A	0.032	0.133	0.058	0.000	0.512	0.000	0.000	11.925

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	52	48	54	38	54	0	0	55
normalized size	1	1.37	1.26	1.42	1.00	1.42	0.00	0.00	1.45
time (sec)	N/A	0.024	0.063	0.049	0.401	0.758	0.000	0.000	1.433

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	75	92	0	79	0	0	303
normalized size	1	1.00	1.42	1.74	0.00	1.49	0.00	0.00	5.72
time (sec)	N/A	0.018	0.079	0.052	0.000	0.619	0.000	0.000	6.959

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	24	39	79	80	0	115	0	0	182
normalized size	1	1.62	3.29	3.33	0.00	4.79	0.00	0.00	7.58
time (sec)	N/A	0.137	0.044	0.056	0.000	0.696	0.000	0.000	2.983

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	64	75	92	0	77	0	0	184
normalized size	1	1.33	1.56	1.92	0.00	1.60	0.00	0.00	3.83
time (sec)	N/A	0.031	0.047	0.056	0.000	0.698	0.000	0.000	3.104

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	A	F	B	F	F	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	35	99	93	91	0	128	0	0	71
normalized size	1	2.83	2.66	2.60	0.00	3.66	0.00	0.00	2.03
time (sec)	N/A	0.041	0.063	0.054	0.000	0.808	0.000	0.000	1.842

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	A	F	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	84	43	53	43	52	0	0	58
normalized size	1	1.53	0.78	0.96	0.78	0.95	0.00	0.00	1.05
time (sec)	N/A	0.037	0.043	0.053	0.368	0.663	0.000	0.000	1.473

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	110	0	138	0	0	602
normalized size	1	1.00	0.83	0.83	0.00	1.05	0.00	0.00	4.56
time (sec)	N/A	0.059	0.078	0.056	0.000	0.652	0.000	0.000	13.421

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	60	63	51	60	0	0	76
normalized size	1	1.00	0.52	0.55	0.44	0.52	0.00	0.00	0.66
time (sec)	N/A	0.052	0.069	0.059	0.333	0.487	0.000	0.000	1.562

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	129	132	0	148	0	0	878
normalized size	1	1.00	0.79	0.81	0.00	0.91	0.00	0.00	5.39
time (sec)	N/A	0.077	0.105	0.065	0.000	0.860	0.000	0.000	34.076

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	76	71	60	69	0	0	95
normalized size	1	1.00	0.52	0.49	0.41	0.47	0.00	0.00	0.65
time (sec)	N/A	0.071	0.089	0.075	0.341	0.495	0.000	0.000	1.675

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	145	152	0	156	0	0	1155
normalized size	1	1.00	0.75	0.78	0.00	0.80	0.00	0.00	5.95
time (sec)	N/A	0.098	0.125	0.087	0.000	0.545	0.000	0.000	38.560

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	111	137	0	116	0	205	521
normalized size	1	1.00	1.00	1.23	0.00	1.05	0.00	1.85	4.69
time (sec)	N/A	0.062	0.193	0.216	0.000	0.498	0.000	0.206	14.445

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	139	114	0	0	0	0	-1
normalized size	1	1.00	1.21	0.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.320	0.055	0.000	0.728	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	58	71	56	60	42	60	0	190	57
normalized size	1	1.22	0.97	1.03	0.72	1.03	0.00	3.28	0.98
time (sec)	N/A	0.041	0.098	0.051	0.400	0.536	0.000	0.172	1.638

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	140	136	0	0	0	0	-1
normalized size	1	1.00	1.25	1.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.445	0.057	0.000	1.734	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	92	112	0	102	0	132	306
normalized size	1	1.00	1.46	1.78	0.00	1.62	0.00	2.10	4.86
time (sec)	N/A	0.037	0.105	0.154	0.000	0.756	0.000	0.173	7.139

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	116	102	0	0	0	0	-1
normalized size	1	1.00	1.73	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.200	0.056	0.000	1.848	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	100	127	0	133	0	0	182
normalized size	1	1.00	1.47	1.87	0.00	1.96	0.00	0.00	2.68
time (sec)	N/A	0.043	0.066	0.188	0.000	1.592	0.000	0.000	3.202

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	135	132	0	0	0	0	-1
normalized size	1	1.00	0.92	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.297	0.066	0.000	0.741	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	93	113	103	0	102	0	252	185
normalized size	1	1.16	1.41	1.29	0.00	1.28	0.00	3.15	2.31
time (sec)	N/A	0.053	0.149	0.160	0.000	0.974	0.000	3.324	3.996

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	104	0	0	0	0	-1
normalized size	1	1.00	1.07	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.198	0.066	0.000	0.773	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	105	129	0	146	0	0	71
normalized size	1	1.00	0.89	1.09	0.00	1.24	0.00	0.00	0.60
time (sec)	N/A	0.061	0.268	0.176	0.000	0.862	0.000	0.000	2.017

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	159	0	0	0	0	0	-1
normalized size	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	2.520	0.094	0.000	2.368	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	159	0	0	0	0	0	-1
normalized size	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	2.391	0.070	0.000	1.634	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	145	0	0	0	0	0	-1
normalized size	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.309	0.044	0.000	0.637	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	139	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.836	0.641	0.000	2.072	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	186	0	0	0	0	0	-1
normalized size	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	5.171	1.085	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	159	0	0	0	0	0	-1
normalized size	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.309	1.061	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	139	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.411	1.069	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	106	96	145	0	102	0	0	-1
normalized size	1	1.22	1.10	1.67	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.157	0.147	0.000	0.700	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	156	0	0	0	0	0	-1
normalized size	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.284	1.071	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	105	123	0	103	0	0	808
normalized size	1	1.00	0.52	0.61	0.00	0.51	0.00	0.00	3.98
time (sec)	N/A	0.700	0.156	0.064	0.000	0.888	0.000	0.000	19.727

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	52	72	42	62	0	0	63
normalized size	1	1.00	0.44	0.62	0.36	0.53	0.00	0.00	0.54
time (sec)	N/A	0.546	0.070	0.047	0.414	1.539	0.000	0.000	1.751

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	86	97	0	87	0	0	420
normalized size	1	1.00	0.51	0.57	0.00	0.51	0.00	0.00	2.49
time (sec)	N/A	0.465	0.076	0.061	0.000	1.533	0.000	0.000	8.956

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	89	89	0	124	0	0	56
normalized size	1	1.00	1.05	1.05	0.00	1.46	0.00	0.00	0.66
time (sec)	N/A	0.434	0.062	0.055	0.000	0.502	0.000	0.000	3.692

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	75	98	0	85	0	0	162
normalized size	1	1.00	1.32	1.72	0.00	1.49	0.00	0.00	2.84
time (sec)	N/A	0.174	0.099	0.061	0.000	1.016	0.000	0.000	4.636

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	97	0	138	0	0	323
normalized size	1	1.00	1.00	1.13	0.00	1.60	0.00	0.00	3.76
time (sec)	N/A	0.446	0.060	0.061	0.000	0.850	0.000	0.000	11.205

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	52	73	46	61	0	122	67
normalized size	1	1.00	0.91	1.28	0.81	1.07	0.00	2.14	1.18
time (sec)	N/A	0.390	0.063	0.058	0.348	1.061	0.000	0.208	1.796

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	121	131	0	146	0	0	885
normalized size	1	1.00	0.82	0.89	0.00	0.99	0.00	0.00	6.02
time (sec)	N/A	0.453	0.126	0.057	0.000	1.016	0.000	0.000	46.991

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	69	84	56	69	0	0	86
normalized size	1	1.00	0.38	0.46	0.31	0.38	0.00	0.00	0.47
time (sec)	N/A	0.505	0.084	0.059	0.397	0.745	0.000	0.000	1.938

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	137	153	0	156	0	0	2480
normalized size	1	1.00	0.51	0.57	0.00	0.58	0.00	0.00	9.29
time (sec)	N/A	0.543	0.147	0.093	0.000	0.549	0.000	0.000	65.189

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	85	92	65	78	0	0	105
normalized size	1	1.00	0.28	0.31	0.22	0.26	0.00	0.00	0.35
time (sec)	N/A	0.576	0.103	0.072	0.397	0.764	0.000	0.000	2.125

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	65	531	0	65	0	0	73
normalized size	1	1.00	0.44	3.61	0.00	0.44	0.00	0.00	0.50
time (sec)	N/A	0.619	0.096	0.362	0.000	1.971	0.000	0.000	2.138

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	97	0	0	95	0	0	795
normalized size	1	1.00	0.60	0.00	0.00	0.58	0.00	0.00	4.88
time (sec)	N/A	0.569	0.129	180.000	0.000	0.782	0.000	0.000	19.825

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	48	269	0	54	0	0	57
normalized size	1	1.00	0.64	3.59	0.00	0.72	0.00	0.00	0.76
time (sec)	N/A	0.513	0.060	0.315	0.000	0.537	0.000	0.000	2.063

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	79	0	0	407
normalized size	1	1.00	0.80	0.00	0.00	0.84	0.00	0.00	4.33
time (sec)	N/A	0.309	0.069	180.000	0.000	0.852	0.000	0.000	9.223

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	72	2612	0	115	0	0	47
normalized size	1	1.00	1.11	40.18	0.00	1.77	0.00	0.00	0.72
time (sec)	N/A	0.166	0.032	0.308	0.000	0.530	0.000	0.000	4.169

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	74	0	0	76	0	0	184
normalized size	1	1.00	1.61	0.00	0.00	1.65	0.00	0.00	4.00
time (sec)	N/A	0.401	0.038	180.000	0.000	1.700	0.000	0.000	3.845

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	92	0	0	128	0	110	323
normalized size	1	1.00	1.28	0.00	0.00	1.78	0.00	1.53	4.49
time (sec)	N/A	0.383	0.060	180.000	0.000	0.514	0.000	0.171	12.053

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	43	0	0	52	0	0	58
normalized size	1	1.00	0.37	0.00	0.00	0.45	0.00	0.00	0.50
time (sec)	N/A	0.424	0.056	180.000	0.000	0.785	0.000	0.000	2.137

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	110	0	0	138	0	0	1511
normalized size	1	1.00	0.55	0.00	0.00	0.69	0.00	0.00	7.56
time (sec)	N/A	0.500	0.109	180.000	0.000	0.629	0.000	0.000	43.714

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	60	0	0	60	0	0	75
normalized size	1	1.00	0.26	0.00	0.00	0.26	0.00	0.00	0.32
time (sec)	N/A	0.500	0.074	180.000	0.000	0.743	0.000	0.000	2.366

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	129	0	0	148	0	0	2479
normalized size	1	1.00	0.40	0.00	0.00	0.46	0.00	0.00	7.75
time (sec)	N/A	0.567	0.141	180.000	0.000	0.831	0.000	0.000	65.404

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	76	0	0	69	0	0	91
normalized size	1	1.00	0.22	0.00	0.00	0.20	0.00	0.00	0.26
time (sec)	N/A	0.605	0.101	180.000	0.000	0.805	0.000	0.000	2.593

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.272	0.674	0.439	0.000	1.415	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	69	69	0	69	0	0	76
normalized size	1	1.00	0.78	0.78	0.00	0.78	0.00	0.00	0.86
time (sec)	N/A	0.181	0.201	0.102	0.000	1.264	0.000	0.000	2.603

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	110	117	0	91	0	0	340
normalized size	1	1.00	1.47	1.56	0.00	1.21	0.00	0.00	4.53
time (sec)	N/A	0.165	0.161	0.080	0.000	1.567	0.000	0.000	8.085

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	52	0	49	0	0	44
normalized size	1	1.00	0.98	1.16	0.00	1.09	0.00	0.00	0.98
time (sec)	N/A	0.125	0.092	0.070	0.000	0.496	0.000	0.000	2.700

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	68	92	0	53	0	0	84
normalized size	1	1.00	1.84	2.49	0.00	1.43	0.00	0.00	2.27
time (sec)	N/A	0.114	0.050	0.076	0.000	0.825	0.000	0.000	3.567

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	73	87	0	92	48	0	59
normalized size	1	1.00	1.03	1.23	0.00	1.30	0.68	0.00	0.83
time (sec)	N/A	0.119	0.043	0.077	0.000	1.914	16.429	0.000	2.918

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	59	61	0	62	0	0	37
normalized size	1	1.00	1.40	1.45	0.00	1.48	0.00	0.00	0.88
time (sec)	N/A	0.143	0.167	0.083	0.000	0.916	0.000	0.000	2.528

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	111	0	156	0	0	331
normalized size	1	1.00	1.00	1.03	0.00	1.44	0.00	0.00	3.06
time (sec)	N/A	0.176	0.159	0.090	0.000	0.854	0.000	0.000	14.525

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	90	86	0	89	0	0	75
normalized size	1	1.00	1.06	1.01	0.00	1.05	0.00	0.00	0.88
time (sec)	N/A	0.165	0.264	0.099	0.000	0.863	0.000	0.000	2.505

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	A	F	A	F	F	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	12	26	28	36	0	35	0	0	76
normalized size	1	2.17	2.33	3.00	0.00	2.92	0.00	0.00	6.33
time (sec)	N/A	1.052	0.275	0.048	0.000	0.629	0.000	0.000	2.962

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	52	104	0	0	0	0	-1
normalized size	1	1.00	0.85	1.70	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.098	0.061	0.341	0.000	0.885	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	84	62	38	283	0	0	56
normalized size	1	1.00	1.47	1.09	0.67	4.96	0.00	0.00	0.98
time (sec)	N/A	0.115	0.213	0.103	0.363	1.754	0.000	0.000	2.992

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	84	0	40	385	0	0	54
normalized size	1	1.00	1.45	0.00	0.69	6.64	0.00	0.00	0.93
time (sec)	N/A	0.113	0.264	0.247	0.381	0.915	0.000	0.000	2.349

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [15] had the largest ratio of [1.200]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	1.00	10	0.700
2	A	7	6	1.00	10	0.600
3	A	6	6	1.00	8	0.750
4	A	4	4	1.00	6	0.667
5	A	14	8	1.00	10	0.800
6	A	5	5	1.00	10	0.500
7	A	7	7	1.00	10	0.700
8	A	8	8	1.00	10	0.800
9	A	17	9	1.00	12	0.750
10	A	11	8	1.00	10	0.800
11	A	8	6	1.00	8	0.750
12	A	17	9	1.00	12	0.750
13	A	12	8	1.00	12	0.667
14	A	23	11	1.00	12	0.917
15	A	16	12	1.00	10	1.200
16	A	10	7	1.00	8	0.875
17	A	20	10	1.00	12	0.833
18	A	14	9	1.00	12	0.750
19	A	32	13	1.00	12	1.083
20	A	4	3	1.00	10	0.300
21	A	4	3	1.00	10	0.300
22	A	4	3	1.00	8	0.375
23	A	3	3	1.00	6	0.500
24	A	7	6	1.00	10	0.600
25	A	5	5	1.00	10	0.500
26	A	6	5	1.00	10	0.500
27	A	7	5	1.00	10	0.500
28	A	3	3	1.00	4	0.750
29	A	7	6	1.00	10	0.600
30	A	7	6	1.00	10	0.600
31	A	7	7	1.00	10	0.700
32	A	5	5	1.30	10	0.500
33	A	5	5	1.00	10	0.500
34	A	3	3	1.37	10	0.300
35	A	4	4	1.00	8	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	3	3	1.62	6	0.500
37	A	5	5	1.33	10	0.500
38	B	6	6	2.83	10	0.600
39	C	5	5	1.53	10	0.500
40	A	8	6	1.00	10	0.600
41	A	7	5	1.00	10	0.500
42	A	10	6	1.00	10	0.600
43	A	9	5	1.00	10	0.500
44	A	12	6	1.00	10	0.600
45	A	6	6	1.00	12	0.500
46	A	5	5	1.00	12	0.417
47	A	4	4	1.22	12	0.333
48	A	7	7	1.00	12	0.583
49	A	5	5	1.00	12	0.417
50	A	4	4	1.00	12	0.333
51	A	6	6	1.00	10	0.600
52	A	8	8	1.00	8	1.000
53	A	5	5	1.16	12	0.417
54	A	5	5	1.00	12	0.417
55	A	7	7	1.00	12	0.583
56	A	4	4	1.00	12	0.333
57	A	4	4	1.00	12	0.333
58	A	4	4	1.00	10	0.400
59	A	5	5	1.00	12	0.417
60	A	4	4	1.00	12	0.333
61	A	4	4	1.00	10	0.400
62	A	4	4	1.00	8	0.500
63	A	6	6	1.22	12	0.500
64	A	4	4	1.00	12	0.333
65	A	9	6	1.00	12	0.500
66	A	8	6	1.00	12	0.500
67	A	7	6	1.00	12	0.500
68	A	8	5	1.00	10	0.500
69	A	7	5	1.00	8	0.625
70	A	5	3	1.00	12	0.250
71	A	4	2	1.00	12	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	5	3	1.00	12	0.250
73	A	4	2	1.00	12	0.167
74	A	5	3	1.00	12	0.250
75	A	4	2	1.00	12	0.167
76	A	8	5	1.00	12	0.417
77	A	7	5	1.00	12	0.417
78	A	6	5	1.00	12	0.417
79	A	5	4	1.00	10	0.400
80	A	6	4	1.00	8	0.500
81	A	5	3	1.00	12	0.250
82	A	5	3	1.00	12	0.250
83	A	4	2	1.00	12	0.167
84	A	5	3	1.00	12	0.250
85	A	4	2	1.00	12	0.167
86	A	5	3	1.00	12	0.250
87	A	4	2	1.00	12	0.167
88	A	5	4	1.00	24	0.167
89	A	8	7	1.00	22	0.318
90	A	7	7	1.00	22	0.318
91	A	4	4	1.00	22	0.182
92	A	5	5	1.00	20	0.250
93	A	8	8	1.00	19	0.421
94	A	5	5	1.00	22	0.227
95	A	9	8	1.00	22	0.364
96	A	8	7	1.00	22	0.318
97	B	7	5	2.17	25	0.200
98	A	8	8	1.00	19	0.421
99	A	5	5	1.00	12	0.417
100	A	5	5	1.00	14	0.357

Chapter 3

Listing of integrals

3.1 $\int x^3\operatorname{sech}^{-1}(a+bx)\,dx$

Optimal. Leaf size=203

$$-\frac{a^4\operatorname{sech}^{-1}(a+bx)}{4b^4}-\frac{\left(17a^2+2\right)\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{12b^4}+\frac{\left(2a^2+1\right)a\tan^{-1}\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{a+bx}\right)}{2b^4}+\frac{a(a+bx)\sqrt{\frac{-a-bx}{a+bx}}}{3b}$$

```
[Out] -1/4*a^4*arcsech(b*x+a)/b^4+1/4*x^4*arcsech(b*x+a)+1/2*a*(2*a^2+1)*arctan((
b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/(b*x+a))/b^4-1/12*(17*a^2+2)*(b*x+a+1
)*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^4-1/12*x^2*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1)
)^(1/2)/b^2+1/3*a*(b*x+a)*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^4
```

Rubi [A] time = 0.16, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6321, 5468, 3782, 4048, 3770, 3767, 8}

$$-\frac{\left(17a^2+2\right)\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{12b^4}+\frac{\left(2a^2+1\right)a\tan^{-1}\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{a+bx}\right)}{2b^4}-\frac{a^4\operatorname{sech}^{-1}(a+bx)}{4b^4}-\frac{x^2\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSech[a + b*x],x]
```

```
[Out] -((2 + 17*a^2)*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(12*b^4) -
(x^2*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(12*b^2) + (a*(a + b*
x)*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(3*b^4) - (a^4*ArcSech[
a + b*x])/(4*b^4) + (x^4*ArcSech[a + b*x])/4 + (a*(1 + 2*a^2)*ArcTan[(Sqrt[
(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(a + b*x)])/(2*b^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^3\text{sech}^{-1}(a+bx)\,dx = -\frac{\text{Subst}\left(\int x\text{sech}(x)(-a+\text{sech}(x))^3\tanh(x)\,dx, x, \text{sech}^{-1}(a+bx)\right)}{b^4}$$

$$= \frac{1}{4}x^4\text{sech}^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\text{sech}(x))^4\,dx, x, \text{sech}^{-1}(a+bx)\right)}{4b^4}$$

$$= -\frac{x^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^2} + \frac{1}{4}x^4\text{sech}^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\text{sech}(x))\left(-3a^3+\left(2+\right.\right.\right.}{}$$

$$= -\frac{x^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^2} + \frac{a(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{3b^4} + \frac{1}{4}x^4\text{sech}^{-1}(a+bx) - \text{Sub}$$

$$= -\frac{x^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^2} + \frac{a(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{3b^4} - \frac{a^4\text{sech}^{-1}(a+bx)}{4b^4} + \frac{1}{4}x^4\text{s}$$

$$= -\frac{x^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^2} + \frac{a(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{3b^4} - \frac{a^4\text{sech}^{-1}(a+bx)}{4b^4} + \frac{1}{4}x^4\text{s}$$

$$= -\frac{\left(2+17a^2\right)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^4} - \frac{x^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{12b^2} + \frac{a(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a}{3b^4}$$

Mathematica [C] time = 0.54, size = 225, normalized size = 1.11

$$-\frac{-3a^4\log(a+bx)+3a^4\log\left(a\sqrt{-\frac{a+bx-1}{a+bx+1}}+bx\sqrt{-\frac{a+bx-1}{a+bx+1}}+\sqrt{-\frac{a+bx-1}{a+bx+1}}+1\right)+6i\left(2a^2+1\right)a\log\left(2\sqrt{-\frac{a+bx-1}{a+bx+1}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSech[a + b*x],x]
```

```
[Out] -1/12*(Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(2 + 2*a + 13*a^2 + 13*a^3 + (
2 - 4*a + 9*a^2)*b*x + (1 - 3*a)*b^2*x^2 + b^3*x^3) - 3*b^4*x^4*ArcSech[a +
 b*x] - 3*a^4*Log[a + b*x] + 3*a^4*Log[1 + Sqrt[-((-1 + a + b*x)/(1 + a + b
*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + b*x*Sqrt[-((-1 + a + b*x)
/(1 + a + b*x))]] + (6*I)*a*(1 + 2*a^2)*Log[(-2*I)*(a + b*x) + 2*Sqrt[-((-1
 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)])/b^4
```

fricas [A] time = 0.48, size = 345, normalized size = 1.70

$$\frac{6\,b^4x^4\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right)-3\,a^4\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{x}\right)+3\,a^4\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}-1}{x}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/24*(6*b^4*x^4*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2
 + 2*a*b*x + a^2)) + 1)/(b*x + a)) - 3*a^4*log(((b*x + a)*sqrt(-(b^2*x^2 +
2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) + 3*a^4*log(((b*x + a
)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) +
12*(2*a^3 + a)*arctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x +
a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(b^3
*x^3 - 3*a*b^2*x^2 + 13*a^3 + (9*a^2 + 2)*b*x + 2*a)*sqrt(-(b^2*x^2 + 2*a*b
*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,\text{arsech}\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsech(b*x + a), x)
```

maple [A] time = 0.10, size = 250, normalized size = 1.23

$$\frac{\frac{\text{arcsech}(bx+a)(bx+a)^4}{4}-\text{arcsech}\,(bx+a)\,(bx+a)^3\,a+\frac{3\,\text{arcsech}(bx+a)(bx+a)^2a^2}{2}-\text{arcsech}\,(bx+a)\,(bx+a)\,a^3+\frac{\text{arcsech}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsech(b*x+a),x)
```

```
[Out] 1/b^4*(1/4*arcsech(b*x+a)*(b*x+a)^4-arcsech(b*x+a)*(b*x+a)^3*a+3/2*arcsech(
b*x+a)*(b*x+a)^2*a^2-arcsech(b*x+a)*(b*x+a)*a^3+1/4*arcsech(b*x+a)*a^4-1/12
*(-(b*x+a-1)/(b*x+a))^(1/2)*(b*x+a)*((b*x+a+1)/(b*x+a))^(1/2)*(3*a^4*arctan
h(1/(1-(b*x+a)^2)^(1/2))+12*a^3*arcsin(b*x+a)+(b*x+a)^2*(1-(b*x+a)^2)^(1/2)
```

```
-6*a*(b*x+a)*(1-(b*x+a)^2)^(1/2)+18*a^2*(1-(b*x+a)^2)^(1/2)+6*a*arcsin(b*x+
a)+2*(1-(b*x+a)^2)^(1/2))/(1-(b*x+a)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,b^4x^4\log\left(\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,bx+\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,a+bx+a\right)-2\,b^4x^4\log\left(bx+a\right)-b^2x^2+6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(2*b^4*x^4*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a
+ 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*b^4*x^4*log(b*x + a) - b^2*x^2 + 6
*a*b*x - (a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1) - 2*(b^4*x^4 - a^
4)*log(b*x + a) - (a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(-b*x - a + 1))/b^4 +
integrate(1/4*(b^2*x^5 + a*b*x^4)/(b^2*x^2 + 2*a*b*x + a^2 + (b^2*x^2 + 2*a
*b*x + a^2 - 1)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a + 1)) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(1/(a + b*x)),x)
```

```
[Out] int(x^3*acosh(1/(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,\mathrm{asech}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asech(b*x+a),x)
```

```
[Out] Integral(x**3*asech(a + b*x), x)
```

3.2 $\int x^2 \operatorname{sech}^{-1}(a+bx)\,dx$

Optimal. Leaf size=153

$$\frac{a^3\operatorname{sech}^{-1}(a+bx)}{3b^3}-\frac{(6a^2+1)\tan^{-1}\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a+bx}\right)}{6b^3}+\frac{5a\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{6b^3}-\frac{x\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{6b^2}+\frac{1}{3}x$$

```
[Out] 1/3*a^3*arcsech(b*x+a)/b^3+1/3*x^3*arcsech(b*x+a)-1/6*(6*a^2+1)*arctan((b*x
+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/(b*x+a))/b^3+5/6*a*(b*x+a+1)*((-b*x-a+1)
/(b*x+a+1))^(1/2)/b^3-1/6*x*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^2
```

Rubi [A] time = 0.10, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6321, 5468, 3782, 3770, 3767, 8}

$$-\frac{(6a^2+1)\tan^{-1}\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a+bx}\right)}{6b^3}+\frac{a^3\operatorname{sech}^{-1}(a+bx)}{3b^3}+\frac{5a\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{6b^3}-\frac{x\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{6b^2}+\frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSech[a + b*x],x]
```

```
[Out] (5*a*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*b^3) - (x*Sqrt[(1
- a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*b^2) + (a^3*ArcSech[a + b*x])/(
3*b^3) + (x^3*ArcSech[a + b*x])/3 - ((1 + 6*a^2)*ArcTan[(Sqrt[(1 - a - b*x)
/(1 + a + b*x)]*(1 + a + b*x))/(a + b*x)])/(6*b^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^2\operatorname{sech}^{-1}(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int x\operatorname{sech}(x)(-a+\operatorname{sech}(x))^2\tanh(x)\,dx,x,\operatorname{sech}^{-1}(a+bx)\right)}{b^3}$$
$$= \frac{1}{3}x^3\operatorname{sech}^{-1}(a+bx) - \frac{\operatorname{Subst}\left(\int(-a+\operatorname{sech}(x))^3\,dx,x,\operatorname{sech}^{-1}(a+bx)\right)}{3b^3}$$
$$= -\frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^2} + \frac{1}{3}x^3\operatorname{sech}^{-1}(a+bx) - \frac{\operatorname{Subst}\left(\int\left(-2a^3+\left(1+6a^2\right)\operatorname{sech}(x) - \right.\right.}{6b^3}$$
$$= -\frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^2} + \frac{a^3\operatorname{sech}^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\operatorname{sech}^{-1}(a+bx) + \frac{(5a)\operatorname{Subst}\left(\int \operatorname{sec}\right.}{}$$
$$= -\frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^2} + \frac{a^3\operatorname{sech}^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\operatorname{sech}^{-1}(a+bx) - \frac{\left(1+6a^2\right)\tan^{-1}\left(\right.}{6b}$$
$$= \frac{5a\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^3} - \frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^2} + \frac{a^3\operatorname{sech}^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\operatorname{sech}^{-1}(a$$

Mathematica [C] time = 0.28, size = 200, normalized size = 1.31

$$\frac{-2a^3\log(a+bx) + 2a^3\log\left(a\sqrt{-\frac{a+bx-1}{a+bx+1}} + bx\sqrt{-\frac{a+bx-1}{a+bx+1}} + \sqrt{-\frac{a+bx-1}{a+bx+1}} + 1\right) + \sqrt{-\frac{a+bx-1}{a+bx+1}}\left(5a^2 + a(4bx+5) - bx(b\right.}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSech[a + b*x],x]
```

```
[Out] (Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(5*a^2 - b*x*(1 + b*x) + a*(5 + 4*b*
x)) + 2*b^3*x^3*ArcSech[a + b*x] - 2*a^3*Log[a + b*x] + 2*a^3*Log[1 + Sqrt[
-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]
+ b*x*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]] + I*(1 + 6*a^2)*Log[(-2*I)*(a
+ b*x) + 2*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)])/(6*b^3)
```

fricas [B] time = 1.92, size = 327, normalized size = 2.14

$$\frac{2\,b^3x^3\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right) + a^3\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{x}\right) - a^3\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}-1}{x}\right) - \left(6\,a^2 +\right.}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2
+ 2*a*b*x + a^2)) + 1)/(b*x + a)) + a^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a
*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - a^3*log(((b*x + a)*sqr
t(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) - (6*a^
```

```
2 + 1)*arctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)
/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (b^2*x^2 - 4*a
*b*x - 5*a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)
))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arsech}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsech(b*x + a), x)
```

maple [A] time = 0.07, size = 189, normalized size = 1.24

$$\frac{\frac{(bx+a)^3\operatorname{arcsech}(bx+a)}{3} - \operatorname{arcsech}(bx+a)(bx+a)^2 a + \operatorname{arcsech}(bx+a)(bx+a)a^2 - \frac{\operatorname{arcsech}(bx+a)a^3}{3} + \frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+}{}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsech(b*x+a),x)
```

```
[Out] 1/b^3*(1/3*(b*x+a)^3*arcsech(b*x+a)-arcsech(b*x+a)*(b*x+a)^2*a+arcsech(b*x+
a)*(b*x+a)*a^2-1/3*arcsech(b*x+a)*a^3+1/6*(-(b*x+a-1)/(b*x+a))^(1/2)*(b*x+a
)*((b*x+a+1)/(b*x+a))^(1/2)*(2*a^3*arctanh(1/(1-(b*x+a)^2)^(1/2))+6*a^2*arc
sin(b*x+a)-(b*x+a)*(1-(b*x+a)^2)^(1/2)+6*a*(1-(b*x+a)^2)^(1/2)+arcsin(b*x+a
))/(1-(b*x+a)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,b^3x^3\log\left(\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,bx+\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,a+bx+a\right)-2\,b^3x^3\log(bx+a)-2\,bx}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*x^3*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a
+ 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*b^3*x^3*log(b*x + a) - 2*b*x + (a^
3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1) - 2*(b^3*x^3 + a^3)*log(b*x + a) + (a
^3 - 3*a^2 + 3*a - 1)*log(-b*x - a + 1))/b^3 + integrate(1/3*(b^2*x^4 + a*b
*x^3)/(b^2*x^2 + 2*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(1/2*log(b
*x + a + 1) + 1/2*log(-b*x - a + 1)) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acosh}\left(\frac{1}{a+b\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(1/(a + b*x)),x)
```

```
[Out] int(x^2*acosh(1/(a + b*x)), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asech}(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asech(b*x+a),x)
```

```
[Out] Integral(x**2*asech(a + b*x), x)
```

3.3 $\int x\text{sech}^{-1}(a+bx)\,dx$

Optimal. Leaf size=107

$$-\frac{a^2\text{sech}^{-1}(a+bx)}{2b^2}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{2b^2}+\frac{a\tan^{-1}\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a+bx}\right)}{b^2}+\frac{1}{2}x^2\text{sech}^{-1}(a+bx)$$

```
[Out] -1/2*a^2*arcsech(b*x+a)/b^2+1/2*x^2*arcsech(b*x+a)+a*arctan((b*x+a+1)*((-b*
x-a+1)/(b*x+a+1))^(1/2)/(b*x+a))/b^2-1/2*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(
1/2)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6321, 5468, 3773, 3770, 3767, 8}

$$-\frac{a^2\text{sech}^{-1}(a+bx)}{2b^2}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{2b^2}+\frac{a\tan^{-1}\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}(a+bx+1)}{a+bx}\right)}{b^2}+\frac{1}{2}x^2\text{sech}^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSech[a + b*x],x]
```

```
[Out] -(Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(2*b^2) - (a^2*ArcSech[a
 + b*x])/(2*b^2) + (x^2*ArcSech[a + b*x])/2 + (a*ArcTan[(Sqrt[(1 - a - b*x)
/(1 + a + b*x)]*(1 + a + b*x))/(a + b*x)])/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3773

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x\text{sech}^{-1}(a+bx)\,dx = -\frac{\text{Subst}\left(\int x\text{sech}(x)(-a+\text{sech}(x))\tanh(x)\,dx, x, \text{sech}^{-1}(a+bx)\right)}{b^2}$$
$$= \frac{1}{2}x^2\text{sech}^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\text{sech}(x))^2\,dx, x, \text{sech}^{-1}(a+bx)\right)}{2b^2}$$
$$= -\frac{a^2\text{sech}^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\text{sech}^{-1}(a+bx) - \frac{\text{Subst}\left(\int \text{sech}^2(x)\,dx, x, \text{sech}^{-1}(a+bx)\right)}{2b^2} + $$
$$= -\frac{a^2\text{sech}^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\text{sech}^{-1}(a+bx) + \frac{a\tan^{-1}\left(\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{a+bx}\right)}{b^2} - \frac{i\,\text{Subst}\left(\int 1\,dx,\right.}{}$$
$$= -\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2b^2} - \frac{a^2\text{sech}^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\text{sech}^{-1}(a+bx) + \frac{a\tan^{-1}\left(\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a}{a+bx}\right.}{b^2}$$

Mathematica [C] time = 0.17, size = 176, normalized size = 1.64

$$\frac{a^2\log(a+bx) - a^2\log\left(a\sqrt{-\frac{a+bx-1}{a+bx+1}} + bx\sqrt{-\frac{a+bx-1}{a+bx+1}} + \sqrt{-\frac{a+bx-1}{a+bx+1}} + 1\right) + b^2x^2\text{sech}^{-1}(a+bx) - \sqrt{-\frac{a+bx-1}{a+bx+1}}(a+bx}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSech[a + b*x],x]
```

```
[Out] (-(Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)) + b^2*x^2*ArcSech[a
 + b*x] + a^2*Log[a + b*x] - a^2*Log[1 + Sqrt[-((-1 + a + b*x)/(1 + a + b*x
))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + b*x*Sqrt[-((-1 + a + b*x)/(
1 + a + b*x))]] - (2*I)*a*Log[(-2*I)*(a + b*x) + 2*Sqrt[-((-1 + a + b*x)/(1
 + a + b*x))]*(1 + a + b*x)])/(2*b^2)
```

fricas [B] time = 1.39, size = 308, normalized size = 2.88

$$\frac{2\,b^2x^2\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right) - a^2\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{x}\right) + a^2\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}-1}{x}\right) + 4\,a\,\text{ar}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2
+ 2*a*b*x + a^2)) + 1)/(b*x + a)) - a^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a
*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) + a^2*log(((b*x + a)*sqr
t(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) + 4*a*a
rctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^
2 + 2*a*b*x + a^2))/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(b*x + a)*sqrt(-(b^2
*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arsech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*arcsech(b*x + a), x)
```

maple [A] time = 0.07, size = 111, normalized size = 1.04

$$\frac{\frac{\operatorname{arcsech}(bx+a)(bx+a)^2}{2} - \operatorname{arcsech}(bx+a)\,a\,(bx+a) - \frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(2a\arcsin(bx+a)+\sqrt{1-(bx+a)^2}\right)}{2\sqrt{1-(bx+a)^2}}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsech(b*x+a),x)
```

```
[Out] 1/b^2*(1/2*arcsech(b*x+a)*(b*x+a)^2-arcsech(b*x+a)*a*(b*x+a)-1/2*(-(b*x+a-1
)/(b*x+a))^(1/2)*(b*x+a)*((b*x+a+1)/(b*x+a))^(1/2)*(2*a*arcsin(b*x+a)+(1-(b
*x+a)^2)^(1/2))/(1-(b*x+a)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,b^2x^2\log\left(\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,bx+\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,a+bx+a\right)-2\,b^2x^2\log\left(bx+a\right)-\left(a^2+\right.}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^2*x^2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a
+ 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*b^2*x^2*log(b*x + a) - (a^2 + 2*a
+ 1)*log(b*x + a + 1) - 2*(b^2*x^2 - a^2)*log(b*x + a) - (a^2 - 2*a + 1)*lo
g(-b*x - a + 1))/b^2 + integrate(1/2*(b^2*x^3 + a*b*x^2)/(b^2*x^2 + 2*a*b*x
 + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b
*x - a + 1)) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(1/(a + b*x)),x)
```

```
[Out] int(x*acosh(1/(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asech(b*x+a),x)
```

```
[Out] Integral(x*asech(a + b*x), x)
```

3.4 $\int \operatorname{sech}^{-1}(a+bx)\,dx$

Optimal. Leaf size=44

$$\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b}-\frac{2\tan^{-1}\left(\sqrt{\frac{-a-bx+1}{a+bx+1}}\right)}{b}$$

```
[Out] (b*x+a)*arcsech(b*x+a)/b-2*arctan(((-b*x-a+1)/(b*x+a+1))^(1/2))/b
```

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6313, 1961, 12, 203}

$$\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b}-\frac{2\tan^{-1}\left(\sqrt{\frac{-a-bx+1}{a+bx+1}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x],x]
```

```
[Out] ((a + b*x)*ArcSech[a + b*x])/b - (2*ArcTan[Sqrt[(1 - a - b*x)/(1 + a + b*x)
]])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 6313

```
Int[ArcSech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcSech[c + d*
x])/d, x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; Free
Q[{c, d}, x]
```

Rubi steps

$$\int \operatorname{sech}^{-1}(a+bx)\,dx = \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b} + \int \frac{\sqrt{\frac{1-a-bx}{1+a+bx}}}{1-a-bx}\,dx$$
$$= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b} - (4b)\operatorname{Subst}\left(\int \frac{1}{2b^2(1+x^2)}\,dx, x, \sqrt{\frac{1-a-bx}{1+a+bx}}\right)$$
$$= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b} - \frac{2\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{1-a-bx}{1+a+bx}}\right)}{b}$$
$$= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b} - \frac{2\tan^{-1}\left(\sqrt{\frac{1-a-bx}{1+a+bx}}\right)}{b}$$

Mathematica [B] time = 0.31, size = 125, normalized size = 2.84

$$x\operatorname{sech}^{-1}(a+bx) - \frac{2b\sqrt{-\frac{a+bx-1}{a+bx+1}}\left(\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right) - a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{-b}\sqrt{\frac{a+bx-1}{a+bx+1}}}{\sqrt{b}}\right)\right)}{(-b)^{5/2}\sqrt{\frac{a+bx-1}{a+bx+1}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a + b*x],x]
```

```
[Out] x*ArcSech[a + b*x] - (2*b*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(Sqrt[-b]*A
rcSinh[Sqrt[-1 + a + b*x]/Sqrt[2]] - a*Sqrt[b]*ArcTanh[(Sqrt[-b]*Sqrt[(-1 +
 a + b*x)/(1 + a + b*x)])/Sqrt[b]]))/((-b)^(5/2)*Sqrt[(-1 + a + b*x)/(1 + a
 + b*x)])
```

fricas [B] time = 0.76, size = 253, normalized size = 5.75

$$\frac{2\,bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right) + a\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{x}\right) - a\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}-1}{x}\right) - 2\,\mathrm{arcta}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*
a*b*x + a^2)) + 1)/(b*x + a)) + a*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x +
 a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - a*log(((b*x + a)*sqrt(-(b^2*
x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) - 2*arctan((b^2
*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*
x + a^2))/(b^2*x^2 + 2*a*b*x + a^2 - 1)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a), x)
```

maple [A] time = 0.05, size = 50, normalized size = 1.14

$$x\,\mathrm{arcsech}\left(bx+a\right)+\frac{\mathrm{arcsech}\left(bx+a\right)a}{b}-\frac{\arctan\left(\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a),x)
```

```
[Out] x*arcsech(b*x+a)+1/b*arcsech(b*x+a)*a-1/b*arctan((1/(b*x+a)-1)^(1/2)*(1/(b*
x+a)+1)^(1/2))
```

maxima [A] time = 0.30, size = 31, normalized size = 0.70

$$\frac{(bx+a)\,\mathrm{arsech}\left(bx+a\right)-\arctan\left(\sqrt{\frac{1}{(bx+a)^2}-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*arcsech(b*x + a) - arctan(sqrt(1/(b*x + a)^2 - 1)))/b
```

mupad [B] time = 2.16, size = 43, normalized size = 0.98

$$\frac{\mathrm{atan}\left(\frac{1}{\sqrt{\frac{1}{a+b\,x}-1}\,\sqrt{\frac{1}{a+b\,x}+1}}\right)+\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)\,(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x)),x)
```

```
[Out] (atan(1/((1/(a + b*x) - 1)^(1/2)*(1/(a + b*x) + 1)^(1/2))) + acosh(1/(a + b
*x))*(a + b*x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{asech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a),x)
```

```
[Out] Integral(asech(a + b*x), x)
```

3.5 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x} dx$

Optimal. Leaf size=170

$$\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}}}{\sqrt{1-a}}\right.$$

```
[Out] -arcsech(b*x+a)*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)
+arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/
(1-(-a^2+1)^(1/2)))+arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1
/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))-1/2*polylog(2,-(1/(b*x+a)+(1/(b*x+a)
-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)+polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2
)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))+polylog(2,a*(1/(b*x+a)+(1/(b*x+a
)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))
```

Rubi [A] time = 0.29, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6321, 5595, 5570, 3718, 2190, 2279, 2391, 5562}

$$\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(a+bx)}\right)+\operatorname{sech}^{-1}(a+bx)\log\left(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]/x,x]
```

```
[Out] ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + ArcS
ech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcSech[
a + b*x]*Log[1 + E^(2*ArcSech[a + b*x])] + PolyLog[2, (a*E^ArcSech[a + b*x]
)/(1 - Sqrt[1 - a^2])] + PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^
2])] - PolyLog[2, -E^(2*ArcSech[a + b*x])]/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5570

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Tanh[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Sinh[c + d*x]*Tanh[c + d*x]^
(n - 1))/(a + b*Cosh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5595

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sech[(c_.) + (d_.)*(x_)]), x_Symbol] :> I
nt[((e + f*x)^m*Cosh[c + d*x]*F[c + d*x]^n*G[c + d*x]^p)/(b + a*Cosh[c + d*
x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G]
 && IntegersQ[m, n, p]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(a+bx)}{x}\,dx &= -\operatorname{Subst}\left(\int \frac{x\operatorname{sech}(x)\tanh(x)}{-a+\operatorname{sech}(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\\
&= -\operatorname{Subst}\left(\int \frac{x\tanh(x)}{1-a\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\\
&= -\left(a\operatorname{Subst}\left(\int \frac{x\sinh(x)}{1-a\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) - \operatorname{Subst}\left(\int x\tanh(x)\,dx, x, \operatorname{sech}^{-1}(\right.\\
&= -\left(2\operatorname{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) - a\operatorname{Subst}\left(\int \frac{e^{x}x}{1-\sqrt{1-a^2}-ae^{x}}\,dx, x, \operatorname{sec}\right.\\
&= \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}\\
&= \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}\\
&= \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}
\end{aligned}$$

Mathematica [C] time = 0.20, size = 332, normalized size = 1.95

$$-\operatorname{Li}_2\left(-\frac{\left(\sqrt{1-a^2}-1\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right)-\operatorname{Li}_2\left(\frac{\left(\sqrt{1-a^2}+1\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right)+\operatorname{sech}^{-1}(a+bx)\log\left(\frac{\left(\sqrt{1-a^2}-1\right)e^{-\operatorname{sech}}}{a}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a + b*x]/x,x]
```

```
[Out] (-4*I)*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*ArcTanh[((1 + a)*Tanh[ArcSech[a + b
*x]/2])/Sqrt[1 - a^2]] - ArcSech[a + b*x]*Log[1 + E^(-2*ArcSech[a + b*x])]
+ ArcSech[a + b*x]*Log[1 + (-1 + Sqrt[1 - a^2])/(a*E^ArcSech[a + b*x])] + (
2*I)*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 + (-1 + Sqrt[1 - a^2])/(a*E^Arc
Sech[a + b*x])] + ArcSech[a + b*x]*Log[1 - (1 + Sqrt[1 - a^2])/(a*E^ArcSech
[a + b*x])] - (2*I)*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 - (1 + Sqrt[1 -
a^2])/(a*E^ArcSech[a + b*x])] + PolyLog[2, -E^(-2*ArcSech[a + b*x])]/2 - Po
lyLog[2, -((-1 + Sqrt[1 - a^2])/(a*E^ArcSech[a + b*x]))] - PolyLog[2, (1 +
Sqrt[1 - a^2])/(a*E^ArcSech[a + b*x])]
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsech}(bx+a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsech(b*x + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(bx+a)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)/x, x)
```

maple [C] time = 0.96, size = 886, normalized size = 5.21

$$\frac{\operatorname{arcsech}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)+\sqrt{-a^2+1}+1}{1+\sqrt{-a^2+1}}\right)}{2}+\frac{\operatorname{arcsech}(bx+a)\ln\left(\frac{a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)+\sqrt{-a^2+}}{-1+\sqrt{-a^2+1}}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)/x,x)
```

```
[Out] 1/2*arcsech(b*x+a)*ln((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2
))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))+1/2*arcsech(b*x+a)*ln((a*(1/(b*x+a
)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(
1/2)))-1/2*(-a^2+1)^(1/2)/(a^2-1)*arcsech(b*x+a)*ln((-a*(1/(b*x+a)+(1/(b*x+
a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))+1/2*
(-a^2+1)^(1/2)/(a^2-1)*arcsech(b*x+a)*ln((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*
(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))+dilog((-a*(1/(b
*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1
)^(1/2)))+dilog((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^
2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))-arcsech(b*x+a)*ln(1+I*(1/(b*x+a)+(1/(b*x
+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-arcsech(b*x+a)*ln(1-I*(1/(b*x+a)+(1/(b*x
+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-dilog(1+I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)
*(1/(b*x+a)+1)^(1/2)))-dilog(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+
1)^(1/2)))+1/2*(a^2-1-(-a^2+1)^(1/2))/a^2/(a^2-1)*arcsech(b*x+a)*(ln((a*(1/
(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^
```

```
2+1)^(1/2)))*a^2+ln((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))
+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))*a^2-2*ln((-a*(1/(b*x+a)+(1/(b*x+a)-1
)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))+2*ln((-a
*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(
-a^2+1)^(1/2)))*(-a^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}\left(bx+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsech(b*x + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))/x,x)
```

```
[Out] int(acosh(1/(a + b*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)/x,x)
```

```
[Out] Integral(asech(a + b*x)/x, x)
```

3.6 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=70

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a+1}\,\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}-\frac{b\operatorname{sech}^{-1}(a+bx)}{a}-\frac{\operatorname{sech}^{-1}(a+bx)}{x}$$

```
[Out] -b*arcsech(b*x+a)/a-arcsech(b*x+a)/x+2*b*arctanh((1+a)^(1/2)*tanh(1/2*arcse
ch(b*x+a))/(1-a)^(1/2))/a/(-a^2+1)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6321, 5468, 3783, 2659, 208}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a+1}\,\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}-\frac{b\operatorname{sech}^{-1}(a+bx)}{a}-\frac{\operatorname{sech}^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]/x^2,x]
```

```
[Out] -((b*ArcSech[a + b*x])/a) - ArcSech[a + b*x]/x + (2*b*ArcTanh[(Sqrt[1 + a]*
Tanh[ArcSech[a + b*x]/2])/Sqrt[1 - a]])/(a*Sqrt[1 - a^2])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\text{sech}^{-1}(a+bx)}{x^2}\,dx = -\left(b\,\text{Subst}\left(\int \frac{x\text{sech}(x)\tanh(x)}{(-a+\text{sech}(x))^2}\,dx, x, \text{sech}^{-1}(a+bx)\right)\right)$$
$$= -\frac{\text{sech}^{-1}(a+bx)}{x} + b\,\text{Subst}\left(\int \frac{1}{-a+\text{sech}(x)}\,dx, x, \text{sech}^{-1}(a+bx)\right)$$
$$= -\frac{b\text{sech}^{-1}(a+bx)}{a} - \frac{\text{sech}^{-1}(a+bx)}{x} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-a\cosh(x)}\,dx, x, \text{sech}^{-1}(a+bx)\right)}{a}$$
$$= -\frac{b\text{sech}^{-1}(a+bx)}{a} - \frac{\text{sech}^{-1}(a+bx)}{x} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{1-a-(1+a)x^2}\,dx, x, \tanh\left(\frac{1}{2}\text{sech}^{-1}(a+\right.\right.}{a}$$
$$= -\frac{b\text{sech}^{-1}(a+bx)}{a} - \frac{\text{sech}^{-1}(a+bx)}{x} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{1+a}\,\tanh\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}$$

Mathematica [B] time = 0.26, size = 244, normalized size = 3.49

$$\frac{b\left(\sqrt{1-a^2}\log(a+bx) - \sqrt{1-a^2}\log\left(a\sqrt{-\frac{a+bx-1}{a+bx+1}} + bx\sqrt{-\frac{a+bx-1}{a+bx+1}} + \sqrt{-\frac{a+bx-1}{a+bx+1}} + 1\right) + \log\left(\sqrt{1-a^2}\,a\sqrt{-\frac{a+bx-1}{a+bx+1}}\right.\right.}{a\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a + b*x]/x^2,x]
```

```
[Out] -(ArcSech[a + b*x]/x) + (b*(-Log[x] + Sqrt[1 - a^2]*Log[a + b*x] - Sqrt[1 -
 a^2]*Log[1 + Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqrt[-((-1 + a + b*
x)/(1 + a + b*x))] + b*x*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]] + Log[1 - a
^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqrt[1
 - a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + Sqrt[1 - a^2]*b*x*Sqrt[-((-
1 + a + b*x)/(1 + a + b*x))]]))/(a*Sqrt[1 - a^2])
```

fricas [B] time = 0.71, size = 651, normalized size = 9.30

$$\left[-\frac{\left(a^2-1\right)bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{x}\right) - \left(a^2-1\right)bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}-1}{x}\right) + \sqrt{-a^2+1}\,bx\log\left(\frac{\left(2a^2-1\right)b^2\cdots}{\cdots}\right.}{2\left(a^3 \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2 - 1)*b*x*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^
2*x^2 + 2*a*b*x + a^2)) + 1)/x) - (a^2 - 1)*b*x*log(((b*x + a)*sqrt(-(b^2*x
^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) + sqrt(-a^2 + 1)
*b*x*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 - 2*(a*b^2*
x^2 + a^3 + (2*a^2 - 1)*b*x - a)*sqrt(-a^2 + 1)*sqrt(-(b^2*x^2 + 2*a*b*x +
a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 2)/x^2) + 2*(a^3 - a)*log(((b*x + a)*
sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x +
a)))/((a^3 - a)*x), -1/2*((a^2 - 1)*b*x*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a
*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - (a^2 - 1)*b*x*log(((b*
x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/
```

```
x) + 2*sqrt(a^2 - 1)*b*x*arctan((a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x - a)*sqr
t(a^2 - 1)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))/(
(a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + 2*(a^3 - a)*log((
(b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) +
1)/(b*x + a)))/((a^3 - a)*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)/x^2, x)
```

maple [B] time = 0.08, size = 542, normalized size = 7.74

$$-\frac{\operatorname{arcsech}(bx+a)}{x}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,\sqrt{\frac{bx+a+1}{bx+a}}\,a\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(bx+a)^2}}\right)x\,b^2}{\sqrt{1-(bx+a)^2}\,(a-1)(1+a)}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,\sqrt{\frac{bx+a+1}{bx+a}}\,\sqrt{-a^2+1}\,\ln\left(\frac{2\sqrt{-a^2}}{}\right.}{\sqrt{1-(bx+a)^2}\,a\,(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)/x^2,x)
```

```
[Out] -arcsech(b*x+a)/x-(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(1-(
b*x+a)^2)^(1/2)*a/(a-1)/(1+a)*arctanh(1/(1-(b*x+a)^2)^(1/2))*x*b^2-(-(b*x+a
-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(1-(b*x+a)^2)^(1/2)/a/(a-1)/(1
+a)*(-a^2+1)^(1/2)*ln(2*((-a^2+1)^(1/2)*(1-(b*x+a)^2)^(1/2)-a*(b*x+a)+1)/b/
x)*x*b^2-b*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(1-(b*x+a)^
2)^(1/2)*a^2/(a-1)/(1+a)*arctanh(1/(1-(b*x+a)^2)^(1/2))-b*(-(b*x+a-1)/(b*x+
a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(1-(b*x+a)^2)^(1/2)/(a-1)/(1+a)*(-a^2+1
)^(1/2)*ln(2*((-a^2+1)^(1/2)*(1-(b*x+a)^2)^(1/2)-a*(b*x+a)+1)/b/x)+(-(b*x+a
-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(1-(b*x+a)^2)^(1/2)/a/(a-1)/(1
+a)*arctanh(1/(1-(b*x+a)^2)^(1/2))*x*b^2+b*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x
+a+1)/(b*x+a))^(1/2)/(1-(b*x+a)^2)^(1/2)/(a-1)/(1+a)*arctanh(1/(1-(b*x+a)^2
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\log(x)}{a^3-a}-\frac{\left(a^2b-ab\right)x\log(bx+a+1)+\left(a^2b+ab\right)x\log(-bx-a+1)+2\left(a^3-a\right)\log\left(\sqrt{bx+a+1}\,\sqrt{-bx-a}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x)/(a^3 - a) - 1/2*((a^2*b - a*b)*x*log(b*x + a + 1) + (a^2*b + a*b)*
x*log(-b*x - a + 1) + 2*(a^3 - a)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*
b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*(a^3 + (a^2*b -
 b)*x - a)*log(b*x + a) - 2*(a^3 - a)*log(b*x + a))/((a^3 - a)*x) - integra
te((b^2*x + a*b)/(b^2*x^3 + 2*a*b*x^2 + (a^2 - 1)*x + (b^2*x^3 + 2*a*b*x^2
+ (a^2 - 1)*x)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))/x^2,x)
```

```
[Out] int(acosh(1/(a + b*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)/x**2,x)
```

```
[Out] Integral(asech(a + b*x)/x**2, x)
```

3.7 $\int \frac{\text{sech}^{-1}(a+bx)}{x^3} \, dx$

Optimal. Leaf size=133

$$\frac{b^2\text{sech}^{-1}(a+bx)}{2a^2}-\frac{\left(1-2a^2\right)b^2\tanh^{-1}\left(\frac{\sqrt{a+1}\tanh\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a^2\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{2a\left(1-a^2\right)x}-\frac{\text{sech}^{-1}(a+bx)}{2x^2}$$

```
[Out] 1/2*b^2*arcsech(b*x+a)/a^2-1/2*arcsech(b*x+a)/x^2-(-2*a^2+1)*b^2*arctanh((1
+a)^(1/2)*tanh(1/2*arcsech(b*x+a))/(1-a)^(1/2))/a^2/(-a^2+1)^(3/2)+1/2*b*(b
*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/a/(-a^2+1)/x
```

Rubi [A] time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6321, 5468, 3785, 3919, 3831, 2659, 208}

$$\frac{b^2\text{sech}^{-1}(a+bx)}{2a^2}-\frac{\left(1-2a^2\right)b^2\tanh^{-1}\left(\frac{\sqrt{a+1}\tanh\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a^2\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{2a\left(1-a^2\right)x}-\frac{\text{sech}^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]/x^3,x]
```

```
[Out] (b*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(2*a*(1 - a^2)*x) + (b^
2*ArcSech[a + b*x])/(2*a^2) - ArcSech[a + b*x]/(2*x^2) - ((1 - 2*a^2)*b^2*A
rcTanh[(Sqrt[1 + a]*Tanh[ArcSech[a + b*x]/2])/Sqrt[1 - a]])/(a^2*(1 - a^2)^
(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^3}\,dx = -\left(b^2\operatorname{Subst}\left(\int \frac{x\operatorname{sech}(x)\tanh(x)}{(-a+\operatorname{sech}(x))^3}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)}{2x^2} + \frac{1}{2}b^2\operatorname{Subst}\left(\int \frac{1}{(-a+\operatorname{sech}(x))^2}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)$$

$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} - \frac{\operatorname{sech}^{-1}(a+bx)}{2x^2} - \frac{b^2\operatorname{Subst}\left(\int \frac{1-a^2-a\operatorname{sech}(x)}{-a+\operatorname{sech}(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{2a(1-a^2)}$$

$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} + \frac{b^2\operatorname{sech}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)}{2x^2} - \frac{\left(\left(1-2a^2\right)b^2\right)\operatorname{Subst}\left(\int \ldots\right.}{2a\ldots}$$

$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} + \frac{b^2\operatorname{sech}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)}{2x^2} - \frac{\left(\left(1-2a^2\right)b^2\right)\operatorname{Subst}\left(\int \ldots\right.}{2a\ldots}$$

$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} + \frac{b^2\operatorname{sech}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)}{2x^2} - \frac{\left(\left(1-2a^2\right)b^2\right)\operatorname{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} + \frac{b^2\operatorname{sech}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)}{2x^2} - \frac{\left(1-2a^2\right)b^2\tanh^{-1}\left(\frac{\sqrt{1+\ldots}}{\ldots}\right.}{a^2\left(1-a^{\ldots}\right.}$$

Mathematica [B] time = 0.97, size = 315, normalized size = 2.37

$$\frac{1}{2}\left(-\frac{\left(2a^2-1\right)b^2\log(x)}{a^2\left(1-a^2\right)^{3/2}} - \frac{b^2\log(a+bx)}{a^2} + \frac{b^2\log\left(a\sqrt{-\frac{a+bx-1}{a+bx+1}} + bx\sqrt{-\frac{a+bx-1}{a+bx+1}} + \sqrt{-\frac{a+bx-1}{a+bx+1}} + 1\right)}{a^2} + \frac{\left(2a^2-1\right)b^2\ldots}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a + b*x]/x^3,x]
```

```
[Out] (-((b*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x))/((-1 + a)*a*(1 +
 a)*x)) - ArcSech[a + b*x]/x^2 - ((-1 + 2*a^2)*b^2*Log[x])/(a^2*(1 - a^2)^(
3/2)) - (b^2*Log[a + b*x])/a^2 + (b^2*Log[1 + Sqrt[-((-1 + a + b*x)/(1 + a
+ b*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + b*x*Sqrt[-((-1 + a + b
*x)/(1 + a + b*x))]])/a^2 + ((-1 + 2*a^2)*b^2*Log[1 - a^2 - a*b*x + Sqrt[1
- a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqrt[1 - a^2]*Sqrt[-((-1 +
 a + b*x)/(1 + a + b*x))] + Sqrt[1 - a^2]*b*x*Sqrt[-((-1 + a + b*x)/(1 + a
+ b*x))]])/(a^2*(1 - a^2)^(3/2)))/2
```

fricas [B] time = 0.69, size = 865, normalized size = 6.50

$$\left[-\frac{\left(2a^2-1\right)\sqrt{-a^2+1}\,b^2x^2\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-4a^2+2\left(ab^2x^2+a^3+\left(2a^2-1\right)bx-a\right)\sqrt{-a^2+1}\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+2}{x^2}\right)-}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/4*((2*a^2 - 1)*sqrt(-a^2 + 1)*b^2*x^2*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4
+ 4*(a^3 - a)*b*x - 4*a^2 + 2*(a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x - a)*sqrt(
-a^2 + 1)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) +
2)/x^2) - (a^4 - 2*a^2 + 1)*b^2*x^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x
 + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) + (a^4 - 2*a^2 + 1)*b^2*x^2*
log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2
)) - 1)/x) + 2*(a^6 - 2*a^4 + a^2)*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x
+ a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)) + 2*((a^3 - a)*b^2*x^
2 + (a^4 - a^2)*b*x)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x
 + a^2)))/((a^6 - 2*a^4 + a^2)*x^2), 1/4*(2*(2*a^2 - 1)*sqrt(a^2 - 1)*b^2*x
^2*arctan((a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x - a)*sqrt(a^2 - 1)*sqrt(-(b^2*
x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))/((a^2 - 1)*b^2*x^2 + a^
4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + (a^4 - 2*a^2 + 1)*b^2*x^2*log(((b*x + a
)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) -
(a^4 - 2*a^2 + 1)*b^2*x^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1
)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) - 2*(a^6 - 2*a^4 + a^2)*log(((b*x + a)
*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x +
 a)) - 2*((a^3 - a)*b^2*x^2 + (a^4 - a^2)*b*x)*sqrt(-(b^2*x^2 + 2*a*b*x + a
^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/((a^6 - 2*a^4 + a^2)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(bx+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)/x^3, x)
```

maple [B] time = 0.09, size = 1233, normalized size = 9.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)/x^3,x)
```

```
[Out] -1/2*arcsech(b*x+a)/x^2+1/2*b^3*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+
a))^(1/2)*x/(a^2-1)/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)*a^2*arctanh(1/(1-(b*x+a
)^2)^(1/2))+b^3*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)*x/(a^2
-1)/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)*(-a^2+1)^(1/2)*ln(2*((-a^2+1)^(1/2)*(1-
(b*x+a)^2)^(1/2)-a*(b*x+a)+1)/b/x)+1/2*b^2*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x
+a+1)/(b*x+a))^(1/2)/(a^2-1)/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)*a^3*arctanh(1/
(1-(b*x+a)^2)^(1/2))+b^2*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/
2)/(a^2-1)/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)*a*(-a^2+1)^(1/2)*ln(2*((-a^2+1)^
(1/2)*(1-(b*x+a)^2)^(1/2)-a*(b*x+a)+1)/b/x)-b^3*(-(b*x+a-1)/(b*x+a))^(1/2)*
((b*x+a+1)/(b*x+a))^(1/2)*x/(a^2-1)/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)*arctanh
(1/(1-(b*x+a)^2)^(1/2))-1/2*b^3*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+
a))^(1/2)*x/(a^2-1)/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)/a^2*(-a^2+1)^(1/2)*ln(2
*((-a^2+1)^(1/2)*(1-(b*x+a)^2)^(1/2)-a*(b*x+a)+1)/b/x)-b^2*(-(b*x+a-1)/(b*x
+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(a^2-1)/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2
)*a*arctanh(1/(1-(b*x+a)^2)^(1/2))-1/2*b^2*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x
+a+1)/(b*x+a))^(1/2)/(a^2-1)/(1+a)/(a-1)*a-1/2*b^2*(-(b*x+a-1)/(b*x+a))^(1/
2)*((b*x+a+1)/(b*x+a))^(1/2)/(a^2-1)/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)/a*(-a^
2+1)^(1/2)*ln(2*((-a^2+1)^(1/2)*(1-(b*x+a)^2)^(1/2)-a*(b*x+a)+1)/b/x)+1/2*b
^3*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)*x/(a^2-1)/(1+a)/(a-
1)/(1-(b*x+a)^2)^(1/2)/a^2*arctanh(1/(1-(b*x+a)^2)^(1/2))-1/2*b*(-(b*x+a-1)
/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/x/(a^2-1)/(1+a)/(a-1)*a^2+1/2*b^2
*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(a^2-1)/(1+a)/(a-1)/(
1-(b*x+a)^2)^(1/2)/a*arctanh(1/(1-(b*x+a)^2)^(1/2))+1/2*b^2*(-(b*x+a-1)/(b*
x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(a^2-1)/(1+a)/(a-1)/a+1/2*b*(-(b*x+a-
1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/x/(a^2-1)/(1+a)/(a-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(3\,a^2b^2-b^2\right)\log(x)}{2\left(a^6-2\,a^4+a^2\right)}+\frac{\left(a^4b^2-2\,a^3b^2+a^2b^2\right)x^2\log\left(bx+a+1\right)+\left(a^4b^2+2\,a^3b^2+a^2b^2\right)x^2\log\left(-bx-a+1\right)-2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*a^2*b^2 - b^2)*log(x)/(a^6 - 2*a^4 + a^2) + 1/4*((a^4*b^2 - 2*a^3*b
^2 + a^2*b^2)*x^2*log(b*x + a + 1) + (a^4*b^2 + 2*a^3*b^2 + a^2*b^2)*x^2*lo
g(-b*x - a + 1) - 2*(a^3*b - a*b)*x - 2*(a^6 - 2*a^4 + a^2)*log(sqrt(b*x +
a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*
x + a) + 2*(a^6 - 2*a^4 - (a^4*b^2 - 2*a^2*b^2 + b^2)*x^2 + a^2)*log(b*x +
a) + 2*(a^6 - 2*a^4 + a^2)*log(b*x + a))/((a^6 - 2*a^4 + a^2)*x^2) - integr
ate(1/2*(b^2*x + a*b)/(b^2*x^4 + 2*a*b*x^3 + (a^2 - 1)*x^2 + (b^2*x^4 + 2*a
*b*x^3 + (a^2 - 1)*x^2)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a + 1))),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))/x^3,x)
```

```
[Out] int(acosh(1/(a + b*x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}{\left(a+bx \right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)/x**3,x)
```

```
[Out] Integral(asech(a + b*x)/x**3, x)
```

3.8 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=197

$$-\frac{b^3\operatorname{sech}^{-1}(a+bx)}{3a^3}-\frac{\left(2-5a^2\right)b^2\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{6a^2\left(1-a^2\right)^2x}+\frac{b\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{6a\left(1-a^2\right)x^2}+\frac{\left(6a^4-5a^2+2\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+\dots}}{\dots}\right)}{3a^3\left(1-a^2\right)^{5\dots}}$$

```
[Out] -1/3*b^3*arcsech(b*x+a)/a^3-1/3*arcsech(b*x+a)/x^3+1/3*(6*a^4-5*a^2+2)*b^3*
arctanh((1+a)^(1/2)*tanh(1/2*arcsech(b*x+a))/(1-a)^(1/2))/a^3/(-a^2+1)^(5/2
)+1/6*b*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/a/(-a^2+1)/x^2-1/6*(-5*a^2+2
)*b^2*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/a^2/(-a^2+1)^2/x
```

Rubi [A] time = 0.32, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6321, 5468, 3785, 4060, 3919, 3831, 2659, 208}

$$-\frac{\left(2-5a^2\right)b^2\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{6a^2\left(1-a^2\right)^2x}-\frac{b^3\operatorname{sech}^{-1}(a+bx)}{3a^3}+\frac{\left(6a^4-5a^2+2\right)b^3\tanh^{-1}\left(\frac{\sqrt{a+1}\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{3a^3\left(1-a^2\right)^{5/2}}+\frac{b\sqrt{\dots}}{\dots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]/x^4,x]
```

```
[Out] (b*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*a*(1 - a^2)*x^2) - (
(2 - 5*a^2)*b^2*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*a^2*(1
- a^2)^2*x) - (b^3*ArcSech[a + b*x])/(3*a^3) - ArcSech[a + b*x]/(3*x^3) + (
(2 - 5*a^2 + 6*a^4)*b^3*ArcTanh[(Sqrt[1 + a]*Tanh[ArcSech[a + b*x]/2])/Sqrt
[1 - a]])/(3*a^3*(1 - a^2)^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\text{sech}^{-1}(a+bx)}{x^4}\,dx = -\left(b^3\,\text{Subst}\left(\int \frac{x\text{sech}(x)\tanh(x)}{(-a+\text{sech}(x))^4}\,dx, x, \text{sech}^{-1}(a+bx)\right)\right)$$
$$= -\frac{\text{sech}^{-1}(a+bx)}{3x^3} + \frac{1}{3}b^3\,\text{Subst}\left(\int \frac{1}{(-a+\text{sech}(x))^3}\,dx, x, \text{sech}^{-1}(a+bx)\right)$$
$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{\text{sech}^{-1}(a+bx)}{3x^3} - \frac{b^3\,\text{Subst}\left(\int \frac{2(1-a^2)-2a\text{sech}(x)-\text{sech}^2(x)}{(-a+\text{sech}(x))^2}\,dx, x, \text{sec}\right.}{6a(1-a^2)}$$
$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a^2(1-a^2)^2x} - \frac{\text{sech}^{-1}(a+bx)}{3x^3} + \frac{b^3\,\text{Subst}}{}$$
$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a^2(1-a^2)^2x} - \frac{b^3\text{sech}^{-1}(a+bx)}{3a^3} - \frac{\text{sech}^{-1}}{3}$$
$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a^2(1-a^2)^2x} - \frac{b^3\text{sech}^{-1}(a+bx)}{3a^3} - \frac{\text{sech}^{-1}}{3}$$
$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a^2(1-a^2)^2x} - \frac{b^3\text{sech}^{-1}(a+bx)}{3a^3} - \frac{\text{sech}^{-1}}{3}$$
$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6a^2(1-a^2)^2x} - \frac{b^3\text{sech}^{-1}(a+bx)}{3a^3} - \frac{\text{sech}^{-1}}{3}$$

Mathematica [**A**] time = 0.39, size = 368, normalized size = 1.87

$$\frac{1}{6}\left(\frac{2b^3\log(a+bx)}{a^3} - \frac{2b^3\log\left(a\sqrt{-\frac{a+bx-1}{a+bx+1}} + bx\sqrt{-\frac{a+bx-1}{a+bx+1}} + \sqrt{-\frac{a+bx-1}{a+bx+1}} + 1\right)}{a^3} - \frac{(6a^4-5a^2+2)b^3\log(x)}{a^3(1-a^2)^{5/2}} + \frac{(6a^4}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a + b*x]/x^4,x]
```

```
[Out] ((b*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(a - a^4 - a*b*x - 2*b*x*(1 + b*x
) + a^3*(-1 + 4*b*x) + a^2*(1 + 5*b*x + 5*b^2*x^2)))/((-1 + a)^2*a^2*(1 + a
)^2*x^2) - (2*ArcSech[a + b*x])/x^3 - ((2 - 5*a^2 + 6*a^4)*b^3*Log[x])/(a^3
*(1 - a^2)^(5/2)) + (2*b^3*Log[a + b*x])/a^3 - (2*b^3*Log[1 + Sqrt[-((-1 +
a + b*x)/(1 + a + b*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + b*x*Sq
rt[-((-1 + a + b*x)/(1 + a + b*x))]])/a^3 + ((2 - 5*a^2 + 6*a^4)*b^3*Log[1
- a^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqr
t[1 - a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + Sqrt[1 - a^2]*b*x*Sqrt[-
((-1 + a + b*x)/(1 + a + b*x))]])/(a^3*(1 - a^2)^(5/2)))/6
```

fricas [**B**] time = 0.69, size = 987, normalized size = 5.01

$$\left[-\frac{(6a^4-5a^2+2)\sqrt{-a^2+1}\,b^3x^3\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-4a^2-2(ab^2x^2+a^3+(2a^2-1)bx-a)\sqrt{-a^2+1}\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+2}{x^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*((6*a^4 - 5*a^2 + 2)*sqrt(-a^2 + 1)*b^3*x^3*log(((2*a^2 - 1)*b^2*x^2
 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 - 2*(a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x -
 a)*sqrt(-a^2 + 1)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x +
 a^2)) + 2)/x^2) + 2*(a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*log(((b*x + a)*sqrt(
-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - 2*(a^6
- 3*a^4 + 3*a^2 - 1)*b^3*x^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2
- 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) + 4*(a^9 - 3*a^7 + 3*a^5 - a^3)*log
(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))
+ 1)/(b*x + a)) - 2*((5*a^5 - 7*a^3 + 2*a)*b^3*x^3 + (4*a^6 - 5*a^4 + a^2)*
b^2*x^2 - (a^7 - 2*a^5 + a^3)*b*x)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2
*x^2 + 2*a*b*x + a^2)))/((a^9 - 3*a^7 + 3*a^5 - a^3)*x^3), -1/6*((6*a^4 - 5
*a^2 + 2)*sqrt(a^2 - 1)*b^3*x^3*arctan((a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x -
 a)*sqrt(a^2 - 1)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x +
a^2))/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + (a^6 - 3*a
^4 + 3*a^2 - 1)*b^3*x^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/
(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - (a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*log(
((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) -
 1)/x) + 2*(a^9 - 3*a^7 + 3*a^5 - a^3)*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*
b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)) - ((5*a^5 - 7*a^3
 + 2*a)*b^3*x^3 + (4*a^6 - 5*a^4 + a^2)*b^2*x^2 - (a^7 - 2*a^5 + a^3)*b*x)*
sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/((a^9 - 3*a
^7 + 3*a^5 - a^3)*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(bx+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)/x^4, x)
```

maple [B] time = 0.10, size = 1946, normalized size = 9.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)/x^4,x)
```

```
[Out] -1/3*arcsech(b*x+a)/x^3-1/3*b^4*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+
a))^(1/2)*x/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)*a^3*arctanh(1/(1-(b*x
+a)^2)^(1/2))-b^4*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)*x/(a
^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)*a*ln(2*((-a^2+1)^(1/2)*(1-(b*x+a)^2
)^(1/2)-a*(b*x+a)+1)/b/x)*(-a^2+1)^(1/2)-1/3*b^3*(-(b*x+a-1)/(b*x+a))^(1/2)
*((b*x+a+1)/(b*x+a))^(1/2)/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)*a^4*ar
ctanh(1/(1-(b*x+a)^2)^(1/2))-b^3*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x
+a))^(1/2)/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)*a^2*ln(2*((-a^2+1)^(1/
2)*(1-(b*x+a)^2)^(1/2)-a*(b*x+a)+1)/b/x)*(-a^2+1)^(1/2)+b^4*(-(b*x+a-1)/(b*
x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)*x/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)
^(1/2)*a*arctanh(1/(1-(b*x+a)^2)^(1/2))+5/6*b^4*(-(b*x+a-1)/(b*x+a))^(1/2)*
((b*x+a+1)/(b*x+a))^(1/2)*x/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)/a*ln(
2*((-a^2+1)^(1/2)*(1-(b*x+a)^2)^(1/2)-a*(b*x+a)+1)/b/x)*(-a^2+1)^(1/2)+b^3*
(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(a^2-1)^2/(1+a)/(a-1)/
(1-(b*x+a)^2)^(1/2)*a^2*arctanh(1/(1-(b*x+a)^2)^(1/2))+5/6*b^3*(-(b*x+a-1)/
(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(a^2-1)^2/(1+a)/(a-1)*a^2+5/6*b^3*
```

```
(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(a^2-1)^2/(1+a)/(a-1)/
(1-(b*x+a)^2)^(1/2)*ln(2*((-a^2+1)^(1/2)*(1-(b*x+a)^2)^(1/2)-a*(b*x+a)+1)/b
/x)*(-a^2+1)^(1/2)-b^4*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)
*x/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)/a*arctanh(1/(1-(b*x+a)^2)^(1/2
))+2/3*b^2*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/x/(a^2-1)^2
/(1+a)/(a-1)*a^3-1/3*b^4*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/
2)*x/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)/a^3*ln(2*((-a^2+1)^(1/2)*(1-
(b*x+a)^2)^(1/2)-a*(b*x+a)+1)/b/x)*(-a^2+1)^(1/2)-b^3*(-(b*x+a-1)/(b*x+a))^
(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)*a
rctanh(1/(1-(b*x+a)^2)^(1/2))-7/6*b^3*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)
/(b*x+a))^(1/2)/(a^2-1)^2/(1+a)/(a-1)-1/6*b*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*
x+a+1)/(b*x+a))^(1/2)/x^2/(a^2-1)^2/(1+a)/(a-1)*a^4-1/3*b^3*(-(b*x+a-1)/(b*
x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(a^2-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^(
1/2)/a^2*ln(2*((-a^2+1)^(1/2)*(1-(b*x+a)^2)^(1/2)-a*(b*x+a)+1)/b/x)*(-a^2+1
)^(1/2)+1/3*b^4*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)*x/(a^2
-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)/a^3*arctanh(1/(1-(b*x+a)^2)^(1/2))-5/
6*b^2*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/x/(a^2-1)^2/(1+a
)/(a-1)*a+1/3*b^3*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(a^2
-1)^2/(1+a)/(a-1)/(1-(b*x+a)^2)^(1/2)/a^2*arctanh(1/(1-(b*x+a)^2)^(1/2))+1/
3*b^3*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/(a^2-1)^2/(1+a)/
(a-1)/a^2+1/3*b*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)/x^2/(a
^2-1)^2/(1+a)/(a-1)*a^2+1/6*b^2*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+
a))^(1/2)/x/(a^2-1)^2/(1+a)/(a-1)/a-1/6*b*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+
a+1)/(b*x+a))^(1/2)/x^2/(a^2-1)^2/(1+a)/(a-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(6\,a^4b^3-3\,a^2b^3+b^3\right)\log(x)}{3\left(a^9-3\,a^7+3\,a^5-a^3\right)}-\frac{\left(a^6b^3-3\,a^5b^3+3\,a^4b^3-a^3b^3\right)x^3\log\left(bx+a+1\right)+\left(a^6b^3+3\,a^5b^3+3\,a^4b^3+a^3b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(6*a^4*b^3 - 3*a^2*b^3 + b^3)*log(x)/(a^9 - 3*a^7 + 3*a^5 - a^3) - 1/6*
((a^6*b^3 - 3*a^5*b^3 + 3*a^4*b^3 - a^3*b^3)*x^3*log(b*x + a + 1) + (a^6*b^
3 + 3*a^5*b^3 + 3*a^4*b^3 + a^3*b^3)*x^3*log(-b*x - a + 1) - 2*(3*a^5*b^2 -
 4*a^3*b^2 + a*b^2)*x^2 + (a^6*b - 2*a^4*b + a^2*b)*x + 2*(a^9 - 3*a^7 + 3*
a^5 - a^3)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)
*sqrt(-b*x - a + 1)*a + b*x + a) - 2*(a^9 - 3*a^7 + 3*a^5 + (a^6*b^3 - 3*a^
4*b^3 + 3*a^2*b^3 - b^3)*x^3 - a^3)*log(b*x + a) - 2*(a^9 - 3*a^7 + 3*a^5 -
 a^3)*log(b*x + a))/((a^9 - 3*a^7 + 3*a^5 - a^3)*x^3) - integrate(1/3*(b^2*
x + a*b)/(b^2*x^5 + 2*a*b*x^4 + (a^2 - 1)*x^3 + (b^2*x^5 + 2*a*b*x^4 + (a^2
 - 1)*x^3)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))/x^4,x)
```

```
[Out] int(acosh(1/(a + b*x))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}\left(a+bx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)/x**4,x)
```

```
[Out] Integral(asech(a + b*x)/x**4, x)
```

3.9 $\int x^2\operatorname{sech}^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=279

$$\frac{a^3\operatorname{sech}^{-1}(a+bx)^2}{3b^3}+\frac{2ia^2\text{Li}_2\left(-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^3}-\frac{2ia^2\text{Li}_2\left(ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^3}-\frac{4a^2\operatorname{sech}^{-1}(a+bx)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^3}+\frac{i\text{L}}{}$$

```
[Out] -1/3*x/b^2+1/3*a^3*arcsech(b*x+a)^2/b^3+1/3*x^3*arcsech(b*x+a)^2-2/3*arcsec
h(b*x+a)*arctan(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/b^3-4*a^
2*arcsech(b*x+a)*arctan(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/
b^3+2*a*ln(b*x+a)/b^3+1/3*I*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/
(b*x+a)+1)^(1/2)))/b^3+2*I*a^2*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*
(1/(b*x+a)+1)^(1/2)))/b^3-1/3*I*polylog(2,I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*
(1/(b*x+a)+1)^(1/2)))/b^3-2*I*a^2*polylog(2,I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2
)*(1/(b*x+a)+1)^(1/2)))/b^3+2*a*(b*x+a+1)*arcsech(b*x+a)*((-b*x-a+1)/(b*x+a
+1))^(1/2)/b^3-1/3*(b*x+a)*(b*x+a+1)*arcsech(b*x+a)*((-b*x-a+1)/(b*x+a+1))^
(1/2)/b^3
```

Rubi [A] time = 0.24, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6321, 5468, 4190, 4180, 2279, 2391, 4184, 3475, 4185}

$$\frac{2ia^2\text{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^3}-\frac{2ia^2\text{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b^3}+\frac{i\text{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{3b^3}-\frac{i\text{PolyLog}\left(2,ie^{\text{se}}\right.}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSech[a + b*x]^2,x]
```

```
[Out] -x/(3*b^2) + (2*a*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a
 + b*x])/b^3 - ((a + b*x)*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*A
rcSech[a + b*x])/(3*b^3) + (a^3*ArcSech[a + b*x]^2)/(3*b^3) + (x^3*ArcSech[
a + b*x]^2)/3 - (2*ArcSech[a + b*x]*ArcTan[E^ArcSech[a + b*x]])/(3*b^3) - (
4*a^2*ArcSech[a + b*x]*ArcTan[E^ArcSech[a + b*x]])/b^3 + (2*a*Log[a + b*x])
/b^3 + ((I/3)*PolyLog[2, (-I)*E^ArcSech[a + b*x]])/b^3 + ((2*I)*a^2*PolyLog
[2, (-I)*E^ArcSech[a + b*x]])/b^3 - ((I/3)*PolyLog[2, I*E^ArcSech[a + b*x]]
)/b^3 - ((2*I)*a^2*PolyLog[2, I*E^ArcSech[a + b*x]])/b^3
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^2\text{sech}^{-1}(a+bx)^2\,dx = -\frac{\text{Subst}\left(\int x^2\text{sech}(x)(-a+\text{sech}(x))^2\tanh(x)\,dx, x, \text{sech}^{-1}(a+bx)\right)}{b^3}$$
$$= \frac{1}{3}x^3\text{sech}^{-1}(a+bx)^2 - \frac{2\,\text{Subst}\left(\int x(-a+\text{sech}(x))^3\,dx, x, \text{sech}^{-1}(a+bx)\right)}{3b^3}$$
$$= \frac{1}{3}x^3\text{sech}^{-1}(a+bx)^2 - \frac{2\,\text{Subst}\left(\int \left(-a^3x+3a^2x\text{sech}(x)-3ax\text{sech}^2(x)+x\text{sech}^3(x)\right) d\right.}{3b^3}$$
$$= \frac{a^3\text{sech}^{-1}(a+bx)^2}{3b^3} + \frac{1}{3}x^3\text{sech}^{-1}(a+bx)^2 - \frac{2\,\text{Subst}\left(\int x\text{sech}^3(x)\,dx, x, \text{sech}^{-1}(a+bx\right.}{3b^3}$$
$$= -\frac{x}{3b^2} + \frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}(a+bx)}{b^3} - \frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}}{3b^3}$$
$$= -\frac{x}{3b^2} + \frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}(a+bx)}{b^3} - \frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}}{3b^3}$$
$$= -\frac{x}{3b^2} + \frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}(a+bx)}{b^3} - \frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}}{3b^3}$$
$$= -\frac{x}{3b^2} + \frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}(a+bx)}{b^3} - \frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)\text{sech}^{-1}}{3b^3}$$

Mathematica [A] time = 2.00, size = 305, normalized size = 1.09

$$-\frac{-\left(6a^2+1\right)\left(2i\text{Li}_2\left(-ie^{\text{sech}^{-1}(a+bx)}\right)-2i\text{Li}_2\left(ie^{\text{sech}^{-1}(a+bx)}\right)-2i\text{sech}^{-1}(a+bx)\log\left(1-ie^{\text{sech}^{-1}(a+bx)}\right)+\pi\log\left(1-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcSech[a + b*x]^2,x]
```

```
[Out] -1/6*(2*(a + b*x)*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)*ArcSe
ch[a + b*x] + 6*a*(a + b*x)^2*ArcSech[a + b*x]^2 - 2*(a + b*x)^3*ArcSech[a
+ b*x]^2 + 2*(a + b*x - 6*a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a +
b*x)*ArcSech[a + b*x] - 3*a^2*(a + b*x)*ArcSech[a + b*x]^2) + 12*a*Log[(a +
 b*x)^(-1)] - (1 + 6*a^2)*(Pi*Log[1 - I*E^ArcSech[a + b*x]] - (2*I)*ArcSech
[a + b*x]*Log[1 - I*E^ArcSech[a + b*x]] - Pi*Log[1 + I*E^ArcSech[a + b*x]]
+ (2*I)*ArcSech[a + b*x]*Log[1 + I*E^ArcSech[a + b*x]] - Pi*Log[Cot[(Pi + (
2*I)*ArcSech[a + b*x])/4]] + (2*I)*PolyLog[2, (-I)*E^ArcSech[a + b*x]] - (2
*I)*PolyLog[2, I*E^ArcSech[a + b*x]]))/b^3
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\,\text{arsech}\,(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*arcsech(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\text{arsech}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsech(b*x + a)^2, x)
```

maple [A] time = 1.68, size = 655, normalized size = 2.35

$$-\frac{a}{3b^3}-\frac{x}{3b^2}-\frac{2a\ln\left(1+\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)^2\right)}{b^3}+\frac{4a\ln\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)}{b^3}+\frac{2ia^2\operatorname{dilog}\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsech(b*x+a)^2,x)
```

```
[Out] -1/3/b^3*a-1/3*x/b^2-2/b^3*a*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)
+1)^(1/2))^2)+4/b^3*a*ln(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))
-2*I/b^3*a^2*dilog(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))
+1/3*a^3*arcsech(b*x+a)^2/b^3+4/3/b^2*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)
/(b*x+a))^(1/2)*arcsech(b*x+a)*x*a-2/b^3*a*arcsech(b*x+a)-1/3*I/b^3*dilog(1
-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-1/3*I/b^3*arcsech(b
*x+a)*ln(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I/b^3*a
^2*arcsech(b*x+a)*ln(1+I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)
))+1/3*x^3*arcsech(b*x+a)^2-2*I/b^3*a^2*arcsech(b*x+a)*ln(1-I*(1/(b*x+a)+(1
/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+1/3*I/b^3*dilog(1+I*(1/(b*x+a)+(1/(
b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+5/3/b^3*(-(b*x+a-1)/(b*x+a))^(1/2)*((
b*x+a+1)/(b*x+a))^(1/2)*arcsech(b*x+a)*a^2+2*I/b^3*a^2*dilog(1+I*(1/(b*x+a)
+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+1/3*I/b^3*arcsech(b*x+a)*ln(1+I*
(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-1/3/b*(-(b*x+a-1)/(b*x
+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)*arcsech(b*x+a)*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,x^3\log\left(\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,bx+\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,a+bx+a\right)^2-\int-\frac{2\left(6\left(b^3x^5+3\,ab^2x^4+\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sq
rt(-b*x - a + 1)*a + b*x + a)^2 - integrate(-2/3*(6*(b^3*x^5 + 3*a*b^2*x^4
+ (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*l
og(b*x + a)^2 + 6*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^
2)*log(b*x + a)^2 - (b^3*x^5 + 2*a*b^2*x^4 + (a^2*b - b)*x^3 + 6*(b^3*x^5 +
 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*log(b*x + a) + (3*(b^3*x^
5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a + 1)*log(
b*x + a) + (2*b^3*x^5 + 4*a*b^2*x^4 + (2*a^2*b - b)*x^3 + 3*(b^3*x^5 + 3*a*
b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*log(b*x + a))*sqrt(b*x + a + 1
))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(
b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2*x^2 + a^3
+ (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqr
t(-b*x - a + 1) + (3*a^2*b - b)*x - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(1/(a + b*x))^2,x)
```

```
[Out] int(x^2*acosh(1/(a + b*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asech}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asech(b*x+a)**2,x)
[Out] Integral(x**2*asech(a + b*x)**2, x)
```

3.10 $\int x\,\mathrm{sech}^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=149

$$-\frac{a^2\mathrm{sech}^{-1}(a+bx)^2}{2b^2}-\frac{2ia\mathrm{Li}_2\left(-ie^{\mathrm{sech}^{-1}(a+bx)}\right)}{b^2}+\frac{2ia\mathrm{Li}_2\left(ie^{\mathrm{sech}^{-1}(a+bx)}\right)}{b^2}-\frac{\log(a+bx)}{b^2}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)\mathrm{sech}}{b^2}$$

```
[Out] -1/2*a^2*arcsech(b*x+a)^2/b^2+1/2*x^2*arcsech(b*x+a)^2+4*a*arcsech(b*x+a)*a
rctan(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/b^2-ln(b*x+a)/b^2-
2*I*a*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2
+2*I*a*polylog(2,I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2
-(b*x+a+1)*arcsech(b*x+a)*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^2
```

Rubi [A] time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6321, 5468, 4190, 4180, 2279, 2391, 4184, 3475}

$$-\frac{2ia\mathrm{PolyLog}\left(2,-ie^{\mathrm{sech}^{-1}(a+bx)}\right)}{b^2}+\frac{2ia\mathrm{PolyLog}\left(2,ie^{\mathrm{sech}^{-1}(a+bx)}\right)}{b^2}-\frac{a^2\mathrm{sech}^{-1}(a+bx)^2}{2b^2}-\frac{\log(a+bx)}{b^2}-\sqrt{\frac{-a-bx+1}{a+bx+1}}\,($$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSech[a + b*x]^2,x]
```

```
[Out] -((Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x])/b^2) -
 (a^2*ArcSech[a + b*x]^2)/(2*b^2) + (x^2*ArcSech[a + b*x]^2)/2 + (4*a*ArcSe
ch[a + b*x]*ArcTan[E^ArcSech[a + b*x]])/b^2 - Log[a + b*x]/b^2 - ((2*I)*a*P
olyLog[2, (-I)*E^ArcSech[a + b*x]])/b^2 + ((2*I)*a*PolyLog[2, I*E^ArcSech[a
 + b*x]])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
```

```
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x\operatorname{sech}^{-1}(a+bx)^2\,dx &= -\frac{\text{Subst}\left(\int x^2\operatorname{sech}(x)(-a+\operatorname{sech}(x))\tanh(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b^2}\\
&= \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int x(-a+\operatorname{sech}(x))^2\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b^2}\\
&= \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int \left(a^2x-2ax\operatorname{sech}(x)+x\operatorname{sech}^2(x)\right)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right.}{b^2}\\
&= -\frac{a^2\operatorname{sech}^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int x\operatorname{sech}^2(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right.}{b^2}\\
&= -\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{b^2} - \frac{a^2\operatorname{sech}^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx)^2 + \frac{4a}{\ }\\
&= -\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{b^2} - \frac{a^2\operatorname{sech}^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx)^2 + \frac{4a}{\ }\\
&= -\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{b^2} - \frac{a^2\operatorname{sech}^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx)^2 + \frac{4a}{\ }
\end{aligned}$$

Mathematica [A] time = 0.43, size = 172, normalized size = 1.15

$$\frac{-4ia\left(\text{Li}_2\left(-ie^{-\operatorname{sech}^{-1}(a+bx)}\right)-\text{Li}_2\left(ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right)+2\log\left(\frac{1}{a+bx}\right)+(a+bx)^2\operatorname{sech}^{-1}(a+bx)^2-2a(a+bx)\operatorname{sech}^{-1}(}{\ }$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcSech[a + b*x]^2,x]
```

```
[Out] (-2*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)*ArcSech[a + b*x] -
2*a*(a + b*x)*ArcSech[a + b*x]^2 + (a + b*x)^2*ArcSech[a + b*x]^2 - (4*I)*a
*ArcSech[a + b*x]*(Log[1 - I/E^ArcSech[a + b*x]] - Log[1 + I/E^ArcSech[a +
```

```
b*x]]) + 2*Log[(a + b*x)^(-1)] - (4*I)*a*(PolyLog[2, (-I)/E^ArcSech[a + b*x
]] - PolyLog[2, I/E^ArcSech[a + b*x]]))/(2*b^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{arsech}\,(bx+a)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x*arcsech(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\text{arsech}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arcsech(b*x + a)^2, x)
```

maple [A] time = 1.31, size = 396, normalized size = 2.66

$$\frac{x^2\text{arcsech}\,(bx+a)^2}{2}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,\sqrt{\frac{bx+a+1}{bx+a}}\,\text{arcsech}\,(bx+a)\,x}{b}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,\sqrt{\frac{bx+a+1}{bx+a}}\,\text{arcsech}\,(bx+a)\,a}{b^2}-\frac{a^2\text{arcs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsech(b*x+a)^2,x)
```

```
[Out] 1/2*x^2*arcsech(b*x+a)^2-1/b*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))
^(1/2)*arcsech(b*x+a)*x-1/b^2*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a)
)^(1/2)*arcsech(b*x+a)*a-1/2*a^2*arcsech(b*x+a)^2/b^2+1/b^2*arcsech(b*x+a)-
2/b^2*ln(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+1/b^2*ln(1+(1/(
b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)-2*I/b^2*a*arcsech(b*x+a)
*ln(1+I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I/b^2*a*arcs
ech(b*x+a)*ln(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*I/
b^2*a*dilog(1+I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I/b^
2*a*dilog(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x^2\log\left(\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,bx+\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,a+bx+a\right)^2-\int-\frac{4\left(b^3x^4+3\,ab^2x^3+(3\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sq
rt(-b*x - a + 1)*a + b*x + a)^2 - integrate(-(4*(b^3*x^4 + 3*a*b^2*x^3 + (3
*a^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x
 + a)^2 + 4*(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b
*x + a)^2 - (b^3*x^4 + 2*a*b^2*x^3 + (a^2*b - b)*x^2 + 4*(b^3*x^4 + 3*a*b^2
*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x + a) + (2*(b^3*x^4 + 3*a*b^
2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*log(b*x + a) + (
2*b^3*x^4 + 4*a*b^2*x^3 + (2*a^2*b - b)*x^2 + 2*(b^3*x^4 + 3*a*b^2*x^3 + (3
*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x -
```

```
 a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*s
qrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (b^3*x^3 + 3
*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1
) + (3*a^2*b - b)*x - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(1/(a + b*x))^2,x)
```

```
[Out] int(x*acosh(1/(a + b*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asech(b*x+a)**2,x)
```

```
[Out] Integral(x*asech(a + b*x)**2, x)
```

3.11 $\int \operatorname{sech}^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=80

$$\frac{2i\operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}-\frac{2i\operatorname{Li}_2\left(ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}+\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b}-\frac{4\operatorname{sech}^{-1}(a+bx)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}$$

```
[Out] (b*x+a)*arcsech(b*x+a)^2/b-4*arcsech(b*x+a)*arctan(1/(b*x+a)+(1/(b*x+a)-1)^
(1/2)*(1/(b*x+a)+1)^(1/2))/b+2*I*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2
)*(1/(b*x+a)+1)^(1/2)))/b-2*I*polylog(2,I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1
/(b*x+a)+1)^(1/2)))/b
```

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6315, 6279, 5418, 4180, 2279, 2391}

$$\frac{2i\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}-\frac{2i\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}+\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b}-\frac{4\operatorname{sech}^{-1}(a+bx)\tan^{-1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*ArcSech[a + b*x]^2)/b - (4*ArcSech[a + b*x]*ArcTan[E^ArcSech[a +
 b*x]])/b + ((2*I)*PolyLog[2, (-I)*E^ArcSech[a + b*x]])/b - ((2*I)*PolyLog[
2, I*E^ArcSech[a + b*x]])/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 6279

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[c^(-1), Su
bst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
```

```
b, c, n}, x] && IGtQ[n, 0]
```

Rule 6315

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSech[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^{-1}(a+bx)^2\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{sech}^{-1}(x)^2\,dx, x, a+bx\right)}{b}\\ &= -\frac{\operatorname{Subst}\left(\int x^2\operatorname{sech}(x)\tanh(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b}\\ &= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{2\operatorname{Subst}\left(\int x\operatorname{sech}(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b}\\ &= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{4\operatorname{sech}^{-1}(a+bx)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{(2i)\operatorname{Subst}\left(\int \log(1-\right.}{}\\ &= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{4\operatorname{sech}^{-1}(a+bx)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{(2i)\operatorname{Subst}\left(\int \frac{\log(1-ix)}{x}\right.}{b}\\ &= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{4\operatorname{sech}^{-1}(a+bx)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{2i\operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.23, size = 105, normalized size = 1.31

$$\frac{i\left(2\operatorname{Li}_2\left(-ie^{-\operatorname{sech}^{-1}(a+bx)}\right) - 2\operatorname{Li}_2\left(ie^{-\operatorname{sech}^{-1}(a+bx)}\right) + \operatorname{sech}^{-1}(a+bx)\left(-i(a+bx)\operatorname{sech}^{-1}(a+bx) + 2\log\left(1-ie^{-\operatorname{sech}^{-1}(\right.\right.\right.}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a + b*x]^2,x]
```

```
[Out] (I*(ArcSech[a + b*x]*((-I)*(a + b*x)*ArcSech[a + b*x] + 2*Log[1 - I/E^ArcSe
ch[a + b*x]] - 2*Log[1 + I/E^ArcSech[a + b*x]]) + 2*PolyLog[2, (-I)/E^ArcSe
ch[a + b*x]] - 2*PolyLog[2, I/E^ArcSech[a + b*x]]))/b
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\operatorname{arsech}(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(arcsech(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsech}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)^2, x)
```

maple [A] time = 0.34, size = 209, normalized size = 2.61

$$x\mathrm{arcsech}\,(bx+a)^2+\frac{\mathrm{arcsech}\,(bx+a)^2\,a}{b}+\frac{2i\mathrm{arcsech}\,(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)\right)}{b}-\frac{2i\mathrm{arcs}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^2,x)
```

```
[Out] x*arcsech(b*x+a)^2+1/b*arcsech(b*x+a)^2*a+2*I/b*arcsech(b*x+a)*ln(1+I*(1/(b
*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*I/b*arcsech(b*x+a)*ln(1-I
*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I/b*dilog(1+I*(1/(b
*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*I/b*dilog(1-I*(1/(b*x+a)+
(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\log\left(\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,bx+\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,a+bx+a\right)^2-\int-\frac{2\left(2\left(b^3x^3+3\,ab^2x^2+a^3+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*
x - a + 1)*a + b*x + a)^2 - integrate(-2*(2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
(3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 +
2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2 - (b^3
*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^
2*b - b)*x - a)*log(b*x + a) + ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b
)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) + (2*b^3*x^3 + 4*a*b^2*x^2 + (2*a^2
*b - b)*x + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a
))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x -
 a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 +
 3*a*b^2*x^2 + a^3 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sq
rt(b*x + a + 1)*sqrt(-b*x - a + 1) + (3*a^2*b - b)*x - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acosh}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^2,x)
```

```
[Out] int(acosh(1/(a + b*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{asech}^2\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**2,x)
```

```
[Out] Integral(asech(a + b*x)**2, x)
```

3.12 $\int \frac{\text{sech}^{-1}(a+bx)^2}{x}\,dx$

Optimal. Leaf size=274

$$2\text{sech}^{-1}(a+bx)\text{Li}_2\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+2\text{sech}^{-1}(a+bx)\text{Li}_2\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)-2\text{Li}_3\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-2\text{Li}_3\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)$$

```
[Out] -arcsech(b*x+a)^2*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^
2)+arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/
2))/(1-(-a^2+1)^(1/2)))+arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1
/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))-arcsech(b*x+a)*polylog(2,-(1/(
b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)+2*arcsech(b*x+a)*polylog
(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2))
)+2*arcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)
^(1/2))/(1+(-a^2+1)^(1/2)))+1/2*polylog(3,-(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(
1/(b*x+a)+1)^(1/2))^2)-2*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x
+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))-2*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1
/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))
```

Rubi [A] time = 0.46, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6321, 5595, 5570, 3718, 2190, 2531, 2282, 6589, 5562}

$$2\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+2\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)-2\text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^2/x,x]
```

```
[Out] ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + Ar
cSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcS
ech[a + b*x]^2*Log[1 + E^(2*ArcSech[a + b*x])] + 2*ArcSech[a + b*x]*PolyLog
[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + 2*ArcSech[a + b*x]*PolyLo
g[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcSech[a + b*x]*PolyLog
[2, -E^(2*ArcSech[a + b*x])] - 2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqr
t[1 - a^2])] - 2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + P
olyLog[3, -E^(2*ArcSech[a + b*x])]/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5570

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Tanh[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Sinh[c + d*x]*Tanh[c + d*x]^
(n - 1))/(a + b*Cosh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5595

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sech[(c_.) + (d_.)*(x_)]), x_Symbol] :> I
nt[((e + f*x)^m*Cosh[c + d*x]*F[c + d*x]^n*G[c + d*x]^p)/(b + a*Cosh[c + d*
x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G]
 && IntegersQ[m, n, p]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{sech}^{-1}(a+bx)^2}{x}\,dx = -\text{Subst}\left(\int \frac{x^2\text{sech}(x)\tanh(x)}{-a+\text{sech}(x)}\,dx, x, \text{sech}^{-1}(a+bx)\right)$$
$$= -\text{Subst}\left(\int \frac{x^2\tanh(x)}{1-a\cosh(x)}\,dx, x, \text{sech}^{-1}(a+bx)\right)$$
$$= -\left(a\,\text{Subst}\left(\int \frac{x^2\sinh(x)}{1-a\cosh(x)}\,dx, x, \text{sech}^{-1}(a+bx)\right)\right) - \text{Subst}\left(\int x^2\tanh(x)\,dx, x, \text{sech}^{-1}\right.$$
$$= -\left(2\,\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx, x, \text{sech}^{-1}(a+bx)\right)\right) - a\,\text{Subst}\left(\int \frac{e^x x^2}{1-\sqrt{1-a^2}-ae^x}\,dx, x, \text{se}\right.$$
$$= \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \text{sec}$$
$$= \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \text{sec}$$
$$= \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \text{sec}$$
$$= \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \text{sec}$$

Mathematica [A] time = 0.30, size = 280, normalized size = 1.02

$$2\text{sech}^{-1}(a+bx)\text{Li}_2\left(-\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}-1}\right)+2\text{sech}^{-1}(a+bx)\text{Li}_2\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)-2\text{Li}_3\left(-\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}-1}\right)-2\text{Li}_3\left(\frac{ae^{\text{sech}^{-1}(a+b}}{\sqrt{1-a^2}+}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a + b*x]^2/x,x]
```

```
[Out] (-2*ArcSech[a + b*x]^3)/3 - ArcSech[a + b*x]^2*Log[1 + E^(-2*ArcSech[a + b*
x])] + ArcSech[a + b*x]^2*Log[1 + (a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2
])] + ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])
] + ArcSech[a + b*x]*PolyLog[2, -E^(-2*ArcSech[a + b*x])] + 2*ArcSech[a + b
*x]*PolyLog[2, -((a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2]))] + 2*ArcSech[
a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + PolyLog[3
, -E^(-2*ArcSech[a + b*x])]/2 - 2*PolyLog[3, -((a*E^ArcSech[a + b*x])/(-1 +
 Sqrt[1 - a^2]))] - 2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])
]
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsech}\,(bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arcsech(b*x + a)^2/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsech}\,(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)^2/x, x)
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{arcsech}\left(bx+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^2/x,x)
```

```
[Out] int(arcsech(b*x+a)^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{arsech}\left(bx+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsech(b*x + a)^2/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^2/x,x)
```

```
[Out] int(acosh(1/(a + b*x))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{asech}^{2}\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**2/x,x)
```

```
[Out] Integral(asech(a + b*x)**2/x, x)
```

3.13 $\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^2} \, dx$

Optimal. Leaf size=224

$$\frac{2b\mathrm{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{2b\mathrm{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}+\frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{se}}}{\sqrt{}}\right.}{a\sqrt{1-a^2}}$$

```
[Out] -b*arcsech(b*x+a)^2/a-arcsech(b*x+a)^2/x+2*b*arcsech(b*x+a)*ln(1-a*(1/(b*x+
a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^
(1/2)-2*b*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1
)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+2*b*polylog(2,a*(1/(b*x+a)+(1
/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
-2*b*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a
^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6321, 5468, 4191, 3320, 2264, 2190, 2279, 2391}

$$\frac{2b\mathrm{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{2b\mathrm{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}+\frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{2b\operatorname{sech}^{-1}(a+}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^2/x^2,x]
```

```
[Out] -((b*ArcSech[a + b*x]^2)/a) - ArcSech[a + b*x]^2/x + (2*b*ArcSech[a + b*x]*
Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (2
*b*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a
*Sqrt[1 - a^2]) + (2*b*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2]
)])/(a*Sqrt[1 - a^2]) - (2*b*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1
- a^2])])/(a*Sqrt[1 - a^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^2}\,dx = -\left(b\,\text{Subst}\left(\int \frac{x^2\operatorname{sech}(x)\tanh(x)}{(-a+\operatorname{sech}(x))^2}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + (2b)\,\text{Subst}\left(\int \frac{x}{-a+\operatorname{sech}(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + (2b)\,\text{Subst}\left(\int \left(-\frac{x}{a} + \frac{x}{a(1-a\cosh(x))}\right)dx, x, \operatorname{sech}^{-1}(a+bx)\right)$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{(2b)\,\text{Subst}\left(\int \frac{x}{1-a\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{a}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{(4b)\,\text{Subst}\left(\int \frac{e^x x}{-a+2e^x-ae^{2x}}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{a}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} - \frac{(4b)\,\text{Subst}\left(\int \frac{e^x x}{2-2\sqrt{1-a^2}-2ae^x}\,dx, x, \operatorname{sech}^{-1}(a+b\right.}{\sqrt{1-a^2}}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{2b\text{se}}{}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{2b\text{se}}{}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{2b\text{se}}{}$$

Mathematica [C] time = 2.69, size = 678, normalized size = 3.03

$$-\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{2b\left(i\left(\text{Li}_2\left(\frac{\left(-i\sqrt{a^2-1}-1\right)\left(a-i\sqrt{a^2-1}\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)-1\right)}{a\left(a+i\sqrt{a^2-1}\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)-1\right)}\right)-\text{Li}_2\left(\frac{\left(\sqrt{a^2-1}+i\right)\left(a-i\sqrt{a^2-1}\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)-1\right)}{a\left(\sqrt{a^2-1}\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)-i(a-1)\right)}\right)\right)+2\operatorname{sech}^{-1}(a+bx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a + b*x]^2/x^2,x]
```

```
[Out] (-(((a + b*x)*ArcSech[a + b*x]^2)/x) + (2*b*(2*ArcSech[a + b*x]*ArcTan[((-1
 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] - (2*I)*ArcCos[a^(-1)]*ArcT
an[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a^(-1)] + 2
*(ArcTan[((-1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + ArcTan[((1 +
 a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[Sqrt[-1 + a^2]/(Sqrt[2]
*Sqrt[a]*E^(ArcSech[a + b*x]/2)*Sqrt[-((b*x)/(a + b*x))])] + (ArcCos[a^(-1)
] - 2*(ArcTan[((-1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + ArcTan[
((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[(Sqrt[-1 + a^2]*E^
(ArcSech[a + b*x]/2))/(Sqrt[2]*Sqrt[a]*Sqrt[-((b*x)/(a + b*x))])] - (ArcCos
[a^(-1)] + 2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log
[-(((-1 + a)*(1 + a - I*Sqrt[-1 + a^2])*(-1 + Tanh[ArcSech[a + b*x]/2]))/(a
*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2])))] - (ArcCos[a^(-1)]
- 2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[((-1 + a
)*(1 + a + I*Sqrt[-1 + a^2])*(1 + Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a + I
*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] + I*(PolyLog[2, ((-1 - I*Sqrt[-
1 + a^2])*(-1 + a - I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a
+ I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] - PolyLog[2, ((I + Sqrt[-1 +
```

```
a^2])*(-1 + a - I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*((-I)*(-1 +
a) + Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))]))))/Sqrt[-1 + a^2])/a
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsech}(bx+a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(arcsech(b*x + a)^2/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x^2,x, algorithm="giac")
[Out] integrate(arcsech(b*x + a)^2/x^2, x)
```

maple [A] time = 0.75, size = 367, normalized size = 1.64

$$-\frac{b\operatorname{arcsech}(bx+a)^2}{a}-\frac{\operatorname{arcsech}(bx+a)^2}{x}+\frac{2b\sqrt{-a^2+1}\,\operatorname{arcsech}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)+\sqrt{-a^2+1}+1}{1+\sqrt{-a^2+1}}\right)}{a\left(a^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^2/x^2,x)
[Out] -b*arcsech(b*x+a)^2/a-arcsech(b*x+a)^2/x+2*b*(-a^2+1)^(1/2)/a/(a^2-1)*arcse
ch(b*x+a)*ln((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+
1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))-2*b*(-a^2+1)^(1/2)/a/(a^2-1)*arcsech(b*x+a)
*ln((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1
)/(-1+(-a^2+1)^(1/2)))+2*b*(-a^2+1)^(1/2)/a/(a^2-1)*dilog((-a*(1/(b*x+a)+(1
/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2))
)-2*b*(-a^2+1)^(1/2)/a/(a^2-1)*dilog((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(
b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,bx+\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,a+bx+a\right)^2}{x}-\int -\frac{2\left(2\left(b^3x^3+3\,ab^2x^2+a^3+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x^2,x, algorithm="maxima")
[Out] -log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x
 - a + 1)*a + b*x + a)^2/x - integrate(-2*(2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
 (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 +
 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2 + (b^
3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a
^2*b - b)*x - a)*log(b*x + a) - ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b -
b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 + 4*a*b^2*x^2 + (2*a^
```

```
2*b - b)*x - (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x +
a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x
- a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^5
+ 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2 + (b^3*x^5 + 3*a*b^2*x^4
+ (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^2}{x^2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^2/x^2,x)
```

```
[Out] int(acosh(1/(a + b*x))^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^{2}{\left(a + bx \right)}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**2/x**2,x)
```

```
[Out] Integral(asech(a + b*x)**2/x**2, x)
```

3.14 $\int \frac{\text{sech}^{-1}(a+bx)^2}{x^3} dx$

Optimal. Leaf size=537

$$-\frac{2b^2\text{Li}_2\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}}+\frac{b^2\text{Li}_2\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\left(1-a^2\right)^{3/2}}+\frac{2b^2\text{Li}_2\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a^2\sqrt{1-a^2}}-\frac{b^2\text{Li}_2\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a^2\left(1-a^2\right)^{3/2}}+\frac{b^2\log\left(\frac{x}{a+bx}\right)}{a^2\left(1-a^2\right)}+\frac{b^2\text{sech}}{}$$

```
[Out] 1/2*b^2*arcsech(b*x+a)^2/a^2-1/2*arcsech(b*x+a)^2/x^2+b^2*ln(x/(b*x+a))/a^2
/(-a^2+1)+b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+
a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)-b^2*arcsech(b*x+a)*ln(1
-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/
a^2/(-a^2+1)^(3/2)+b^2*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a
)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)-b^2*polylog(2,a*(1/(b*x+
a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1
)^(3/2)-2*b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+
a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)+2*b^2*arcsech(b*x+a)*ln
(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2))
)/a^2/(-a^2+1)^(1/2)-2*b^2*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b
*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)+2*b^2*polylog(2,a*(1
/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(
-a^2+1)^(1/2)+b^2*(b*x+a+1)*arcsech(b*x+a)*((-b*x-a+1)/(b*x+a+1))^(1/2)/a/(
-a^2+1)/(b*x+a)/(1-a/(b*x+a))
```

Rubi [A] time = 0.75, antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6321, 5468, 4191, 3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$-\frac{2b^2\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}}+\frac{b^2\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\left(1-a^2\right)^{3/2}}+\frac{2b^2\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a^2\sqrt{1-a^2}}-\frac{b^2\text{PolyLog}\left(2,\frac{ae^{\text{se}}}{\sqrt{}}\right)}{a^2\left(1-a^2\right)^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^2/x^3,x]
```

```
[Out] (b^2*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x])/(a*(
1 - a^2)*(a + b*x)*(1 - a/(a + b*x))) + (b^2*ArcSech[a + b*x]^2)/(2*a^2) -
ArcSech[a + b*x]^2/(2*x^2) + (b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a +
 b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) - (2*b^2*ArcSech[a + b*x
]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2])
- (b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])]
)/(a^2*(1 - a^2)^(3/2)) + (2*b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a +
b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) + (b^2*Log[x/(a + b*x)])/(a
^2*(1 - a^2)) + (b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])]
)/(a^2*(1 - a^2)^(3/2)) - (2*b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqr
t[1 - a^2])])/(a^2*Sqrt[1 - a^2]) - (b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/
(1 + Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) + (2*b^2*PolyLog[2, (a*E^ArcSec
h[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
```

```
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^3}\,dx = -\left(b^2\,\mathrm{Subst}\left(\int \frac{x^2\operatorname{sech}(x)\tanh(x)}{(-a+\operatorname{sech}(x))^3}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + b^2\,\mathrm{Subst}\left(\int \frac{x}{(-a+\operatorname{sech}(x))^2}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + b^2\,\mathrm{Subst}\left(\int \left(\frac{x}{a^2} + \frac{x}{a^2(-1+a\cosh(x))^2} + \frac{2x}{a^2(-1+a\cosh(x))}\right)\right.$$

$$= \frac{b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{x}{(-1+a\cosh(x))^2}\,dx, x, \operatorname{sech}^{-1}(a+b\right.}{a^2}$$

$$= \frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + \frac{(4}{}$$

$$= \frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + \frac{b^2}{}$$

$$= \frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} - \frac{2b}{}$$

$$= \frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + \frac{b^2}{}$$

$$= \frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + \frac{b^2}{}$$

$$= \frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2} + \frac{b^2}{}$$

Mathematica [C] time = 7.58, size = 1439, normalized size = 2.68

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a + b*x]^2/x^3,x]
```

```
[Out] -1/2*((a + b*x)^2*ArcSech[a + b*x]^2)/(a^2*x^2) + (b*ArcSech[a + b*x]*(-(a*
Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)) + (-1 + a^2)*(a + b*x)
*ArcSech[a + b*x]))/((-1 + a)*a^2*(1 + a)*x) + (b^2*Log[(b*x)/(a + b*x)])/(
a^2 - a^4) - (2*b^2*(2*ArcSech[a + b*x]*ArcTan[((-1 + a)*Coth[ArcSech[a + b
*x]/2])/Sqrt[-1 + a^2]] - (2*I)*ArcCos[a^(-1)]*ArcTan[((1 + a)*Tanh[ArcSech
[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a^(-1)] + 2*(ArcTan[((-1 + a)*Coth[
ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + ArcTan[((1 + a)*Tanh[ArcSech[a + b*x
]/2])/Sqrt[-1 + a^2]]))*Log[Sqrt[-1 + a^2]/(Sqrt[2]*Sqrt[a]*E^(ArcSech[a +
b*x]/2)*Sqrt[-((b*x)/(a + b*x))])] + (ArcCos[a^(-1)] - 2*(ArcTan[((-1 + a)*
Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + ArcTan[((1 + a)*Tanh[ArcSech[a
+ b*x]/2])/Sqrt[-1 + a^2]]))*Log[(Sqrt[-1 + a^2]*E^(ArcSech[a + b*x]/2))/(S
qrt[2]*Sqrt[a]*Sqrt[-((b*x)/(a + b*x))])] - (ArcCos[a^(-1)] + 2*ArcTan[((1
+ a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[-(((-1 + a)*(1 + a - I*
Sqrt[-1 + a^2])*(-1 + Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a + I*Sqrt[-1 + a
^2]*Tanh[ArcSech[a + b*x]/2])))] - (ArcCos[a^(-1)] - 2*ArcTan[((1 + a)*Tanh
[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[((-1 + a)*(1 + a + I*Sqrt[-1 + a
^2])*(1 + Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[Arc
Sech[a + b*x]/2]))] + I*(PolyLog[2, ((-1 - I*Sqrt[-1 + a^2])*(-1 + a - I*Sq
rt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[
ArcSech[a + b*x]/2]))] - PolyLog[2, ((I + Sqrt[-1 + a^2])*(-1 + a - I*Sqrt[
-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*((-I)*(-1 + a) + Sqrt[-1 + a^2]*Tan
h[ArcSech[a + b*x]/2]))])))/(-1 + a^2)^(3/2) + (b^2*(2*ArcSech[a + b*x]*Arc
Tan[((-1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] - (2*I)*ArcCos[a^(-
1)]*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a^(
-1)] + 2*(ArcTan[((-1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + ArcT
an[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[Sqrt[-1 + a^2]/
(Sqrt[2]*Sqrt[a]*E^(ArcSech[a + b*x]/2)*Sqrt[-((b*x)/(a + b*x))])] + (ArcCo
s[a^(-1)] - 2*(ArcTan[((-1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] +
 ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[(Sqrt[-1 +
 a^2]*E^(ArcSech[a + b*x]/2))/(Sqrt[2]*Sqrt[a]*Sqrt[-((b*x)/(a + b*x))])] -
 (ArcCos[a^(-1)] + 2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^
2]])*Log[-(((-1 + a)*(1 + a - I*Sqrt[-1 + a^2])*(-1 + Tanh[ArcSech[a + b*x]
/2]))/(a*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2])))] - (ArcCos[
a^(-1)] - 2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[
((-1 + a)*(1 + a + I*Sqrt[-1 + a^2])*(1 + Tanh[ArcSech[a + b*x]/2]))/(a*(-1
 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] + I*(PolyLog[2, ((-1 -
I*Sqrt[-1 + a^2])*(-1 + a - I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*
(-1 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] - PolyLog[2, ((I + S
qrt[-1 + a^2])*(-1 + a - I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*((-
I)*(-1 + a) + Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))])))/(a^2*(-1 + a^2)
^(3/2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsech}(bx+a)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsech(b*x + a)^2/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(bx+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)^2/x^3, x)
```

maple [A] time = 1.29, size = 1026, normalized size = 1.91

$$\frac{b^2 \operatorname{arcsech}(bx+a)^2}{2a^2-2} - \frac{b^2 \operatorname{arcsech}(bx+a)\sqrt{-\frac{bx+a-1}{bx+a}}\sqrt{\frac{bx+a+1}{bx+a}}}{a\left(a^2-1\right)} - \frac{b \operatorname{arcsech}(bx+a)\sqrt{-\frac{bx+a-1}{bx+a}}\sqrt{\frac{bx+a+1}{bx+a}}}{\left(a^2-1\right)x} - \frac{b^2 \operatorname{arcs}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^2/x^3,x)
```

```
[Out] 1/2*b^2*arcsech(b*x+a)^2/(a^2-1)-b^2*arcsech(b*x+a)/a/(a^2-1)*(-(b*x+a-1)/(
b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)-b*arcsech(b*x+a)/(a^2-1)/x*(-(b*x+a
-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)-1/2*b^2*arcsech(b*x+a)^2/a^2/(
a^2-1)-1/2*arcsech(b*x+a)^2*a^2/(a^2-1)/x^2-b^2*arcsech(b*x+a)/a^2/(a^2-1)+
1/2*arcsech(b*x+a)^2/(a^2-1)/x^2+2*b^2/a^2/(a^2-1)*ln(1/(b*x+a)+(1/(b*x+a)-
1)^(1/2)*(1/(b*x+a)+1)^(1/2))-b^2/a^2/(a^2-1)*ln(a*(1/(b*x+a)+(1/(b*x+a)-1)
^(1/2)*(1/(b*x+a)+1)^(1/2))^2+a-2/(b*x+a)-2*(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+
1)^(1/2))+b^2*(-a^2+1)^(1/2)/a^2/(a^2-1)^2*arcsech(b*x+a)*ln((-a*(1/(b*x+a)
+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/
2)))-b^2*(-a^2+1)^(1/2)/a^2/(a^2-1)^2*arcsech(b*x+a)*ln((a*(1/(b*x+a)+(1/(b
*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))+
b^2*(-a^2+1)^(1/2)/a^2/(a^2-1)^2*dilog((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(
1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))-b^2*(-a^2+1)^(1/2
)/a^2/(a^2-1)^2*dilog((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)
)+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))-2*b^2*(-a^2+1)^(1/2)/(a^2-1)^2*arc
sech(b*x+a)*ln((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^
2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))+2*b^2*(-a^2+1)^(1/2)/(a^2-1)^2*arcsech(b*
x+a)*ln((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/
2)-1)/(-1+(-a^2+1)^(1/2)))-2*b^2*(-a^2+1)^(1/2)/(a^2-1)^2*dilog((-a*(1/(b*x
+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^
(1/2)))+2*b^2*(-a^2+1)^(1/2)/(a^2-1)^2*dilog((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1
/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(\sqrt{bx+a+1}\sqrt{-bx-a+1}\,bx+\sqrt{bx+a+1}\sqrt{-bx-a+1}\,a+bx+a\right)^2}{2\,x^2} - \int -\frac{4\left(b^3x^3+3\,ab^2x^2+a^3+(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(
-b*x - a + 1)*a + b*x + a)^2/x^2 - integrate(-(4*(b^3*x^3 + 3*a*b^2*x^2 + a
^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)
^2 + 4*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2 +
 (b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 4*(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
(3*a^2*b - b)*x - a)*log(b*x + a) - (2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^
2*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 + 4*a*b^2*x^2 +
 (2*a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*lo
g(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sq
rt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(
b^3*x^6 + 3*a*b^2*x^5 + (3*a^2*b - b)*x^4 + (a^3 - a)*x^3 + (b^3*x^6 + 3*a*
b^2*x^5 + (3*a^2*b - b)*x^4 + (a^3 - a)*x^3)*sqrt(b*x + a + 1)*sqrt(-b*x -
a + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^2/x^3,x)
```

```
[Out] int(acosh(1/(a + b*x))^2/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^2(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**2/x**3,x)
```

```
[Out] Integral(asech(a + b*x)**2/x**3, x)
```

3.15 $\int x\,\text{sech}^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=260

$$-\frac{a^2\text{sech}^{-1}(a+bx)^3}{2b^2}-\frac{6ia\,\text{sech}^{-1}(a+bx)\text{Li}_2\left(-ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}+\frac{6ia\,\text{sech}^{-1}(a+bx)\text{Li}_2\left(ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}+\frac{3\text{Li}_2\left(-e^{2s}\right.}{2}$$

```
[Out] -3/2*arcsech(b*x+a)^2/b^2-1/2*a^2*arcsech(b*x+a)^3/b^2+1/2*x^2*arcsech(b*x+
a)^3+6*a*arcsech(b*x+a)^2*arctan(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1
)^(1/2))/b^2+3*arcsech(b*x+a)*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a
)+1)^(1/2))^2)/b^2-6*I*a*arcsech(b*x+a)*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-
1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2+6*I*a*arcsech(b*x+a)*polylog(2,I*(1/(b*x
+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2+3/2*polylog(2,-(1/(b*x+a)
+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)/b^2+6*I*a*polylog(3,-I*(1/(b*x
+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2-6*I*a*polylog(3,I*(1/(b*x
+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2-3/2*(b*x+a+1)*arcsech(b*x
+a)^2*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^2
```

Rubi [A] time = 0.26, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {6321, 5468, 4190, 4180, 2531, 2282, 6589, 4184, 3718, 2190, 2279, 2391}

$$-\frac{6ia\,\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}+\frac{6ia\,\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}+\frac{3\text{PolyLog}\left(2,\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSech[a + b*x]^3,x]
```

```
[Out] (-3*ArcSech[a + b*x]^2)/(2*b^2) - (3*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 +
 a + b*x)*ArcSech[a + b*x]^2)/(2*b^2) - (a^2*ArcSech[a + b*x]^3)/(2*b^2) +
(x^2*ArcSech[a + b*x]^3)/2 + (6*a*ArcSech[a + b*x]^2*ArcTan[E^ArcSech[a + b
*x]])/b^2 + (3*ArcSech[a + b*x]*Log[1 + E^(2*ArcSech[a + b*x])])/b^2 - ((6*
I)*a*ArcSech[a + b*x]*PolyLog[2, (-I)*E^ArcSech[a + b*x]])/b^2 + ((6*I)*a*A
rcSech[a + b*x]*PolyLog[2, I*E^ArcSech[a + b*x]])/b^2 + (3*PolyLog[2, -E^(2
*ArcSech[a + b*x])])/(2*b^2) + ((6*I)*a*PolyLog[3, (-I)*E^ArcSech[a + b*x]]
)/b^2 - ((6*I)*a*PolyLog[3, I*E^ArcSech[a + b*x]])/b^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\text{sech}^{-1}(a+bx)^3\,dx = -\frac{\text{Subst}\left(\int x^3\text{sech}(x)(-a+\text{sech}(x))\tanh(x)\,dx,x,\text{sech}^{-1}(a+bx)\right)}{b^2}$$

$$= \frac{1}{2}x^2\text{sech}^{-1}(a+bx)^3 - \frac{3\,\text{Subst}\left(\int x^2(-a+\text{sech}(x))^2\,dx,x,\text{sech}^{-1}(a+bx)\right)}{2b^2}$$

$$= \frac{1}{2}x^2\text{sech}^{-1}(a+bx)^3 - \frac{3\,\text{Subst}\left(\int \left(a^2x^2-2ax^2\text{sech}(x)+x^2\text{sech}^2(x)\right)\,dx,x,\text{sech}^{-1}(\right.}{2b^2}$$

$$= -\frac{a^2\text{sech}^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\text{sech}^{-1}(a+bx)^3 - \frac{3\,\text{Subst}\left(\int x^2\text{sech}^2(x)\,dx,x,\text{sech}^{-1}(a\right.}{2b^2}$$

$$= -\frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2\text{sech}^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\text{sech}^{-1}(a+bx)^3$$

$$= -\frac{3\text{sech}^{-1}(a+bx)^2}{2b^2} - \frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2\text{sech}^{-1}(a+bx)^3}{2b^2} +$$

$$= -\frac{3\text{sech}^{-1}(a+bx)^2}{2b^2} - \frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2\text{sech}^{-1}(a+bx)^3}{2b^2} +$$

$$= -\frac{3\text{sech}^{-1}(a+bx)^2}{2b^2} - \frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2\text{sech}^{-1}(a+bx)^3}{2b^2} +$$

$$= -\frac{3\text{sech}^{-1}(a+bx)^2}{2b^2} - \frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2\text{sech}^{-1}(a+bx)^3}{2b^2} +$$

Mathematica [A] time = 0.54, size = 254, normalized size = 0.98

$$\frac{-3\text{Li}_2\left(-e^{-2\text{sech}^{-1}(a+bx)}\right) + 6ia\left(-2\text{sech}^{-1}(a+bx)\left(\text{Li}_2\left(-ie^{-\text{sech}^{-1}(a+bx)}\right) - \text{Li}_2\left(ie^{-\text{sech}^{-1}(a+bx)}\right)\right) - 2\text{Li}_3\left(-ie^{-\text{sech}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcSech[a + b*x]^3,x]
```

```
[Out] (-3*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)*ArcSech[a + b*x]^2
- 2*a*(a + b*x)*ArcSech[a + b*x]^3 + (a + b*x)^2*ArcSech[a + b*x]^3 + 3*Arc
Sech[a + b*x]*(ArcSech[a + b*x] + 2*Log[1 + E^(-2*ArcSech[a + b*x])]) - 3*P
olyLog[2, -E^(-2*ArcSech[a + b*x])] + (6*I)*a*(-(ArcSech[a + b*x]^2*(Log[1
- I/E^ArcSech[a + b*x]] - Log[1 + I/E^ArcSech[a + b*x]])) - 2*ArcSech[a + b
*x]*(PolyLog[2, (-I)/E^ArcSech[a + b*x]] - PolyLog[2, I/E^ArcSech[a + b*x]]
) - 2*PolyLog[3, (-I)/E^ArcSech[a + b*x]] + 2*PolyLog[3, I/E^ArcSech[a + b*
x]]))/(2*b^2)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{arsech}\left(bx+a\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x*arcsech(b*x + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arsech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arcsech(b*x + a)^3, x)
```

maple [F] time = 1.88, size = 0, normalized size = 0.00

$$\int x \operatorname{arcsech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsech(b*x+a)^3,x)
```

```
[Out] int(x*arcsech(b*x+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2\log\left(\sqrt{bx+a+1}\sqrt{-bx-a+1}\,bx+\sqrt{bx+a+1}\sqrt{-bx-a+1}\,a+bx+a\right)^3-\int \frac{16\left(b^3x^4+3\,ab^2x^3+\left(3\,a^2b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sq
rt(-b*x - a + 1)*a + b*x + a)^3 - integrate(1/2*(16*(b^3*x^4 + 3*a*b^2*x^3
+ (3*a^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log
(b*x + a)^3 + 16*(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*
log(b*x + a)^3 + 3*(b^3*x^4 + 2*a*b^2*x^3 + (a^2*b - b)*x^2 + 4*(b^3*x^4 +
3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x + a) + (2*(b^3*x^4 +
 3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*log(b*x +
 a) + (2*b^3*x^4 + 4*a*b^2*x^3 + (2*a^2*b - b)*x^2 + 2*(b^3*x^4 + 3*a*b^2*x
^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt
(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a
 + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2 - 24*((b^3*x^4 + 3*a*b^2*x^3 + (3*a
^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x +
 a)^2 + (b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x +
 a)^2)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqr
t(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (b^3*x^3 + 3*a
*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)
+ (3*a^2*b - b)*x - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(1/(a + b*x))^3,x)
```

```
[Out] int(x*acosh(1/(a + b*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asech}^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asech(b*x+a)**3,x)
```

```
[Out] Integral(x*asech(a + b*x)**3, x)
```

3.16 $\int \operatorname{sech}^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=136

$$\frac{6i\operatorname{sech}^{-1}(a+bx)\mathrm{Li}_2\left(-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}-\frac{6i\operatorname{sech}^{-1}(a+bx)\mathrm{Li}_2\left(ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}-\frac{6i\mathrm{Li}_3\left(-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}+\frac{6i\mathrm{Li}_3\left(ie^{\operatorname{sech}^{-1}(}\right.}{b}$$

```
[Out] (b*x+a)*arcsech(b*x+a)^3/b-6*arcsech(b*x+a)^2*arctan(1/(b*x+a)+(1/(b*x+a)-1
)^(1/2)*(1/(b*x+a)+1)^(1/2))/b+6*I*arcsech(b*x+a)*polylog(2,-I*(1/(b*x+a)+(
1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b-6*I*arcsech(b*x+a)*polylog(2,I*(
1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b-6*I*polylog(3,-I*(1/(
b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b+6*I*polylog(3,I*(1/(b*x+
a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b
```

Rubi [A] time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6315, 6279, 5418, 4180, 2531, 2282, 6589}

$$\frac{6i\operatorname{sech}^{-1}(a+bx)\mathrm{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}-\frac{6i\operatorname{sech}^{-1}(a+bx)\mathrm{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}-\frac{6i\mathrm{PolyLog}\left(3,-ie^{\operatorname{sech}}\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^3,x]
```

```
[Out] ((a + b*x)*ArcSech[a + b*x]^3)/b - (6*ArcSech[a + b*x]^2*ArcTan[E^ArcSech[a
 + b*x]])/b + ((6*I)*ArcSech[a + b*x]*PolyLog[2, (-I)*E^ArcSech[a + b*x]])/
b - ((6*I)*ArcSech[a + b*x]*PolyLog[2, I*E^ArcSech[a + b*x]])/b - ((6*I)*Po
lyLog[3, (-I)*E^ArcSech[a + b*x]])/b + ((6*I)*PolyLog[3, I*E^ArcSech[a + b*
x]])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
```

```
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 6279

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[c^(-1), Su
bst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rule 6315

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSech[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \operatorname{sech}^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \operatorname{sech}^{-1}(x)^3\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int x^3\operatorname{sech}(x)\tanh(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b}$$
$$= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{3\,\text{Subst}\left(\int x^2\operatorname{sech}(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b}$$
$$= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{6\operatorname{sech}^{-1}(a+bx)^2\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{(6i)\,\text{Subst}\left(\int x\,\text{lo}\right.}{}$$
$$= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{6\operatorname{sech}^{-1}(a+bx)^2\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{6i\operatorname{sech}^{-1}(a+bx)}{}$$
$$= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{6\operatorname{sech}^{-1}(a+bx)^2\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{6i\operatorname{sech}^{-1}(a+bx)}{}$$
$$= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{6\operatorname{sech}^{-1}(a+bx)^2\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{6i\operatorname{sech}^{-1}(a+bx)}{}$$

Mathematica [A] time = 0.14, size = 153, normalized size = 1.12

$$-\frac{-(a+bx)\operatorname{sech}^{-1}(a+bx)^3 + 3i\left(-2\operatorname{sech}^{-1}(a+bx)\left(\text{Li}_2\left(-ie^{-\operatorname{sech}^{-1}(a+bx)}\right) - \text{Li}_2\left(ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right) - 2\left(\text{Li}_3\left(-ie^{-}\right.\right.\right.}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a + b*x]^3,x]
```

```
[Out] -((-((a + b*x)*ArcSech[a + b*x]^3) + (3*I)*(-(ArcSech[a + b*x]^2*(Log[1 - I
/E^ArcSech[a + b*x]] - Log[1 + I/E^ArcSech[a + b*x]])) - 2*ArcSech[a + b*x]
*(PolyLog[2, (-I)/E^ArcSech[a + b*x]] - PolyLog[2, I/E^ArcSech[a + b*x]]) -
```

```
 2*(PolyLog[3, (-I)/E^ArcSech[a + b*x]] - PolyLog[3, I/E^ArcSech[a + b*x]])
))/b)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{arsech}\,(bx+a)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(arcsech(b*x + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{arsech}\,(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)^3, x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \text{arcsech}\,(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^3,x)
```

```
[Out] int(arcsech(b*x+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\log\left(\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,bx+\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,a+bx+a\right)^3-\int\frac{8\left(b^3x^3+3\,ab^2x^2+a^3+\left(3\,a^2b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*
x - a + 1)*a + b*x + a)^3 - integrate((8*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*
a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^3 + 8*(
b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^3 + 3*(b^3*
x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2
*b - b)*x - a)*log(b*x + a) + ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)
*x - a)*sqrt(b*x + a + 1)*log(b*x + a) + (2*b^3*x^3 + 4*a*b^2*x^2 + (2*a^2*
b - b)*x + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)
)*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x -
a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2 - 12*((b^3
*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x
 - a + 1)*log(b*x + a)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x -
 a)*log(b*x + a)^2)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x
 + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (
b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-
b*x - a + 1) + (3*a^2*b - b)*x - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \text{acosh}\left(\frac{1}{a+b\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^3,x)
```

```
[Out] int(acosh(1/(a + b*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**3,x)
```

```
[Out] Integral(asech(a + b*x)**3, x)
```

3.17 $\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x}\,dx$

Optimal. Leaf size=378

$$3\operatorname{sech}^{-1}(a+bx)^2\mathrm{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+3\operatorname{sech}^{-1}(a+bx)^2\mathrm{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)-6\operatorname{sech}^{-1}(a+bx)\mathrm{Li}_3\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-6\text{sec}$$

```
[Out] -arcsech(b*x+a)^3*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^
2)+arcsech(b*x+a)^3*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/
2))/(1-(-a^2+1)^(1/2)))+arcsech(b*x+a)^3*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1
/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))-3/2*arcsech(b*x+a)^2*polylog(2
,-(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)+3*arcsech(b*x+a)^2
*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1
)^(1/2)))+3*arcsech(b*x+a)^2*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/
(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))+3/2*arcsech(b*x+a)*polylog(3,-(1/(b*x
+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)-6*arcsech(b*x+a)*polylog(3,
a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))-6
*arcsech(b*x+a)*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1
/2))/(1+(-a^2+1)^(1/2)))-3/4*polylog(4,-(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(
b*x+a)+1)^(1/2))^2)+6*polylog(4,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)
+1)^(1/2))/(1-(-a^2+1)^(1/2)))+6*polylog(4,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)
*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))
```

Rubi [A] time = 0.51, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6321, 5595, 5570, 3718, 2190, 2531, 6609, 2282, 6589, 5562}

$$3\operatorname{sech}^{-1}(a+bx)^2\text{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+3\operatorname{sech}^{-1}(a+bx)^2\text{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)-6\operatorname{sech}^{-1}(a+bx)\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^3/x,x]
```

```
[Out] ArcSech[a + b*x]^3*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + Ar
cSech[a + b*x]^3*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcS
ech[a + b*x]^3*Log[1 + E^(2*ArcSech[a + b*x])] + 3*ArcSech[a + b*x]^2*PolyL
og[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + 3*ArcSech[a + b*x]^2*Po
lyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - (3*ArcSech[a + b*x]^
2*PolyLog[2, -E^(2*ArcSech[a + b*x])])/2 - 6*ArcSech[a + b*x]*PolyLog[3, (a
*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] - 6*ArcSech[a + b*x]*PolyLog[3, (
a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + (3*ArcSech[a + b*x]*PolyLog[3,
 -E^(2*ArcSech[a + b*x])])/2 + 6*PolyLog[4, (a*E^ArcSech[a + b*x])/(1 - Sqr
t[1 - a^2])] + 6*PolyLog[4, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - (
3*PolyLog[4, -E^(2*ArcSech[a + b*x])])/4
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5570

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Tanh[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Sinh[c + d*x]*Tanh[c + d*x]^
(n - 1))/(a + b*Cosh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5595

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sech[(c_.) + (d_.)*(x_)]), x_Symbol] :> I
nt[((e + f*x)^m*Cosh[c + d*x]*F[c + d*x]^n*G[c + d*x]^p)/(b + a*Cosh[c + d*
x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G]
 && IntegersQ[m, n, p]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
```

```
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\text{sech}^{-1}(a+bx)^3}{x}\,dx = -\text{Subst}\left(\int \frac{x^3\text{sech}(x)\tanh(x)}{-a+\text{sech}(x)}\,dx, x, \text{sech}^{-1}(a+bx)\right)$$
$$= -\text{Subst}\left(\int \frac{x^3\tanh(x)}{1-a\cosh(x)}\,dx, x, \text{sech}^{-1}(a+bx)\right)$$
$$= -\left(a\,\text{Subst}\left(\int \frac{x^3\sinh(x)}{1-a\cosh(x)}\,dx, x, \text{sech}^{-1}(a+bx)\right)\right) - \text{Subst}\left(\int x^3\tanh(x)\,dx, x, \text{sech}^{-1}\right.$$
$$= -\left(2\,\text{Subst}\left(\int \frac{e^{2x}x^3}{1+e^{2x}}\,dx, x, \text{sech}^{-1}(a+bx)\right)\right) - a\,\text{Subst}\left(\int \frac{e^x x^3}{1-\sqrt{1-a^2}-ae^x}\,dx, x, \text{se}\right.$$
$$= \text{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \text{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \text{sec}$$
$$= \text{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \text{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \text{sec}$$
$$= \text{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \text{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \text{sec}$$
$$= \text{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \text{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \text{sec}$$
$$= \text{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \text{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \text{sec}$$

Mathematica [A] time = 0.32, size = 384, normalized size = 1.02

$$3\text{sech}^{-1}(a+bx)^2\text{Li}_2\left(-\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}-1}\right)+3\text{sech}^{-1}(a+bx)^2\text{Li}_2\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)-6\text{sech}^{-1}(a+bx)\text{Li}_3\left(-\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}-1}\right)-6$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a + b*x]^3/x,x]
```

```
[Out] -1/2*ArcSech[a + b*x]^4 - ArcSech[a + b*x]^3*Log[1 + E^(-2*ArcSech[a + b*x]
)] + ArcSech[a + b*x]^3*Log[1 + (a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2])
] + ArcSech[a + b*x]^3*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])]
+ (3*ArcSech[a + b*x]^2*PolyLog[2, -E^(-2*ArcSech[a + b*x])])/2 + 3*ArcSech
[a + b*x]^2*PolyLog[2, -((a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2]))] + 3*
ArcSech[a + b*x]^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] +
 (3*ArcSech[a + b*x]*PolyLog[3, -E^(-2*ArcSech[a + b*x])])/2 - 6*ArcSech[a
+ b*x]*PolyLog[3, -((a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2]))] - 6*ArcSe
ch[a + b*x]*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + (3*Pol
yLog[4, -E^(-2*ArcSech[a + b*x])])/4 + 6*PolyLog[4, -((a*E^ArcSech[a + b*x]
)/(-1 + Sqrt[1 - a^2]))] + 6*PolyLog[4, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1
- a^2])]
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsech}(bx+a)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(arcsech(b*x + a)^3/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x,x, algorithm="giac")
[Out] integrate(arcsech(b*x + a)^3/x, x)
```

maple [F] time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsech}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^3/x,x)
[Out] int(arcsech(b*x+a)^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x,x, algorithm="maxima")
[Out] integrate(arcsech(b*x + a)^3/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^3/x,x)
[Out] int(acosh(1/(a + b*x))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^{3}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**3/x,x)
[Out] Integral(asech(a + b*x)**3/x, x)
```

3.18 $\int \frac{\text{sech}^{-1}(a+bx)^3}{x^2}\,dx$

Optimal. Leaf size=330

$$\frac{6b\text{sech}^{-1}(a+bx)\text{Li}_2\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{6b\text{sech}^{-1}(a+bx)\text{Li}_2\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{6b\text{Li}_3\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{6b\text{Li}_3\left(\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}$$

```
[Out] -b*arcsech(b*x+a)^3/a-arcsech(b*x+a)^3/x+3*b*arcsech(b*x+a)^2*ln(1-a*(1/(b*
x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1
)^(1/2)-3*b*arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+
a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+6*b*arcsech(b*x+a)*polylo
g(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)
))/a/(-a^2+1)^(1/2)-6*b*arcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)
^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)-6*b*polylo
g(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)
))/a/(-a^2+1)^(1/2)+6*b*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+
a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6321, 5468, 4191, 3320, 2264, 2190, 2531, 2282, 6589}

$$\frac{6b\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{6b\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{6b\text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^3/x^2,x]
```

```
[Out] -((b*ArcSech[a + b*x]^3)/a) - ArcSech[a + b*x]^3/x + (3*b*ArcSech[a + b*x]^
2*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) -
(3*b*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])]
)/(a*Sqrt[1 - a^2]) + (6*b*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x
])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (6*b*ArcSech[a + b*x]*PolyLog[
2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (6*b*Po
lyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (
6*b*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2
])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2}\,dx = -\left(b\,\text{Subst}\left(\int \frac{x^3\operatorname{sech}(x)\tanh(x)}{(-a+\operatorname{sech}(x))^2}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + (3b)\,\text{Subst}\left(\int \frac{x^2}{-a+\operatorname{sech}(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + (3b)\,\text{Subst}\left(\int \left(-\frac{x^2}{a} + \frac{x^2}{a(1-a\cosh(x))}\right) dx, x, \operatorname{sech}^{-1}(a+bx)\right)$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{(3b)\,\text{Subst}\left(\int \frac{x^2}{1-a\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{a}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{(6b)\,\text{Subst}\left(\int \frac{e^x x^2}{-a+2e^x-ae^{2x}}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{a}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} - \frac{(6b)\,\text{Subst}\left(\int \frac{e^x x^2}{2-2\sqrt{1-a^2}-2ae^x}\,dx, x, \operatorname{sech}^{-1}(a+b\right.}{\sqrt{1-a^2}}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{3bs}{}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{3bs}{}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{3bs}{}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} - \frac{3bs}{}$$

Mathematica [C] time = 46.92, size = 1849, normalized size = 5.60

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a + b*x]^3/x^2,x]
```

```
[Out] -((-3*b*x*ArcCos[-a^(-1)]*ArcSech[a + b*x]^2 + a*Sqrt[-1 + a^2]*ArcSech[a +
 b*x]^3 + Sqrt[-1 + a^2]*b*x*ArcSech[a + b*x]^3 + 12*b*x*ArcCos[-a^(-1)]*Ar
cSech[a + b*x]*ArcTanh[Tanh[ArcSech[a + b*x]/2]] - 12*b*x*ArcCos[-a^(-1)]*A
rcTanh[Tanh[ArcSech[a + b*x]/2]]^2 - 6*b*x*ArcCos[-a^(-1)]*ArcSech[a + b*x]
*Log[Sqrt[-1 + a^2]/(Sqrt[2]*Sqrt[a]*E^(I*ArcTan[((1 + a)*Tanh[ArcSech[a +
b*x]/2])/Sqrt[-1 + a^2]])*Sqrt[1 + a*Cos[2*ArcTan[((1 + a)*Tanh[ArcSech[a +
 b*x]/2])/Sqrt[-1 + a^2]]]])] + (12*I)*b*x*ArcSech[a + b*x]*ArcTanh[Coth[Ar
cSech[a + b*x]/2]]*Log[Sqrt[-1 + a^2]/(Sqrt[2]*Sqrt[a]*E^(I*ArcTan[((1 + a)
*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Sqrt[1 + a*Cos[2*ArcTan[((1 + a
)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]]])] - (12*I)*b*x*ArcSech[a + b*
x]*ArcTanh[Tanh[ArcSech[a + b*x]/2]]*Log[Sqrt[-1 + a^2]/(Sqrt[2]*Sqrt[a]*E^
(I*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Sqrt[1 + a*Co
s[2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]]])] - 6*b*x*A
rcCos[-a^(-1)]*ArcSech[a + b*x]*Log[(Sqrt[-1 + a^2]*E^(I*ArcTan[((1 + a)*Ta
nh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))/(Sqrt[2]*Sqrt[a]*Sqrt[1 + a*Cos[2
*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]]])] - (12*I)*b*x
*ArcSech[a + b*x]*ArcTanh[Coth[ArcSech[a + b*x]/2]]*Log[(Sqrt[-1 + a^2]*E^(
```

```
I*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))/(Sqrt[2]*Sqrt
[a]*Sqrt[1 + a*Cos[2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^
2]]]]])] + (12*I)*b*x*ArcSech[a + b*x]*ArcTanh[Tanh[ArcSech[a + b*x]/2]]*Log
[(Sqrt[-1 + a^2]*E^(I*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a
^2]]))/(Sqrt[2]*Sqrt[a]*Sqrt[1 + a*Cos[2*ArcTan[((1 + a)*Tanh[ArcSech[a + b
*x]/2])/Sqrt[-1 + a^2]]]]])] + 6*b*x*ArcCos[-a^(-1)]*ArcSech[a + b*x]*Log[(S
qrt[-1 + a^2] + I*(1 + a)*Tanh[ArcSech[a + b*x]/2])/(2*Sqrt[a]*Sqrt[-(((-1
+ a^2)*(a + b*x))/(b*x))]*Sqrt[-((b*x)/((-1 + a)*(1 + a + b*x)))])] + (12*I
)*b*x*ArcSech[a + b*x]*ArcTanh[Coth[ArcSech[a + b*x]/2]]*Log[(Sqrt[-1 + a^2
] + I*(1 + a)*Tanh[ArcSech[a + b*x]/2])/(2*Sqrt[a]*Sqrt[-(((-1 + a^2)*(a +
b*x))/(b*x))]*Sqrt[-((b*x)/((-1 + a)*(1 + a + b*x)))])] - (12*I)*b*x*ArcSec
h[a + b*x]*ArcTanh[Tanh[ArcSech[a + b*x]/2]]*Log[(Sqrt[-1 + a^2] + I*(1 + a
)*Tanh[ArcSech[a + b*x]/2])/(2*Sqrt[a]*Sqrt[-(((-1 + a^2)*(a + b*x))/(b*x))
]*Sqrt[-((b*x)/((-1 + a)*(1 + a + b*x)))])] + 6*b*x*ArcCos[-a^(-1)]*ArcSech
[a + b*x]*Log[((-I)*(-1 + a^2)*Sqrt[-((b*x)/((-1 + a)*(1 + a + b*x)))])/(Sq
rt[a]*Sqrt[-(((-1 + a^2)*(a + b*x))/(b*x))]*((-I)*Sqrt[-1 + a^2] + (1 + a)*
Tanh[ArcSech[a + b*x]/2]))] - (12*I)*b*x*ArcSech[a + b*x]*ArcTanh[Coth[ArcS
ech[a + b*x]/2]]*Log[((-I)*(-1 + a^2)*Sqrt[-((b*x)/((-1 + a)*(1 + a + b*x))
)])/(Sqrt[a]*Sqrt[-(((-1 + a^2)*(a + b*x))/(b*x))]*((-I)*Sqrt[-1 + a^2] + (
1 + a)*Tanh[ArcSech[a + b*x]/2]))] + (12*I)*b*x*ArcSech[a + b*x]*ArcTanh[Ta
nh[ArcSech[a + b*x]/2]]*Log[((-I)*(-1 + a^2)*Sqrt[-((b*x)/((-1 + a)*(1 + a
+ b*x)))])/(Sqrt[a]*Sqrt[-(((-1 + a^2)*(a + b*x))/(b*x))]*((-I)*Sqrt[-1 + a
^2] + (1 + a)*Tanh[ArcSech[a + b*x]/2]))] - (3*I)*b*x*ArcSech[a + b*x]^2*Lo
g[((-I + I*a + Sqrt[-1 + a^2])*(-I + ((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqr
t[-1 + a^2]))/(a*(1 + Tanh[ArcSech[a + b*x]/2]))] + (3*I)*b*x*ArcSech[a + b
*x]^2*Log[((I - I*a + Sqrt[-1 + a^2])*(I + ((1 + a)*Tanh[ArcSech[a + b*x]/2
])/Sqrt[-1 + a^2]))/(a*(1 + Tanh[ArcSech[a + b*x]/2]))] - (6*I)*b*x*ArcSech
[a + b*x]*PolyLog[2, ((1 - I*Sqrt[-1 + a^2])*(1 - Sqrt[-((-1 + a + b*x)/(1
+ a + b*x))]*(1 + a + b*x)))/(a*(a + b*x))] + (6*I)*b*x*ArcSech[a + b*x]*Po
lyLog[2, ((1 + I*Sqrt[-1 + a^2])*(1 - Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]
*(1 + a + b*x)))/(a*(a + b*x))] - (6*I)*b*x*PolyLog[3, ((1 - I*Sqrt[-1 + a^
2])*(1 - Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)))/(a*(a + b*x)
)] + (6*I)*b*x*PolyLog[3, ((1 + I*Sqrt[-1 + a^2])*(1 - Sqrt[-((-1 + a + b*x
)/(1 + a + b*x))]*(1 + a + b*x)))/(a*(a + b*x))])/(a*Sqrt[-1 + a^2]*x))
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsech}(bx+a)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsech(b*x + a)^3/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)^3/x^2, x)
```

maple [F] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsech}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(b*x+a)^3/x^2,x)

[Out] int(arcsech(b*x+a)^3/x^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(\sqrt{bx+a+1}\sqrt{-bx-a+1}\,bx+\sqrt{bx+a+1}\sqrt{-bx-a+1}\,a+bx+a\right)^3}{x}-\int\frac{8\left(b^3x^3+3\,ab^2x^2+a^3+\left(3\,a^2b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(b*x+a)^3/x^2,x, algorithm="maxima")

[Out] -log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^3/x - integrate((8*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^3 + 8*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^3 - 3*(b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a) - ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 + 4*a*b^2*x^2 + (2*a^2*b - b)*x - (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2 - 12*((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2 + (b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(1/(a + b*x))^3/x^2,x)

[Out] int(acosh(1/(a + b*x))^3/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(b*x+a)**3/x**2,x)

[Out] Integral(asech(a + b*x)**3/x**2, x)

3.19 $\int \frac{\text{sech}^{-1}(a+bx)^3}{x^3} dx$

Optimal. Leaf size=965

$$\frac{b^2\text{sech}^{-1}(a+bx)^3}{2a^2}-\frac{\text{sech}^{-1}(a+bx)^3}{2x^2}-\frac{3b^2\text{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)}-\frac{3b^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\text{sech}^{-1}(a+bx)^2}{a^2\sqrt{1-a^2}}+\frac{3b^2\log}{}$$

```
[Out] -3/2*b^2*arcsech(b*x+a)^2/a^2/(-a^2+1)+1/2*b^2*arcsech(b*x+a)^3/a^2-1/2*arc
sech(b*x+a)^3/x^2+3*b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2
)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)+3/2*b^2*arcsech(b*x
+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1
)^(1/2)))/a^2/(-a^2+1)^(3/2)+3*b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x
+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)-3/2*b^2*
arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))
/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)+3*b^2*polylog(2,a*(1/(b*x+a)+(1/(b*
x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)+3*b^2*a
rcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2
))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)+3*b^2*polylog(2,a*(1/(b*x+a)+(1/(
b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)-3*b^2
*arcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1
/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)-3*b^2*polylog(3,a*(1/(b*x+a)+(1
/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/
2)+3*b^2*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1
+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)-3*b^2*arcsech(b*x+a)^2*ln(1-a*(1/(b*x+
a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1
)^(1/2)+3*b^2*arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*
x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)-6*b^2*arcsech(b*x+a)*
polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)
^(1/2)))/a^2/(-a^2+1)^(1/2)+6*b^2*arcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/
(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2
)+6*b^2*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-
(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)-6*b^2*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)
-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)+3/2*b
^2*(b*x+a+1)*arcsech(b*x+a)^2*((-b*x-a+1)/(b*x+a+1))^(1/2)/a/(-a^2+1)/(b*x+
a)/(1-a/(b*x+a))
```

Rubi [A] time = 1.34, antiderivative size = 965, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {6321, 5468, 4191, 3324, 3320, 2264, 2190, 2531, 2282, 6589, 5562, 2279, 2391}

$$\frac{b^2\text{sech}^{-1}(a+bx)^3}{2a^2}-\frac{\text{sech}^{-1}(a+bx)^3}{2x^2}-\frac{3b^2\text{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)}-\frac{3b^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)\text{sech}^{-1}(a+bx)^2}{a^2\sqrt{1-a^2}}+\frac{3b^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^3/x^3,x]
```

```
[Out] (-3*b^2*ArcSech[a + b*x]^2)/(2*a^2*(1 - a^2)) + (3*b^2*Sqrt[(1 - a - b*x)/(
1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x]^2)/(2*a*(1 - a^2)*(a + b*x)*(1
 - a/(a + b*x))) + (b^2*ArcSech[a + b*x]^3)/(2*a^2) - ArcSech[a + b*x]^3/(2
*x^2) + (3*b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1
- a^2])])/(a^2*(1 - a^2)) + (3*b^2*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[
a + b*x])/(1 - Sqrt[1 - a^2])])/(2*a^2*(1 - a^2)^(3/2)) - (3*b^2*ArcSech[a
+ b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 -
 a^2]) + (3*b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1
```

```
 - a^2])])/(a^2*(1 - a^2)) - (3*b^2*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech
[a + b*x])/(1 + Sqrt[1 - a^2])])/(2*a^2*(1 - a^2)^(3/2)) + (3*b^2*ArcSech[a
 + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1
- a^2]) + (3*b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a
^2*(1 - a^2)) + (3*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(
1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) - (6*b^2*ArcSech[a + b*x]*PolyLo
g[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) + (3*
b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*(1 - a^2))
 - (3*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 -
a^2])])/(a^2*(1 - a^2)^(3/2)) + (6*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^Arc
Sech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) - (3*b^2*PolyLog[3
, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) + (6*b
^2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^
2]) + (3*b^2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*(
1 - a^2)^(3/2)) - (6*b^2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^
2])])/(a^2*Sqrt[1 - a^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5468

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e
 + f*x)^m*(a + b*Sech[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*
d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6321

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> -Dist[(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3}\,dx = -\left(b^2\,\mathrm{Subst}\left(\int \frac{x^3\operatorname{sech}(x)\tanh(x)}{(-a+\operatorname{sech}(x))^3}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} + \frac{1}{2}\left(3b^2\right)\mathrm{Subst}\left(\int \frac{x^2}{(-a+\operatorname{sech}(x))^2}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} + \frac{1}{2}\left(3b^2\right)\mathrm{Subst}\left(\int \left(\frac{x^2}{a^2} + \frac{x^2}{a^2(-1+a\cosh(x))^2} + \frac{2x^2}{a^2(-1+a\cosh(x)}\right.\right.$$

$$= \frac{b^2\operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} + \frac{\left(3b^2\right)\mathrm{Subst}\left(\int \frac{x^2}{(-1+a\cosh(x))^2}\,dx, x, \operatorname{sech}^{-1}(a+b\right.}{2a^2}$$

$$= \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} + \frac{(6b}{}$$

$$= -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)} + \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^3}{2a^2}$$

$$= -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)} + \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^3}{2a^2}$$

$$= -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)} + \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^3}{2a^2}$$

$$= -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)} + \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^3}{2a^2}$$

$$= -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)} + \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^3}{2a^2}$$

$$= -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)} + \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^3}{2a^2}$$

Mathematica [F] time = 9.95, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSech[a + b*x]^3/x^3,x]
```

```
[Out] Integrate[ArcSech[a + b*x]^3/x^3, x]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\operatorname{arsech}(bx+a)^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x^3,x, algorithm="fricas")
[Out] integral(arcsech(b*x + a)^3/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(bx+a)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x^3,x, algorithm="giac")
[Out] integrate(arcsech(b*x + a)^3/x^3, x)
```

maple [F] time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsech}(bx+a)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^3/x^3,x)
[Out] int(arcsech(b*x+a)^3/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,bx+\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,a+bx+a\right)^3}{2\,x^2}-\int \frac{16\left(b^3x^3+3\,ab^2x^2+a^3+(3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x^3,x, algorithm="maxima")
[Out] -1/2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(
-b*x - a + 1)*a + b*x + a)^3/x^2 - integrate(1/2*(16*(b^3*x^3 + 3*a*b^2*x^2
 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x
+ a)^3 + 16*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a
)^3 - 3*(b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 4*(b^3*x^3 + 3*a*b^2*x^2 +
 a^3 + (3*a^2*b - b)*x - a)*log(b*x + a) - (2*(b^3*x^3 + 3*a*b^2*x^2 + a^3
+ (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 + 4*a*b^
2*x^2 + (2*a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x
- a)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a
+ 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x
+ a)^2 - 24*((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x +
 a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 +
(3*a^2*b - b)*x - a)*log(b*x + a)^2)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a +
1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^6 + 3*a*
b^2*x^5 + (3*a^2*b - b)*x^4 + (a^3 - a)*x^3 + (b^3*x^6 + 3*a*b^2*x^5 + (3*a
^2*b - b)*x^4 + (a^3 - a)*x^3)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^3/x^3,x)
[Out] int(acosh(1/(a + b*x))^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^3(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**3/x**3,x)
```

```
[Out] Integral(asech(a + b*x)**3/x**3, x)
```

3.20 $\int x^3 \operatorname{sech}^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=164

$$\frac{1}{4}x^4\operatorname{sech}^{-1}\left(\sqrt{x}\right)+\frac{(1-x)^4}{28\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{3(1-x)^3}{20\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{(1-x)^2}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\quad}{4\sqrt{\frac{1}{\sqrt{x}}\cdots}}$$

```
[Out] 1/4*x^4*arcsech(x^(1/2))+1/4*(-1+x)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/
2))^(1/2)+1/4*(1-x)^2/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)-3/20
*(1-x)^3/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)+1/28*(1-x)^4/x^(1
/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6345, 12, 43}

$$\frac{1}{4}x^4\operatorname{sech}^{-1}\left(\sqrt{x}\right)+\frac{(1-x)^4}{28\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{3(1-x)^3}{20\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{(1-x)^2}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\quad}{4\sqrt{\frac{1}{\sqrt{x}}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSech[Sqrt[x]],x]
```

```
[Out] -(1 - x)/(4*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (1 - x)^2/(
4*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) - (3*(1 - x)^3)/(20*Sqr
t[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (1 - x)^4/(28*Sqrt[-1 + 1/
Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (x^4*ArcSech[Sqrt[x]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6345

```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSech[u]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1
- u^2])/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]), Int[SimplifyIntegrand[(
(c + d*x)^(m + 1)*D[u, x])/(u*Sqrt[1 - u^2]), x], x], x] /; FreeQ[{a, b, c,
 d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
+ d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\int x^3\operatorname{sech}^{-1}\left(\sqrt{x}\right)dx = \frac{1}{4}x^4\operatorname{sech}^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{1-x}\int \frac{x^3}{2\sqrt{1-x}}\,dx}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= \frac{1}{4}x^4\operatorname{sech}^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{1-x}\int \frac{x^3}{\sqrt{1-x}}\,dx}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= \frac{1}{4}x^4\operatorname{sech}^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{1-x}\int \left(\frac{1}{\sqrt{1-x}} - 3\sqrt{1-x} + 3(1-x)^{3/2} - (1-x)^{5/2}\right)dx}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= -\frac{1-x}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{(1-x)^2}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} - \frac{3(1-x)^3}{20\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 0.51

$$\frac{1}{4}x^4\operatorname{sech}^{-1}\left(\sqrt{x}\right) - \frac{1}{140}\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}}\left(5x^{7/2}+6x^{5/2}+8x^{3/2}+5x^3+6x^2+8x+16\sqrt{x}+16\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSech[Sqrt[x]],x]
```

```
[Out] -1/140*(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(16 + 16*Sqrt[x] + 8*x + 8*x^(3/2
) + 6*x^2 + 6*x^(5/2) + 5*x^3 + 5*x^(7/2))) + (x^4*ArcSech[Sqrt[x]])/4
```

fricas [A] time = 0.64, size = 57, normalized size = 0.35

$$\frac{1}{4}x^4\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right) - \frac{1}{140}\left(5x^3+6x^2+8x+16\right)\sqrt{x}\sqrt{-\frac{x-1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - 1/140*(5*x^3 + 6*x^2 + 8*x
+ 16)*sqrt(x)*sqrt(-(x - 1)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\operatorname{arsech}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsech(sqrt(x)), x)
```

maple [A] time = 0.07, size = 54, normalized size = 0.33

$$\frac{x^4\operatorname{arcsech}\left(\sqrt{x}\right)}{4} - \frac{\sqrt{-\frac{-1+\sqrt{x}}{\sqrt{x}}}\sqrt{x}\sqrt{\frac{1+\sqrt{x}}{\sqrt{x}}}\left(5x^3+6x^2+8x+16\right)}{140}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsech(x^(1/2)),x)
```

```
[Out] 1/4*x^4*arcsech(x^(1/2))-1/140*(-(-1+x^(1/2))/x^(1/2))^(1/2)*x^(1/2)*((1+x^
(1/2))/x^(1/2))^(1/2)*(5*x^3+6*x^2+8*x+16)
```

maxima [A] time = 0.32, size = 58, normalized size = 0.35

$$\frac{1}{28}\,x^{\frac{7}{2}}\left(\frac{1}{x}-1\right)^{\frac{7}{2}}-\frac{3}{20}\,x^{\frac{5}{2}}\left(\frac{1}{x}-1\right)^{\frac{5}{2}}+\frac{1}{4}\,x^{4}\,\operatorname{arsech}\left(\sqrt{x}\right)+\frac{1}{4}\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}}-\frac{1}{4}\,\sqrt{x}\sqrt{\frac{1}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/28*x^(7/2)*(1/x - 1)^(7/2) - 3/20*x^(5/2)*(1/x - 1)^(5/2) + 1/4*x^4*arcse
ch(sqrt(x)) + 1/4*x^(3/2)*(1/x - 1)^(3/2) - 1/4*sqrt(x)*sqrt(1/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(1/x^(1/2)),x)
```

```
[Out] int(x^3*acosh(1/x^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\operatorname{asech}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asech(x**(1/2)),x)
```

```
[Out] Integral(x**3*asech(sqrt(x)), x)
```

3.21 $\int x^2 \text{sech}^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=126

$$\frac{1}{3}x^3\text{sech}^{-1}\left(\sqrt{x}\right)-\frac{(1-x)^3}{15\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{2(1-x)^2}{9\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{1-x}{3\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

```
[Out] 1/3*x^3*arcsech(x^(1/2))+1/3*(-1+x)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/
2))^(1/2)+2/9*(1-x)^2/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)-1/15
*(1-x)^3/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6345, 12, 43}

$$\frac{1}{3}x^3\text{sech}^{-1}\left(\sqrt{x}\right)-\frac{(1-x)^3}{15\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{2(1-x)^2}{9\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{1-x}{3\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSech[Sqrt[x]],x]
```

```
[Out] -(1 - x)/(3*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (2*(1 - x)^
2)/(9*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) - (1 - x)^3/(15*Sqr
t[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (x^3*ArcSech[Sqrt[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6345

```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSech[u]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1
- u^2])/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]), Int[SimplifyIntegrand[(
(c + d*x)^(m + 1)*D[u, x])/(u*Sqrt[1 - u^2]), x], x], x] /; FreeQ[{a, b, c,
 d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
+ d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\int x^2\operatorname{sech}^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{3}x^3\operatorname{sech}^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{1-x}\int\frac{x^2}{2\sqrt{1-x}}\,dx}{3\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= \frac{1}{3}x^3\operatorname{sech}^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{1-x}\int\frac{x^2}{\sqrt{1-x}}\,dx}{6\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= \frac{1}{3}x^3\operatorname{sech}^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{1-x}\int\left(\frac{1}{\sqrt{1-x}}-2\sqrt{1-x}+(1-x)^{3/2}\right)dx}{6\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= -\frac{1-x}{3\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{2(1-x)^2}{9\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} - \frac{(1-x)^3}{15\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.57

$$\frac{1}{3}x^3\operatorname{sech}^{-1}\left(\sqrt{x}\right) - \frac{1}{45}\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}}\left(3x^{5/2}+4x^{3/2}+3x^2+4x+8\sqrt{x}+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSech[Sqrt[x]],x]
[Out] -1/45*(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(8 + 8*Sqrt[x] + 4*x + 4*x^(3/2) +
 3*x^2 + 3*x^(5/2))) + (x^3*ArcSech[Sqrt[x]])/3
```

fricas [A] time = 0.52, size = 52, normalized size = 0.41

$$\frac{1}{3}\,x^3\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right) - \frac{1}{45}\left(3\,x^2+4\,x+8\right)\sqrt{x}\sqrt{-\frac{x-1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(x^(1/2)),x, algorithm="fricas")
[Out] 1/3*x^3*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - 1/45*(3*x^2 + 4*x + 8)*sqrt
(x)*sqrt(-(x - 1)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{arsech}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(x^(1/2)),x, algorithm="giac")
[Out] integrate(x^2*arcsech(sqrt(x)), x)
```

maple [A] time = 0.07, size = 49, normalized size = 0.39

$$\frac{x^3\operatorname{arcsech}\left(\sqrt{x}\right)}{3} - \frac{\sqrt{-\frac{-1+\sqrt{x}}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{1+\sqrt{x}}{\sqrt{x}}}\,\left(3x^2+4x+8\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsech(x^(1/2)),x)
```

```
[Out] 1/3*x^3*arcsech(x^(1/2))-1/45*(-(-1+x^(1/2))/x^(1/2))^(1/2)*x^(1/2)*((1+x^(
1/2))/x^(1/2))^(1/2)*(3*x^2+4*x+8)
```

maxima [A] time = 0.34, size = 46, normalized size = 0.37

$$-\frac{1}{15}\,x^{\frac{5}{2}}\left(\frac{1}{x}-1\right)^{\frac{5}{2}}+\frac{1}{3}\,x^{3}\,\mathrm{arsech}\left(\sqrt{x}\right)+\frac{2}{9}\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}}-\frac{1}{3}\,\sqrt{x}\sqrt{\frac{1}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(x^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/15*x^(5/2)*(1/x - 1)^(5/2) + 1/3*x^3*arcsech(sqrt(x)) + 2/9*x^(3/2)*(1/x
 - 1)^(3/2) - 1/3*sqrt(x)*sqrt(1/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acosh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(1/x^(1/2)),x)
```

```
[Out] int(x^2*acosh(1/x^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\,\mathrm{asech}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asech(x**(1/2)),x)
```

```
[Out] Integral(x**2*asech(sqrt(x)), x)
```

3.22 $\int x\text{sech}^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=88

$$\frac{1}{2}x^2\text{sech}^{-1}\left(\sqrt{x}\right)+\frac{(1-x)^2}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{1-x}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

```
[Out] 1/2*x^2*arcsech(x^(1/2))+1/2*(-1+x)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/
2))^(1/2)+1/6*(1-x)^2/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6345, 12, 43}

$$\frac{1}{2}x^2\text{sech}^{-1}\left(\sqrt{x}\right)+\frac{(1-x)^2}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{1-x}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSech[Sqrt[x]],x]
```

```
[Out] -(1 - x)/(2*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (1 - x)^2/(
6*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (x^2*ArcSech[Sqrt[x]]
)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6345

```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSech[u]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1
- u^2])/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]), Int[SimplifyIntegrand[(
(c + d*x)^(m + 1)*D[u, x])/(u*Sqrt[1 - u^2]), x], x], x] /; FreeQ[{a, b, c,
 d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
+ d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\int x\operatorname{sech}^{-1}\left(\sqrt{x}\right)dx = \frac{1}{2}x^2\operatorname{sech}^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{1-x}\int \frac{x}{2\sqrt{1-x}}dx}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$
$$= \frac{1}{2}x^2\operatorname{sech}^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{1-x}\int \frac{x}{\sqrt{1-x}}dx}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$
$$= \frac{1}{2}x^2\operatorname{sech}^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{1-x}\int \left(\frac{1}{\sqrt{1-x}}-\sqrt{1-x}\right)dx}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$
$$= -\frac{1-x}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{(1-x)^2}{6\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{1}{2}x^2\operatorname{sech}^{-1}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.64

$$\frac{1}{2}x^2\operatorname{sech}^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}}\left(x^{3/2}+x+2\sqrt{x}+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSech[Sqrt[x]],x]
```

```
[Out] -1/6*(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(2 + 2*Sqrt[x] + x + x^(3/2))) + (x
^2*ArcSech[Sqrt[x]])/2
```

fricas [A] time = 0.63, size = 45, normalized size = 0.51

$$\frac{1}{2}x^2\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right) - \frac{1}{6}(x+2)\sqrt{x}\sqrt{-\frac{x-1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - 1/6*(x + 2)*sqrt(x)*sqrt(-(
x - 1)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{arsech}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x*arcsech(sqrt(x)), x)
```

maple [A] time = 0.06, size = 42, normalized size = 0.48

$$\frac{x^2\operatorname{arcsech}\left(\sqrt{x}\right)}{2} - \frac{\sqrt{-\frac{-1+\sqrt{x}}{\sqrt{x}}}\sqrt{x}\sqrt{\frac{1+\sqrt{x}}{\sqrt{x}}}(x+2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsech(x^(1/2)),x)
```

```
[Out] 1/2*x^2*arcsech(x^(1/2))-1/6*(-(-1+x^(1/2))/x^(1/2))^(1/2)*x^(1/2)*((1+x^(1
/2))/x^(1/2))^(1/2)*(x+2)
```

maxima [A] time = 0.38, size = 34, normalized size = 0.39

$$\frac{1}{6}x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}}+\frac{1}{2}x^2\operatorname{arsech}\left(\sqrt{x}\right)-\frac{1}{2}\sqrt{x}\sqrt{\frac{1}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/6*x^(3/2)*(1/x - 1)^(3/2) + 1/2*x^2*arcsech(sqrt(x)) - 1/2*sqrt(x)*sqrt(1
/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(1/x^(1/2)),x)
```

```
[Out] int(x*acosh(1/x^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{asech}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asech(x**(1/2)),x)
```

```
[Out] Integral(x*asech(sqrt(x)), x)
```

3.23 $\int \operatorname{sech}^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=43

$$x\operatorname{sech}^{-1}\left(\sqrt{x}\right)-\frac{1-x}{\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

```
[Out] x*arcsech(x^(1/2))+(-1+x)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6343, 12, 32}

$$x\operatorname{sech}^{-1}\left(\sqrt{x}\right)-\frac{1-x}{\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[Sqrt[x]],x]
```

```
[Out] -((1 - x)/(Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x])) + x*ArcSech[S
qrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6343

```
Int[ArcSech[u_], x_Symbol] :> Simp[x*ArcSech[u], x] + Dist[Sqrt[1 - u^2]/(u
*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]), Int[SimplifyIntegrand[(x*D[u, x])/(u*Sqrt[1
 - u^2]), x], x], x] /; InverseFunctionFreeQ[u, x] &&  !FunctionOfExponenti
alQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^{-1}\left(\sqrt{x}\right) dx &= x\operatorname{sech}^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{1-x}\int \frac{1}{2\sqrt{1-x}}\,dx}{\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}\\ &= x\operatorname{sech}^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}}\,dx}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}\\ &= -\frac{1-x}{\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}+x\operatorname{sech}^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.10, size = 67, normalized size = 1.56

$$x\text{sech}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}}\sqrt{\sqrt{x}+1}\sqrt{1-x}}{\sqrt{1-\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[Sqrt[x]],x]
```

```
[Out] -((Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*Sqrt[1 + Sqrt[x]]*Sqrt[1 - x])/Sqrt[1
- Sqrt[x]]) + x*ArcSech[Sqrt[x]]
```

fricas [A] time = 0.56, size = 39, normalized size = 0.91

$$x\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)-\sqrt{x}\sqrt{-\frac{x-1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2)),x, algorithm="fricas")
```

```
[Out] x*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - sqrt(x)*sqrt(-(x - 1)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\text{arsech}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(arcsech(sqrt(x)), x)
```

maple [A] time = 0.06, size = 36, normalized size = 0.84

$$x\,\text{arcsech}\left(\sqrt{x}\right)-\sqrt{-\frac{-1+\sqrt{x}}{\sqrt{x}}}\,\sqrt{\frac{1+\sqrt{x}}{\sqrt{x}}}\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(x^(1/2)),x)
```

```
[Out] x*arcsech(x^(1/2))-(-(-1+x^(1/2))/x^(1/2))^(1/2)*((1+x^(1/2))/x^(1/2))^(1/2
)*x^(1/2)
```

maxima [A] time = 0.39, size = 19, normalized size = 0.44

$$x\,\text{arsech}\left(\sqrt{x}\right)-\sqrt{x}\sqrt{\frac{1}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arcsech(sqrt(x)) - sqrt(x)*sqrt(1/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\text{acosh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/x^(1/2)),x)
```

```
[Out] int(acosh(1/x^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asech}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(x**(1/2)),x)
```

```
[Out] Integral(asech(sqrt(x)), x)
```

3.24 $\int \frac{\text{sech}^{-1}\left(\sqrt{x}\right)}{x}\,dx$

Optimal. Leaf size=46

$$-\text{Li}_2\left(-e^{2\text{sech}^{-1}\left(\sqrt{x}\right)}\right)+\text{sech}^{-1}\left(\sqrt{x}\right)^2-2\text{sech}^{-1}\left(\sqrt{x}\right)\log\left(e^{2\text{sech}^{-1}\left(\sqrt{x}\right)}+1\right)$$

```
[Out] arcsech(x^(1/2))^2-2*arcsech(x^(1/2))*ln(1+(1/x^(1/2)+(-1+1/x^(1/2))^(1/2)*
(1+1/x^(1/2))^(1/2))^2)-polylog(2,-(1/x^(1/2)+(-1+1/x^(1/2))^(1/2)*(1+1/x^(
1/2))^(1/2))^2)
```

Rubi [A] time = 0.10, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6281, 5660, 3718, 2190, 2279, 2391}

$$-\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}\left(\sqrt{x}\right)}\right)+\text{sech}^{-1}\left(\sqrt{x}\right)^2-2\text{sech}^{-1}\left(\sqrt{x}\right)\log\left(e^{2\text{sech}^{-1}\left(\sqrt{x}\right)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[Sqrt[x]]/x,x]
```

```
[Out] ArcSech[Sqrt[x]]^2 - 2*ArcSech[Sqrt[x]]*Log[1 + E^(2*ArcSech[Sqrt[x]])] - P
olyLog[2, -E^(2*ArcSech[Sqrt[x]])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6281

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{\operatorname{sech}^{-1}(x)}{x}\,dx, x, \sqrt{x}\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\left(2\,\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)\right)\right)\\
&= \cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 4\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)\right)\\
&= \cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) + 2\,\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{c}\right.\\
&= \cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) + \text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\,\text{cosl}}\right.\\
&= \cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) - \text{Li}_2\left(-e^{2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.98

$$\text{Li}_2\left(-e^{-2\operatorname{sech}^{-1}\left(\sqrt{x}\right)}\right) - \operatorname{sech}^{-1}\left(\sqrt{x}\right)\left(\operatorname{sech}^{-1}\left(\sqrt{x}\right) + 2\log\left(e^{-2\operatorname{sech}^{-1}\left(\sqrt{x}\right)}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[Sqrt[x]]/x,x]
[Out] -(ArcSech[Sqrt[x]]*(ArcSech[Sqrt[x]] + 2*Log[1 + E^(-2*ArcSech[Sqrt[x]])]))
 + PolyLog[2, -E^(-2*ArcSech[Sqrt[x]])]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsech}\left(\sqrt{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x,x, algorithm="fricas")
[Out] integral(arcsech(sqrt(x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsech}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x,x, algorithm="giac")
[Out] integrate(arcsech(sqrt(x))/x, x)
```

maple [A] time = 0.10, size = 65, normalized size = 1.41

$$\operatorname{arcsech}\left(\sqrt{x}\right)^{2}-2\operatorname{arcsech}\left(\sqrt{x}\right)\ln\left(1+\left(\frac{1}{\sqrt{x}}+\sqrt{-1+\frac{1}{\sqrt{x}}}\,\sqrt{1+\frac{1}{\sqrt{x}}}\right)^{2}\right)-\operatorname{polylog}\left(2,-\left(\frac{1}{\sqrt{x}}+\sqrt{-1+\frac{1}{\sqrt{x}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(x^(1/2))/x,x)
```

```
[Out] arcsech(x^(1/2))^2-2*arcsech(x^(1/2))*ln(1+(1/x^(1/2)+(-1+1/x^(1/2))^(1/2)*
(1+1/x^(1/2))^(1/2))^2)-polylog(2,-(1/x^(1/2)+(-1+1/x^(1/2))^(1/2)*(1+1/x^(
1/2))^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\log(x)^{2}+\log(x)\log\left(\sqrt{\sqrt{x}+1}\,\sqrt{-\sqrt{x}+1}+1\right)-\log\left(\sqrt{x}+1\right)\log\left(\sqrt{x}\right)-\log\left(\sqrt{x}\right)\log\left(-\sqrt{x}+1\right)-\operatorname{Li}_2\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] -1/4*log(x)^2 + log(x)*log(sqrt(sqrt(x) + 1)*sqrt(-sqrt(x) + 1) + 1) - log(
sqrt(x) + 1)*log(sqrt(x)) - log(sqrt(x))*log(-sqrt(x) + 1) - dilog(-sqrt(x)
) - dilog(sqrt(x)) + integrate(1/2*log(x)/((x - 1)*e^(1/2*log(sqrt(x) + 1)
+ 1/2*log(-sqrt(x) + 1)) + x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/x^(1/2))/x,x)
```

```
[Out] int(acosh(1/x^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(x**(1/2))/x,x)
```

```
[Out] Integral(asech(sqrt(x))/x, x)
```

3.25 $\int \frac{\mathrm{sech}^{-1}(\sqrt{x})}{x^2}\,dx$

Optimal. Leaf size=98

$$\frac{1-x}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{3/2}}+\frac{\sqrt{1-x}\tanh^{-1}\left(\sqrt{1-x}\right)}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,\sqrt{x}}-\frac{\mathrm{sech}^{-1}\left(\sqrt{x}\right)}{x}$$

```
[Out] -arcsech(x^(1/2))/x+1/2*(1-x)/x^(3/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1
/2)+1/2*arctanh((1-x)^(1/2))*(1-x)^(1/2)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/
x^(1/2))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6345, 12, 51, 63, 206}

$$\frac{1-x}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{3/2}}+\frac{\sqrt{1-x}\tanh^{-1}\left(\sqrt{1-x}\right)}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,\sqrt{x}}-\frac{\mathrm{sech}^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[Sqrt[x]]/x^2,x]
```

```
[Out] (1 - x)/(2*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(3/2)) - ArcSech[Sqrt
[x]]/x + (Sqrt[1 - x]*ArcTanh[Sqrt[1 - x]])/(2*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1
+ 1/Sqrt[x]]*Sqrt[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6345

```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSech[u]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1
- u^2])/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]), Int[SimplifyIntegrand[(
(c + d*x)^(m + 1)*D[u, x])/(u*Sqrt[1 - u^2]), x], x], x] /; FreeQ[{a, b, c,
 d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
+ d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x}-\frac{\sqrt{1-x}\int \frac{1}{2\sqrt{1-x}\,x^2}\,dx}{\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}\\
&= -\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x}-\frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}\,x^2}\,dx}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}\\
&= \frac{1-x}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x}-\frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}\,x}\,dx}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}\\
&= \frac{1-x}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{1-x}\,\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x}\right)}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}\\
&= \frac{1-x}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{1-x}\tanh^{-1}\left(\sqrt{1-x}\right)}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 1.13

$$\frac{\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}}\left(\sqrt{x}+1\right)+x\log\left(\sqrt{x}\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}}+\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}}+1\right)-\frac{1}{2}x\log(x)-2\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[Sqrt[x]]/x^2,x]
```

```
[Out] (Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(1 + Sqrt[x]) - 2*ArcSech[Sqrt[x]] + x*L
og[1 + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])] + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])
]*Sqrt[x]] - (x*Log[x])/2)/(2*x)
```

fricas [A] time = 0.61, size = 45, normalized size = 0.46

$$\frac{(x-2)\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)+\sqrt{x}\sqrt{-\frac{x-1}{x}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x - 2)*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) + sqrt(x)*sqrt(-(x - 1)/
x))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsech(sqrt(x))/x^2, x)
```

maple [A] time = 0.07, size = 64, normalized size = 0.65

$$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{-\frac{-1+\sqrt{x}}{\sqrt{x}}}\,\sqrt{\frac{1+\sqrt{x}}{\sqrt{x}}}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x+\sqrt{1-x}\right)}{2\sqrt{x}\,\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(x^(1/2))/x^2,x)
```

```
[Out] -arcsech(x^(1/2))/x+1/2*(-(-1+x^(1/2))/x^(1/2))^(1/2)/x^(1/2)*((1+x^(1/2))/
x^(1/2))^(1/2)*(arctanh(1/(1-x)^(1/2))*x+(1-x)^(1/2))/(1-x)^(1/2)
```

maxima [A] time = 0.30, size = 65, normalized size = 0.66

$$-\frac{\sqrt{x}\sqrt{\frac{1}{x}-1}}{2\left(x\left(\frac{1}{x}-1\right)-1\right)}-\frac{\operatorname{arsech}\left(\sqrt{x}\right)}{x}+\frac{1}{4}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}+1\right)-\frac{1}{4}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(x)*sqrt(1/x - 1)/(x*(1/x - 1) - 1) - arcsech(sqrt(x))/x + 1/4*log
(sqrt(x)*sqrt(1/x - 1) + 1) - 1/4*log(sqrt(x)*sqrt(1/x - 1) - 1)
```

mupad [B] time = 2.15, size = 40, normalized size = 0.41

$$\frac{\sqrt{\frac{1}{\sqrt{x}}-1}\,\sqrt{\frac{1}{\sqrt{x}}+1}}{2\sqrt{x}}-\frac{2\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)\left(\frac{1}{2\sqrt{x}}-\frac{\sqrt{x}}{4}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/x^(1/2))/x^2,x)
```

```
[Out] ((1/x^(1/2) - 1)^(1/2)*(1/x^(1/2) + 1)^(1/2))/(2*x^(1/2)) - (2*acosh(1/x^(1
/2))*(1/(2*x^(1/2)) - x^(1/2)/4))/x^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(x**(1/2))/x**2,x)
```

```
[Out] Integral(asech(sqrt(x))/x**2, x)
```

3.26 $\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^3} \, dx$

Optimal. Leaf size=136

$$\frac{3(1-x)}{16\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{3/2}}+\frac{1-x}{8\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{5/2}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3\sqrt{1-x}\tanh^{-1}\left(\sqrt{1-x}\right)}{16\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

```
[Out] -1/2*arcsech(x^(1/2))/x^2+1/8*(1-x)/x^(5/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/
2))^(1/2)+3/16*(1-x)/x^(3/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)+3/16*
arctanh((1-x)^(1/2))*(1-x)^(1/2)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))
^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6345, 12, 51, 63, 206}

$$\frac{3(1-x)}{16\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{3/2}}+\frac{1-x}{8\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{5/2}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3\sqrt{1-x}\tanh^{-1}\left(\sqrt{1-x}\right)}{16\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[Sqrt[x]]/x^3,x]
```

```
[Out] (1 - x)/(8*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(5/2)) + (3*(1 - x))/
(16*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(3/2)) - ArcSech[Sqrt[x]]/(2
*x^2) + (3*Sqrt[1 - x]*ArcTanh[Sqrt[1 - x]])/(16*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[
1 + 1/Sqrt[x]]*Sqrt[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6345

```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSech[u]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1
- u^2])/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]), Int[SimplifyIntegrand[(
(c + d*x)^(m + 1)*D[u, x])/(u*Sqrt[1 - u^2]), x], x], x] /; FreeQ[{a, b, c,
 d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
+ d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{\sqrt{1-x}\int \frac{1}{2\sqrt{1-x}\,x^3}\,dx}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= -\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}\,x^3}\,dx}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= \frac{1-x}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} - \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{\left(3\sqrt{1-x}\right)\int \frac{1}{\sqrt{1-x}\,x^2}\,dx}{16\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= \frac{1-x}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} + \frac{3(1-x)}{16\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}} - \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{\left(3\sqrt{1-x}\right)}{32\sqrt{-1+\frac{1}{\sqrt{x}}}}$$

$$= \frac{1-x}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} + \frac{3(1-x)}{16\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}} - \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{\left(3\sqrt{1-x}\right)\mathrm{Su}}{16\sqrt{-1}}$$

$$= \frac{1-x}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} + \frac{3(1-x)}{16\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}} - \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{3\sqrt{1-x}\,\tanh}{16\sqrt{-1+\frac{1}{\sqrt{x}}}}$$

Mathematica [A] time = 0.12, size = 125, normalized size = 0.92

$$\frac{1}{16}\left(-\frac{8\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x^2} + \frac{\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}}\left(3x^{3/2}+3x+2\sqrt{x}+2\right)}{x^2} + 3\log\left(\sqrt{x}\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}} + \sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}} + 1\right) - \frac{3\log(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[Sqrt[x]]/x^3,x]
```

```
[Out] ((Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(2 + 2*Sqrt[x] + 3*x + 3*x^(3/2)))/x^2
- (8*ArcSech[Sqrt[x]])/x^2 + 3*Log[1 + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])] +
Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]] - (3*Log[x])/2)/16
```

fricas [A] time = 1.09, size = 54, normalized size = 0.40

$$\frac{(3x+2)\sqrt{x}\sqrt{-\frac{x-1}{x}} + \left(3x^2-8\right)\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*((3*x + 2)*sqrt(x)*sqrt(-(x - 1)/x) + (3*x^2 - 8)*log((x*sqrt(-(x - 1)
/x) + sqrt(x))/x))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsech(sqrt(x))/x^3, x)
```

maple [A] time = 0.07, size = 79, normalized size = 0.58

$$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{-\frac{-1+\sqrt{x}}{\sqrt{x}}}\,\sqrt{\frac{1+\sqrt{x}}{\sqrt{x}}}\,\left(3\operatorname{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x^2+3x\sqrt{1-x}\,+2\sqrt{1-x}\right)}{16x^{\frac{3}{2}}\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(x^(1/2))/x^3,x)
```

```
[Out] -1/2*arcsech(x^(1/2))/x^2+1/16*(-(-1+x^(1/2))/x^(1/2))^(1/2)/x^(3/2)*((1+x^
(1/2))/x^(1/2))^(1/2)*(3*arctanh(1/(1-x)^(1/2))*x^2+3*x*(1-x)^(1/2)+2*(1-x)
^(1/2))/(1-x)^(1/2)
```

maxima [A] time = 0.34, size = 92, normalized size = 0.68

$$-\frac{3\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}}-5\,\sqrt{x}\,\sqrt{\frac{1}{x}-1}}{16\left(x^2\left(\frac{1}{x}-1\right)^2-2\,x\left(\frac{1}{x}-1\right)+1\right)}-\frac{\operatorname{arsech}\left(\sqrt{x}\right)}{2\,x^2}+\frac{3}{32}\,\log\left(\sqrt{x}\,\sqrt{\frac{1}{x}-1}\,+1\right)-\frac{3}{32}\,\log\left(\sqrt{x}\,\sqrt{\frac{1}{x}-1}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*x^(3/2)*(1/x - 1)^(3/2) - 5*sqrt(x)*sqrt(1/x - 1))/(x^2*(1/x - 1)^
2 - 2*x*(1/x - 1) + 1) - 1/2*arcsech(sqrt(x))/x^2 + 3/32*log(sqrt(x)*sqrt(1
/x - 1) + 1) - 3/32*log(sqrt(x)*sqrt(1/x - 1) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/x^(1/2))/x^3,x)
```

```
[Out] int(acosh(1/x^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(x**(1/2))/x**3,x)
```

```
[Out] Integral(asech(sqrt(x))/x**3, x)
```

3.27 $\int \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x^4} \, dx$

Optimal. Leaf size=172

$$\frac{5(1-x)}{48\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{3/2}}+\frac{5(1-x)}{72\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{5/2}}+\frac{1-x}{18\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{7/2}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5\sqrt{1-x}\,\mathrm{t}}{48\sqrt{\frac{1}{\sqrt{x}}-}}$$

```
[Out] -1/3*arcsech(x^(1/2))/x^3+1/18*(1-x)/x^(7/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1
/2))^(1/2)+5/72*(1-x)/x^(5/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)+5/48
*(1-x)/x^(3/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)+5/48*arctanh((1-x)^
(1/2))*(1-x)^(1/2)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6345, 12, 51, 63, 206}

$$\frac{5(1-x)}{48\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{3/2}}+\frac{5(1-x)}{72\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{5/2}}+\frac{1-x}{18\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{7/2}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5\sqrt{1-x}\,\mathrm{t}}{48\sqrt{\frac{1}{\sqrt{x}}-}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[Sqrt[x]]/x^4,x]
```

```
[Out] (1 - x)/(18*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(7/2)) + (5*(1 - x))
/(72*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(5/2)) + (5*(1 - x))/(48*Sq
rt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(3/2)) - ArcSech[Sqrt[x]]/(3*x^3)
+ (5*Sqrt[1 - x]*ArcTanh[Sqrt[1 - x]])/(48*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/
Sqrt[x]]*Sqrt[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6345

```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcSech[u]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1
- u^2])/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]), Int[SimplifyIntegrand[(
(c + d*x)^(m + 1)*D[u, x])/(u*Sqrt[1 - u^2]), x], x], x] /; FreeQ[{a, b, c,
 d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
+ d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x^4}\,dx = -\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{3x^3} - \frac{\sqrt{1-x}\int \frac{1}{2\sqrt{1-x}\,x^4}\,dx}{3\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= -\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{3x^3} - \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}\,x^4}\,dx}{6\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{7/2}} - \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{3x^3} - \frac{\left(5\sqrt{1-x}\right)\int \frac{1}{\sqrt{1-x}\,x^3}\,dx}{36\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{7/2}} + \frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} - \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{3x^3} - \frac{\left(5\sqrt{1-x}\right)}{48\sqrt{-1+\frac{1}{\sqrt{x}}}}$$

$$= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{7/2}} + \frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} + \frac{5(1-x)}{48\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}}$$

$$= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{7/2}} + \frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} + \frac{5(1-x)}{48\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}}$$

$$= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{7/2}} + \frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} + \frac{5(1-x)}{48\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}}$$

Mathematica [A] time = 0.13, size = 140, normalized size = 0.81

$$\frac{15x^3\log\left(\sqrt{x}\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}}+\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}}+1\right)-\frac{15}{2}x^3\log(x)+\sqrt{\frac{1-\sqrt{x}}{\sqrt{x}+1}}\left(15x^{5/2}+10x^{3/2}+15x^2+10x+8\sqrt{x}+8\right)-48\text{se}}{144x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[Sqrt[x]]/x^4,x]
```

```
[Out] (Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(8 + 8*Sqrt[x] + 10*x + 10*x^(3/2) + 15*
x^2 + 15*x^(5/2)) - 48*ArcSech[Sqrt[x]] + 15*x^3*Log[1 + Sqrt[(1 - Sqrt[x])
/(1 + Sqrt[x])] + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]] - (15*x^3*Log[
x])/2)/(144*x^3)
```

fricas [A] time = 1.39, size = 60, normalized size = 0.35

$$\frac{\left(15\,x^{2}+10\,x+8\right)\sqrt{x}\sqrt{-\frac{x-1}{x}}+3\left(5\,x^{3}-16\right)\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)}{144\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/144*((15*x^2 + 10*x + 8)*sqrt(x)*sqrt(-(x - 1)/x) + 3*(5*x^3 - 16)*log((x
*sqrt(-(x - 1)/x) + sqrt(x))/x))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}\left(\sqrt{x}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsech(sqrt(x))/x^4, x)
```

maple [A] time = 0.07, size = 91, normalized size = 0.53

$$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{-\frac{-1+\sqrt{x}}{\sqrt{x}}}\,\sqrt{\frac{1+\sqrt{x}}{\sqrt{x}}}\,\left(15\operatorname{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x^{3}+15\sqrt{1-x}\,x^{2}+10x\sqrt{1-x}+8\sqrt{1-x}\right)}{144x^{\frac{5}{2}}\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(x^(1/2))/x^4,x)
```

```
[Out] -1/3*arcsech(x^(1/2))/x^3+1/144*(-(-1+x^(1/2))/x^(1/2))^(1/2)/x^(5/2)*((1+x
^(1/2))/x^(1/2))^(1/2)*(15*arctanh(1/(1-x)^(1/2))*x^3+15*(1-x)^(1/2)*x^2+10
*x*(1-x)^(1/2)+8*(1-x)^(1/2))/(1-x)^(1/2)
```

maxima [A] time = 0.32, size = 116, normalized size = 0.67

$$-\frac{15\,x^{\frac{5}{2}}\left(\frac{1}{x}-1\right)^{\frac{5}{2}}-40\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}}+33\,\sqrt{x}\sqrt{\frac{1}{x}-1}}{144\left(x^{3}\left(\frac{1}{x}-1\right)^{3}-3\,x^{2}\left(\frac{1}{x}-1\right)^{2}+3\,x\left(\frac{1}{x}-1\right)-1\right)}-\frac{\operatorname{arsech}\left(\sqrt{x}\right)}{3\,x^{3}}+\frac{5}{96}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}+1\right)-\frac{5}{96}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/144*(15*x^(5/2)*(1/x - 1)^(5/2) - 40*x^(3/2)*(1/x - 1)^(3/2) + 33*sqrt(x
)*sqrt(1/x - 1))/(x^3*(1/x - 1)^3 - 3*x^2*(1/x - 1)^2 + 3*x*(1/x - 1) - 1)
- 1/3*arcsech(sqrt(x))/x^3 + 5/96*log(sqrt(x)*sqrt(1/x - 1) + 1) - 5/96*log
(sqrt(x)*sqrt(1/x - 1) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/x^(1/2))/x^4,x)
```

```
[Out] int(acosh(1/x^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}\left(\sqrt{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(x**(1/2))/x**4,x)
```

```
[Out] Integral(asech(sqrt(x))/x**4, x)
```

3.28 $\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right) dx$

Optimal. Leaf size=21

$$x\cosh^{-1}(x)-\sqrt{x-1}\sqrt{x+1}$$

```
[Out] x*arccosh(x)-(-1+x)^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6327, 5654, 74}

$$x\cosh^{-1}(x)-\sqrt{x-1}\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[x^(-1)],x]
```

```
[Out] -(Sqrt[-1 + x]*Sqrt[1 + x]) + x*ArcCosh[x]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 6327

```
Int[ArcSech[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCosh[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right) dx &= \int \cosh^{-1}(x)\,dx\\ &= x\cosh^{-1}(x)-\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}}\,dx\\ &= -\sqrt{-1+x}\sqrt{1+x}+x\cosh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.19

$$x\operatorname{sech}^{-1}\left(\frac{1}{x}\right)-\frac{x-1}{\sqrt{\frac{x-1}{x+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[x^(-1)],x]
```

```
[Out] -((-1 + x)/Sqrt[(-1 + x)/(1 + x)]) + x*ArcSech[x^(-1)]
```

fricas [A]   time = 0.63, size = 22, normalized size = 1.05

$$x \log\left(x + \sqrt{x^2 - 1}\right) - \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(1/x),x, algorithm="fricas")
[Out] x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsech}\left(\frac{1}{x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(1/x),x, algorithm="giac")
[Out] integrate(arcsech(1/x), x)
```

maple [A]   time = 0.06, size = 29, normalized size = 1.38

$$x \operatorname{arcsech}\left(\frac{1}{x}\right) - \sqrt{-\left(\frac{1}{x} - 1\right)x}\,\sqrt{\left(\frac{1}{x} + 1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(1/x),x)
[Out] x*arcsech(1/x)-(-(1/x-1)*x)^(1/2)*((1/x+1)*x)^(1/2)
```

maxima [A]   time = 0.32, size = 16, normalized size = 0.76

$$x \operatorname{arsech}\left(\frac{1}{x}\right) - \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(1/x),x, algorithm="maxima")
[Out] x*arcsech(1/x) - sqrt(x^2 - 1)
```

mupad [B]   time = 1.35, size = 17, normalized size = 0.81

$$x \operatorname{acosh}(x) - \sqrt{x - 1}\,\sqrt{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x),x)
[Out] x*acosh(x) - (x - 1)^(1/2)*(x + 1)^(1/2)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asech}\left(\frac{1}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(1/x),x)
[Out] Integral(asech(1/x), x)
```

3.29 $\int \frac{\operatorname{sech}^{-1}(ax^n)}{x} dx$

Optimal. Leaf size=61

$$-\frac{\text{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax^n)}\right)}{2n}+\frac{\operatorname{sech}^{-1}(ax^n)^2}{2n}-\frac{\operatorname{sech}^{-1}(ax^n)\log\left(e^{2\operatorname{sech}^{-1}(ax^n)}+1\right)}{n}$$

```
[Out] 1/2*arcsech(a*x^n)^2/n-arcsech(a*x^n)*ln(1+(1/a/(x^n)+(1/a/(x^n)-1)^(1/2)*(
1/a/(x^n)+1)^(1/2))^2)/n-1/2*polylog(2,-(1/a/(x^n)+(1/a/(x^n)-1)^(1/2)*(1/a
/(x^n)+1)^(1/2))^2)/n
```

Rubi [A] time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6281, 5660, 3718, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(ax^n)}\right)}{2n}+\frac{\operatorname{sech}^{-1}(ax^n)^2}{2n}-\frac{\operatorname{sech}^{-1}(ax^n)\log\left(e^{2\operatorname{sech}^{-1}(ax^n)}+1\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x^n]/x,x]
```

```
[Out] ArcSech[a*x^n]^2/(2*n) - (ArcSech[a*x^n]*Log[1 + E^(2*ArcSech[a*x^n])])/n -
 PolyLog[2, -E^(2*ArcSech[a*x^n])]/(2*n)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6281

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(ax^n)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\operatorname{sech}^{-1}(ax)}{x}\,dx, x, x^n\right)}{n}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)}{x}\,dx, x, x^{-n}\right)}{n}\\
&= -\frac{\operatorname{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n}\\
&= \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{2\operatorname{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n}\\
&= \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} + \frac{\operatorname{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \cosh^{-1}\right.}{n}\\
&= \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} + \frac{\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cosh^{-1}\left(\frac{x^{-n}}{a}\right.}\right.}{2n}\\
&= \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} - \frac{\operatorname{Li}_2\left(-e^{2\cosh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{2n}
\end{aligned}$$

Mathematica [B] time = 0.98, size = 219, normalized size = 3.59

$$\frac{\sqrt{\frac{1-ax^n}{ax^n+1}}\left(\sqrt{1-a^2x^{2n}}\left(-4\operatorname{Li}_2\left(\frac{1}{2}-\frac{1}{2}\sqrt{1-a^2x^{2n}}\right)+\log^2\left(a^2x^{2n}\right)+2\log^2\left(\frac{1}{2}\left(\sqrt{1-a^2x^{2n}}+1\right)\right)-4\log\left(\frac{1}{2}\left(\sqrt{1-a\right.\right.\right.\right.}{8\left(n-anx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a*x^n]/x,x]
```

```
[Out] ArcSech[a*x^n]*Log[x] + (Sqrt[(1 - a*x^n)/(1 + a*x^n)]*(4*Sqrt[-1 + a^2*x^(
2*n)]*ArcTan[Sqrt[-1 + a^2*x^(2*n)]]*(2*n*Log[x] - Log[a^2*x^(2*n)]) + Sqrt
[1 - a^2*x^(2*n)]*(Log[a^2*x^(2*n)]^2 - 4*Log[a^2*x^(2*n)]*Log[(1 + Sqrt[1
- a^2*x^(2*n)])/2] + 2*Log[(1 + Sqrt[1 - a^2*x^(2*n)])/2]^2 - 4*PolyLog[2,
1/2 - Sqrt[1 - a^2*x^(2*n)]/2])))/(8*(n - a*n*x^n))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x^n)/x, x)
```

maple [A] time = 0.15, size = 116, normalized size = 1.90

$$\frac{\operatorname{arcsech}(a x^n)^2}{2n} - \frac{\operatorname{arcsech}(a x^n)\ln\left(1+\left(\frac{x^{-n}}{a}+\sqrt{\frac{x^{-n}}{a}-1}\,\sqrt{\frac{x^{-n}}{a}+1}\right)^2\right)}{n} - \frac{\operatorname{polylog}\left(2,-\left(\frac{x^{-n}}{a}+\sqrt{\frac{x^{-n}}{a}-1}\,\sqrt{\frac{x^{-n}}{a}+\right.\right.}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x^n)/x,x)
```

```
[Out] 1/2*arcsech(a*x^n)^2/n-arcsech(a*x^n)*ln(1+(1/a/(x^n)+(1/a/(x^n)-1)^(1/2)*(
1/a/(x^n)+1)^(1/2))^2)/n-1/2*polylog(2,-(1/a/(x^n)+(1/a/(x^n)-1)^(1/2)*(1/a
/(x^n)+1)^(1/2))^2)/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 n \int \frac{x^{2n}\log(x)}{a^2 x x^{2n} + \left(a^2 x x^{2n} - x\right)\sqrt{a x^n + 1}\,\sqrt{-a x^n + 1} - x}\,dx + n\int \frac{\log(x)}{2\,(a x x^n + x)}\,dx - n\int \frac{\log(x)}{2\,(a x x^n - x)}\,dx + \log\left(\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] a^2*n*integrate(x^(2*n)*log(x)/(a^2*x*x^(2*n) + (a^2*x*x^(2*n) - x)*sqrt(a*
x^n + 1)*sqrt(-a*x^n + 1) - x), x) + n*integrate(1/2*log(x)/(a*x*x^n + x),
x) - n*integrate(1/2*log(x)/(a*x*x^n - x), x) + log(sqrt(a*x^n + 1)*sqrt(-a
*x^n + 1) + 1)*log(x) - log(a)*log(x) - log(x)*log(x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a\,x^n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a*x^n))/x,x)
```

```
[Out] int(acosh(1/(a*x^n))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}(a x^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x**n)/x,x)
```

```
[Out] Integral(asech(a*x**n)/x, x)
```

3.30 $\int \frac{\text{sech}^{-1}(ax^5)}{x}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{10}\text{Li}_2\left(-e^{2\text{sech}^{-1}(ax^5)}\right)+\frac{1}{10}\text{sech}^{-1}\left(ax^5\right)^2-\frac{1}{5}\text{sech}^{-1}\left(ax^5\right)\log\left(e^{2\text{sech}^{-1}(ax^5)}+1\right)$$

```
[Out] 1/10*arcsech(a*x^5)^2-1/5*arcsech(a*x^5)*ln(1+(1/a/x^5+(1/a/x^5-1)^(1/2)*(1
/a/x^5+1)^(1/2))^2)-1/10*polylog(2,-(1/a/x^5+(1/a/x^5-1)^(1/2)*(1/a/x^5+1)^
(1/2))^2)
```

Rubi [A] time = 0.11, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6281, 5660, 3718, 2190, 2279, 2391}

$$-\frac{1}{10}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax^5)}\right)+\frac{1}{10}\text{sech}^{-1}\left(ax^5\right)^2-\frac{1}{5}\text{sech}^{-1}\left(ax^5\right)\log\left(e^{2\text{sech}^{-1}(ax^5)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x^5]/x,x]
```

```
[Out] ArcSech[a*x^5]^2/10 - (ArcSech[a*x^5]*Log[1 + E^(2*ArcSech[a*x^5])])/5 - Po
lyLog[2, -E^(2*ArcSech[a*x^5])]/10
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6281

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}^{-1}\left(ax^5\right)}{x}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{\text{sech}^{-1}(ax)}{x}\,dx, x, x^5\right)\\
&= -\left(\frac{1}{5}\,\text{Subst}\left(\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)}{x}\,dx, x, \frac{1}{x^5}\right)\right)\\
&= -\left(\frac{1}{5}\,\text{Subst}\left(\int x\tanh(x)\,dx, x, \text{sech}^{-1}\left(ax^5\right)\right)\right)\\
&= \frac{1}{10}\text{sech}^{-1}\left(ax^5\right)^2 - \frac{2}{5}\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \text{sech}^{-1}\left(ax^5\right)\right)\\
&= \frac{1}{10}\text{sech}^{-1}\left(ax^5\right)^2 - \frac{1}{5}\text{sech}^{-1}\left(ax^5\right)\log\left(1+e^{2\text{sech}^{-1}\left(ax^5\right)}\right) + \frac{1}{5}\,\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx,\right.\\
&= \frac{1}{10}\text{sech}^{-1}\left(ax^5\right)^2 - \frac{1}{5}\text{sech}^{-1}\left(ax^5\right)\log\left(1+e^{2\text{sech}^{-1}\left(ax^5\right)}\right) + \frac{1}{10}\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x\right.\\
&= \frac{1}{10}\text{sech}^{-1}\left(ax^5\right)^2 - \frac{1}{5}\text{sech}^{-1}\left(ax^5\right)\log\left(1+e^{2\text{sech}^{-1}\left(ax^5\right)}\right) - \frac{1}{10}\text{Li}_2\left(-e^{2\text{sech}^{-1}\left(ax^5\right)}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.91

$$\frac{1}{10}\left(\text{Li}_2\left(-e^{-2\text{sech}^{-1}\left(ax^5\right)}\right) - \text{sech}^{-1}\left(ax^5\right)\left(\text{sech}^{-1}\left(ax^5\right) + 2\log\left(e^{-2\text{sech}^{-1}\left(ax^5\right)}+1\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a*x^5]/x,x]
[Out] (-(ArcSech[a*x^5]*(ArcSech[a*x^5] + 2*Log[1 + E^(-2*ArcSech[a*x^5])])) + Po
lyLog[2, -E^(-2*ArcSech[a*x^5])])/10
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsech}\left(ax^5\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x^5)/x,x, algorithm="fricas")
[Out] integral(arcsech(a*x^5)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsech}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x^5)/x,x, algorithm="giac")
[Out] integrate(arcsech(a*x^5)/x, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\text{arcsech}\left(a\,x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x^5)/x,x)
[Out] int(arcsech(a*x^5)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsech}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x^5)/x,x, algorithm="maxima")
[Out] integrate(arcsech(a*x^5)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a\,x^5}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a*x^5))/x,x)
[Out] int(acosh(1/(a*x^5))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x**5)/x,x)
[Out] Integral(asech(a*x**5)/x, x)
```

3.31 $\int \text{sech}^{-1}\left(ce^{a+bx}\right) dx$

Optimal. Leaf size=77

$$-\frac{\text{Li}_2\left(-e^{2\text{sech}^{-1}\left(ce^{a+bx}\right)}\right)}{2b}+\frac{\text{sech}^{-1}\left(ce^{a+bx}\right)^2}{2b}-\frac{\text{sech}^{-1}\left(ce^{a+bx}\right)\log\left(e^{2\text{sech}^{-1}\left(ce^{a+bx}\right)}+1\right)}{b}$$

```
[Out] 1/2*arcsech(c*exp(b*x+a))^2/b-arcsech(c*exp(b*x+a))*ln(1+(1/c/exp(b*x+a)+(1
/c/exp(b*x+a)-1)^(1/2)*(1/c/exp(b*x+a)+1)^(1/2))^2)/b-1/2*polylog(2,-(1/c/e
xp(b*x+a)+(1/c/exp(b*x+a)-1)^(1/2)*(1/c/exp(b*x+a)+1)^(1/2))^2)/b
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2282, 6281, 5660, 3718, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}\left(ce^{a+bx}\right)}\right)}{2b}+\frac{\text{sech}^{-1}\left(ce^{a+bx}\right)^2}{2b}-\frac{\text{sech}^{-1}\left(ce^{a+bx}\right)\log\left(e^{2\text{sech}^{-1}\left(ce^{a+bx}\right)}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[c*E^(a + b*x)],x]
```

```
[Out] ArcSech[c*E^(a + b*x)]^2/(2*b) - (ArcSech[c*E^(a + b*x)]*Log[1 + E^(2*ArcSe
ch[c*E^(a + b*x)])])/b - PolyLog[2, -E^(2*ArcSech[c*E^(a + b*x)])]/(2*b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6281

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \operatorname{sech}^{-1}\left(ce^{a+bx}\right)dx = \frac{\text{Subst}\left(\int \frac{\operatorname{sech}^{-1}(cx)}{x}dx, x, e^{a+bx}\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\cosh^{-1}\left(\frac{x}{c}\right)}{x}dx, x, e^{-a-bx}\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\right)}{b}$$
$$= \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}dx, x, \cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\right)}{b}$$
$$= \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{\text{Subst}\left(\int \log\left(1+e^{2x}\right)dx,\right.}{b}$$
$$= \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}dx, x, e^{2\,\text{c}}\right.}{2b}$$
$$= \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} - \frac{\text{Li}_2\left(-e^{2\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{2b}$$

Mathematica [B] time = 1.17, size = 249, normalized size = 3.23

$$x\operatorname{sech}^{-1}\left(ce^{a+bx}\right) - \frac{\sqrt{\frac{1-ce^{a+bx}}{ce^{a+bx}+1}}\sqrt{ce^{a+bx}+1}\left(4\text{Li}_2\left(\frac{1}{2}\left(1-\sqrt{1-c^2e^{2(a+bx)}}\right)\right)-\log^2\left(c^2e^{2(a+bx)}\right)-2\log^2\left(\frac{1}{2}\left(\sqrt{1-c^2e^{2}}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[c*E^(a + b*x)],x]
```

```
[Out] x*ArcSech[c*E^(a + b*x)] - (Sqrt[(1 - c*E^(a + b*x))/(1 + c*E^(a + b*x))]*S
qrt[1 + c*E^(a + b*x)]*(ArcTanh[Sqrt[1 - c^2*E^(2*(a + b*x))]]*(8*b*x - 4*L
og[c^2*E^(2*(a + b*x))]) - Log[c^2*E^(2*(a + b*x))]^2 + 4*Log[c^2*E^(2*(a +
 b*x))]*Log[(1 + Sqrt[1 - c^2*E^(2*(a + b*x))])/2] - 2*Log[(1 + Sqrt[1 - c^
2*E^(2*(a + b*x))])/2]^2 + 4*PolyLog[2, (1 - Sqrt[1 - c^2*E^(2*(a + b*x))])
/2]))/(8*b*Sqrt[1 - c*E^(a + b*x)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(c*exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsech}\left(ce^{(bx+a)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(c*exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arcsech(c*e^(b*x + a)), x)
```

maple [A] time = 0.15, size = 140, normalized size = 1.82

$$\frac{\operatorname{arcsech}\left(c\,\mathrm{e}^{bx+a}\right)^2}{2b}-\frac{\operatorname{arcsech}\left(c\,\mathrm{e}^{bx+a}\right)\ln\left(1+\left(\frac{\mathrm{e}^{-bx-a}}{c}+\sqrt{\frac{\mathrm{e}^{-bx-a}}{c}-1}\,\sqrt{\frac{\mathrm{e}^{-bx-a}}{c}+1}\right)^2\right)}{b}-\frac{\operatorname{polylog}\left(2,-\left(\frac{\mathrm{e}^{-bx-a}}{c}+\sqrt{\right.\right.}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(c*exp(b*x+a)),x)
```

```
[Out] 1/2*arcsech(c*exp(b*x+a))^2/b-arcsech(c*exp(b*x+a))*ln(1+(1/c/exp(b*x+a)+(1
/c/exp(b*x+a)-1)^(1/2)*(1/c/exp(b*x+a)+1)^(1/2))^2)/b-1/2*polylog(2,-(1/c/e
xp(b*x+a)+(1/c/exp(b*x+a)-1)^(1/2)*(1/c/exp(b*x+a)+1)^(1/2))^2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$bc^2\int \frac{xe^{(2\,bx+2\,a)}}{c^2e^{(2\,bx+2\,a)}+\left(c^2e^{(2\,bx+2\,a)}-1\right)e^{\left(\frac{1}{2}\,\log\left(ce^{(bx+a)}+1\right)+\frac{1}{2}\,\log\left(-ce^{(bx+a)}+1\right)\right)}-1}\,dx-\frac{1}{2}\,bx^2-\left(a+\log(c)\right)x+x\log\left(\sqrt{ce\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(c*exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] b*c^2*integrate(x*e^(2*b*x + 2*a)/(c^2*e^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2*
a) - 1)*e^(1/2*log(c*e^(b*x + a) + 1) + 1/2*log(-c*e^(b*x + a) + 1)) - 1),
x) - 1/2*b*x^2 - (a + log(c))*x + x*log(sqrt(c*e^(b*x + a) + 1)*sqrt(-c*e^(
b*x + a) + 1) + 1) - 1/2*(b*x*log(c*e^(b*x + a) + 1) + dilog(-c*e^(b*x + a)
))/b - 1/2*(b*x*log(-c*e^(b*x + a) + 1) + dilog(c*e^(b*x + a)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}\left(\frac{\mathrm{e}^{-a-b\,x}}{c}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(exp(- a - b*x)/c),x)
```

```
[Out] int(acosh(exp(- a - b*x)/c), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asech}\left(ce^{a+bx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(c*exp(b*x+a)),x)
```

```
[Out] Integral(asech(c*exp(a + b*x)), x)
```

3.32 $\int e^{\operatorname{sech}^{-1}(ax)} x^4 \, dx$

Optimal. Leaf size=64

$$-\frac{2xe^{\operatorname{sech}^{-1}(ax)}}{15a^4}+\frac{x^2}{15a^3}-\frac{x^3e^{\operatorname{sech}^{-1}(ax)}}{15a^2}+\frac{1}{5}x^5e^{\operatorname{sech}^{-1}(ax)}+\frac{x^4}{20a}$$

```
[Out] -2/15*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x/a^4+1/15*x^2/a^3-1/15*(1/a/
x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3/a^2+1/20*x^4/a+1/5*(1/a/x+(1/a/x-1)^
(1/2)*(1+1/a/x)^(1/2))*x^5
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6335, 30, 100, 12, 74}

$$-\frac{x^2\sqrt{1-ax}}{15a^3\sqrt{\frac{1}{ax+1}}}-\frac{2\sqrt{1-ax}}{15a^5\sqrt{\frac{1}{ax+1}}}+\frac{x^4}{20a}+\frac{1}{5}x^5e^{\operatorname{sech}^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x]*x^4,x]
```

```
[Out] x^4/(20*a) + (E^ArcSech[a*x]*x^5)/5 - (2*Sqrt[1 - a*x])/(15*a^5*Sqrt[(1 + a
*x)^(-1)]) - (x^2*Sqrt[1 - a*x])/(15*a^3*Sqrt[(1 + a*x)^(-1)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}
\int e^{\operatorname{sech}^{-1}(ax)}x^4\,dx &= \frac{1}{5}e^{\operatorname{sech}^{-1}(ax)}x^5+\frac{\int x^3\,dx}{5a}+\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int\frac{x^3}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{5a}\\
&= \frac{x^4}{20a}+\frac{1}{5}e^{\operatorname{sech}^{-1}(ax)}x^5-\frac{x^2\sqrt{1-ax}}{15a^3\sqrt{\frac{1}{1+ax}}}-\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int-\frac{2x}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{15a^3}\\
&= \frac{x^4}{20a}+\frac{1}{5}e^{\operatorname{sech}^{-1}(ax)}x^5-\frac{x^2\sqrt{1-ax}}{15a^3\sqrt{\frac{1}{1+ax}}}+\frac{\left(2\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int\frac{x}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{15a^3}\\
&= \frac{x^4}{20a}+\frac{1}{5}e^{\operatorname{sech}^{-1}(ax)}x^5-\frac{2\sqrt{1-ax}}{15a^5\sqrt{\frac{1}{1+ax}}}-\frac{x^2\sqrt{1-ax}}{15a^3\sqrt{\frac{1}{1+ax}}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 65, normalized size = 1.02

$$\frac{15a^4x^4+4\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^2\left(3a^3x^3-3a^2x^2+2ax-2\right)}{60a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]*x^4,x]
```

```
[Out] (15*a^4*x^4 + 4*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-2 + 2*a*x - 3*a^2*x
^2 + 3*a^3*x^3))/(60*a^5)
```

fricas [A] time = 0.55, size = 65, normalized size = 1.02

$$\frac{15\,a^3x^4+4\left(3\,a^4x^5-a^2x^3-2\,x\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}}{60\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x, algorithm="fricas"
)
```

```
[Out] 1/60*(15*a^3*x^4 + 4*(3*a^4*x^5 - a^2*x^3 - 2*x)*sqrt((a*x + 1)/(a*x))*sqrt
(-(a*x - 1)/(a*x)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at param
eters values [86,-97]Warning, choosing root of [1,0,%%%{4,[1,1]%%%},0,%%%{4
,[4,4]%%%}] at parameters values [-82,7]Warning, choosing root of [1,0,%%%{
-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-27,26]Warning, choos
ing root of [1,0,%%%{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-
```

```
89,63]Unable to divide, perhaps due to rounding error%%%{1,[3,2,2,0,0]%%%}+
%%%{1,[2,0,1,1,1]%%%} / %%%{1,[0,2,3,0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.06, size = 64, normalized size = 1.00

$$\frac{\sqrt{-\frac{ax-1}{ax}}\,x\sqrt{\frac{ax+1}{ax}}\,\left(a^2x^2-1\right)\left(3a^2x^2+2\right)}{15a^4}+\frac{x^4}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x)
```

```
[Out] 1/15*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(a^2*x^2-1)*(3*a^2*x^2+2)/a
^4+1/4*x^4/a
```

maxima [A] time = 0.38, size = 47, normalized size = 0.73

$$\frac{x^4}{4\,a}+\frac{\left(3\,a^4x^4-a^2x^2-2\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{15\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x, algorithm="maxima"
)
```

```
[Out] 1/4*x^4/a + 1/15*(3*a^4*x^4 - a^2*x^2 - 2)*sqrt(a*x + 1)*sqrt(-a*x + 1)/a^5
```

mupad [B] time = 1.47, size = 75, normalized size = 1.17

$$\frac{x^4}{4\,a}-\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,x\,\sqrt{\frac{1}{a\,x}+1}}{15\,a^4}-\frac{x^5\,\sqrt{\frac{1}{a\,x}+1}}{5}+\frac{x^3\,\sqrt{\frac{1}{a\,x}+1}}{15\,a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] x^4/(4*a) - (1/(a*x) - 1)^(1/2)*((2*x*(1/(a*x) + 1)^(1/2))/(15*a^4) - (x^5*
(1/(a*x) + 1)^(1/2))/5 + (x^3*(1/(a*x) + 1)^(1/2))/(15*a^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**4,x)
```

```
[Out] Timed out
```

3.33 $\int e^{\text{sech}^{-1}(ax)} x^3 \, dx$

Optimal. Leaf size=84

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\sin^{-1}(ax)}{8a^4}-\frac{x\sqrt{1-ax}}{8a^3\sqrt{\frac{1}{ax+1}}}+\frac{1}{4}x^4e^{\text{sech}^{-1}(ax)}+\frac{x^3}{12a}$$

```
[Out] 1/12*x^3/a+1/4*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4-1/8*x*(-a*x+1)^(
1/2)/a^3/(1/(a*x+1))^(1/2)+1/8*arcsin(a*x)*(1/(a*x+1))^(1/2)*(a*x+1)^(1/2)/
a^4
```

Rubi [A] time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6335, 30, 90, 41, 216}

$$-\frac{x\sqrt{1-ax}}{8a^3\sqrt{\frac{1}{ax+1}}}+\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\sin^{-1}(ax)}{8a^4}+\frac{x^3}{12a}+\frac{1}{4}x^4e^{\text{sech}^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]*x^3,x]
```

```
[Out] x^3/(12*a) + (E^ArcSech[a*x]*x^4)/4 - (x*Sqrt[1 - a*x])/(8*a^3*Sqrt[(1 + a*
x)^(-1)]) + (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*ArcSin[a*x])/(8*a^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}
\int e^{\operatorname{sech}^{-1}(ax)}x^3\,dx &= \frac{1}{4}e^{\operatorname{sech}^{-1}(ax)}x^4+\frac{\int x^2\,dx}{4a}+\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int\frac{x^2}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{4a}\\
&=\frac{x^3}{12a}+\frac{1}{4}e^{\operatorname{sech}^{-1}(ax)}x^4-\frac{x\sqrt{1-ax}}{8a^3\sqrt{\frac{1}{1+ax}}}+\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{8a^3}\\
&=\frac{x^3}{12a}+\frac{1}{4}e^{\operatorname{sech}^{-1}(ax)}x^4-\frac{x\sqrt{1-ax}}{8a^3\sqrt{\frac{1}{1+ax}}}+\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a^3}\\
&=\frac{x^3}{12a}+\frac{1}{4}e^{\operatorname{sech}^{-1}(ax)}x^4-\frac{x\sqrt{1-ax}}{8a^3\sqrt{\frac{1}{1+ax}}}+\frac{\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\sin^{-1}(ax)}{8a^4}
\end{aligned}$$

Mathematica [C] time = 0.13, size = 97, normalized size = 1.15

$$\frac{8a^3x^3-3a\sqrt{\frac{1-ax}{ax+1}}\left(-2a^3x^4-2a^2x^3+ax^2+x\right)+3i\log\left(2\sqrt{\frac{1-ax}{ax+1}}(ax+1)-2iax\right)}{24a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]*x^3,x]
```

```
[Out] (8*a^3*x^3 - 3*a*Sqrt[(1 - a*x)/(1 + a*x)]*(x + a*x^2 - 2*a^2*x^3 - 2*a^3*x
^4) + (3*I)*Log[(-2*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/(24*a^
4)
```

fricas [A] time = 0.51, size = 95, normalized size = 1.13

$$\frac{8\,a^3x^3+3\left(2\,a^4x^4-a^2x^2\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-3\,\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x, algorithm="fricas"
)
```

```
[Out] 1/24*(8*a^3*x^3 + 3*(2*a^4*x^4 - a^2*x^2)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x
- 1)/(a*x)) - 3*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at param
eters values [86,-97]Warning, choosing root of [1,0,%%%{4,[1,1]%%%},0,%%%{4
,[4,4]%%%}] at parameters values [-82,7]Warning, choosing root of [1,0,%%%{
-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-27,26]Warning, choos
```

```
ing root of [1,0,%%%{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-
89,63]Unable to divide, perhaps due to rounding error%%%{1,[2,2,2,0,0]%%%}+
%%%{1,[1,0,1,1,1]%%%} / %%%{1,[0,2,3,0,0]%%%} Error: Bad Argument Value
```

maple [C] time = 0.06, size = 118, normalized size = 1.40

$$\frac{\sqrt{-\frac{ax-1}{ax}}\, x\sqrt{\frac{ax+1}{ax}}\left(2\,\mathrm{csgn}(a)x^3a^3\sqrt{-a^2x^2+1}\;-x\sqrt{-a^2x^2+1}\;\mathrm{csgn}(a)a+\arctan\left(\frac{\mathrm{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)\right)\mathrm{csgn}(a)}{8\sqrt{-a^2x^2+1}\;a^3}+\frac{x^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x)
```

```
[Out] 1/8*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(2*csgn(a)*x^3*a^3*(-a^2*x^2
+1)^(1/2)-x*(-a^2*x^2+1)^(1/2)*csgn(a)*a+arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1
/2)))*csgn(a)/(-a^2*x^2+1)^(1/2)/a^3+1/3*x^3/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^3}{3\,a}+\frac{-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{4\,a^2}+\frac{\sqrt{-a^2x^2+1}\,x}{8\,a^2}+\frac{\arcsin(ax)}{8\,a^3}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x, algorithm="maxima"
)
```

```
[Out] 1/3*x^3/a + integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^2, x)/a
```

mupad [B] time = 11.93, size = 521, normalized size = 6.20

$$\frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)1\mathrm{i}}{8\,a^4}-\frac{\frac{1\mathrm{i}}{1024\,a^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{128\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4 11\mathrm{i}}{512\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6 7\mathrm{i}}{256\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8 239\mathrm{i}}{1024\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}+\cdots}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{6\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{12}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] (log((((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)*1i)/(8*a
^4) - (1i/(1024*a^4) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(128*a^4*((1/(a*x)
 + 1)^(1/2) - 1)^2) + (((1/(a*x) - 1)^(1/2) - 1i)^4*11i)/(512*a^4*((1/(a*x)
 + 1)^(1/2) - 1)^4) + (((1/(a*x) - 1)^(1/2) - 1i)^6*7i)/(256*a^4*((1/(a*x)
+ 1)^(1/2) - 1)^6) - (((1/(a*x) - 1)^(1/2) - 1i)^8*239i)/(1024*a^4*((1/(a*x
) + 1)^(1/2) - 1)^8) + (((1/(a*x) - 1)^(1/2) - 1i)^10*1i)/(256*a^4*((1/(a*x
) + 1)^(1/2) - 1)^10))/(((1/(a*x) - 1)^(1/2) - 1i)^4/((1/(a*x) + 1)^(1/2) -
 1)^4 + (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (6*(
(1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 + (4*((1/(a*x) - 1
)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + ((1/(a*x) - 1)^(1/2) - 1i)
^12/((1/(a*x) + 1)^(1/2) - 1)^12) - (log((((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*
x) + 1)^(1/2) - 1))*1i)/(8*a^4) + x^3/(3*a) - ((((1/(a*x) - 1)^(1/2) - 1i)^2
*1i)/(256*a^4*((1/(a*x) + 1)^(1/2) - 1)^2) - ((((1/(a*x) - 1)^(1/2) - 1i)^4*
1i)/(1024*a^4*((1/(a*x) + 1)^(1/2) - 1)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**3,x)
```

```
[Out] Timed out
```

3.34 $\int e^{\text{sech}^{-1}(ax)} x^2 \, dx$

Optimal. Leaf size=38

$$-\frac{x e^{\text{sech}^{-1}(ax)}}{3a^2} + \frac{1}{3} x^3 e^{\text{sech}^{-1}(ax)} + \frac{x^2}{6a}$$

```
[Out] -1/3*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x/a^2+1/6*x^2/a+1/3*(1/a/x+(1/
a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.37, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6335, 30, 74}

$$-\frac{\sqrt{1-ax}}{3a^3\sqrt{\frac{1}{ax+1}}} + \frac{x^2}{6a} + \frac{1}{3} x^3 e^{\text{sech}^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x]*x^2,x]
```

```
[Out] x^2/(6*a) + (E^ArcSech[a*x]*x^3)/3 - Sqrt[1 - a*x]/(3*a^3*Sqrt[(1 + a*x)^(-
1)])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}
\int e^{\text{sech}^{-1}(ax)} x^2 \, dx &= \frac{1}{3} e^{\text{sech}^{-1}(ax)} x^3 + \frac{\int x \, dx}{3a} + \frac{\left(\sqrt{\frac{1}{1+ax}} \sqrt{1+ax}\right) \int \frac{x}{\sqrt{1-ax}\sqrt{1+ax}} \, dx}{3a} \\
&= \frac{x^2}{6a} + \frac{1}{3} e^{\text{sech}^{-1}(ax)} x^3 - \frac{\sqrt{1-ax}}{3a^3\sqrt{\frac{1}{1+ax}}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 1.26

$$\frac{3a^2x^2 + 2(ax-1)\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]*x^2,x]
```

```
[Out] (3*a^2*x^2 + 2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(6*a^3)
```

fricas [A] time = 0.76, size = 54, normalized size = 1.42

$$\frac{3\,ax^2 + 2\left(a^2x^3 - x\right)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x, algorithm="fricas"
)
```

```
[Out] 1/6*(3*a*x^2 + 2*(a^2*x^3 - x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))
)/a^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at param
eters values [86,-97]Warning, choosing root of [1,0,%%%{4,[1,1]%%%},0,%%%{4
,[4,4]%%%}] at parameters values [-82,7]Warning, choosing root of [1,0,%%%{
-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-27,26]Warning, choos
ing root of [1,0,%%%{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-
89,63]Unable to divide, perhaps due to rounding error%%%{1,[1,2,2,0,0]%%%}+
%%%{1,[0,0,1,1,1]%%%} / %%%{1,[0,2,3,0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.05, size = 54, normalized size = 1.42

$$\frac{\sqrt{-\frac{ax-1}{ax}}\,x\sqrt{\frac{ax+1}{ax}}\,\left(a^2x^2-1\right)}{3a^2}+\frac{x^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x)
```

```
[Out] 1/3*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(a^2*x^2-1)/a^2+1/2*x^2/a
```

maxima [A] time = 0.40, size = 38, normalized size = 1.00

$$\frac{x^2}{2\,a}+\frac{\left(a^2x^2-1\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*x^2/a + 1/3*(a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)/a^3
```

mupad [B] time = 1.43, size = 55, normalized size = 1.45

$$\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{x^3\,\sqrt{\frac{1}{a\,x}+1}}{3}-\frac{x\,\sqrt{\frac{1}{a\,x}+1}}{3\,a^2}\right)+\frac{x^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] (1/(a*x) - 1)^(1/2)*((x^3*(1/(a*x) + 1)^(1/2))/3 - (x*(1/(a*x) + 1)^(1/2))/
(3*a^2)) + x^2/(2*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x\,dx + \int ax^2\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**2,x)
```

```
[Out] (Integral(x, x) + Integral(a*x**2*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))
/a
```

3.35 $\int e^{\operatorname{sech}^{-1}(ax)} x \, dx$

Optimal. Leaf size=53

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\sin^{-1}(ax)}{2a^2}+\frac{1}{2}x^2e^{\operatorname{sech}^{-1}(ax)}+\frac{x}{2a}$$

```
[Out] 1/2*x/a+1/2*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2+1/2*arcsin(a*x)*(1/
(a*x+1))^(1/2)*(a*x+1)^(1/2)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6335, 8, 41, 216}

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\sin^{-1}(ax)}{2a^2}+\frac{1}{2}x^2e^{\operatorname{sech}^{-1}(ax)}+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]*x,x]
```

```
[Out] x/(2*a) + (E^ArcSech[a*x]*x^2)/2 + (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*ArcS
in[a*x])/(2*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}
\int e^{\operatorname{sech}^{-1}(ax)} x \, dx &= \frac{1}{2}e^{\operatorname{sech}^{-1}(ax)}x^2+\frac{\int 1\,dx}{2a}+\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2a}\\
&= \frac{x}{2a}+\frac{1}{2}e^{\operatorname{sech}^{-1}(ax)}x^2+\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a}\\
&= \frac{x}{2a}+\frac{1}{2}e^{\operatorname{sech}^{-1}(ax)}x^2+\frac{\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\sin^{-1}(ax)}{2a^2}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 75, normalized size = 1.42

$$\frac{2ax + ax\sqrt{\frac{1-ax}{ax+1}}(ax+1) + i\log\left(2\sqrt{\frac{1-ax}{ax+1}}(ax+1) - 2iax\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]*x,x]
```

```
[Out] (2*a*x + a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + I*Log[(-2*I)*a*x + 2*Sqr
t[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/(2*a^2)
```

fricas [A] time = 0.62, size = 79, normalized size = 1.49

$$\frac{a^2x^2\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 2\,ax - \arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 2*a*x - arctan(
sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1} + \frac{1}{ax}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x,x, algorithm="giac")
```

```
[Out] integrate(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

maple [C] time = 0.05, size = 92, normalized size = 1.74

$$\frac{\sqrt{-\frac{ax-1}{ax}}\,x\sqrt{\frac{ax+1}{ax}}\left(x\sqrt{-a^2x^2+1}\,\operatorname{csgn}(a)a + \arctan\left(\frac{\operatorname{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)\right)\operatorname{csgn}(a)}{2\sqrt{-a^2x^2+1}\,a} + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x,x)
```

```
[Out] 1/2*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(x*(-a^2*x^2+1)^(1/2)*csgn(a
)*a+arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1/2)))/(-a^2*x^2+1)^(1/2)*csgn(a)/a+x/
a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{a} + \frac{\frac{1}{2}\sqrt{-a^2x^2+1}\,x + \frac{\arcsin(ax)}{2\,a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x,x, algorithm="maxima")
```

```
[Out] x/a + integrate(sqrt(a*x + 1)*sqrt(-a*x + 1), x)/a
```

mupad [B] time = 6.96, size = 303, normalized size = 5.72

$$\frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)\,1\mathrm{i}}{2\,a^2}-\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\,1\mathrm{i}}{2\,a^2}+\frac{\frac{1\mathrm{i}}{32\,a^2}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{16\,a^2\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{32\,a^2\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{2\,\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}}+\frac{x}{a}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\,a^2\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] (log(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)*1i)/(2*a
^2) - (log(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/(2*a^2
) + (1i/(32*a^2) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(16*a^2*((1/(a*x) + 1)
^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/2) - 1i)^4*15i)/(32*a^2*((1/(a*x) + 1)^
(1/2) - 1)^4))/(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 +
(2*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a*x) -
1)^(1/2) - 1i)^6/((1/(a*x) + 1)^(1/2) - 1)^6) + x/a + (((1/(a*x) - 1)^(1/2)
 - 1i)^2*1i)/(32*a^2*((1/(a*x) + 1)^(1/2) - 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x,x)
```

```
[Out] Timed out
```

3.36 $\int e^{\text{sech}^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\frac{\log(x)}{a}+xe^{\text{sech}^{-1}(ax)}-\frac{\text{sech}^{-1}(ax)}{a}$$

```
[Out] (1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x-arcsech(a*x)/a+ln(x)/a
```

Rubi [A] time = 0.14, antiderivative size = 39, normalized size of antiderivative = 1.62, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6329, 1962, 208}

$$\frac{\log(x)}{a}-\frac{2\tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a}+xe^{\text{sech}^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x],x]
```

```
[Out] E^ArcSech[a*x]*x - (2*ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]])/a + Log[x]/a
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1962

```
Int[(u_)^(r_.)*(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)
*(x_)^(n_.)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c
- a*d))/n, Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^
((m + 1)/n - 1)*(u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(
b*e - d*x^q)^((m + 1)/n + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q
)], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] &
& IntegerQ[1/n] && IntegersQ[m, r]
```

Rule 6329

```
Int[E^ArcSech[(a_.)*(x_)], x_Symbol] :> Simp[x*E^ArcSech[a*x], x] + (Dist[1
/a, Int[(1*Sqrt[(1 - a*x)/(1 + a*x)])/(x*(1 - a*x)), x], x] + Simp[Log[x]/a
, x]) /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int e^{\text{sech}^{-1}(ax)}\,dx &= e^{\text{sech}^{-1}(ax)}x+\frac{\log(x)}{a}+\frac{\int\frac{\sqrt{\frac{1-ax}{1+ax}}}{x(1-ax)}\,dx}{a}\\
&= e^{\text{sech}^{-1}(ax)}x+\frac{\log(x)}{a}-4\,\text{Subst}\left(\int\frac{1}{2a-2ax^2}\,dx,x,\sqrt{\frac{1-ax}{1+ax}}\right)\\
&= e^{\text{sech}^{-1}(ax)}x-\frac{2\tanh^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{a}+\frac{\log(x)}{a}
\end{aligned}$$

Mathematica [B] time = 0.04, size = 79, normalized size = 3.29

$$\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)+2\log(ax)-\log\left(ax\sqrt{\frac{1-ax}{ax+1}}+\sqrt{\frac{1-ax}{ax+1}}+1\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x],x]
```

```
[Out] (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + 2*Log[a*x] - Log[1 + Sqrt[(1 - a*x)/
(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/a
```

fricas [B] time = 0.70, size = 115, normalized size = 4.79

$$\frac{2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)+\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+2\,\log\left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - log(a*x*sqrt((a*x
 + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) + log(a*x*sqrt((a*x + 1)/(a*x))*sq
rt(-(a*x - 1)/(a*x)) - 1) + 2*log(x))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x), x)
```

maple [A] time = 0.06, size = 80, normalized size = 3.33

$$\frac{\ln\left(x\right)}{a}-\frac{\sqrt{-\frac{ax-1}{ax}}\,x\sqrt{\frac{ax+1}{ax}}\,\left(-\sqrt{-a^{2}x^{2}+1}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)\right)}{\sqrt{-a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x)
```

```
[Out] ln(x)/a-(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(-(-a^2*x^2+1)^(1/2)+arc
tanh(1/(-a^2*x^2+1)^(1/2)))/(-a^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x), x)
```

mupad [B] time = 2.98, size = 182, normalized size = 7.58

$$\frac{\ln\left(x\right)}{a}-\frac{4\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)}{a}+\frac{\frac{5\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}+1}{\frac{4\,a\,\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x}+1}-1}+\frac{4\,a\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^3}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^3}}+\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{4\,a\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x),x)
```

```
[Out] log(x)/a - (4*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))/
a + ((5*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)/((4*
a*((1/(a*x) - 1)^(1/2) - 1i))/((1/(a*x) + 1)^(1/2) - 1) + (4*a*((1/(a*x) -
1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) - 1)^3) + ((1/(a*x) - 1)^(1/2) - 1i)
/(4*a*((1/(a*x) + 1)^(1/2) - 1))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x}\, dx + \int a\sqrt{-1 + \frac{1}{ax}}\,\sqrt{1 + \frac{1}{ax}}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2),x)
```

```
[Out] (Integral(1/x, x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))/a
```

3.37 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=48

$$2\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{2}{1-\sqrt{\frac{1-ax}{ax+1}}}$$

```
[Out] 2*arctan(((-a*x+1)/(a*x+1))^(1/2))-2/(1-((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6334, 97, 12, 41, 216}

$$-\frac{\sqrt{1-ax}}{ax\sqrt{\frac{1}{ax+1}}}-\frac{1}{ax}-\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\sin^{-1}(ax)$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x]/x,x]
```

```
[Out] -(1/(a*x)) - Sqrt[1 - a*x]/(a*x*Sqrt[(1 + a*x)^(-1)]) - Sqrt[(1 + a*x)^(-1)
]*Sqrt[1 + a*x]*ArcSin[a*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6334

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]/(x_), x_Symbol] :> -Simp[(a*p*x^p)^(-1), x]
 + Dist[(Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/a, Int[(Sqrt[1 + a*x^p]*Sqrt[
1 - a*x^p])/x^(p + 1), x], x] /; FreeQ[{a, p}, x]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x}\,dx = -\frac{1}{ax} + \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{\sqrt{1-ax}\,\sqrt{1+ax}}{x^2}\,dx}{a}$$
$$= -\frac{1}{ax} - \frac{\sqrt{1-ax}}{ax\sqrt{\frac{1}{1+ax}}} - \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{a^2}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a}$$
$$= -\frac{1}{ax} - \frac{\sqrt{1-ax}}{ax\sqrt{\frac{1}{1+ax}}} - \left(a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx$$
$$= -\frac{1}{ax} - \frac{\sqrt{1-ax}}{ax\sqrt{\frac{1}{1+ax}}} - \left(a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{1}{ax} - \frac{\sqrt{1-ax}}{ax\sqrt{\frac{1}{1+ax}}} - \sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\,\sin^{-1}(ax)$$

Mathematica [**C**] time = 0.05, size = 75, normalized size = 1.56

$$\sqrt{\frac{1-ax}{ax+1}}\left(-\frac{1}{ax}-1\right)-\frac{1}{ax}-i\log\left(2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)-2iax\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x,x]
```

```
[Out] -(1/(a*x)) + (-1 - 1/(a*x))*Sqrt[(1 - a*x)/(1 + a*x)] - I*Log[(-2*I)*a*x +
2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)]
```

fricas [**A**] time = 0.70, size = 77, normalized size = 1.60

$$-\frac{ax\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-ax\arctan\left(\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\right)+1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] -(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - a*x*arctan(sqrt((a*x +
 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))) + 1)/(a*x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{ax}+1}\,\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x, x)
```

maple [**C**] time = 0.06, size = 92, normalized size = 1.92

$$-\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\left(\arctan\left(\frac{\operatorname{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)xa+\sqrt{-a^2x^2+1}\,\operatorname{csgn}(a)\right)\operatorname{csgn}(a)}{\sqrt{-a^2x^2+1}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x)
```

```
[Out] -(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*(arctan(csgn(a)*a*x/(-a^2*x^2+1)^
(1/2))*x*a+(-a^2*x^2+1)^(1/2)*csgn(a))*csgn(a)/(-a^2*x^2+1)^(1/2)-1/a/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-a \arcsin(ax) - \frac{\sqrt{-a^2x^2+1}}{x}}{a} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^2, x)/a - 1/(a*x)
```

mupad [B] time = 3.10, size = 184, normalized size = 3.83

$$-\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)1\mathrm{i}+\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)1\mathrm{i}-\frac{1}{a\,x}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 8\mathrm{i}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2\left(1+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x,x)
```

```
[Out] log(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i - log(((1/(a*x
) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)*1i - 1/(a*x) + (((1/(
a*x) - 1)^(1/2) - 1i)^2*8i)/(((1/(a*x) + 1)^(1/2) - 1)^2*(((1/(a*x) - 1)^(1
/2) - 1i)^4/((1/(a*x) + 1)^(1/2) - 1)^4 - (2*((1/(a*x) - 1)^(1/2) - 1i)^2)/
((1/(a*x) + 1)^(1/2) - 1)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2}\, dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x,x)
```

```
[Out] (Integral(x**(-2), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x,
x))/a
```

3.38 $\int \frac{e^{\mathbf{sech}^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=35

$$a\tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{e^{\operatorname{sech}^{-1}(ax)}}{2x}$$

```
[Out] -1/2*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x+a*arctanh(((-a*x+1)/(a*x+1))
^(1/2))
```

Rubi [B] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 2.83, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6335, 30, 103, 12, 92, 208}

$$\frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{ax+1}}}+\frac{1}{2ax^2}+\frac{1}{2}a\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)-\frac{e^{\operatorname{sech}^{-1}(ax)}}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x]/x^2,x]
```

```
[Out] 1/(2*a*x^2) - E^ArcSech[a*x]/x + Sqrt[1 - a*x]/(2*a*x^2*Sqrt[(1 + a*x)^(-1)
]) + (a*Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a
*x]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^2}\,dx &= -\frac{e^{\operatorname{sech}^{-1}(ax)}}{x} - \frac{\int \frac{1}{x^3}\,dx}{a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a}\\
&= \frac{1}{2ax^2} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} + \frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{1+ax}}} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{a^2}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2a}\\
&= \frac{1}{2ax^2} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} + \frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{2}\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx\\
&= \frac{1}{2ax^2} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} + \frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{2}\left(a^2\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\operatorname{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-\right.\\
&= \frac{1}{2ax^2} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} + \frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{2}a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)\end{aligned}$$

Mathematica [B] time = 0.06, size = 93, normalized size = 2.66

$$\frac{1}{2}\left(-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax^2} - \frac{1}{ax^2} - a\log(x) + a\log\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x^2,x]
```

```
[Out] (-(1/(a*x^2)) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*x^2) - a*Log[x] +
a*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/2
```

fricas [B] time = 0.81, size = 128, normalized size = 3.66

$$\frac{a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right) - a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right) - 2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 2}{4\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="fricas"
)
```

```
[Out] 1/4*(a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a^
2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 2*a*x*sqr
t((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2)/(a*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^2, x)
```

maple [A] time = 0.05, size = 91, normalized size = 2.60

$$\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\left(a^2x^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\sqrt{-a^2x^2+1}\right)}{2x\sqrt{-a^2x^2+1}}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x)
```

```
[Out] 1/2*(-(a*x-1)/a/x)^(1/2)/x*((a*x+1)/a/x)^(1/2)*(a^2*x^2*arctanh(1/(-a^2*x^2
+1)^(1/2))-(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/2/a/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{1}{2}\,a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{1}{2}\sqrt{-a^2x^2+1}\,a^2-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{2\,x^2}}{a}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^3, x)/a - 1/2/(a*x^2)
```

mupad [B] time = 1.84, size = 71, normalized size = 2.03

$$\frac{a\,\ln\left(\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}+\frac{1}{a\,x}\right)}{2}-\frac{1}{2\,a\,x^2}-\frac{\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^2,x)
```

```
[Out] (a*log((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)))/2 - 1/(2*a*x^2)
- ((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2))/(2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{1}{x^3}\,dx+\int\frac{a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**2,x)
```

```
[Out] (Integral(x**(-3), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
2, x))/a
```

3.39 $\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=55

$$-\frac{8a^2\left(\frac{1-ax}{ax+1}\right)^{3/2}}{3\left(1-\frac{1-ax}{ax+1}\right)^3}-\frac{1}{3ax^3}$$

```
[Out] -1/3/a/x^3-8/3*a^2*((-a*x+1)/(a*x+1))^(3/2)/(1+(a*x-1)/(a*x+1))^3
```

Rubi **[C]** time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.53, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6335, 30, 103, 12, 95}

$$\frac{\sqrt{1-ax}}{6ax^3\sqrt{\frac{1}{ax+1}}}+\frac{1}{6ax^3}-\frac{e^{\operatorname{sech}^{-1}(ax)}}{2x^2}+\frac{a\sqrt{1-ax}}{3x\sqrt{\frac{1}{ax+1}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x]/x^3,x]
```

```
[Out] 1/(6*a*x^3) - E^ArcSech[a*x]/(2*x^2) + Sqrt[1 - a*x]/(6*a*x^3*Sqrt[(1 + a*x
)^(-1)]) + (a*Sqrt[1 - a*x])/(3*x*Sqrt[(1 + a*x)^(-1)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
```

]

Rubi steps

$$\begin{aligned}\int \frac{e^{\text{sech}^{-1}(ax)}}{x^3}\,dx &= -\frac{e^{\text{sech}^{-1}(ax)}}{2x^2} - \frac{\int \frac{1}{x^4}\,dx}{2a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2a}\\
&= \frac{1}{6ax^3} - \frac{e^{\text{sech}^{-1}(ax)}}{2x^2} + \frac{\sqrt{1-ax}}{6ax^3\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int -\frac{2a^2}{x^2\sqrt{1-ax}\sqrt{1+ax}}\,dx}{6a}\\
&= \frac{1}{6ax^3} - \frac{e^{\text{sech}^{-1}(ax)}}{2x^2} + \frac{\sqrt{1-ax}}{6ax^3\sqrt{\frac{1}{1+ax}}} - \frac{1}{3}\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^2\sqrt{1-ax}\sqrt{1+ax}}\,dx\\
&= \frac{1}{6ax^3} - \frac{e^{\text{sech}^{-1}(ax)}}{2x^2} + \frac{\sqrt{1-ax}}{6ax^3\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{3x\sqrt{\frac{1}{1+ax}}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.78

$$\frac{(ax-1)\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2-1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSech[a*x]/x^3,x]
[Out] (-1 + (-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(3*a*x^3)
```

fricas [A] time = 0.66, size = 52, normalized size = 0.95

$$\frac{\left(a^3x^3-ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="fricas"
)
[Out] 1/3*((a^3*x^3 - a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1)/(a*x
^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="giac")
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^3, x)
```

maple [A] time = 0.05, size = 53, normalized size = 0.96

$$\frac{\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\left(a^2x^2-1\right)}{3x^2}-\frac{1}{3x^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x)
```

```
[Out] 1/3*(-(a*x-1)/a/x)^(1/2)/x^2*((a*x+1)/a/x)^(1/2)*(a^2*x^2-1)-1/3/x^3/a
```

maxima [A] time = 0.37, size = 43, normalized size = 0.78

$$\frac{\left(a^2x^3 - x\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{3\,ax^4} - \frac{1}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="maxima"
)
```

```
[Out] 1/3*(a^2*x^3 - x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a*x^4) - 1/3/(a*x^3)
```

mupad [B] time = 1.47, size = 58, normalized size = 1.05

$$-\frac{1}{3\,a\,x^3} - \frac{\left(\frac{\sqrt{\frac{1}{a\,x}+1}}{3} - \frac{a^2\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{3}\right)\sqrt{\frac{1}{a\,x}-1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^3,x)
```

```
[Out] - 1/(3*a*x^3) - (((1/(a*x) + 1)^(1/2)/3 - (a^2*x^2*(1/(a*x) + 1)^(1/2))/3)*
(1/(a*x) - 1)^(1/2))/x^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^4}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^3}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**3,x)
```

```
[Out] (Integral(x**(-4), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
3, x))/a
```

3.40 $\int \frac{e^{\mathbf{sech}^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=132

$$\frac{1}{8}a^3\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)+\frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{ax+1}}}+\frac{1}{12ax^4}-\frac{e^{\text{sech}^{-1}(ax)}}{3x^3}+\frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{ax+1}}}$$

```
[Out] 1/12/a/x^4-1/3*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3+1/12*(-a*x+1)^(1
/2)/a/x^4/(1/(a*x+1))^(1/2)+1/8*a*(-a*x+1)^(1/2)/x^2/(1/(a*x+1))^(1/2)+1/8*
a^3*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(1/(a*x+1))^(1/2)*(a*x+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6335, 30, 103, 12, 92, 208}

$$\frac{1}{8}a^3\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)+\frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{ax+1}}}+\frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{ax+1}}}+\frac{1}{12ax^4}-\frac{e^{\text{sech}^{-1}(ax)}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]/x^4,x]
```

```
[Out] 1/(12*a*x^4) - E^ArcSech[a*x]/(3*x^3) + Sqrt[1 - a*x]/(12*a*x^4*Sqrt[(1 + a
*x)^(-1)]) + (a*Sqrt[1 - a*x])/(8*x^2*Sqrt[(1 + a*x)^(-1)]) + (a^3*Sqrt[(1
+ a*x)^(-1)]*Sqrt[1 + a*x]*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^4}\,dx &= -\frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} - \frac{\int \frac{1}{x^5}\,dx}{3a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^5\sqrt{1-ax}\sqrt{1+ax}}\,dx}{3a}\\
&= \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int -\frac{3a^2}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{12a}\\
&= \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} - \frac{1}{4}\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx\\
&= \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{8}\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{a^2}{x\sqrt{1-ax}\sqrt{1}}\\
&= \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{8}\left(a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{}}\\
&= \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{8}\left(a^4\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\operatorname{Subst}\left(\int \frac{1}{a-}\right.\\
&= \frac{1}{12ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{8}a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\tanh^{-1}\left(\sqrt{1-ax}\right.
\end{aligned}$$

Mathematica [A] time = 0.08, size = 110, normalized size = 0.83

$$\frac{-a^4x^4\log(x) + a^4x^4\log\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1\right) + \sqrt{\frac{1-ax}{ax+1}}\left(a^3x^3 + a^2x^2 - 2ax - 2\right) - 2}{8ax^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x^4,x]
```

```
[Out] (-2 + Sqrt[(1 - a*x)/(1 + a*x)]*(-2 - 2*a*x + a^2*x^2 + a^3*x^3) - a^4*x^4*
Log[x] + a^4*x^4*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1
+ a*x)]])/(8*a*x^4)
```

fricas [A] time = 0.65, size = 138, normalized size = 1.05

$$\frac{a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 1\right) - a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 1\right) + 2\left(a^3x^3 - 2\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 4}{16\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="fricas"
)
```

```
[Out] 1/16*(a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a
^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*(a^3*x
^3 - 2*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 4)/(a*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^4, x)
```

maple [A] time = 0.06, size = 110, normalized size = 0.83

$$\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)x^4a^4+a^2x^2\sqrt{-a^2x^2+1}-2\sqrt{-a^2x^2+1}\right)}{8x^3\sqrt{-a^2x^2+1}}-\frac{1}{4x^4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x)
```

```
[Out] 1/8*(-(a*x-1)/a/x)^(1/2)/x^3*((a*x+1)/a/x)^(1/2)*(arctanh(1/(-a^2*x^2+1)^(1
/2))*x^4*a^4+a^2*x^2*(-a^2*x^2+1)^(1/2)-2*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^
(1/2)-1/4/x^4/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{1}{8}a^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{1}{8}\sqrt{-a^2x^2+1}\,a^4-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2}{8\,x^2}-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{4\,x^4}}{a}-\frac{1}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^5, x)/a - 1/4/(a*x^4)
```

mupad [B] time = 13.42, size = 602, normalized size = 4.56

$$\frac{a^3\operatorname{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)}{2}-\frac{\frac{35\,a^3\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{273\,a^3\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{715\,a^3\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{715\,a^3\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^9}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^9}+\frac{273\,a^3\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{11}}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{11}}}{1+\frac{28\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{56\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{70\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{56\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}+\frac{28\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{12}}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^4,x)
```

```
[Out] (a^3*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))/2 - ((35*
a^3*((1/(a*x) - 1)^(1/2) - 1i)^3)/(2*((1/(a*x) + 1)^(1/2) - 1)^3) + (273*a^
3*((1/(a*x) - 1)^(1/2) - 1i)^5)/(2*((1/(a*x) + 1)^(1/2) - 1)^5) + (715*a^3*
((1/(a*x) - 1)^(1/2) - 1i)^7)/(2*((1/(a*x) + 1)^(1/2) - 1)^7) + (715*a^3*((
1/(a*x) - 1)^(1/2) - 1i)^9)/(2*((1/(a*x) + 1)^(1/2) - 1)^9) + (273*a^3*((1/
```

```
(a*x) - 1)^(1/2) - 1i)^11)/(2*((1/(a*x) + 1)^(1/2) - 1)^11) + (35*a^3*((1/(
a*x) - 1)^(1/2) - 1i)^13)/(2*((1/(a*x) + 1)^(1/2) - 1)^13) + (a^3*((1/(a*x)
 - 1)^(1/2) - 1i)^15)/(2*((1/(a*x) + 1)^(1/2) - 1)^15) + (a^3*((1/(a*x) - 1
)^(1/2) - 1i))/(2*((1/(a*x) + 1)^(1/2) - 1)))/((28*((1/(a*x) - 1)^(1/2) - 1
i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (8*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a
*x) + 1)^(1/2) - 1)^2 - (56*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1
/2) - 1)^6 + (70*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8
- (56*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + (28*((1
/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (8*((1/(a*x) - 1
)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + ((1/(a*x) - 1)^(1/2) - 1i)
^16/((1/(a*x) + 1)^(1/2) - 1)^16 + 1) - 1/(4*a*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^5}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^4}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**4,x)
```

```
[Out] (Integral(x**(-5), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
4, x))/a
```

3.41 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=115

$$\frac{2a^3\sqrt{1-ax}}{15x\sqrt{\frac{1}{ax+1}}}+\frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{ax+1}}}+\frac{1}{20ax^5}-\frac{e^{\text{sech}^{-1}(ax)}}{4x^4}+\frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{ax+1}}}$$

```
[Out] 1/20/a/x^5-1/4*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4+1/20*(-a*x+1)^(1
/2)/a/x^5/(1/(a*x+1))^(1/2)+1/15*a*(-a*x+1)^(1/2)/x^3/(1/(a*x+1))^(1/2)+2/1
5*a^3*(-a*x+1)^(1/2)/x/(1/(a*x+1))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6335, 30, 103, 12, 95}

$$\frac{2a^3\sqrt{1-ax}}{15x\sqrt{\frac{1}{ax+1}}}+\frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{ax+1}}}+\frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{ax+1}}}+\frac{1}{20ax^5}-\frac{e^{\text{sech}^{-1}(ax)}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]/x^5,x]
```

```
[Out] 1/(20*a*x^5) - E^ArcSech[a*x]/(4*x^4) + Sqrt[1 - a*x]/(20*a*x^5*Sqrt[(1 + a
*x)^(-1)]) + (a*Sqrt[1 - a*x])/(15*x^3*Sqrt[(1 + a*x)^(-1)]) + (2*a^3*Sqrt[
1 - a*x])/(15*x*Sqrt[(1 + a*x)^(-1)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
```

```
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^5}\,dx = -\frac{e^{\text{sech}^{-1}(ax)}}{4x^4} - \frac{\int \frac{1}{x^6}\,dx}{4a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^6\sqrt{1-ax}\sqrt{1+ax}}\,dx}{4a}$$

$$= \frac{1}{20ax^5} - \frac{e^{\text{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int -\frac{4a^2}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{20a}$$

$$= \frac{1}{20ax^5} - \frac{e^{\text{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} - \frac{1}{5}\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx$$

$$= \frac{1}{20ax^5} - \frac{e^{\text{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{1+ax}}} + \frac{1}{15}\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int -\frac{2}{x^2\sqrt{1-}}$$

$$= \frac{1}{20ax^5} - \frac{e^{\text{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{1+ax}}} - \frac{1}{15}\left(2a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{}{x^2\sqrt{1-}}$$

$$= \frac{1}{20ax^5} - \frac{e^{\text{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{1+ax}}} + \frac{2a^3\sqrt{1-ax}}{15x\sqrt{\frac{1}{1+ax}}}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.52

$$\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^2\left(2a^3x^3-2a^2x^2+3ax-3\right)-3}{15ax^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x^5,x]
```

```
[Out] (-3 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-3 + 3*a*x - 2*a^2*x^2 + 2*a^3
*x^3))/(15*a*x^5)
```

fricas [A] time = 0.49, size = 60, normalized size = 0.52

$$\frac{\left(2\,a^5x^5+a^3x^3-3\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-3}{15\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="fricas"
)
```

```
[Out] 1/15*((2*a^5*x^5 + a^3*x^3 - 3*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(
a*x)) - 3)/(a*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^5, x)
```

maple [A] time = 0.06, size = 63, normalized size = 0.55

$$\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\left(a^{2}x^{2}-1\right)\left(2a^{2}x^{2}+3\right)}{15x^{4}}-\frac{1}{5a\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x)
```

```
[Out] 1/15*(-(a*x-1)/a/x)^(1/2)/x^4*((a*x+1)/a/x)^(1/2)*(a^2*x^2-1)*(2*a^2*x^2+3)
-1/5/a/x^5
```

maxima [A] time = 0.33, size = 51, normalized size = 0.44

$$\frac{\left(2\,a^{4}x^{5}+a^{2}x^{3}-3\,x\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{15\,ax^{6}}-\frac{1}{5\,ax^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="maxima"
)
```

```
[Out] 1/15*(2*a^4*x^5 + a^2*x^3 - 3*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a*x^6) - 1/5
/(a*x^5)
```

mupad [B] time = 1.56, size = 76, normalized size = 0.66

$$\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{a^2\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{15}-\frac{\sqrt{\frac{1}{a\,x}+1}}{5}+\frac{2\,a^4\,x^4\,\sqrt{\frac{1}{a\,x}+1}}{15}\right)}{x^4}-\frac{1}{5\,a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^5,x)
```

```
[Out] ((1/(a*x) - 1)^(1/2)*((a^2*x^2*(1/(a*x) + 1)^(1/2))/15 - (1/(a*x) + 1)^(1/2
)/5 + (2*a^4*x^4*(1/(a*x) + 1)^(1/2))/15))/x^4 - 1/(5*a*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^{6}}\, dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^{5}}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**5,x)
```

```
[Out] (Integral(x**(-6), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
5, x))/a
```

3.42 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^6}\,dx$

Optimal. Leaf size=163

$$\frac{1}{16}a^5\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)+\frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{ax+1}}}+\frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{ax+1}}}+\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}+\frac{a\sqrt{1-a}}{24x^4\sqrt{\frac{1}{ax}}}$$

```
[Out] 1/30/a/x^6-1/5*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5+1/30*(-a*x+1)^(1
/2)/a/x^6/(1/(a*x+1))^(1/2)+1/24*a*(-a*x+1)^(1/2)/x^4/(1/(a*x+1))^(1/2)+1/1
6*a^3*(-a*x+1)^(1/2)/x^2/(1/(a*x+1))^(1/2)+1/16*a^5*arctanh((-a*x+1)^(1/2)*
(a*x+1)^(1/2))*(1/(a*x+1))^(1/2)*(a*x+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6335, 30, 103, 12, 92, 208}

$$\frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{ax+1}}}+\frac{1}{16}a^5\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)+\frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{ax+1}}}+\frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{ax+1}}}+\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]/x^6,x]
```

```
[Out] 1/(30*a*x^6) - E^ArcSech[a*x]/(5*x^5) + Sqrt[1 - a*x]/(30*a*x^6*Sqrt[(1 + a
*x)^(-1)]) + (a*Sqrt[1 - a*x])/(24*x^4*Sqrt[(1 + a*x)^(-1)]) + (a^3*Sqrt[1
- a*x])/(16*x^2*Sqrt[(1 + a*x)^(-1)]) + (a^5*Sqrt[(1 + a*x)^(-1)]*Sqrt[1 +
a*x]*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^6}\,dx = -\frac{e^{\text{sech}^{-1}(ax)}}{5x^5} - \frac{\int \frac{1}{x^7}\,dx}{5a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^7\sqrt{1-ax}\sqrt{1+ax}}\,dx}{5a}$$

$$= \frac{1}{30ax^6} - \frac{e^{\text{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int -\frac{5a^2}{x^5\sqrt{1-ax}\sqrt{1+ax}}\,dx}{30a}$$

$$= \frac{1}{30ax^6} - \frac{e^{\text{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} - \frac{1}{6}\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^5\sqrt{1-ax}\sqrt{1+ax}}\,dx$$

$$= \frac{1}{30ax^6} - \frac{e^{\text{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{1}{24}\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int -\frac{3a^2}{x^3\sqrt{1-ax}\,\cdots}$$

$$= \frac{1}{30ax^6} - \frac{e^{\text{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} - \frac{1}{8}\left(a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^3\sqrt{1-ax}\,\sqrt{\cdots}}$$

$$= \frac{1}{30ax^6} - \frac{e^{\text{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{16}\left(a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\cdots$$

$$= \frac{1}{30ax^6} - \frac{e^{\text{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{16}\left(a^5\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\cdots$$

$$= \frac{1}{30ax^6} - \frac{e^{\text{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{16}\left(a^6\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\cdots$$

$$= \frac{1}{30ax^6} - \frac{e^{\text{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{16}a^5\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\;\text{ta}\cdots$$

Mathematica [A] time = 0.10, size = 129, normalized size = 0.79

$$\frac{-3a^6x^6\log(x) + 3a^6x^6\log\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1\right) + \sqrt{\frac{1-ax}{ax+1}}\left(3a^5x^5 + 3a^4x^4 + 2a^3x^3 + 2a^2x^2 - 8ax - 8\right) - 8}{48ax^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x^6,x]
```

```
[Out] (-8 + Sqrt[(1 - a*x)/(1 + a*x)]*(-8 - 8*a*x + 2*a^2*x^2 + 2*a^3*x^3 + 3*a^4
*x^4 + 3*a^5*x^5) - 3*a^6*x^6*Log[x] + 3*a^6*x^6*Log[1 + Sqrt[(1 - a*x)/(1
+ a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(48*a*x^6)
```

fricas [A] time = 0.86, size = 148, normalized size = 0.91

$$\frac{3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+2\left(3\,a^5x^5+2\,a^3x^3-8\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-}}{96\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="fricas"
)
```

```
[Out] 1/96*(3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) -
 3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*(3
*a^5*x^5 + 2*a^3*x^3 - 8*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))
- 16)/(a*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^6, x)
```

maple [A] time = 0.06, size = 132, normalized size = 0.81

$$\frac{\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\left(3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)x^6a^6+3\sqrt{-a^2x^2+1}\,x^4a^4+2a^2x^2\sqrt{-a^2x^2+1}-8\sqrt{-a^2x^2+1}\right)}{48x^5\sqrt{-a^2x^2+1}}-\frac{1}{6x^6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x)
```

```
[Out] 1/48*(-(a*x-1)/a/x)^(1/2)/x^5*((a*x+1)/a/x)^(1/2)*(3*arctanh(1/(-a^2*x^2+1)
^(1/2))*x^6*a^6+3*(-a^2*x^2+1)^(1/2)*x^4*a^4+2*a^2*x^2*(-a^2*x^2+1)^(1/2)-8
*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/6/x^6/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{1}{16}\,a^6\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{1}{16}\sqrt{-a^2x^2+1}\,a^6-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}a^4}{16\,x^2}-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2}{8\,x^4}-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{6\,x^6}}{a}-\frac{1}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^7, x)/a - 1/6/(a*x^6)
```

mupad [B] time = 34.08, size = 878, normalized size = 5.39

$$\frac{\frac{35\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{12\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{757\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{7339\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{41929\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^9}{6\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^9}+\frac{25661\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{11}}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{11}}+\frac{25661\,a^5\left(\sqrt{\frac{1}{a\,x}-}\right.}{2\left(\sqrt{\frac{1}{a\,x}+1-}\right.}}{1+\frac{66\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{220\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{495\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{792\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}+\frac{924\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{12}}-\frac{792\left(\sqrt{\frac{1}{a\,x}-}\right.}{\left(\sqrt{\frac{1}{a\,x}+1-}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^6,x)
```

```
[Out] ((35*a^5*((1/(a*x) - 1)^(1/2) - 1i)^3)/(12*((1/(a*x) + 1)^(1/2) - 1)^3) + (
757*a^5*((1/(a*x) - 1)^(1/2) - 1i)^5)/(4*((1/(a*x) + 1)^(1/2) - 1)^5) + (73
39*a^5*((1/(a*x) - 1)^(1/2) - 1i)^7)/(4*((1/(a*x) + 1)^(1/2) - 1)^7) + (419
29*a^5*((1/(a*x) - 1)^(1/2) - 1i)^9)/(6*((1/(a*x) + 1)^(1/2) - 1)^9) + (256
61*a^5*((1/(a*x) - 1)^(1/2) - 1i)^11)/(2*((1/(a*x) + 1)^(1/2) - 1)^11) + (2
5661*a^5*((1/(a*x) - 1)^(1/2) - 1i)^13)/(2*((1/(a*x) + 1)^(1/2) - 1)^13) +
(41929*a^5*((1/(a*x) - 1)^(1/2) - 1i)^15)/(6*((1/(a*x) + 1)^(1/2) - 1)^15)
+ (7339*a^5*((1/(a*x) - 1)^(1/2) - 1i)^17)/(4*((1/(a*x) + 1)^(1/2) - 1)^17)
 + (757*a^5*((1/(a*x) - 1)^(1/2) - 1i)^19)/(4*((1/(a*x) + 1)^(1/2) - 1)^19)
 + (35*a^5*((1/(a*x) - 1)^(1/2) - 1i)^21)/(12*((1/(a*x) + 1)^(1/2) - 1)^21)
 - (a^5*((1/(a*x) - 1)^(1/2) - 1i)^23)/(4*((1/(a*x) + 1)^(1/2) - 1)^23) - (
a^5*((1/(a*x) - 1)^(1/2) - 1i))/(4*((1/(a*x) + 1)^(1/2) - 1)))/((66*((1/(a*
x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (12*((1/(a*x) - 1)^(1/
2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (220*((1/(a*x) - 1)^(1/2) - 1i)^6
)/((1/(a*x) + 1)^(1/2) - 1)^6 + (495*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x
) + 1)^(1/2) - 1)^8 - (792*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1
/2) - 1)^10 + (924*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)
^12 - (792*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + (4
95*((1/(a*x) - 1)^(1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) - 1)^16 - (220*((1/(
a*x) - 1)^(1/2) - 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 + (66*((1/(a*x) - 1)
^(1/2) - 1i)^20)/((1/(a*x) + 1)^(1/2) - 1)^20 - (12*((1/(a*x) - 1)^(1/2) -
1i)^22)/((1/(a*x) + 1)^(1/2) - 1)^22 + ((1/(a*x) - 1)^(1/2) - 1i)^24/((1/(a
*x) + 1)^(1/2) - 1)^24 + 1) + (a^5*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*
x) + 1)^(1/2) - 1)))/4 - 1/(6*a*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^7}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^6}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**6,x)
```

```
[Out] (Integral(x**(-7), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
6, x))/a
```

3.43 $\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^7}\,dx$

Optimal. Leaf size=146

$$\frac{8a^5\sqrt{1-ax}}{105x\sqrt{\frac{1}{ax+1}}}+\frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{ax+1}}}+\frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{ax+1}}}+\frac{1}{42ax^7}-\frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6}+\frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{ax+1}}}$$

```
[Out] 1/42/a/x^7-1/6*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6+1/42*(-a*x+1)^(1
/2)/a/x^7/(1/(a*x+1))^(1/2)+1/35*a*(-a*x+1)^(1/2)/x^5/(1/(a*x+1))^(1/2)+4/1
05*a^3*(-a*x+1)^(1/2)/x^3/(1/(a*x+1))^(1/2)+8/105*a^5*(-a*x+1)^(1/2)/x/(1/(
a*x+1))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6335, 30, 103, 12, 95}

$$\frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{ax+1}}}+\frac{8a^5\sqrt{1-ax}}{105x\sqrt{\frac{1}{ax+1}}}+\frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{ax+1}}}+\frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{ax+1}}}+\frac{1}{42ax^7}-\frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]/x^7,x]
```

```
[Out] 1/(42*a*x^7) - E^ArcSech[a*x]/(6*x^6) + Sqrt[1 - a*x]/(42*a*x^7*Sqrt[(1 + a
*x)^(-1)]) + (a*Sqrt[1 - a*x])/(35*x^5*Sqrt[(1 + a*x)^(-1)]) + (4*a^3*Sqrt[
1 - a*x])/(105*x^3*Sqrt[(1 + a*x)^(-1)]) + (8*a^5*Sqrt[1 - a*x])/(105*x*Sqr
t[(1 + a*x)^(-1)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^7}\,dx = -\frac{e^{\text{sech}^{-1}(ax)}}{6x^6} - \frac{\int \frac{1}{x^8}\,dx}{6a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^8\sqrt{1-ax}\sqrt{1+ax}}\,dx}{6a}$$

$$= \frac{1}{42ax^7} - \frac{e^{\text{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int -\frac{6a^2}{x^6\sqrt{1-ax}\sqrt{1+ax}}\,dx}{42a}$$

$$= \frac{1}{42ax^7} - \frac{e^{\text{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} - \frac{1}{7}\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^6\sqrt{1-ax}\sqrt{1+ax}}\,dx$$

$$= \frac{1}{42ax^7} - \frac{e^{\text{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} + \frac{1}{35}\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int -\frac{4a^2}{x^4\sqrt{1-ax}\,\cdots}$$

$$= \frac{1}{42ax^7} - \frac{e^{\text{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} - \frac{1}{35}\left(4a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^4\sqrt{1-ax}\,\cdots}$$

$$= \frac{1}{42ax^7} - \frac{e^{\text{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} + \frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{1+ax}}} + \frac{1}{105}\left(4a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+\cdots}\right.$$

$$= \frac{1}{42ax^7} - \frac{e^{\text{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} + \frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{1+ax}}} - \frac{1}{105}\left(8a^5\sqrt{\frac{1}{1+ax}}\sqrt{1+\cdots}\right.$$

$$= \frac{1}{42ax^7} - \frac{e^{\text{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} + \frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{1+ax}}} + \frac{8a^5\sqrt{1-ax}}{105x\sqrt{\frac{1}{1+ax}}}$$

Mathematica [A] time = 0.09, size = 76, normalized size = 0.52

$$\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2\left(8a^5x^5-8a^4x^4+12a^3x^3-12a^2x^2+15ax-15\right)-15}{105ax^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x^7,x]
```

```
[Out] (-15 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-15 + 15*a*x - 12*a^2*x^2 + 1
2*a^3*x^3 - 8*a^4*x^4 + 8*a^5*x^5))/(105*a*x^7)
```

fricas [A] time = 0.49, size = 69, normalized size = 0.47

$$\frac{\left(8\,a^7x^7+4\,a^5x^5+3\,a^3x^3-15\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-15}{105\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="fricas"
)
```

```
[Out] 1/105*((8*a^7*x^7 + 4*a^5*x^5 + 3*a^3*x^3 - 15*a*x)*sqrt((a*x + 1)/(a*x))*s
qrt(-(a*x - 1)/(a*x)) - 15)/(a*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^7, x)
```

maple [A] time = 0.08, size = 71, normalized size = 0.49

$$\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\left(a^2x^2-1\right)\left(8x^4a^4+12a^2x^2+15\right)}{105x^6}-\frac{1}{7a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x)
```

```
[Out] 1/105*(-(a*x-1)/a/x)^(1/2)/x^6*((a*x+1)/a/x)^(1/2)*(a^2*x^2-1)*(8*a^4*x^4+1
2*a^2*x^2+15)-1/7/a/x^7
```

maxima [A] time = 0.34, size = 60, normalized size = 0.41

$$\frac{\left(8\,a^6x^7+4\,a^4x^5+3\,a^2x^3-15\,x\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{105\,ax^8}-\frac{1}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="maxima"
)
```

```
[Out] 1/105*(8*a^6*x^7 + 4*a^4*x^5 + 3*a^2*x^3 - 15*x)*sqrt(a*x + 1)*sqrt(-a*x +
1)/(a*x^8) - 1/7/(a*x^7)
```

mupad [B] time = 1.68, size = 95, normalized size = 0.65

$$\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{a^2\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{35}-\frac{\sqrt{\frac{1}{a\,x}+1}}{7}+\frac{4\,a^4\,x^4\,\sqrt{\frac{1}{a\,x}+1}}{105}+\frac{8\,a^6\,x^6\,\sqrt{\frac{1}{a\,x}+1}}{105}\right)}{x^6}-\frac{1}{7\,a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^7,x)
```

```
[Out] ((1/(a*x) - 1)^(1/2)*((a^2*x^2*(1/(a*x) + 1)^(1/2))/35 - (1/(a*x) + 1)^(1/2
)/7 + (4*a^4*x^4*(1/(a*x) + 1)^(1/2))/105 + (8*a^6*x^6*(1/(a*x) + 1)^(1/2))
/105))/x^6 - 1/(7*a*x^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^8}\,dx+\int \frac{a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^7}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**7,x)
```

```
[Out] (Integral(x**(-8), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
7, x))/a
```

3.44 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^8}\,dx$

Optimal. Leaf size=194

$$\frac{5}{128}a^7\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)+\frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{ax+1}}}+\frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{ax+1}}}+\frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{ax+1}}}+\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}}}{7x^7}$$

```
[Out] 1/56/a/x^8-1/7*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7+1/56*(-a*x+1)^(1
/2)/a/x^8/(1/(a*x+1))^(1/2)+1/48*a*(-a*x+1)^(1/2)/x^6/(1/(a*x+1))^(1/2)+5/1
92*a^3*(-a*x+1)^(1/2)/x^4/(1/(a*x+1))^(1/2)+5/128*a^5*(-a*x+1)^(1/2)/x^2/(1
/(a*x+1))^(1/2)+5/128*a^7*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(1/(a*x+1))
^(1/2)*(a*x+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6335, 30, 103, 12, 92, 208}

$$\frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{ax+1}}}+\frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{ax+1}}}+\frac{5}{128}a^7\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)+\frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{ax+1}}}+\frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{ax+1}}}+$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]/x^8,x]
```

```
[Out] 1/(56*a*x^8) - E^ArcSech[a*x]/(7*x^7) + Sqrt[1 - a*x]/(56*a*x^8*Sqrt[(1 + a
*x)^(-1)]) + (a*Sqrt[1 - a*x])/(48*x^6*Sqrt[(1 + a*x)^(-1)]) + (5*a^3*Sqrt[
1 - a*x])/(192*x^4*Sqrt[(1 + a*x)^(-1)]) + (5*a^5*Sqrt[1 - a*x])/(128*x^2*S
qrt[(1 + a*x)^(-1)]) + (5*a^7*Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*ArcTanh[Sq
rt[1 - a*x]*Sqrt[1 + a*x]])/128
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^8}\,dx = -\frac{e^{\text{sech}^{-1}(ax)}}{7x^7} - \frac{\int \frac{1}{x^9}\,dx}{7a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{x^9\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{7a}$$

$$= \frac{1}{56ax^8} - \frac{e^{\text{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int -\frac{7a^2}{x^7\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{56a}$$

$$= \frac{1}{56ax^8} - \frac{e^{\text{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} - \frac{1}{8}\left(a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{x^7\sqrt{1-ax}\,\sqrt{1+ax}}\,dx$$

$$= \frac{1}{56ax^8} - \frac{e^{\text{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{1}{48}\left(a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int -\frac{\cdots}{x^5\sqrt{1-\cdots}}$$

$$= \frac{1}{56ax^8} - \frac{e^{\text{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} - \frac{1}{48}\left(5a^3\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{\cdots}{x^5\sqrt{1-\cdots}}$$

$$= \frac{1}{56ax^8} - \frac{e^{\text{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{1}{192}\left(5a^3\sqrt{\frac{1}{1+ax}}\,\sqrt{1\cdots}\right.$$

$$= \frac{1}{56ax^8} - \frac{e^{\text{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} - \frac{1}{64}\left(5a^5\sqrt{\frac{1}{1+ax}}\,\sqrt{1+\cdots}\right.$$

$$= \frac{1}{56ax^8} - \frac{e^{\text{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{128}\left(5\cdots\right.$$

$$= \frac{1}{56ax^8} - \frac{e^{\text{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{128}\left(5\cdots\right.$$

$$= \frac{1}{56ax^8} - \frac{e^{\text{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{128}\left(5\cdots\right.$$

$$= \frac{1}{56ax^8} - \frac{e^{\text{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{1+ax}}} + \frac{5}{128}a^7\cdots$$

Mathematica [A] time = 0.13, size = 145, normalized size = 0.75

$$\frac{-15a^8x^8\log(x) + 15a^8x^8\log\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1\right) + \sqrt{\frac{1-ax}{ax+1}}\left(15a^7x^7 + 15a^6x^6 + 10a^5x^5 + 10a^4x^4 + 8a^3x^3 \cdots\right.}{384ax^8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x^8,x]
```

```
[Out] (-48 + Sqrt[(1 - a*x)/(1 + a*x)]*(-48 - 48*a*x + 8*a^2*x^2 + 8*a^3*x^3 + 10
*a^4*x^4 + 10*a^5*x^5 + 15*a^6*x^6 + 15*a^7*x^7) - 15*a^8*x^8*Log[x] + 15*a
^8*x^8*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/
(384*a*x^8)
```

fricas [A] time = 0.55, size = 156, normalized size = 0.80

$$\frac{15\,a^8x^8\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-15\,a^8x^8\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+2\left(15\,a^7x^7+10\,a^5x^5+8\,a^3x^3-48\,ax\right.}{768\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x, algorithm="fricas"
)
```

```
[Out] 1/768*(15*a^8*x^8*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1)
 - 15*a^8*x^8*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2
*(15*a^7*x^7 + 10*a^5*x^5 + 8*a^3*x^3 - 48*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(
-(a*x - 1)/(a*x)) - 96)/(a*x^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^8, x)
```

maple [A] time = 0.09, size = 152, normalized size = 0.78

$$\frac{\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\left(15\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)x^8a^8+15\sqrt{-a^2x^2+1}\,x^6a^6+10\sqrt{-a^2x^2+1}\,x^4a^4+8a^2x^2\sqrt{-a^2x^2+1}-\right.}{384x^7\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x)
```

```
[Out] 1/384*(-(a*x-1)/a/x)^(1/2)/x^7*((a*x+1)/a/x)^(1/2)*(15*arctanh(1/(-a^2*x^2+
1)^(1/2))*x^8*a^8+15*(-a^2*x^2+1)^(1/2)*x^6*a^6+10*(-a^2*x^2+1)^(1/2)*x^4*a
^4+8*a^2*x^2*(-a^2*x^2+1)^(1/2)-48*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1
/8/a/x^8
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{5}{128}\,a^8\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{5}{128}\sqrt{-a^2x^2+1}\,a^8-\frac{5\left(-a^2x^2+1\right)^{\frac{3}{2}}a^6}{128\,x^2}-\frac{5\left(-a^2x^2+1\right)^{\frac{3}{2}}a^4}{64\,x^4}-\frac{5\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2}{48\,x^6}-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{8\,x^8}}{a}-\frac{1}{8\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^9, x)/a - 1/8/(a*x^8)
```

mupad [B] time = 38.56, size = 1155, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^8,x)
```

```
[Out] (5*a^7*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))/32 - ((
1723*a^7*((1/(a*x) - 1)^(1/2) - 1i)^5)/(96*((1/(a*x) + 1)^(1/2) - 1)^5) - (
235*a^7*((1/(a*x) - 1)^(1/2) - 1i)^3)/(96*((1/(a*x) + 1)^(1/2) - 1)^3) + (7
2283*a^7*((1/(a*x) - 1)^(1/2) - 1i)^7)/(32*((1/(a*x) + 1)^(1/2) - 1)^7) + (
848801*a^7*((1/(a*x) - 1)^(1/2) - 1i)^9)/(32*((1/(a*x) + 1)^(1/2) - 1)^9) +
 (4181067*a^7*((1/(a*x) - 1)^(1/2) - 1i)^11)/(32*((1/(a*x) + 1)^(1/2) - 1)^
11) + (10994181*a^7*((1/(a*x) - 1)^(1/2) - 1i)^13)/(32*((1/(a*x) + 1)^(1/2)
 - 1)^13) + (17457599*a^7*((1/(a*x) - 1)^(1/2) - 1i)^15)/(32*((1/(a*x) + 1)
^(1/2) - 1)^15) + (17457599*a^7*((1/(a*x) - 1)^(1/2) - 1i)^17)/(32*((1/(a*x
) + 1)^(1/2) - 1)^17) + (10994181*a^7*((1/(a*x) - 1)^(1/2) - 1i)^19)/(32*((
1/(a*x) + 1)^(1/2) - 1)^19) + (4181067*a^7*((1/(a*x) - 1)^(1/2) - 1i)^21)/(
32*((1/(a*x) + 1)^(1/2) - 1)^21) + (848801*a^7*((1/(a*x) - 1)^(1/2) - 1i)^2
3)/(32*((1/(a*x) + 1)^(1/2) - 1)^23) + (72283*a^7*((1/(a*x) - 1)^(1/2) - 1i
)^25)/(32*((1/(a*x) + 1)^(1/2) - 1)^25) + (1723*a^7*((1/(a*x) - 1)^(1/2) -
1i)^27)/(96*((1/(a*x) + 1)^(1/2) - 1)^27) - (235*a^7*((1/(a*x) - 1)^(1/2) -
 1i)^29)/(96*((1/(a*x) + 1)^(1/2) - 1)^29) + (5*a^7*((1/(a*x) - 1)^(1/2) -
1i)^31)/(32*((1/(a*x) + 1)^(1/2) - 1)^31) + (5*a^7*((1/(a*x) - 1)^(1/2) - 1
i))/(32*((1/(a*x) + 1)^(1/2) - 1)))/((120*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1
/(a*x) + 1)^(1/2) - 1)^4 - (16*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)
^(1/2) - 1)^2 - (560*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1
)^6 + (1820*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (43
68*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + (8008*((1/
(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (11440*((1/(a*x)
- 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + (12870*((1/(a*x) - 1)^(
1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) - 1)^16 - (11440*((1/(a*x) - 1)^(1/2) -
 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 + (8008*((1/(a*x) - 1)^(1/2) - 1i)^20
)/((1/(a*x) + 1)^(1/2) - 1)^20 - (4368*((1/(a*x) - 1)^(1/2) - 1i)^22)/((1/(
a*x) + 1)^(1/2) - 1)^22 + (1820*((1/(a*x) - 1)^(1/2) - 1i)^24)/((1/(a*x) +
1)^(1/2) - 1)^24 - (560*((1/(a*x) - 1)^(1/2) - 1i)^26)/((1/(a*x) + 1)^(1/2)
 - 1)^26 + (120*((1/(a*x) - 1)^(1/2) - 1i)^28)/((1/(a*x) + 1)^(1/2) - 1)^28
 - (16*((1/(a*x) - 1)^(1/2) - 1i)^30)/((1/(a*x) + 1)^(1/2) - 1)^30 + ((1/(a
*x) - 1)^(1/2) - 1i)^32/((1/(a*x) + 1)^(1/2) - 1)^32 + 1) - 1/(8*a*x^8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^9}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^8}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**8,x)
```

```
[Out] (Integral(x**(-9), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
8, x))/a
```

3.45 $\int e^{\text{sech}^{-1}(ax^2)}x^7\,dx$

Optimal. Leaf size=111

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,\sin^{-1}(ax^2)}{16a^4}-\frac{x^2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{16a^3}+\frac{x^6}{24a}+\frac{1}{8}x^8e^{\text{sech}^{-1}(ax^2)}$$

```
[Out] 1/24*x^6/a+1/8*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^8+1/16*arcsi
n(a*x^2)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^4-1/16*x^2*(1/(a*x^2+1))^(1/
2)*(a*x^2+1)^(1/2)*(-a^2*x^4+1)^(1/2)/a^3
```

Rubi [A] time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6335, 30, 259, 275, 321, 216}

$$-\frac{x^2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{16a^3}+\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,\sin^{-1}(ax^2)}{16a^4}+\frac{x^6}{24a}+\frac{1}{8}x^8e^{\text{sech}^{-1}(ax^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x^7,x]
```

```
[Out] x^6/(24*a) + (E^ArcSech[a*x^2]*x^8)/8 - (x^2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1
+ a*x^2]*Sqrt[1 - a^2*x^4])/(16*a^3) + (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x
^2]*ArcSin[a*x^2])/(16*a^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^7\,dx = \frac{1}{8}e^{\operatorname{sech}^{-1}(ax^2)}x^8+\frac{\int x^5\,dx}{4a}+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{x^5}{\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{4a}$$
$$=\frac{x^6}{24a}+\frac{1}{8}e^{\operatorname{sech}^{-1}(ax^2)}x^8+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{x^5}{\sqrt{1-a^2x^4}}\,dx}{4a}$$
$$=\frac{x^6}{24a}+\frac{1}{8}e^{\operatorname{sech}^{-1}(ax^2)}x^8+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\operatorname{Subst}\left(\int\frac{x^2}{\sqrt{1-a^2x^2}}\,dx,x,x^2\right)}{8a}$$
$$=\frac{x^6}{24a}+\frac{1}{8}e^{\operatorname{sech}^{-1}(ax^2)}x^8-\frac{x^2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{16a^3}+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\operatorname{Subst}\left(\int\ldots\right.}{16a^3}$$
$$=\frac{x^6}{24a}+\frac{1}{8}e^{\operatorname{sech}^{-1}(ax^2)}x^8-\frac{x^2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{16a^3}+\frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sin^{-1}\left(ax^2\right)}{16a^4}$$

Mathematica [C] time = 0.19, size = 111, normalized size = 1.00

$$\frac{8a^3x^6-3a\sqrt{\frac{1-ax^2}{ax^2+1}}\left(-2a^3x^8-2a^2x^6+ax^4+x^2\right)+3i\log\left(2\sqrt{\frac{1-ax^2}{ax^2+1}}\left(ax^2+1\right)-2iax^2\right)}{48a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^7,x]
```

```
[Out] (8*a^3*x^6 - 3*a*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x^2 + a*x^4 - 2*a^2*x^6 - 2
*a^3*x^8) + (3*I)*Log[(-2*I)*a*x^2 + 2*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a
*x^2)])/(48*a^4)
```

fricas [A] time = 0.50, size = 116, normalized size = 1.05

$$\frac{8\,a^3x^6+3\left(2\,a^4x^8-a^2x^4\right)\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}-6\,\arctan\left(\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}-1}{ax^2}\right)}{48\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7,x, algorithm="f
ricas")
```

```
[Out] 1/48*(8*a^3*x^6 + 3*(2*a^4*x^8 - a^2*x^4)*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(
a*x^2 - 1)/(a*x^2)) - 6*arctan((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^
2 - 1)/(a*x^2)) - 1)/(a*x^2)))/a^4
```

giac [B] time = 0.21, size = 205, normalized size = 1.85

$$\frac{8\,a^2x^6+4\sqrt{a^2x^2+a}\sqrt{-a^2x^2+a}\left(\left(a^2x^2+a\right)\left(\frac{2\left(a^2x^2+a\right)}{a^4}-\frac{7}{a^3}\right)+\frac{9}{a^2}\right)+\left(\sqrt{a^2x^2+a}\sqrt{-a^2x^2+a}\left(\left(a^2x^2+a\right)\left(2\left(\ldots\right.\right.\right.\right.}{48\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7,x, algorithm="g
iac")
```

```
[Out] 1/48*(8*a^2*x^6 + 4*sqrt(a^2*x^2 + a)*sqrt(-a^2*x^2 + a)*((a^2*x^2 + a)*(2*
(a^2*x^2 + a)/a^4 - 7/a^3) + 9/a^2) + (sqrt(a^2*x^2 + a)*sqrt(-a^2*x^2 + a)
*((a^2*x^2 + a)*(2*(a^2*x^2 + a)*(3*(a^2*x^2 + a)/a^6 - 13/a^5) + 43/a^4) -
 39/a^3) - 18*arcsin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a))/a^2)*a + 24*arc
sin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a))/a)/a^3
```

maple [A] time = 0.22, size = 137, normalized size = 1.23

$$\frac{\sqrt{-\frac{a x^2-1}{a x^2}}\, x^2 \sqrt{\frac{a x^2+1}{a x^2}} \left(2x^6\sqrt{-\frac{a^2x^4-1}{a^2}}\, a^4 - x^2\sqrt{-\frac{a^2x^4-1}{a^2}}\, a^2 + \arctan\left(\frac{x^2}{\sqrt{-\frac{a^2x^4-1}{a^2}}}\right)\right)}{16\sqrt{-\frac{a^2x^4-1}{a^2}}\, a^4} + \frac{x^6}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7,x)
```

```
[Out] 1/16*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(2*x^6*(-(a^2*x^4
-1)/a^2)^(1/2)*a^4-x^2*(-(a^2*x^4-1)/a^2)^(1/2)*a^2+arctan(x^2/(-(a^2*x^4-1
)/a^2)^(1/2)))/(-(a^2*x^4-1)/a^2)^(1/2)/a^4+1/6*x^6/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^6}{6\,a} + \frac{-\frac{\left(-a^2x^4+1\right)^{\frac{3}{2}} x^2}{8\,a^2} + \frac{\sqrt{-a^2x^4+1}\,x^2}{16\,a^2} + \frac{\arcsin\left(ax^2\right)}{16\,a^3}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7,x, algorithm="m
axima")
```

```
[Out] 1/6*x^6/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)*x^5, x)/a
```

mupad [B] time = 14.45, size = 521, normalized size = 4.69

$$\frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+1\right)1\mathrm{i}}{16\,a^4} - \frac{\frac{1\mathrm{i}}{2048\,a^4} + \frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{256\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2} + \frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4 11\mathrm{i}}{1024\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4} + \frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^6 7\mathrm{i}}{512\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^6} - \frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^8 239\mathrm{i}}{2048\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)}}{\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4} + \frac{4\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^6} + \frac{6\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^8} + \frac{4\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^{10}} + \frac{\left(\sqrt{\frac{1}{a\,x^2}-\right.}{\left(\sqrt{\frac{1}{a\,x^2}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] (log(((1/(a*x^2) - 1)^(1/2) - 1i)^2/((1/(a*x^2) + 1)^(1/2) - 1)^2 + 1)*1i)/
(16*a^4) - (1i/(2048*a^4) + (((1/(a*x^2) - 1)^(1/2) - 1i)^2*1i)/(256*a^4*((
1/(a*x^2) + 1)^(1/2) - 1)^2) + (((1/(a*x^2) - 1)^(1/2) - 1i)^4*11i)/(1024*a
^4*((1/(a*x^2) + 1)^(1/2) - 1)^4) + (((1/(a*x^2) - 1)^(1/2) - 1i)^6*7i)/(51
2*a^4*((1/(a*x^2) + 1)^(1/2) - 1)^6) - (((1/(a*x^2) - 1)^(1/2) - 1i)^8*239i
)/(2048*a^4*((1/(a*x^2) + 1)^(1/2) - 1)^8) + (((1/(a*x^2) - 1)^(1/2) - 1i)^
10*1i)/(512*a^4*((1/(a*x^2) + 1)^(1/2) - 1)^10))/(((1/(a*x^2) - 1)^(1/2) -
```

```
1i)^4/((1/(a*x^2) + 1)^(1/2) - 1)^4 + (4*((1/(a*x^2) - 1)^(1/2) - 1i)^6)/((
1/(a*x^2) + 1)^(1/2) - 1)^6 + (6*((1/(a*x^2) - 1)^(1/2) - 1i)^8)/((1/(a*x^2
) + 1)^(1/2) - 1)^8 + (4*((1/(a*x^2) - 1)^(1/2) - 1i)^10)/((1/(a*x^2) + 1)^
(1/2) - 1)^10 + ((1/(a*x^2) - 1)^(1/2) - 1i)^12/((1/(a*x^2) + 1)^(1/2) - 1)
^12) - (log(((1/(a*x^2) - 1)^(1/2) - 1i)/((1/(a*x^2) + 1)^(1/2) - 1))*1i)/(
16*a^4) + x^6/(6*a) - (((1/(a*x^2) - 1)^(1/2) - 1i)^2*1i)/(512*a^4*((1/(a*x
^2) + 1)^(1/2) - 1)^2) - (((1/(a*x^2) - 1)^(1/2) - 1i)^4*1i)/(2048*a^4*((1/
(a*x^2) + 1)^(1/2) - 1)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**7,x)
```

```
[Out] Timed out
```

3.46 $\int e^{\text{sech}^{-1}\left(ax^2\right)} x^6 \, dx$

Optimal. Leaf size=115

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{21a^{7/2}}-\frac{2x\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{21a^3}+\frac{2x^5}{35a}+\frac{1}{7}x^7e^{\text{sech}^{-1}\left(ax^2\right)}$$

```
[Out] 2/35*x^5/a+1/7*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7+2/21*Ellip
ticF(x*a^(1/2),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(7/2)-2/21*x*(1/(a*
x^2+1))^(1/2)*(a*x^2+1)^(1/2)*(-a^2*x^4+1)^(1/2)/a^3
```

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6335, 30, 259, 321, 221}

$$-\frac{2x\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{21a^3}+\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{21a^{7/2}}+\frac{2x^5}{35a}+\frac{1}{7}x^7e^{\text{sech}^{-1}\left(ax^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x^6,x]
```

```
[Out] (2*x^5)/(35*a) + (E^ArcSech[a*x^2]*x^7)/7 - (2*x*Sqrt[(1 + a*x^2)^(-1)]*Sqr
t[1 + a*x^2]*Sqrt[1 - a^2*x^4])/(21*a^3) + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1
 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/(21*a^(7/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\text{sech}^{-1}\left(ax^2\right)}x^6\,dx = \frac{1}{7}e^{\text{sech}^{-1}\left(ax^2\right)}x^7 + \frac{2\int x^4\,dx}{7a} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{x^4}{\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{7a}$$

$$= \frac{2x^5}{35a} + \frac{1}{7}e^{\text{sech}^{-1}\left(ax^2\right)}x^7 + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{x^4}{\sqrt{1-a^2x^4}}\,dx}{7a}$$

$$= \frac{2x^5}{35a} + \frac{1}{7}e^{\text{sech}^{-1}\left(ax^2\right)}x^7 - \frac{2x\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{21a^3} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{\sqrt{1-a^2x^4}}}{21a^3}$$

$$= \frac{2x^5}{35a} + \frac{1}{7}e^{\text{sech}^{-1}\left(ax^2\right)}x^7 - \frac{2x\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{21a^3} + \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,F\left(\sin^{-1}\left(\sqrt{}\right.\right.}{21a^{7/2}}$$

Mathematica [C] time = 0.32, size = 139, normalized size = 1.21

$$-\frac{2i\sqrt{\frac{1-ax^2}{ax^2+1}}\sqrt{1-a^2x^4}\,F\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)}{21(-a)^{7/2}\left(ax^2-1\right)} + \frac{x\sqrt{\frac{1-ax^2}{ax^2+1}}\left(3a^3x^6+3a^2x^4-2ax^2-2\right)}{21a^3} + \frac{x^5}{5a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^6,x]
```

```
[Out] x^5/(5*a) + (x*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(-2 - 2*a*x^2 + 3*a^2*x^4 + 3*
a^3*x^6))/(21*a^3) - (((2*I)/21)*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2
*x^4]*EllipticF[I*ArcSinh[Sqrt[-a]*x], -1])/((-a)^(7/2)*(-1 + a*x^2))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax^6\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+x^4}{a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x, algorithm="f
ricas")
```

```
[Out] integral((a*x^6*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + x^4)
/a, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-4,[1,0]%%%},0,%%%{4,[4,4]%%%}] at param
eters values [86,-97]Warning, choosing root of [1,0,%%%{-4,[1,0]%%%},0,%%%{
4,[4,4]%%%}] at parameters values [-82,7]Warning, choosing root of [1,0,%%%
{-4,[1,0]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-27,26]Warning, choo
```

```
sing root of [1,0,%%%{-4,[1,0]%%%},0,%%%{4,[4,4]%%%}] at parameters values
[-89,63]Unable to divide, perhaps due to rounding error%%%{1,[4,2,1,1,1]%%%
}+%%%{1,[4,0,0,0,2]%%%} / %%%{1,[0,0,0,0,3]%%%} Error: Bad Argument Value
```

maple [A] time = 0.06, size = 114, normalized size = 0.99

$$\frac{\sqrt{-\frac{a\,x^2-1}{a\,x^2}}\;x^2\sqrt{\frac{a\,x^2+1}{a\,x^2}}\;\left(3x^9a^{\frac{9}{2}}-5x^5a^{\frac{5}{2}}-2\,\mathrm{EllipticF}\left(x\sqrt{a}\,,i\right)\sqrt{-a\,x^2+1}\;\sqrt{a\,x^2+1}\;+2x\sqrt{a}\right)}{21a^{\frac{5}{2}}\left(a^2x^4-1\right)}+\frac{x^5}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x)
```

```
[Out] 1/21*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(3*x^9*a^(9/2)-5*
x^5*a^(5/2)-2*EllipticF(x*a^(1/2),I)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)+2*x*a
^(1/2))/a^(5/2)/(a^2*x^4-1)+1/5*x^5/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^5}{5\,a}+\frac{\int\sqrt{ax^2+1}\,\sqrt{-ax^2+1}\,x^4\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x, algorithm="m
axima")
```

```
[Out] 1/5*x^5/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)*x^4, x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\left(\sqrt{\frac{1}{a\,x^2}-1}\;\sqrt{\frac{1}{a\,x^2}+1}\;+\frac{1}{a\,x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] int(x^6*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x^4\,dx+\int ax^6\sqrt{-1+\frac{1}{ax^2}}\,\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**6,x)
```

```
[Out] (Integral(x**4, x) + Integral(a*x**6*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x*
*2)), x))/a
```

3.47 $\int e^{\text{sech}^{-1}\left(ax^2\right)}x^5\,dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{1-ax^2}}{6a^3\sqrt{\frac{1}{ax^2+1}}}+\frac{x^4}{12a}+\frac{1}{6}x^6e^{\text{sech}^{-1}\left(ax^2\right)}$$

```
[Out] 1/12*x^4/a+1/6*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6-1/6*(-a*x^
2+1)^(1/2)/a^3/(1/(a*x^2+1))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6335, 30, 259, 261}

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{6a^3}+\frac{x^4}{12a}+\frac{1}{6}x^6e^{\text{sech}^{-1}\left(ax^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x^2]*x^5,x]
```

```
[Out] x^4/(12*a) + (E^ArcSech[a*x^2]*x^6)/6 - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*
x^2]*Sqrt[1 - a^2*x^4])/(6*a^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^5\,dx = \frac{1}{6}e^{\operatorname{sech}^{-1}(ax^2)}x^6 + \frac{\int x^3\,dx}{3a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{x^3}{\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{3a}$$
$$= \frac{x^4}{12a} + \frac{1}{6}e^{\operatorname{sech}^{-1}(ax^2)}x^6 + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{x^3}{\sqrt{1-a^2x^4}}\,dx}{3a}$$
$$= \frac{x^4}{12a} + \frac{1}{6}e^{\operatorname{sech}^{-1}(ax^2)}x^6 - \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{6a^3}$$

Mathematica [A] time = 0.10, size = 56, normalized size = 0.97

$$\frac{(ax^2-1)\sqrt{\frac{1-ax^2}{ax^2+1}}\,(ax^2+1)^2}{6a^3} + \frac{x^4}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^5,x]
```

```
[Out] x^4/(4*a) + ((-1 + a*x^2)*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)^2)/(6*a
^3)
```

fricas [A] time = 0.54, size = 60, normalized size = 1.03

$$\frac{3\,ax^4 + 2\,(a^2x^6 - x^2)\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}}{12\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x, algorithm="f
ricas")
```

```
[Out] 1/12*(3*a*x^4 + 2*(a^2*x^6 - x^2)*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 -
1)/(a*x^2)))/a^2
```

giac [B] time = 0.17, size = 190, normalized size = 3.28

$$\frac{\left(\sqrt{a^2x^2+a}\sqrt{-a^2x^2+a}\left((a^2x^2+a)\left(\frac{2(a^2x^2+a)}{a^4}-\frac{7}{a^3}\right)+\frac{9}{a^2}\right)+\frac{6\arcsin\left(\frac{\sqrt{2}\sqrt{a^2x^2+a}}{2\sqrt{a}}\right)}{a}\right)a - \frac{3\left(2a^2\arcsin\left(\frac{\sqrt{2}\sqrt{a^2x^2+a}}{2\sqrt{a}}\right)-\sqrt{a^2x^2+a}\right.}{a^2}}{12\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x, algorithm="g
iac")
```

```
[Out] 1/12*((sqrt(a^2*x^2 + a)*sqrt(-a^2*x^2 + a)*((a^2*x^2 + a)*(2*(a^2*x^2 + a)
/a^4 - 7/a^3) + 9/a^2) + 6*arcsin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a))/a)
*a - 3*(2*a^2*arcsin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a)) - sqrt(a^2*x^2
+ a)*(a^2*x^2 - 2*a)*sqrt(-a^2*x^2 + a))/a^2 + 3*((a^2*x^2 + a)^2 - 2*(a^2*
x^2 + a)*a)/a^2)/a^3
```

maple [A] time = 0.05, size = 60, normalized size = 1.03

$$\frac{\sqrt{-\frac{a\,x^2-1}{a\,x^2}}\,x^2\sqrt{\frac{a\,x^2+1}{a\,x^2}}\,(a^2x^4-1)}{6a^2} + \frac{x^4}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x)
```

```
[Out] 1/6*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(a^2*x^4-1)/a^2+1/
4*x^4/a
```

maxima [A] time = 0.40, size = 42, normalized size = 0.72

$$\frac{x^4}{4\,a} + \frac{\left(a^2x^4 - 1\right)\sqrt{ax^2 + 1}\,\sqrt{-ax^2 + 1}}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x, algorithm="m
axima")
```

```
[Out] 1/4*x^4/a + 1/6*(a^2*x^4 - 1)*sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/a^3
```

mupad [B] time = 1.64, size = 57, normalized size = 0.98

$$\sqrt{\frac{1}{a\,x^2} - 1}\,\left(\frac{x^6\,\sqrt{\frac{1}{a\,x^2} + 1}}{6} - \frac{x^2\,\sqrt{\frac{1}{a\,x^2} + 1}}{6\,a^2}\right) + \frac{x^4}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] (1/(a*x^2) - 1)^(1/2)*((x^6*(1/(a*x^2) + 1)^(1/2))/6 - (x^2*(1/(a*x^2) + 1)
^(1/2))/(6*a^2)) + x^4/(4*a)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**5,x)
```

```
[Out] Timed out
```

3.48 $\int e^{\text{sech}^{-1}(ax^2)} x^4 \, dx$

Optimal. Leaf size=112

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{5a^{5/2}}+\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,E\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{5a^{5/2}}+\frac{2x^3}{15a}+\frac{1}{5}x^5e^{\text{sech}^{-1}(ax^2)}$$

```
[Out] 2/15*x^3/a+1/5*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5+2/5*Ellipt
icE(x*a^(1/2),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(5/2)-2/5*EllipticF(
x*a^(1/2),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6335, 30, 259, 307, 221, 1199, 424}

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{5a^{5/2}}+\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,E\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{5a^{5/2}}+\frac{2x^3}{15a}+\frac{1}{5}x^5e^{\text{sech}^{-1}(ax^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x^4,x]
```

```
[Out] (2*x^3)/(15*a) + (E^ArcSech[a*x^2]*x^5)/5 + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[
1 + a*x^2]*EllipticE[ArcSin[Sqrt[a]*x], -1])/(5*a^(5/2)) - (2*Sqrt[(1 + a*x
^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/(5*a^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
```

```
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}\int e^{\operatorname{sech}^{-1}(ax^2)}x^4\,dx &= \frac{1}{5}e^{\operatorname{sech}^{-1}(ax^2)}x^5+\frac{2\int x^2\,dx}{5a}+\frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{x^2}{\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{5a}\\ &= \frac{2x^3}{15a}+\frac{1}{5}e^{\operatorname{sech}^{-1}(ax^2)}x^5+\frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{x^2}{\sqrt{1-a^2x^4}}\,dx}{5a}\\ &= \frac{2x^3}{15a}+\frac{1}{5}e^{\operatorname{sech}^{-1}(ax^2)}x^5-\frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{1}{\sqrt{1-a^2x^4}}\,dx}{5a^2}+\frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{1+}{\sqrt{1-}}}{5a^2}\\ &= \frac{2x^3}{15a}+\frac{1}{5}e^{\operatorname{sech}^{-1}(ax^2)}x^5-\frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{5a^{5/2}}+\frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)}{5a^2}\\ &= \frac{2x^3}{15a}+\frac{1}{5}e^{\operatorname{sech}^{-1}(ax^2)}x^5+\frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,E\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{5a^{5/2}}-\frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,F(}{5a^{5/}}\end{aligned}$$

Mathematica [C] time = 0.44, size = 140, normalized size = 1.25

$$\frac{1}{15}\left(\frac{6i\sqrt{\frac{1-ax^2}{ax^2+1}}\sqrt{1-a^2x^4}\left(E\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)\right)}{(-a)^{5/2}\left(ax^2-1\right)}+\frac{5x^3}{a}+\frac{3\sqrt{\frac{1-ax^2}{ax^2+1}}\left(ax^5+x^3\right)}{a}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^4,x]
```

```
[Out] ((5*x^3)/a + (3*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x^3 + a*x^5))/a + ((6*I)*Sqr
t[(1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2*x^4]*(EllipticE[I*ArcSinh[Sqrt[-a]*
x], -1] - EllipticF[I*ArcSinh[Sqrt[-a]*x], -1]))/((-a)^(5/2)*(-1 + a*x^2)))
/15
```

fricas [F] time = 1.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax^4\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+x^2}{a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x, algorithm="f
ricas")
```

```
[Out] integral((a*x^4*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + x^2)
/a, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-4,[1,0]%%%},0,%%%{4,[4,4]%%%}] at param
eters values [86,-97]Warning, choosing root of [1,0,%%%{-4,[1,0]%%%},0,%%%{
4,[4,4]%%%}] at parameters values [-82,7]Warning, choosing root of [1,0,%%%
{-4,[1,0]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-27,26]Warning, choo
sing root of [1,0,%%%{-4,[1,0]%%%},0,%%%{4,[4,4]%%%}] at parameters values
[-89,63]Unable to divide, perhaps due to rounding error%%%{1,[2,2,1,1,1]%%%
}+%%%{1,[2,0,0,0,2]%%%} / %%%{1,[0,0,0,0,3]%%%} Error: Bad Argument Value
```

maple [A] time = 0.06, size = 136, normalized size = 1.21

$$\frac{\sqrt{-\frac{a x^2-1}{a x^2}}\, x^2 \sqrt{\frac{a x^2+1}{a x^2}}\, \left(a^{\frac{7}{2}} x^7 - x^3 a^{\frac{3}{2}} + 2\,\mathrm{EllipticF}\left(x\sqrt{a}\,,i\right)\sqrt{-a x^2+1}\,\sqrt{a x^2+1} - 2\sqrt{-a x^2+1}\,\sqrt{a x^2+1}\,\mathrm{Ellipti}}{5\left(a^2 x^4-1\right) a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x)
```

```
[Out] 1/5*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(a^(7/2)*x^7-x^3*a
^(3/2)+2*EllipticF(x*a^(1/2),I)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)-2*(-a*x^2+
1)^(1/2)*(a*x^2+1)^(1/2)*EllipticE(x*a^(1/2),I))/(a^2*x^4-1)/a^(3/2)+1/3*x^
3/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^3}{3\,a} + \frac{\int \sqrt{ax^2+1}\,\sqrt{-ax^2+1}\,x^2\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x, algorithm="m
axima")
```

```
[Out] 1/3*x^3/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)*x^2, x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] int(x^4*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x^2\,dx + \int ax^4\sqrt{-1+\frac{1}{ax^2}}\,\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**4,x)
```

```
[Out] (Integral(x**2, x) + Integral(a*x**4*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x*
*2)), x))/a
```

3.49 $\int e^{\text{sech}^{-1}(ax^2)} x^3\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,\sin^{-1}(ax^2)}{4a^2}+\frac{x^2}{4a}+\frac{1}{4}x^4e^{\text{sech}^{-1}(ax^2)}$$

```
[Out] 1/4*x^2/a+1/4*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4+1/4*arcsin(
a*x^2)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6335, 30, 259, 275, 216}

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,\sin^{-1}(ax^2)}{4a^2}+\frac{x^2}{4a}+\frac{1}{4}x^4e^{\text{sech}^{-1}(ax^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x^3,x]
```

```
[Out] x^2/(4*a) + (E^ArcSech[a*x^2]*x^4)/4 + (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x
^2]*ArcSin[a*x^2])/(4*a^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}\int e^{\operatorname{sech}^{-1}(ax^2)}x^3\,dx &= \frac{1}{4}e^{\operatorname{sech}^{-1}(ax^2)}x^4 + \frac{\int x\,dx}{2a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{x}{\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{2a}\\ &= \frac{x^2}{4a} + \frac{1}{4}e^{\operatorname{sech}^{-1}(ax^2)}x^4 + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{x}{\sqrt{1-a^2x^4}}\,dx}{2a}\\ &= \frac{x^2}{4a} + \frac{1}{4}e^{\operatorname{sech}^{-1}(ax^2)}x^4 + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^2}}\,dx, x, x^2\right)}{4a}\\ &= \frac{x^2}{4a} + \frac{1}{4}e^{\operatorname{sech}^{-1}(ax^2)}x^4 + \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,\sin^{-1}\left(ax^2\right)}{4a^2}\end{aligned}$$

Mathematica [C] time = 0.11, size = 92, normalized size = 1.46

$$\frac{2ax^2 + i\log\left(2\sqrt{\frac{1-ax^2}{ax^2+1}}\left(ax^2+1\right) - 2iax^2\right) + a\sqrt{\frac{1-ax^2}{ax^2+1}}\left(ax^4+x^2\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^3,x]
```

```
[Out] (2*a*x^2 + a*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x^2 + a*x^4) + I*Log[(-2*I)*a*x
^2 + 2*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)])/(4*a^2)
```

fricas [A] time = 0.76, size = 102, normalized size = 1.62

$$\frac{a^2x^4\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}} + 2\,ax^2 - 2\,\arctan\left(\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}} - 1}{ax^2}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x, algorithm="f
ricas")
```

```
[Out] 1/4*(a^2*x^4*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 2*a*x^2
 - 2*arctan((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 1
)/(a*x^2)))/a^2
```

giac [B] time = 0.17, size = 132, normalized size = 2.10

$$\frac{2\,a^2x^2 + 4\,a\arcsin\left(\frac{\sqrt{2}\sqrt{a^2x^2+a}}{2\sqrt{a}}\right) + 2\sqrt{a^2x^2+a}\sqrt{-a^2x^2+a} + 2\,a - \frac{2\,a^2\arcsin\left(\frac{\sqrt{2}\sqrt{a^2x^2+a}}{2\sqrt{a}}\right) - \sqrt{a^2x^2+a}\left(a^2x^2-2\,a\right)\sqrt{-a^2x^2+a}}{a}}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x, algorithm="g
iac")
```

```
[Out] 1/4*(2*a^2*x^2 + 4*a*arcsin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a)) + 2*sqrt
(a^2*x^2 + a)*sqrt(-a^2*x^2 + a) + 2*a - (2*a^2*arcsin(1/2*sqrt(2)*sqrt(a^2
*x^2 + a)/sqrt(a)) - sqrt(a^2*x^2 + a)*(a^2*x^2 - 2*a)*sqrt(-a^2*x^2 + a))/
a)/a^3
```

maple [A] time = 0.15, size = 112, normalized size = 1.78

$$\frac{\sqrt{-\frac{a x^2-1}{a x^2}}\, x^2 \sqrt{\frac{a x^2+1}{a x^2}}\left(x^2\sqrt{-\frac{a^2x^4-1}{a^2}}\, a^2+\arctan\left(\frac{x^2}{\sqrt{-\frac{a^2x^4-1}{a^2}}}\right)\right)}{4\sqrt{-\frac{a^2x^4-1}{a^2}}\, a^2}+\frac{x^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x)
```

```
[Out] 1/4*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(x^2*(-(a^2*x^4-1)
/a^2)^(1/2)*a^2+arctan(x^2/(-(a^2*x^4-1)/a^2)^(1/2)))/(-(a^2*x^4-1)/a^2)^(1
/2)/a^2+1/2*x^2/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^2}{2a}+\frac{\frac{1}{4}\sqrt{-a^2x^4+1}\,x^2+\frac{\arcsin\left(ax^2\right)}{4a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x, algorithm="m
axima")
```

```
[Out] 1/2*x^2/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)*x, x)/a
```

mupad [B] time = 7.14, size = 306, normalized size = 4.86

$$\frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a x^2}+1}-1\right)^2}+1\right)1\mathrm{i}}{4a^2}-\frac{\ln\left(\frac{\sqrt{\frac{1}{a x^2}-1}-\mathrm{i}}{\sqrt{\frac{1}{a x^2}+1}-1}\right)1\mathrm{i}}{4a^2}+\frac{\frac{1\mathrm{i}}{64a^2}+\frac{\left(\sqrt{\frac{1}{a x^2}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{32a^2\left(\sqrt{\frac{1}{a x^2}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a x^2}-1}-\mathrm{i}\right)^4 15\mathrm{i}}{64a^2\left(\sqrt{\frac{1}{a x^2}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{a x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a x^2}+1}-1\right)^2}+\frac{2\left(\sqrt{\frac{1}{a x^2}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a x^2}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a x^2}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a x^2}+1}-1\right)^6}}+\frac{x^2}{2a}+\frac{\left(\sqrt{\frac{1}{a x^2}-1}-\mathrm{i}\right)}{64a^2\left(\sqrt{\frac{1}{a x^2}+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] (log((((1/(a*x^2) - 1)^(1/2) - 1i)^2/((1/(a*x^2) + 1)^(1/2) - 1)^2 + 1)*1i)/
(4*a^2) - (log(((1/(a*x^2) - 1)^(1/2) - 1i)/((1/(a*x^2) + 1)^(1/2) - 1))*1i
)/(4*a^2) + (1i/(64*a^2) + (((1/(a*x^2) - 1)^(1/2) - 1i)^2*1i)/(32*a^2*((1/
(a*x^2) + 1)^(1/2) - 1)^2) - (((1/(a*x^2) - 1)^(1/2) - 1i)^4*15i)/(64*a^2*(
(1/(a*x^2) + 1)^(1/2) - 1)^4))/((((1/(a*x^2) - 1)^(1/2) - 1i)^2/((1/(a*x^2)
+ 1)^(1/2) - 1)^2 + (2*((1/(a*x^2) - 1)^(1/2) - 1i)^4)/((1/(a*x^2) + 1)^(1/
2) - 1)^4 + ((1/(a*x^2) - 1)^(1/2) - 1i)^6/((1/(a*x^2) + 1)^(1/2) - 1)^6) +
 x^2/(2*a) + (((1/(a*x^2) - 1)^(1/2) - 1i)^2*1i)/(64*a^2*((1/(a*x^2) + 1)^(
1/2) - 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**3,x)
```

```
[Out] Timed out
```

3.50 $\int e^{\text{sech}^{-1}(ax^2)}x^2\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{3a^{3/2}}+\frac{1}{3}x^3e^{\text{sech}^{-1}(ax^2)}+\frac{2x}{3a}$$

```
[Out] 2/3*x/a+1/3*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3+2/3*EllipticF
(x*a^(1/2),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6335, 8, 248, 221}

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{3a^{3/2}}+\frac{1}{3}x^3e^{\text{sech}^{-1}(ax^2)}+\frac{2x}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x^2,x]
```

```
[Out] (2*x)/(3*a) + (E^ArcSech[a*x^2]*x^3)/3 + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 +
 a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/(3*a^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^2\,dx = \frac{1}{3}e^{\operatorname{sech}^{-1}(ax^2)}x^3 + \frac{2\int 1\,dx}{3a} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{3a}$$
$$= \frac{2x}{3a} + \frac{1}{3}e^{\operatorname{sech}^{-1}(ax^2)}x^3 + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{\sqrt{1-a^2x^4}}\,dx}{3a}$$
$$= \frac{2x}{3a} + \frac{1}{3}e^{\operatorname{sech}^{-1}(ax^2)}x^3 + \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{3a^{3/2}}$$

Mathematica [C] time = 0.20, size = 116, normalized size = 1.73

$$-\frac{2i\sqrt{\frac{1-ax^2}{ax^2+1}}\sqrt{1-a^2x^4}\,F\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)}{3(-a)^{3/2}\left(ax^2-1\right)} + \frac{\sqrt{\frac{1-ax^2}{ax^2+1}}\left(ax^3+x\right)}{3a} + \frac{x}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^2,x]
```

```
[Out] x/a + (Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x + a*x^3))/(3*a) - (((2*I)/3)*Sqrt[(
1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2*x^4]*EllipticF[I*ArcSinh[Sqrt[-a]*x],
-1])/((-a)^(3/2)*(-1 + a*x^2))
```

fricas [F] time = 1.85, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+1}{a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x, algorithm="f
ricas")
```

```
[Out] integral((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1)/a
, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-4,[1,0]%%%},0,%%%{4,[4,4]%%%}] at param
eters values [86,-97]Warning, choosing root of [1,0,%%%{-4,[1,0]%%%},0,%%%{
4,[4,4]%%%}] at parameters values [-82,7]Warning, choosing root of [1,0,%%%
{-4,[1,0]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-27,26]Warning, choo
sing root of [1,0,%%%{-4,[1,0]%%%},0,%%%{4,[4,4]%%%}] at parameters values
[-89,63]Unable to divide, perhaps due to rounding error%%%{1,[0,2,1,1,1]%%%
}+%%%{1,[0,0,0,0,2]%%%} / %%%{1,[0,0,0,0,3]%%%} Error: Bad Argument Value
```

maple [A] time = 0.06, size = 102, normalized size = 1.52

$$\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\,x^{2}\sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\,\left(x^{5}a^{\frac{5}{2}}-2\,\mathrm{EllipticF}\left(x\sqrt{a},i\right)\sqrt{-a\,x^{2}+1}\,\sqrt{a\,x^{2}+1}\,-x\sqrt{a}\right)}{3\left(a^{2}x^{4}-1\right)\sqrt{a}}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x)
```

```
[Out] 1/3*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(x^5*a^(5/2)-2*Ell
ipticF(x*a^(1/2),I)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)-x*a^(1/2))/(a^2*x^4-1)
/a^(1/2)+x/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{a}+\frac{\int\sqrt{ax^{2}+1}\,\sqrt{-ax^{2}+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x, algorithm="m
axima")
```

```
[Out] x/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1), x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2}\left(\sqrt{\frac{1}{a\,x^{2}}-1}\,\sqrt{\frac{1}{a\,x^{2}}+1}\,+\frac{1}{a\,x^{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] int(x^2*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int 1\,dx+\int ax^{2}\sqrt{-1+\frac{1}{ax^{2}}}\,\sqrt{1+\frac{1}{ax^{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**2,x)
```

```
[Out] (Integral(1, x) + Integral(a*x**2*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2)
), x))/a
```

3.51 $\int e^{\text{sech}^{-1}(ax^2)} x \, dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\tanh^{-1}\left(\sqrt{1-a^2x^4}\right)}{2a}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^2)}+\frac{\log(x)}{a}$$

```
[Out] 1/2*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2+ln(x)/a-1/2*arctanh((
-a^2*x^4+1)^(1/2))*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6335, 29, 259, 266, 63, 208}

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\tanh^{-1}\left(\sqrt{1-a^2x^4}\right)}{2a}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^2)}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x,x]
```

```
[Out] (E^ArcSech[a*x^2]*x^2)/2 - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*ArcTanh[
Sqrt[1 - a^2*x^4]])/(2*a) + Log[x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
```

```
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}
\int e^{\text{sech}^{-1}(ax^2)} x\,dx &= \frac{1}{2} e^{\text{sech}^{-1}(ax^2)} x^2 + \frac{\int \frac{1}{x}\,dx}{a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{x\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{a}\\
&= \frac{1}{2} e^{\text{sech}^{-1}(ax^2)} x^2 + \frac{\log(x)}{a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{x\sqrt{1-a^2x^4}}\,dx}{a}\\
&= \frac{1}{2} e^{\text{sech}^{-1}(ax^2)} x^2 + \frac{\log(x)}{a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^4\right)}{4a}\\
&= \frac{1}{2} e^{\text{sech}^{-1}(ax^2)} x^2 + \frac{\log(x)}{a} - \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^4}\right)}{2a^3}\\
&= \frac{1}{2} e^{\text{sech}^{-1}(ax^2)} x^2 - \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\tanh^{-1}\left(\sqrt{1-a^2x^4}\right)}{2a} + \frac{\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 100, normalized size = 1.47

$$\frac{\sqrt{\frac{1-ax^2}{ax^2+1}}\left(ax^2+1\right)+2\log\left(ax^2\right)-\log\left(ax^2\sqrt{\frac{1-ax^2}{ax^2+1}}+\sqrt{\frac{1-ax^2}{ax^2+1}}+1\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x,x]
[Out] (Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2) + 2*Log[a*x^2] - Log[1 + Sqrt[(1
 - a*x^2)/(1 + a*x^2)] + a*x^2*Sqrt[(1 - a*x^2)/(1 + a*x^2)]])/(2*a)
```

fricas [B] time = 1.59, size = 133, normalized size = 1.96

$$\frac{2\,ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}-\log\left(ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+1\right)+\log\left(ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}-1\right)+4\,\log(x)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x,x, algorithm="fri
cas")
[Out] 1/4*(2*a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - log(a*x
^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1) + log(a*x^2*sq
rt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 1) + 4*log(x))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
schur row 1 2.33984e-10Francis algorithm not precise enough for[1.0,-1117.2
2141279,260038.267747,-22596024.9566,676199006.929]Warning, choosing root o
f [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4]%
%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%}]
 at parameters values [54.1277311612,-82]schur row 1 3.80414e-10Francis alg
orithm not precise enough for[1.0,-439.975588666,40328.8580463,-1380066.571
27,16264167.9132]Bad conditionned root j= 2 value 36.6628221508 ratio 0.000
412274208284 mindist 0.00165644519952Bad conditionned root j= 0 value 36.66
 ratio 0.00026134143357 mindist 0.0110443353806Bad conditionned root j= 2 v
alue 36.67-0.004688*i ratio 0.00158404473284 mindist 0.009376Bad conditionn
ed root j= 3 value 36.67+0.004688*i ratio 0.00158404473284 mindist 0.009376
Warning, choosing root of [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[
2,0]%%%},0,%%%{16,[5,4]%%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6
,4]%%%}+%%%{9,[4,0]%%%}] at parameters values [82.1195442914,-89]schur row
1 2.26297e-10Francis algorithm not precise enough for[1.0,-310.806973653,20
125.2030982,-486504.158708,4050237.99743]Unable to isolate roots number Vec
tor [0,1][0.259008132109614e2,0.259012999453233e2]Bad conditionned root j=
2 value 25.8996302569 ratio 0.000423663040072 mindist 0.00118295401739Warni
ng, choosing root of [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%
%%},0,%%%{16,[5,4]%%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%
%}+%%%{9,[4,0]%%%}] at parameters values [35.2935628123,-64]schur row 1 3.6
7828e-10Francis algorithm not precise enough for[1.0,-1024.27388138,218570.
205017,-17412558.5081,477729345.21]Bad conditionned root j= 2 value 85.3520
228111 ratio 0.000286084735534 mindist 0.0052898804365Warning, choosing roo
t of [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,
4]%%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%
%}] at parameters values [78.6493344628,42]schur row 1 1.40127e-11Francis a
lgorithm not precise enough for[1.0,-550.918251291,63231.4415845,-2709416.5
1745,39982152.0485]Bad conditionned root j= 2 value 45.9094216765 ratio 0.0
00696541081041 mindist 0.00106311994459Warning, choosing root of [1,0,%%%{-
12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4]%%%}+%%%{-28,
[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%}] at paramete
rs values [62.4600259969,46]schur row 1 3.2626e-10Francis algorithm not pre
cise enough for[1.0,-897.25122063,167720.781859,-11704597.0415,281302606.67
4]Unable to isolate roots number Vector [0,1][0.747726202076933e2,0.7477267
16167272e2]Bad conditionned root j= 2 value 74.7675133332 ratio 0.001011418
11991 mindist 0.00510687449423schur row 1 1.16125e-10Francis algorithm not
precise enough for[1.0,-1092.17002563,248507.367685,-21109845.4104,61755911
7.938]Bad conditionned root j= 2 value 91.0116865143 ratio 0.00109001405383
 mindist 0.00219671084823Warning, choosing root of [1,0,%%%{-12,[1,0]%%%},0
,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4]%%%}+%%%{-28,[3,0]%%%},0,%%
%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%}] at parameters values [33.
9285577983,-49]schur row 1 7.18728e-11Francis algorithm not precise enough
for[1.0,-185.418596232,7162.51163099,-103293.777387,513015.728641]Warning,
choosing root of [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},
0,%%%{16,[5,4]%%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%
%%{9,[4,0]%%%}] at parameters values [18.4052062202,63]schur row 1 2.75123e
-11Francis algorithm not precise enough for[1.0,-619.731778616,80014.057797
2,-3856786.44967,64022494.4518]Bad conditionned root j= 2 value 51.64364277
69 ratio 0.00161773509906 mindist 0.00178986679927Warning, choosing root of
 [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4]%%
%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%}]
at parameters values [10.4309062702,-37]Warning, choosing root of [1,0,%%%{
-4,[1,0]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-23,65]schur row 1 1.
68784e-10Francis algorithm not precise enough for[1.0,-96.6277521998,1945.1
921865,-14619.0760005,37837.726424]Warning, choosing root of [1,0,%%%{-12,[
```

```
1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4]%%%}+%%%{-28,[3,0
]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%}] at parameters v
alues [39.1803401988,-44]schur row 1 3.85284e-10Francis algorithm not preci
se enough for[1.0,-1161.32542683,280974.322293,-25379093.0373,789465697.88]
Bad conditionned root j= 2 value 96.7723338924 ratio 0.000375252022965 mind
ist 0.00603026989475Warning, choosing root of [1,0,%%%{-12,[1,0]%%%},0,%%%{
8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4]%%%}+%%%{-28,[3,0]%%%},0,%%%{16,
[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%}] at parameters values [39.98282
99829,31]schur row 1 3.46041e-10Francis algorithm not precise enough for[1.
0,-1129.51443638,265792.262915,-23350148.7365,706455270.256]Bad conditionne
d root j= 2 value 94.1217752457 ratio 0.000267509068199 mindist 0.005750406
63189Warning, choosing root of [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%
{30,[2,0]%%%},0,%%%{16,[5,4]%%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-
24,[6,4]%%%}+%%%{9,[4,0]%%%}] at parameters values [83.4865739918,-66]schur
 row 1 3.76847e-10Francis algorithm not precise enough for[1.0,-637.3497375
72,84628.0599964,-4195152.25343,71619085.3875]Warning, choosing root of [1,
0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4]%%%}+%
%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%}] at p
arameters values [6.82230772497,79]Warning, choosing root of [1,0,%%%{-8,[1
,2]%%%}+%%%{-4,[1,0]%%%},0,%%%{8,[4,0]%%%}+%%%{16,[2,4]%%%}+%%%{8,[2,2]%%%}
+%%%{6,[2,0]%%%},0,%%%{-32,[5,2]%%%}+%%%{48,[5,0]%%%}+%%%{-32,[3,4]%%%}+%%%
{8,[3,2]%%%}+%%%{-4,[3,0]%%%},0,%%%{16,[8,0]%%%}+%%%{-32,[6,2]%%%}+%%%{8,[6
,0]%%%}+%%%{16,[4,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,[4,0]%%%}] at parameters va
lues [55.0343274642,0]Warning, choosing root of [1,0,%%%{-8,[1,2]%%%}+%%%{-
4,[1,0]%%%},0,%%%{8,[4,0]%%%}+%%%{16,[2,4]%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,0]%
%%},0,%%%{-32,[5,2]%%%}+%%%{48,[5,0]%%%}+%%%{-32,[3,4]%%%}+%%%{8,[3,2]%%%}+
%%%{-4,[3,0]%%%},0,%%%{16,[8,0]%%%}+%%%{-32,[6,2]%%%}+%%%{8,[6,0]%%%}+%%%{1
6,[4,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,[4,0]%%%}] at parameters values [66.0382
199469,-8]Warning, choosing root of [1,0,%%%{-8,[1,2]%%%}+%%%{-4,[1,0]%%%},
0,%%%{8,[4,0]%%%}+%%%{16,[2,4]%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,0]%%%},0,%%%{-3
2,[5,2]%%%}+%%%{48,[5,0]%%%}+%%%{-32,[3,4]%%%}+%%%{8,[3,2]%%%}+%%%{-4,[3,0]
%%%},0,%%%{16,[8,0]%%%}+%%%{-32,[6,2]%%%}+%%%{8,[6,0]%%%}+%%%{16,[4,4]%%%}+
%%%{-8,[4,2]%%%}+%%%{1,[4,0]%%%}] at parameters values [4.66774101928,97]Wa
rning, choosing root of [1,0,%%%{-8,[1,2]%%%}+%%%{-4,[1,0]%%%},0,%%%{8,[4,0
]%%%}+%%%{16,[2,4]%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,0]%%%},0,%%%{-32,[5,2]%%%}+
%%%{48,[5,0]%%%}+%%%{-32,[3,4]%%%}+%%%{8,[3,2]%%%}+%%%{-4,[3,0]%%%},0,%%%{1
6,[8,0]%%%}+%%%{-32,[6,2]%%%}+%%%{8,[6,0]%%%}+%%%{16,[4,4]%%%}+%%%{-8,[4,2]
%%%}+%%%{1,[4,0]%%%}] at parameters values [70.9232513234,-17]Warning, choo
sing root of [1,0,%%%{-8,[1,2]%%%}+%%%{-4,[1,0]%%%},0,%%%{8,[4,0]%%%}+%%%{1
6,[2,4]%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,0]%%%},0,%%%{-32,[5,2]%%%}+%%%{48,[5,0
]%%%}+%%%{-32,[3,4]%%%}+%%%{8,[3,2]%%%}+%%%{-4,[3,0]%%%},0,%%%{16,[8,0]%%%}
+%%%{-32,[6,2]%%%}+%%%{8,[6,0]%%%}+%%%{16,[4,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,
[4,0]%%%}] at parameters values [82.4264548342,0]Warning, choosing root of
[1,0,%%%{-8,[1,2]%%%}+%%%{-4,[1,0]%%%},0,%%%{8,[4,0]%%%}+%%%{16,[2,4]%%%}+%
%%{8,[2,2]%%%}+%%%{6,[2,0]%%%},0,%%%{-32,[5,2]%%%}+%%%{48,[5,0]%%%}+%%%{-32
,[3,4]%%%}+%%%{8,[3,2]%%%}+%%%{-4,[3,0]%%%},0,%%%{16,[8,0]%%%}+%%%{-32,[6,2
]%%%}+%%%{8,[6,0]%%%}+%%%{16,[4,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,[4,0]%%%}] at
 parameters values [59.4272477375,89]schur row 3 1.36691e-10Warning, choosi
ng root of [1,0,%%%{-8,[1,2]%%%}+%%%{-4,[1,0]%%%},0,%%%{8,[4,0]%%%}+%%%{16,
[2,4]%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,0]%%%},0,%%%{-32,[5,2]%%%}+%%%{48,[5,0]%
%%}+%%%{-32,[3,4]%%%}+%%%{8,[3,2]%%%}+%%%{-4,[3,0]%%%},0,%%%{16,[8,0]%%%}+%
%%{-32,[6,2]%%%}+%%%{8,[6,0]%%%}+%%%{16,[4,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,[4
,0]%%%}] at parameters values [61.7431004322,-65]Warning, choosing root of
[1,0,%%%{-8,[1,2]%%%}+%%%{-4,[1,0]%%%},0,%%%{8,[4,0]%%%}+%%%{16,[2,4]%%%}+%
%%{8,[2,2]%%%}+%%%{6,[2,0]%%%},0,%%%{-32,[5,2]%%%}+%%%{48,[5,0]%%%}+%%%{-32
,[3,4]%%%}+%%%{8,[3,2]%%%}+%%%{-4,[3,0]%%%},0,%%%{16,[8,0]%%%}+%%%{-32,[6,2
]%%%}+%%%{8,[6,0]%%%}+%%%{16,[4,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,[4,0]%%%}] at
 parameters values [58.4409598615,-10]Warning, choosing root of [1,0,%%%{-8
,[1,2]%%%}+%%%{-4,[1,0]%%%},0,%%%{8,[4,0]%%%}+%%%{16,[2,4]%%%}+%%%{8,[2,2]%
```

```
%}+%%%{6,[2,0]%%%},0,%%%{-32,[5,2]%%%}+%%%{48,[5,0]%%%}+%%%{-32,[3,4]%%%}+
%%%{8,[3,2]%%%}+%%%{-4,[3,0]%%%},0,%%%{16,[8,0]%%%}+%%%{-32,[6,2]%%%}+%%%{8
,[6,0]%%%}+%%%{16,[4,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,[4,0]%%%}] at parameters
 values [18.9804396471,0]Warning, choosing root of [1,0,%%%{-8,[1,2]%%%}+%%
%{-4,[1,0]%%%},0,%%%{8,[4,0]%%%}+%%%{16,[2,4]%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,
0]%%%},0,%%%{-32,[5,2]%%%}+%%%{48,[5,0]%%%}+%%%{-32,[3,4]%%%}+%%%{8,[3,2]%%
%}+%%%{-4,[3,0]%%%},0,%%%{16,[8,0]%%%}+%%%{-32,[6,2]%%%}+%%%{8,[6,0]%%%}+%%
%{16,[4,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,[4,0]%%%}] at parameters values [70.2
045348478,0]Warning, choosing root of [1,0,%%%{-8,[2,1]%%%}+%%%{-4,[0,1]%%%
},0,%%%{16,[4,2]%%%}+%%%{8,[2,2]%%%}+%%%{8,[0,4]%%%}+%%%{6,[0,2]%%%},0,%%%{
-32,[4,3]%%%}+%%%{-32,[2,5]%%%}+%%%{8,[2,3]%%%}+%%%{48,[0,5]%%%}+%%%{-4,[0,
3]%%%},0,%%%{16,[4,4]%%%}+%%%{-32,[2,6]%%%}+%%%{-8,[2,4]%%%}+%%%{16,[0,8]%%
%}+%%%{8,[0,6]%%%}+%%%{1,[0,4]%%%}] at parameters values [0,57.2153722499]s
chur row 3 2.56736e-11Warning, choosing root of [1,0,%%%{-8,[2,1]%%%}+%%%{-
4,[0,1]%%%},0,%%%{16,[4,2]%%%}+%%%{8,[2,2]%%%}+%%%{8,[0,4]%%%}+%%%{6,[0,2]%
%%},0,%%%{-32,[4,3]%%%}+%%%{-32,[2,5]%%%}+%%%{8,[2,3]%%%}+%%%{48,[0,5]%%%}+
%%%{-4,[0,3]%%%},0,%%%{16,[4,4]%%%}+%%%{-32,[2,6]%%%}+%%%{-8,[2,4]%%%}+%%%{
16,[0,8]%%%}+%%%{8,[0,6]%%%}+%%%{1,[0,4]%%%}] at parameters values [-58,54.
6372379069]Warning, choosing root of [1,0,%%%{-8,[2,1]%%%}+%%%{-4,[0,1]%%%}
,0,%%%{16,[4,2]%%%}+%%%{8,[2,2]%%%}+%%%{8,[0,4]%%%}+%%%{6,[0,2]%%%},0,%%%{-
32,[4,3]%%%}+%%%{-32,[2,5]%%%}+%%%{8,[2,3]%%%}+%%%{48,[0,5]%%%}+%%%{-4,[0,3
]%%%},0,%%%{16,[4,4]%%%}+%%%{-32,[2,6]%%%}+%%%{-8,[2,4]%%%}+%%%{16,[0,8]%%%
}+%%%{8,[0,6]%%%}+%%%{1,[0,4]%%%}] at parameters values [71,86.2839511861]W
arning, choosing root of [1,0,%%%{-8,[2,1]%%%}+%%%{-4,[0,1]%%%},0,%%%{16,[4
,2]%%%}+%%%{8,[2,2]%%%}+%%%{8,[0,4]%%%}+%%%{6,[0,2]%%%},0,%%%{-32,[4,3]%%%}
+%%%{-32,[2,5]%%%}+%%%{8,[2,3]%%%}+%%%{48,[0,5]%%%}+%%%{-4,[0,3]%%%},0,%%%{
16,[4,4]%%%}+%%%{-32,[2,6]%%%}+%%%{-8,[2,4]%%%}+%%%{16,[0,8]%%%}+%%%{8,[0,6
]%%%}+%%%{1,[0,4]%%%}] at parameters values [11,80.4553440167]Warning, choo
sing root of [1,0,%%%{-8,[2,1]%%%}+%%%{-4,[0,1]%%%},0,%%%{16,[4,2]%%%}+%%%{
8,[2,2]%%%}+%%%{8,[0,4]%%%}+%%%{6,[0,2]%%%},0,%%%{-32,[4,3]%%%}+%%%{-32,[2,
5]%%%}+%%%{8,[2,3]%%%}+%%%{48,[0,5]%%%}+%%%{-4,[0,3]%%%},0,%%%{16,[4,4]%%%}
+%%%{-32,[2,6]%%%}+%%%{-8,[2,4]%%%}+%%%{16,[0,8]%%%}+%%%{8,[0,6]%%%}+%%%{1,
[0,4]%%%}] at parameters values [0,45.716705855]Warning, choosing root of [
1,0,%%%{-8,[2,1]%%%}+%%%{-4,[0,1]%%%},0,%%%{16,[4,2]%%%}+%%%{8,[2,2]%%%}+%%
%{8,[0,4]%%%}+%%%{6,[0,2]%%%},0,%%%{-32,[4,3]%%%}+%%%{-32,[2,5]%%%}+%%%{8,[
2,3]%%%}+%%%{48,[0,5]%%%}+%%%{-4,[0,3]%%%},0,%%%{16,[4,4]%%%}+%%%{-32,[2,6]
%%%}+%%%{-8,[2,4]%%%}+%%%{16,[0,8]%%%}+%%%{8,[0,6]%%%}+%%%{1,[0,4]%%%}] at
parameters values [81,87.5126850624]Warning, choosing root of [1,0,%%%{-8,[
2,1]%%%}+%%%{-4,[0,1]%%%},0,%%%{16,[4,2]%%%}+%%%{8,[2,2]%%%}+%%%{8,[0,4]%%%
}+%%%{6,[0,2]%%%},0,%%%{-32,[4,3]%%%}+%%%{-32,[2,5]%%%}+%%%{8,[2,3]%%%}+%%%
{48,[0,5]%%%}+%%%{-4,[0,3]%%%},0,%%%{16,[4,4]%%%}+%%%{-32,[2,6]%%%}+%%%{-8,
[2,4]%%%}+%%%{16,[0,8]%%%}+%%%{8,[0,6]%%%}+%%%{1,[0,4]%%%}] at parameters v
alues [-11,23.9552401127]Warning, choosing root of [1,0,%%%{-8,[2,1]%%%}+%%
%{-4,[0,1]%%%},0,%%%{16,[4,2]%%%}+%%%{8,[2,2]%%%}+%%%{8,[0,4]%%%}+%%%{6,[0,
2]%%%},0,%%%{-32,[4,3]%%%}+%%%{-32,[2,5]%%%}+%%%{8,[2,3]%%%}+%%%{48,[0,5]%%
%}+%%%{-4,[0,3]%%%},0,%%%{16,[4,4]%%%}+%%%{-32,[2,6]%%%}+%%%{-8,[2,4]%%%}+%
%%{16,[0,8]%%%}+%%%{8,[0,6]%%%}+%%%{1,[0,4]%%%}] at parameters values [93,4
1.1512670754]schur row 1 1.99488e-10Francis algorithm not precise enough fo
r[1.0,-729.896147886,110989.247229,-6300826.31183,123186130.005]Warning, ch
oosing root of [1,0,%%%{-12,[0,1]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[0,2]%%%},0,
%%%{16,[4,5]%%%}+%%%{-28,[0,3]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[4,6]%%%}+%%%
{9,[0,4]%%%}] at parameters values [-26,75.876540896]schur row 1 3.66933e-1
0Francis algorithm not precise enough for[1.0,-1159.70905962,280192.729784,
-25273270.3354,785079658.236]Warning, choosing root of [1,0,%%%{-12,[0,1]%%
%},0,%%%{8,[4,4]%%%}+%%%{30,[0,2]%%%},0,%%%{16,[4,5]%%%}+%%%{-28,[0,3]%%%},
0,%%%{16,[8,8]%%%}+%%%{-24,[4,6]%%%}+%%%{9,[0,4]%%%}] at parameters values
[25,45.0210851603]Sign error (%%%{-2*a,2%%%}+%%%{undef,3%%%})Evaluation tim
e: 35.6Limit: Max order reached or unable to make series expansion Error: B
ad Argument Value
```

maple [C] time = 0.19, size = 127, normalized size = 1.87

$$\frac{\sqrt{-\frac{a x^2-1}{a x^2}}\, x^2 \sqrt{\frac{a x^2+1}{a x^2}}\left(\operatorname{csgn}\left(\frac{1}{a}\right) a\sqrt{-\frac{a^2x^4-1}{a^2}}-\ln\left(\frac{2\operatorname{csgn}\left(\frac{1}{a}\right) a\sqrt{-\frac{a^2x^4-1}{a^2}}+2}{a^2x^2}\right)\right)\operatorname{csgn}\left(\frac{1}{a}\right)}{2a\sqrt{-\frac{a^2x^4-1}{a^2}}}+\frac{\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x,x)
```

```
[Out] 1/2*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(csgn(1/a)*a*(-(a^
2*x^4-1)/a^2)^(1/2)-ln(2*(csgn(1/a)*a*(-(a^2*x^4-1)/a^2)^(1/2)+1)/a^2/x^2))
*csgn(1/a)/a/(-(a^2*x^4-1)/a^2)^(1/2)+ln(x)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{1}{2}\sqrt{-a^2x^4+1}-\frac{1}{2}\log\left(\frac{2\sqrt{-a^2x^4+1}}{x^2}+\frac{2}{x^2}\right)}{a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x, x)/a + log(x)/a
```

mupad [B] time = 3.20, size = 182, normalized size = 2.68

$$\frac{\ln(x)}{a}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x^2}+1}-1}\right)}{a}+\frac{\frac{5\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+1}{\frac{8\,a\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x^2}+1}-1}+\frac{8\,a\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^3}}+\frac{\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}}{8\,a\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] log(x)/a - (2*atanh(((1/(a*x^2) - 1)^(1/2) - 1i)/((1/(a*x^2) + 1)^(1/2) - 1
)))/a + ((5*((1/(a*x^2) - 1)^(1/2) - 1i)^2)/((1/(a*x^2) + 1)^(1/2) - 1)^2 +
 1)/((8*a*((1/(a*x^2) - 1)^(1/2) - 1i))/((1/(a*x^2) + 1)^(1/2) - 1) + (8*a*
((1/(a*x^2) - 1)^(1/2) - 1i)^3)/((1/(a*x^2) + 1)^(1/2) - 1)^3) + ((1/(a*x^2
) - 1)^(1/2) - 1i)/(8*a*((1/(a*x^2) + 1)^(1/2) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x,x)
```

```
[Out] Timed out
```

3.52 $\int e^{\text{sech}^{-1}(ax^2)}\,dx$

Optimal. Leaf size=147

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{ax}+xe^{\text{sech}^{-1}(ax^2)}+\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}F\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)}{\sqrt{a}}-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}E\left(\sin\right.}{\sqrt{a}}$$

```
[Out] -2/a/x+(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x-2*EllipticE(x*a^(1/2
),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(1/2)+2*EllipticF(x*a^(1/2),I)*(
1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(1/2)-2*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^
(1/2)*(-a^2*x^4+1)^(1/2)/a/x
```

Rubi [A] time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6330, 30, 259, 325, 307, 221, 1199, 424}

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{ax}+xe^{\text{sech}^{-1}(ax^2)}+\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}F\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)}{\sqrt{a}}-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}E\left(\sin\right.}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2],x]
```

```
[Out] -2/(a*x) + E^ArcSech[a*x^2]*x - (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*S
qrt[1 - a^2*x^4])/(a*x) - (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Ellipti
cE[ArcSin[Sqrt[a]*x], -1])/Sqrt[a] + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x
^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/Sqrt[a]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6330

```
Int[E^ArcSech[(a_.)*(x_)^(p_)], x_Symbol] :> Simp[x*E^ArcSech[a*x^p], x] +
(Dist[p/a, Int[1/x^p, x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])
/a, Int[1/(x^p*Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, p}, x
]
```

Rubi steps

$$\begin{aligned}
\int e^{\text{sech}^{-1}\left(ax^2\right)}\,dx &= e^{\text{sech}^{-1}\left(ax^2\right)}x+\frac{2\int \frac{1}{x^2}\,dx}{a}+\frac{\left(2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\right)\int \frac{1}{x^2\sqrt{1-ax^2}\,\sqrt{1+ax^2}}\,dx}{a}\\
&= -\frac{2}{ax}+e^{\text{sech}^{-1}\left(ax^2\right)}x+\frac{\left(2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\right)\int \frac{1}{x^2\sqrt{1-a^2x^4}}\,dx}{a}\\
&= -\frac{2}{ax}+e^{\text{sech}^{-1}\left(ax^2\right)}x-\frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,\sqrt{1-a^2x^4}}{ax}-\left(2a\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\right)\int \frac{x^2}{\sqrt{1-a^2 x^4}}
\\
&= -\frac{2}{ax}+e^{\text{sech}^{-1}\left(ax^2\right)}x-\frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,\sqrt{1-a^2x^4}}{ax}+\left(2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\right)\int \frac{1}{\sqrt{1-a^2x^4}}
\\
&= -\frac{2}{ax}+e^{\text{sech}^{-1}\left(ax^2\right)}x-\frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,\sqrt{1-a^2x^4}}{ax}+\frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-\right.}{\sqrt{a}}\\
&= -\frac{2}{ax}+e^{\text{sech}^{-1}\left(ax^2\right)}x-\frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,\sqrt{1-a^2x^4}}{ax}-\frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,E\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-\right.}{\sqrt{a}}
\end{aligned}$$

Mathematica [C] time = 0.30, size = 135, normalized size = 0.92

$$-\frac{2i\sqrt{\frac{1-ax^2}{ax^2+1}}\,\sqrt{1-a^2x^4}\,\left(E\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)\right)}{\sqrt{-a}\,\left(ax^2-1\right)}+\sqrt{\frac{1-ax^2}{ax^2+1}}\,\left(-\frac{1}{ax}-x\right)-\frac{1}{ax}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2],x]
```

```
[Out] -(1/(a*x)) + (-(1/(a*x)) - x)*Sqrt[(1 - a*x^2)/(1 + a*x^2)] - ((2*I)*Sqrt[(
1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2*x^4]*(EllipticE[I*ArcSinh[Sqrt[-a]*x],
 -1] - EllipticF[I*ArcSinh[Sqrt[-a]*x], -1]))/(Sqrt[-a]*(-1 + a*x^2))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+1}{ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1)/(
a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(1/(a*x^2) + 1)*sqrt(1/(a*x^2) - 1) + 1/(a*x^2), x)
```

maple [A] time = 0.07, size = 132, normalized size = 0.90

$$-\frac{1}{ax}-\frac{\sqrt{-\frac{a\,x^2-1}{a\,x^2}}\,x\sqrt{\frac{a\,x^2+1}{a\,x^2}}\,\left(a^2x^4+2\sqrt{-a\,x^2+1}\,\sqrt{a\,x^2+1}\,x\,\mathrm{EllipticF}\left(x\sqrt{a},i\right)\sqrt{a}-2\sqrt{-a\,x^2+1}\,\sqrt{a\,x^2+1}\,x\,\mathrm{Ell}}{a^2x^4-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x)
```

```
[Out] -1/a/x-(-(a*x^2-1)/a/x^2)^(1/2)*x*((a*x^2+1)/a/x^2)^(1/2)*(a^2*x^4+2*(-a*x^
2+1)^(1/2)*(a*x^2+1)^(1/2)*x*EllipticF(x*a^(1/2),I)*a^(1/2)-2*(-a*x^2+1)^(1
/2)*(a*x^2+1)^(1/2)*x*EllipticE(x*a^(1/2),I)*a^(1/2)-1)/(a^2*x^4-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{ax^2+1}\,\sqrt{-ax^2+1}}{x^2}\,dx}{a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^2, x)/a - 1/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2),x)
```

```
[Out] int((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2}\,dx + \int a\sqrt{-1+\frac{1}{ax^2}}\,\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2),x)
```

```
[Out] (Integral(x**(-2), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2
)), x))/a
```

3.53 $\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{1-ax^2}}{2ax^2\sqrt{\frac{1}{ax^2+1}}}-\frac{1}{2ax^2}-\frac{1}{2}\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,\sin^{-1}\left(ax^2\right)$$

```
[Out] -1/2/a/x^2-1/2*(-a*x^2+1)^(1/2)/a/x^2/(1/(a*x^2+1))^(1/2)-1/2*arcsin(a*x^2)
*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6334, 259, 275, 277, 216}

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{2ax^2}-\frac{1}{2ax^2}-\frac{1}{2}\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,\sin^{-1}\left(ax^2\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x^2]/x,x]
```

```
[Out] -1/(2*a*x^2) - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Sqrt[1 - a^2*x^4])/(
2*a*x^2) - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*ArcSin[a*x^2])/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6334

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]/(x_), x_Symbol] :> -Simp[(a*p*x^p)^(-1), x]
 + Dist[(Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/a, Int[(Sqrt[1 + a*x^p]*Sqrt[
1 - a*x^p])/x^(p + 1), x], x] /; FreeQ[{a, p}, x]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x}\,dx = -\frac{1}{2ax^2} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{\sqrt{1-ax^2}\sqrt{1+ax^2}}{x^3}\,dx}{a}$$
$$= -\frac{1}{2ax^2} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{\sqrt{1-a^2x^4}}{x^3}\,dx}{a}$$
$$= -\frac{1}{2ax^2} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-a^2x^2}}{x^2}\,dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2} - \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{2ax^2} - \frac{1}{2}\left(a\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^2}}\,d$$
$$= -\frac{1}{2ax^2} - \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{2ax^2} - \frac{1}{2}\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sin^{-1}\left(ax^2\right)$$

Mathematica [**A**] time = 0.15, size = 113, normalized size = 1.41

$$\frac{\sqrt{\frac{1-ax^2}{ax^2+1}}\left(ax^2+1\right)\tanh^{-1}\left(\frac{ax^2}{\sqrt{a^2x^4-1}}\right)}{2\sqrt{a^2x^4-1}} + \sqrt{\frac{1-ax^2}{ax^2+1}}\left(-\frac{1}{2ax^2}-\frac{1}{2}\right) - \frac{1}{2ax^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]/x,x]
```

```
[Out] -1/2*1/(a*x^2) + (-1/2 - 1/(2*a*x^2))*Sqrt[(1 - a*x^2)/(1 + a*x^2)] + (Sqrt
[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)*ArcTanh[(a*x^2)/Sqrt[-1 + a^2*x^4]])/
(2*Sqrt[-1 + a^2*x^4])
```

fricas [**B**] time = 0.97, size = 102, normalized size = 1.28

$$-\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}} - 2\,ax^2\arctan\left(\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}-1}{ax^2}\right)+1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x, algorithm="fri
cas")
```

```
[Out] -1/2*(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 2*a*x^2*
arctan((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 1)/(a*
x^2)) + 1)/(a*x^2)
```

giac [**B**] time = 3.32, size = 252, normalized size = 3.15

$$-\frac{\left(\pi + 2\arctan\left(\frac{\sqrt{a^2x^2+a}\left(\frac{\left(\sqrt{2}\sqrt{a}-\sqrt{-a^2x^2+a}\right)^2}{a^2x^2+a}-1\right)}{2\left(\sqrt{2}\sqrt{a}-\sqrt{-a^2x^2+a}\right)}\right)\right)a^3 + \frac{4\,a^3\left(\frac{\sqrt{2}\sqrt{a}-\sqrt{-a^2x^2+a}}{\sqrt{a^2x^2+a}}-\frac{\sqrt{a^2x^2+a}}{\sqrt{2}\sqrt{a}-\sqrt{-a^2x^2+a}}\right)}{\left(\frac{\sqrt{2}\sqrt{a}-\sqrt{-a^2x^2+a}}{\sqrt{a^2x^2+a}}-\frac{\sqrt{a^2x^2+a}}{\sqrt{2}\sqrt{a}-\sqrt{-a^2x^2+a}}\right)^2-4} + \frac{a^2}{x^2}}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x, algorithm="gia
c")
```

```
[Out] -1/2*((pi + 2*arctan(1/2*sqrt(a^2*x^2 + a)*((sqrt(2)*sqrt(a) - sqrt(-a^2*x^
2 + a))^2/(a^2*x^2 + a) - 1)/(sqrt(2)*sqrt(a) - sqrt(-a^2*x^2 + a))))*a^3 +
 4*a^3*((sqrt(2)*sqrt(a) - sqrt(-a^2*x^2 + a))/sqrt(a^2*x^2 + a) - sqrt(a^2
*x^2 + a)/(sqrt(2)*sqrt(a) - sqrt(-a^2*x^2 + a)))/(((sqrt(2)*sqrt(a) - sqrt
(-a^2*x^2 + a))/sqrt(a^2*x^2 + a) - sqrt(a^2*x^2 + a)/(sqrt(2)*sqrt(a) - sq
rt(-a^2*x^2 + a)))^2 - 4) + a^2/x^2)/a^3
```

maple [A] time = 0.16, size = 103, normalized size = 1.29

$$-\frac{\sqrt{-\frac{a\,x^2-1}{a\,x^2}}\,\sqrt{\frac{a\,x^2+1}{a\,x^2}}\left(\arctan\left(\frac{x^2}{\sqrt{-\frac{a^2x^4-1}{a^2}}}\right)x^2+\sqrt{-\frac{a^2x^4-1}{a^2}}\right)}{2\sqrt{-\frac{a^2x^4-1}{a^2}}}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x)
```

```
[Out] -1/2*(-(a*x^2-1)/a/x^2)^(1/2)*((a*x^2+1)/a/x^2)^(1/2)*(arctan(x^2/(-(a^2*x^
4-1)/a^2)^(1/2))*x^2+(-(a^2*x^4-1)/a^2)^(1/2))/(-(a^2*x^4-1)/a^2)^(1/2)-1/2
/a/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\frac{1}{2}\,a\arcsin\left(ax^2\right)-\frac{\sqrt{-a^2x^4+1}}{2\,x^2}}{a}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^3, x)/a - 1/2/(a*x^2)
```

mupad [B] time = 4.00, size = 185, normalized size = 2.31

$$-\frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+1\right)1\mathrm{i}}{2}+\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x^2}+1}-1}\right)1\mathrm{i}}{2}-\frac{1}{2\,a\,x^2}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2 8\mathrm{i}}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2\left(2+\frac{2\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4}-\frac{4\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2))/x,x)
```

```
[Out] (log(((1/(a*x^2) - 1)^(1/2) - 1i)/((1/(a*x^2) + 1)^(1/2) - 1))*1i)/2 - (log
(((1/(a*x^2) - 1)^(1/2) - 1i)^2/((1/(a*x^2) + 1)^(1/2) - 1)^2 + 1)*1i)/2 -
1/(2*a*x^2) + (((1/(a*x^2) - 1)^(1/2) - 1i)^2*8i)/(((1/(a*x^2) + 1)^(1/2) -
 1)^2*((2*((1/(a*x^2) - 1)^(1/2) - 1i)^4)/((1/(a*x^2) + 1)^(1/2) - 1)^4 - (
4*((1/(a*x^2) - 1)^(1/2) - 1i)^2)/((1/(a*x^2) + 1)^(1/2) - 1)^2 + 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^3}\,dx+\int \frac{a\sqrt{-1+\frac{1}{ax^2}}\,\sqrt{1+\frac{1}{ax^2}}}{x}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))/x,x)
```

```
[Out] (Integral(x**(-3), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2
))/x, x))/a
```

3.54 $\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^2} dx$

Optimal. Leaf size=115

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{3ax^3}+\frac{2}{3ax^3}-\frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x}-\frac{2}{3}\sqrt{a}\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}F\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)$$

```
[Out] 2/3/a/x^3-(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x-2/3*EllipticF(x*a
^(1/2),I)*a^(1/2)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)+2/3*(1/(a*x^2+1))^(1/
2)*(a*x^2+1)^(1/2)*(-a^2*x^4+1)^(1/2)/a/x^3
```

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6335, 30, 259, 325, 221}

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{3ax^3}+\frac{2}{3ax^3}-\frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x}-\frac{2}{3}\sqrt{a}\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}F\left(\sin^{-1}\left(\sqrt{a}x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]/x^2,x]
```

```
[Out] 2/(3*a*x^3) - E^ArcSech[a*x^2]/x + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2
]*Sqrt[1 - a^2*x^4])/(3*a*x^3) - (2*Sqrt[a]*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 +
 a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
```

]

Rubi steps

$$\int \frac{e^{\text{sech}^{-1}(ax^2)}}{x^2}\,dx = -\frac{e^{\text{sech}^{-1}(ax^2)}}{x} - \frac{2\int \frac{1}{x^4}\,dx}{a} - \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{x^4\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{a}$$

$$= \frac{2}{3ax^3} - \frac{e^{\text{sech}^{-1}(ax^2)}}{x} - \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{x^4\sqrt{1-a^2x^4}}\,dx}{a}$$

$$= \frac{2}{3ax^3} - \frac{e^{\text{sech}^{-1}(ax^2)}}{x} + \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{3ax^3} - \frac{1}{3}\left(2a\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{\sqrt{1-}}$$

$$= \frac{2}{3ax^3} - \frac{e^{\text{sech}^{-1}(ax^2)}}{x} + \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{3ax^3} - \frac{2}{3}\sqrt{a}\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,F\left(\sin^{-1}\left(\right.\right.$$

Mathematica [C] time = 0.20, size = 123, normalized size = 1.07

$$\frac{2i\sqrt{-a}\sqrt{\frac{1-ax^2}{ax^2+1}}\sqrt{1-a^2x^4}\,F\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)}{3ax^2-3} - \frac{1}{3ax^3} - \frac{\sqrt{\frac{1-ax^2}{ax^2+1}}\left(ax^2+1\right)}{3ax^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]/x^2,x]
```

```
[Out] -1/3*1/(a*x^3) - (Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2))/(3*a*x^3) + ((
2*I)*Sqrt[-a]*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2*x^4]*EllipticF[I*A
rcSinh[Sqrt[-a]*x], -1])/(-3 + 3*a*x^2)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+1}{ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x, algorithm="f
ricas")
```

```
[Out] integral((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1)/(
a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{ax^2}+1}\sqrt{\frac{1}{ax^2}-1}+\frac{1}{ax^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x, algorithm="g
iac")
```

```
[Out] integrate((sqrt(1/(a*x^2) + 1)*sqrt(1/(a*x^2) - 1) + 1/(a*x^2))/x^2, x)
```

maple [A] time = 0.07, size = 104, normalized size = 0.90

$$\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\,\sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\,\left(2\sqrt{-a\,x^{2}+1}\,\sqrt{a\,x^{2}+1}\,\operatorname{EllipticF}\left(x\sqrt{a}\,,i\right)x^{3}a^{\frac{3}{2}}-a^{2}x^{4}+1\right)}{3x\left(a^{2}x^{4}-1\right)}-\frac{1}{3x^{3}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x)
```

```
[Out] 1/3*(-(a*x^2-1)/a/x^2)^(1/2)/x*((a*x^2+1)/a/x^2)^(1/2)*(2*(-a*x^2+1)^(1/2)*
(a*x^2+1)^(1/2)*EllipticF(x*a^(1/2),I)*x^3*a^(3/2)-a^2*x^4+1)/(a^2*x^4-1)-1
/3/x^3/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{ax^{2}+1}\,\sqrt{-ax^{2}+1}}{x^{4}}\,dx}{a}-\frac{1}{3\,ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^4, x)/a - 1/3/(a*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{a\,x^{2}}-1}\,\sqrt{\frac{1}{a\,x^{2}}+1}+\frac{1}{a\,x^{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2))/x^2,x)
```

```
[Out] int(((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^{4}}\,dx+\int \frac{a\sqrt{-1+\frac{1}{ax^{2}}}\,\sqrt{1+\frac{1}{ax^{2}}}}{x^{2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))/x**2,x)
```

```
[Out] (Integral(x**(-4), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2
))/x**2, x))/a
```

3.55 $\int \frac{e^{\mathbf{sech}^{-1}(ax^2)}}{x^3} \, dx$

Optimal. Leaf size=118

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{4ax^4}+\frac{1}{4}a\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\tanh^{-1}\left(\sqrt{1-a^2x^4}\right)+\frac{1}{4ax^4}-\frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2}$$

```
[Out] 1/4/a/x^4-1/2*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2+1/4*a*arcta
nh((-a^2*x^4+1)^(1/2))*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)+1/4*(1/(a*x^2+1)
)^(1/2)*(a*x^2+1)^(1/2)*(-a^2*x^4+1)^(1/2)/a/x^4
```

Rubi [A] time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6335, 30, 259, 266, 51, 63, 208}

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{4ax^4}+\frac{1}{4}a\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\tanh^{-1}\left(\sqrt{1-a^2x^4}\right)+\frac{1}{4ax^4}-\frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]/x^3,x]
```

```
[Out] 1/(4*a*x^4) - E^ArcSech[a*x^2]/(2*x^2) + (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a
*x^2]*Sqrt[1 - a^2*x^4])/(4*a*x^4) + (a*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x
^2]*ArcTanh[Sqrt[1 - a^2*x^4]])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
```

```
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}\left(ax^2\right)}}{x^3}\,dx = -\frac{e^{\operatorname{sech}^{-1}\left(ax^2\right)}}{2x^2} - \frac{\int \frac{1}{x^5}\,dx}{a} - \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{x^5\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{a}$$
$$= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}\left(ax^2\right)}}{2x^2} - \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{x^5\sqrt{1-a^2x^4}}\,dx}{a}$$
$$= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}\left(ax^2\right)}}{2x^2} - \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^4\right)}{4a}$$
$$= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}\left(ax^2\right)}}{2x^2} + \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{4ax^4} - \frac{1}{8}\left(a\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\operatorname{Subst}\left(\int \frac{}{x\sqrt{}}\right.$$
$$= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}\left(ax^2\right)}}{2x^2} + \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{4ax^4} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x\right.}{4a}$$
$$= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}\left(ax^2\right)}}{2x^2} + \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{4ax^4} + \frac{1}{4}a\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\tanh^{-1}\left(\sqrt{1-a^2}\right.$$

Mathematica [A] time = 0.27, size = 105, normalized size = 0.89

$$-\frac{-\frac{a^2\sqrt{\frac{1-ax^2}{ax^2+1}}\left(ax^2+1\right)\tan^{-1}\left(\sqrt{a^2x^4-1}\right)}{\sqrt{a^2x^4-1}} + \frac{\sqrt{\frac{1-ax^2}{ax^2+1}}\left(ax^2+1\right)}{x^4} + \frac{1}{x^4}}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]/x^3,x]
```

```
[Out] -1/4*(x^(-4) + (Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2))/x^4 - (a^2*Sqrt[
(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)*ArcTan[Sqrt[-1 + a^2*x^4]])/Sqrt[-1 +
a^2*x^4])/a
```

fricas [A] time = 0.86, size = 146, normalized size = 1.24

$$\frac{a^2x^4\log\left(ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+1\right) - a^2x^4\log\left(ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}-1\right) - 2\,ax^2\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}} - 2}{8\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^3,x, algorithm="f
ricas")
```

```
[Out] 1/8*(a^2*x^4*log(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2))
 + 1) - a^2*x^4*log(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^
2)) - 1) - 2*a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 2
)/(a*x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^3,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
schur row 1 2.33984e-10Francis algorithm not precise enough for[1.0,-1117.2
2141279,260038.267747,-22596024.9566,676199006.929]Warning, choosing root o
f [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4]%
%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%}]
 at parameters values [54.1277311612,-82]schur row 1 3.80414e-10Francis alg
orithm not precise enough for[1.0,-439.975588666,40328.8580463,-1380066.571
27,16264167.9132]Bad conditionned root j= 2 value 36.6628221508 ratio 0.000
412274208284 mindist 0.00165644519952Bad conditionned root j= 0 value 36.66
 ratio 0.00026134143357 mindist 0.0110443353806Bad conditionned root j= 2 v
alue 36.67-0.004688*i ratio 0.00158404473284 mindist 0.009376Bad conditionn
ed root j= 3 value 36.67+0.004688*i ratio 0.00158404473284 mindist 0.009376
Warning, choosing root of [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[
2,0]%%%},0,%%%{16,[5,4]%%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6
,4]%%%}+%%%{9,[4,0]%%%}] at parameters values [82.1195442914,-89]schur row
1 2.26297e-10Francis algorithm not precise enough for[1.0,-310.806973653,20
125.2030982,-486504.158708,4050237.99743]Unable to isolate roots number Vec
tor [0,1][0.259008132109614e2,0.259012999453233e2]Bad conditionned root j=
2 value 25.8996302569 ratio 0.000423663040072 mindist 0.00118295401739Warni
ng, choosing root of [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%
%%},0,%%%{16,[5,4]%%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%
%}+%%%{9,[4,0]%%%}] at parameters values [35.2935628123,-64]schur row 1 3.6
7828e-10Francis algorithm not precise enough for[1.0,-1024.27388138,218570.
205017,-17412558.5081,477729345.21]Bad conditionned root j= 2 value 85.3520
228111 ratio 0.000286084735534 mindist 0.0052898804365Warning, choosing roo
t of [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,
4]%%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%
%}] at parameters values [78.6493344628,42]schur row 1 1.40127e-11Francis a
lgorithm not precise enough for[1.0,-550.918251291,63231.4415845,-2709416.5
1745,39982152.0485]Bad conditionned root j= 2 value 45.9094216765 ratio 0.0
00696541081041 mindist 0.00106311994459Warning, choosing root of [1,0,%%%{-
12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4]%%%}+%%%{-28,
[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%}] at paramete
rs values [62.4600259969,46]schur row 1 3.2626e-10Francis algorithm not pre
cise enough for[1.0,-897.25122063,167720.781859,-11704597.0415,281302606.67
4]Unable to isolate roots number Vector [0,1][0.747726202076933e2,0.7477267
16167272e2]Bad conditionned root j= 2 value 74.7675133332 ratio 0.001011418
11991 mindist 0.00510687449423schur row 1 1.16125e-10Francis algorithm not
precise enough for[1.0,-1092.17002563,248507.367685,-21109845.4104,61755911
7.938]Bad conditionned root j= 2 value 91.0116865143 ratio 0.00109001405383
```

```
 mindist 0.00219671084823Warning, choosing root of [1,0,%%%{-12,[1,0]%%%},0
,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4]%%%}+%%%{-28,[3,0]%%%},0,%%
%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%}] at parameters values [33.
9285577983,-49]schur row 1 7.18728e-11Francis algorithm not precise enough
for[1.0,-185.418596232,7162.51163099,-103293.777387,513015.728641]Warning,
choosing root of [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},
0,%%%{16,[5,4]%%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%
%%{9,[4,0]%%%}] at parameters values [18.4052062202,63]schur row 1 2.75123e
-11Francis algorithm not precise enough for[1.0,-619.731778616,80014.057797
2,-3856786.44967,64022494.4518]Bad conditionned root j= 2 value 51.64364277
69 ratio 0.00161773509906 mindist 0.00178986679927Warning, choosing root of
 [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4]%%
%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%}]
at parameters values [10.4309062702,-37]Warning, choosing root of [1,0,%%%{
-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4]%%%}+%%%{-28
,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%}] at paramet
ers values [-23,65]schur row 1 1.68784e-10Francis algorithm not precise eno
ugh for[1.0,-96.6277521998,1945.1921865,-14619.0760005,37837.726424]Warning
, choosing root of [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%
},0,%%%{16,[5,4]%%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}
+%%%{9,[4,0]%%%}] at parameters values [39.1803401988,-44]schur row 1 3.852
84e-10Francis algorithm not precise enough for[1.0,-1161.32542683,280974.32
2293,-25379093.0373,789465697.88]Bad conditionned root j= 2 value 96.772333
8924 ratio 0.000375252022965 mindist 0.00603026989475Warning, choosing root
 of [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4
]%%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%
}] at parameters values [39.9828299829,31]schur row 1 3.46041e-10Francis al
gorithm not precise enough for[1.0,-1129.51443638,265792.262915,-23350148.7
365,706455270.256]Bad conditionned root j= 2 value 94.1217752457 ratio 0.00
0267509068199 mindist 0.00575040663189Warning, choosing root of [1,0,%%%{-1
2,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[2,0]%%%},0,%%%{16,[5,4]%%%}+%%%{-28,[
3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[6,4]%%%}+%%%{9,[4,0]%%%}] at parameter
s values [83.4865739918,-66]schur row 1 3.76847e-10Francis algorithm not pr
ecise enough for[1.0,-637.349737572,84628.0599964,-4195152.25343,71619085.3
875]Warning, choosing root of [1,0,%%%{-12,[1,0]%%%},0,%%%{8,[4,4]%%%}+%%%{
30,[2,0]%%%},0,%%%{16,[5,4]%%%}+%%%{-28,[3,0]%%%},0,%%%{16,[8,8]%%%}+%%%{-2
4,[6,4]%%%}+%%%{9,[4,0]%%%}] at parameters values [6.82230772497,79]Warning
, choosing root of [1,0,%%%{-8,[1,2]%%%}+%%%{-4,[1,0]%%%},0,%%%{8,[4,0]%%%}
+%%%{16,[2,4]%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,0]%%%},0,%%%{-32,[5,2]%%%}+%%%{4
8,[5,0]%%%}+%%%{-32,[3,4]%%%}+%%%{8,[3,2]%%%}+%%%{-4,[3,0]%%%},0,%%%{16,[8,
0]%%%}+%%%{-32,[6,2]%%%}+%%%{8,[6,0]%%%}+%%%{16,[4,4]%%%}+%%%{-8,[4,2]%%%}+
%%%{1,[4,0]%%%}] at parameters values [55.0343274642,0]Warning, choosing ro
ot of [1,0,%%%{-8,[1,2]%%%}+%%%{-4,[1,0]%%%},0,%%%{8,[4,0]%%%}+%%%{16,[2,4]
%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,0]%%%},0,%%%{-32,[5,2]%%%}+%%%{48,[5,0]%%%}+%
%%{-32,[3,4]%%%}+%%%{8,[3,2]%%%}+%%%{-4,[3,0]%%%},0,%%%{16,[8,0]%%%}+%%%{-3
2,[6,2]%%%}+%%%{8,[6,0]%%%}+%%%{16,[4,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,[4,0]%%
%}] at parameters values [66.0382199469,-8]Warning, choosing root of [1,0,%
%%{-8,[1,2]%%%}+%%%{-4,[1,0]%%%},0,%%%{8,[4,0]%%%}+%%%{16,[2,4]%%%}+%%%{8,[
2,2]%%%}+%%%{6,[2,0]%%%},0,%%%{-32,[5,2]%%%}+%%%{48,[5,0]%%%}+%%%{-32,[3,4]
%%%}+%%%{8,[3,2]%%%}+%%%{-4,[3,0]%%%},0,%%%{16,[8,0]%%%}+%%%{-32,[6,2]%%%}+
%%%{8,[6,0]%%%}+%%%{16,[4,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,[4,0]%%%}] at param
eters values [4.66774101928,97]Warning, choosing root of [1,0,%%%{-8,[1,2]%
%%}+%%%{-4,[1,0]%%%},0,%%%{8,[4,0]%%%}+%%%{16,[2,4]%%%}+%%%{8,[2,2]%%%}+%%%
{6,[2,0]%%%},0,%%%{-32,[5,2]%%%}+%%%{48,[5,0]%%%}+%%%{-32,[3,4]%%%}+%%%{8,[
3,2]%%%}+%%%{-4,[3,0]%%%},0,%%%{16,[8,0]%%%}+%%%{-32,[6,2]%%%}+%%%{8,[6,0]%
%%}+%%%{16,[4,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,[4,0]%%%}] at parameters values
 [70.9232513234,-17]Warning, choosing root of [1,0,%%%{-8,[1,2]%%%}+%%%{-4,
[1,0]%%%},0,%%%{8,[4,0]%%%}+%%%{16,[2,4]%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,0]%%%
},0,%%%{-32,[5,2]%%%}+%%%{48,[5,0]%%%}+%%%{-32,[3,4]%%%}+%%%{8,[3,2]%%%}+%%
```

```
%{-4,[3,0]%%%},0,%%%{16,[8,0]%%%}+%%%{-32,[6,2]%%%}+%%%{8,[6,0]%%%}+%%%{16,
[4,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,[4,0]%%%}] at parameters values [82.426454
8342,0]Warning, choosing root of [1,0,%%%{-8,[1,2]%%%}+%%%{-4,[1,0]%%%},0,%
%%{8,[4,0]%%%}+%%%{16,[2,4]%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,0]%%%},0,%%%{-32,[
5,2]%%%}+%%%{48,[5,0]%%%}+%%%{-32,[3,4]%%%}+%%%{8,[3,2]%%%}+%%%{-4,[3,0]%%%
},0,%%%{16,[8,0]%%%}+%%%{-32,[6,2]%%%}+%%%{8,[6,0]%%%}+%%%{16,[4,4]%%%}+%%%
{-8,[4,2]%%%}+%%%{1,[4,0]%%%}] at parameters values [59.4272477375,89]schur
 row 3 1.36691e-10Warning, choosing root of [1,0,%%%{-8,[1,2]%%%}+%%%{-4,[1
,0]%%%},0,%%%{8,[4,0]%%%}+%%%{16,[2,4]%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,0]%%%},
0,%%%{-32,[5,2]%%%}+%%%{48,[5,0]%%%}+%%%{-32,[3,4]%%%}+%%%{8,[3,2]%%%}+%%%{
-4,[3,0]%%%},0,%%%{16,[8,0]%%%}+%%%{-32,[6,2]%%%}+%%%{8,[6,0]%%%}+%%%{16,[4
,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,[4,0]%%%}] at parameters values [61.74310043
22,-65]Warning, choosing root of [1,0,%%%{-8,[1,2]%%%}+%%%{-4,[1,0]%%%},0,%
%%{8,[4,0]%%%}+%%%{16,[2,4]%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,0]%%%},0,%%%{-32,[
5,2]%%%}+%%%{48,[5,0]%%%}+%%%{-32,[3,4]%%%}+%%%{8,[3,2]%%%}+%%%{-4,[3,0]%%%
},0,%%%{16,[8,0]%%%}+%%%{-32,[6,2]%%%}+%%%{8,[6,0]%%%}+%%%{16,[4,4]%%%}+%%%
{-8,[4,2]%%%}+%%%{1,[4,0]%%%}] at parameters values [58.4409598615,-10]Warn
ing, choosing root of [1,0,%%%{-8,[1,2]%%%}+%%%{-4,[1,0]%%%},0,%%%{8,[4,0]%
%%}+%%%{16,[2,4]%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,0]%%%},0,%%%{-32,[5,2]%%%}+%%
%{48,[5,0]%%%}+%%%{-32,[3,4]%%%}+%%%{8,[3,2]%%%}+%%%{-4,[3,0]%%%},0,%%%{16,
[8,0]%%%}+%%%{-32,[6,2]%%%}+%%%{8,[6,0]%%%}+%%%{16,[4,4]%%%}+%%%{-8,[4,2]%%
%}+%%%{1,[4,0]%%%}] at parameters values [18.9804396471,0]Warning, choosing
 root of [1,0,%%%{-8,[1,2]%%%}+%%%{-4,[1,0]%%%},0,%%%{8,[4,0]%%%}+%%%{16,[2
,4]%%%}+%%%{8,[2,2]%%%}+%%%{6,[2,0]%%%},0,%%%{-32,[5,2]%%%}+%%%{48,[5,0]%%%
}+%%%{-32,[3,4]%%%}+%%%{8,[3,2]%%%}+%%%{-4,[3,0]%%%},0,%%%{16,[8,0]%%%}+%%%
{-32,[6,2]%%%}+%%%{8,[6,0]%%%}+%%%{16,[4,4]%%%}+%%%{-8,[4,2]%%%}+%%%{1,[4,0
]%%%}] at parameters values [70.2045348478,0]Warning, choosing root of [1,0
,%%%{-8,[2,1]%%%}+%%%{-4,[0,1]%%%},0,%%%{16,[4,2]%%%}+%%%{8,[2,2]%%%}+%%%{8
,[0,4]%%%}+%%%{6,[0,2]%%%},0,%%%{-32,[4,3]%%%}+%%%{-32,[2,5]%%%}+%%%{8,[2,3
]%%%}+%%%{48,[0,5]%%%}+%%%{-4,[0,3]%%%},0,%%%{16,[4,4]%%%}+%%%{-32,[2,6]%%%
}+%%%{-8,[2,4]%%%}+%%%{16,[0,8]%%%}+%%%{8,[0,6]%%%}+%%%{1,[0,4]%%%}] at par
ameters values [0,57.2153722499]schur row 3 2.56736e-11Warning, choosing ro
ot of [1,0,%%%{-8,[2,1]%%%}+%%%{-4,[0,1]%%%},0,%%%{16,[4,2]%%%}+%%%{8,[2,2]
%%%}+%%%{8,[0,4]%%%}+%%%{6,[0,2]%%%},0,%%%{-32,[4,3]%%%}+%%%{-32,[2,5]%%%}+
%%%{8,[2,3]%%%}+%%%{48,[0,5]%%%}+%%%{-4,[0,3]%%%},0,%%%{16,[4,4]%%%}+%%%{-3
2,[2,6]%%%}+%%%{-8,[2,4]%%%}+%%%{16,[0,8]%%%}+%%%{8,[0,6]%%%}+%%%{1,[0,4]%%
%}] at parameters values [-58,54.6372379069]Warning, choosing root of [1,0,
%%%{-8,[2,1]%%%}+%%%{-4,[0,1]%%%},0,%%%{16,[4,2]%%%}+%%%{8,[2,2]%%%}+%%%{8,
[0,4]%%%}+%%%{6,[0,2]%%%},0,%%%{-32,[4,3]%%%}+%%%{-32,[2,5]%%%}+%%%{8,[2,3]
%%%}+%%%{48,[0,5]%%%}+%%%{-4,[0,3]%%%},0,%%%{16,[4,4]%%%}+%%%{-32,[2,6]%%%}
+%%%{-8,[2,4]%%%}+%%%{16,[0,8]%%%}+%%%{8,[0,6]%%%}+%%%{1,[0,4]%%%}] at para
meters values [71,86.2839511861]Warning, choosing root of [1,0,%%%{-8,[2,1]
%%%}+%%%{-4,[0,1]%%%},0,%%%{16,[4,2]%%%}+%%%{8,[2,2]%%%}+%%%{8,[0,4]%%%}+%%
%{6,[0,2]%%%},0,%%%{-32,[4,3]%%%}+%%%{-32,[2,5]%%%}+%%%{8,[2,3]%%%}+%%%{48,
[0,5]%%%}+%%%{-4,[0,3]%%%},0,%%%{16,[4,4]%%%}+%%%{-32,[2,6]%%%}+%%%{-8,[2,4
]%%%}+%%%{16,[0,8]%%%}+%%%{8,[0,6]%%%}+%%%{1,[0,4]%%%}] at parameters value
s [11,80.4553440167]Warning, choosing root of [1,0,%%%{-8,[2,1]%%%}+%%%{-4,
[0,1]%%%},0,%%%{16,[4,2]%%%}+%%%{8,[2,2]%%%}+%%%{8,[0,4]%%%}+%%%{6,[0,2]%%%
},0,%%%{-32,[4,3]%%%}+%%%{-32,[2,5]%%%}+%%%{8,[2,3]%%%}+%%%{48,[0,5]%%%}+%%
%{-4,[0,3]%%%},0,%%%{16,[4,4]%%%}+%%%{-32,[2,6]%%%}+%%%{-8,[2,4]%%%}+%%%{16
,[0,8]%%%}+%%%{8,[0,6]%%%}+%%%{1,[0,4]%%%}] at parameters values [0,45.7167
05855]Warning, choosing root of [1,0,%%%{-8,[2,1]%%%}+%%%{-4,[0,1]%%%},0,%%
%{16,[4,2]%%%}+%%%{8,[2,2]%%%}+%%%{8,[0,4]%%%}+%%%{6,[0,2]%%%},0,%%%{-32,[4
,3]%%%}+%%%{-32,[2,5]%%%}+%%%{8,[2,3]%%%}+%%%{48,[0,5]%%%}+%%%{-4,[0,3]%%%}
,0,%%%{16,[4,4]%%%}+%%%{-32,[2,6]%%%}+%%%{-8,[2,4]%%%}+%%%{16,[0,8]%%%}+%%%
{8,[0,6]%%%}+%%%{1,[0,4]%%%}] at parameters values [81,87.5126850624]Warnin
g, choosing root of [1,0,%%%{-8,[2,1]%%%}+%%%{-4,[0,1]%%%},0,%%%{16,[4,2]%%
%}+%%%{8,[2,2]%%%}+%%%{8,[0,4]%%%}+%%%{6,[0,2]%%%},0,%%%{-32,[4,3]%%%}+%%%{
-32,[2,5]%%%}+%%%{8,[2,3]%%%}+%%%{48,[0,5]%%%}+%%%{-4,[0,3]%%%},0,%%%{16,[4
```

```
,4]%%%}+%%%{-32,[2,6]%%%}+%%%{-8,[2,4]%%%}+%%%{16,[0,8]%%%}+%%%{8,[0,6]%%%}
+%%%{1,[0,4]%%%}] at parameters values [-11,23.9552401127]Warning, choosing
 root of [1,0,%%%{-8,[2,1]%%%}+%%%{-4,[0,1]%%%},0,%%%{16,[4,2]%%%}+%%%{8,[2
,2]%%%}+%%%{8,[0,4]%%%}+%%%{6,[0,2]%%%},0,%%%{-32,[4,3]%%%}+%%%{-32,[2,5]%%
%}+%%%{8,[2,3]%%%}+%%%{48,[0,5]%%%}+%%%{-4,[0,3]%%%},0,%%%{16,[4,4]%%%}+%%%
{-32,[2,6]%%%}+%%%{-8,[2,4]%%%}+%%%{16,[0,8]%%%}+%%%{8,[0,6]%%%}+%%%{1,[0,4
]%%%}] at parameters values [93,41.1512670754]schur row 1 1.99488e-10Franci
s algorithm not precise enough for[1.0,-729.896147886,110989.247229,-630082
6.31183,123186130.005]Warning, choosing root of [1,0,%%%{-12,[0,1]%%%},0,%%
%{8,[4,4]%%%}+%%%{30,[0,2]%%%},0,%%%{16,[4,5]%%%}+%%%{-28,[0,3]%%%},0,%%%{1
6,[8,8]%%%}+%%%{-24,[4,6]%%%}+%%%{9,[0,4]%%%}] at parameters values [-26,75
.876540896]schur row 1 3.66933e-10Francis algorithm not precise enough for[
1.0,-1159.70905962,280192.729784,-25273270.3354,785079658.236]Warning, choo
sing root of [1,0,%%%{-12,[0,1]%%%},0,%%%{8,[4,4]%%%}+%%%{30,[0,2]%%%},0,%%
%{16,[4,5]%%%}+%%%{-28,[0,3]%%%},0,%%%{16,[8,8]%%%}+%%%{-24,[4,6]%%%}+%%%{9
,[0,4]%%%}] at parameters values [25,45.0210851603]Sign error (%%%{-2*a,2%%
%}+%%%{undef,3%%%})Evaluation time: 44.85Limit: Max order reached or unable
 to make series expansion Error: Bad Argument Value
```

maple [C] time = 0.18, size = 129, normalized size = 1.09

$$-\frac{\sqrt{-\frac{a x^2-1}{a x^2}}\,\sqrt{\frac{a x^2+1}{a x^2}}\left(-\ln\left(\frac{2\,\mathrm{csgn}\left(\frac{1}{a}\right) a \sqrt{-\frac{a^2x^4-1}{a^2}}+2}{a^2x^2}\right)x^4a+\sqrt{-\frac{a^2x^4-1}{a^2}}\;\mathrm{csgn}\left(\frac{1}{a}\right)\right)\mathrm{csgn}\left(\frac{1}{a}\right)}{4x^2\sqrt{-\frac{a^2x^4-1}{a^2}}}-\frac{1}{4x^4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^3,x)
```

```
[Out] -1/4*(-(a*x^2-1)/a/x^2)^(1/2)/x^2*((a*x^2+1)/a/x^2)^(1/2)*(-ln(2*(csgn(1/a)
*a*(-(a^2*x^4-1)/a^2)^(1/2)+1)/a^2/x^2)*x^4*a+(-(a^2*x^4-1)/a^2)^(1/2)*csgn
(1/a))*csgn(1/a)/(-(a^2*x^4-1)/a^2)^(1/2)-1/4/x^4/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{1}{4}\,a^2\log\left(\frac{2\,\sqrt{-a^2x^4+1}}{x^2}+\frac{2}{x^2}\right)-\frac{1}{4}\,\sqrt{-a^2x^4+1}\,a^2-\frac{\left(-a^2x^4+1\right)^{\frac{3}{2}}}{4\,x^4}}{a}-\frac{1}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^3,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^5, x)/a - 1/4/(a*x^4)
```

mupad [B] time = 2.02, size = 71, normalized size = 0.60

$$\frac{a\,\ln\left(\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\right)}{4}-\frac{1}{4\,a\,x^4}-\frac{\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2))/x^3,x)
```

```
[Out] (a*log((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)))/4 - 1/(4*a
*x^4) - ((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2))/(4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^5}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax^2}}\,\sqrt{1+\frac{1}{ax^2}}}{x^3}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))/x**3,x)
```

```
[Out] (Integral(x**(-5), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2
))/x**3, x))/a
```

3.56 $\int e^{\text{sech}^{-1}\left(ax^3\right)}x^m\,dx$

Optimal. Leaf size=109

$$-\frac{3\sqrt{\frac{1}{ax^3+1}}\sqrt{ax^3+1}\,x^{m-2}\,{}_2F_1\left(\frac{1}{2},\frac{m-2}{6};\frac{m+4}{6};a^2x^6\right)}{a\left(-m^2+m+2\right)}-\frac{3x^{m-2}}{a\left(-m^2+m+2\right)}+\frac{x^{m+1}e^{\text{sech}^{-1}\left(ax^3\right)}}{m+1}$$

```
[Out] -3*x^(-2+m)/a/(-m^2+m+2)+(1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^(1
+m)/(1+m)-3*x^(-2+m)*hypergeom([1/2, -1/3+1/6*m],[2/3+1/6*m],a^2*x^6)*(1/(a
*x^3+1))^(1/2)*(a*x^3+1)^(1/2)/a/(-m^2+m+2)
```

Rubi [A] time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6335, 30, 259, 364}

$$-\frac{3\sqrt{\frac{1}{ax^3+1}}\sqrt{ax^3+1}\,x^{m-2}\,{}_2F_1\left(\frac{1}{2},\frac{m-2}{6};\frac{m+4}{6};a^2x^6\right)}{a\left(-m^2+m+2\right)}-\frac{3x^{m-2}}{a\left(-m^2+m+2\right)}+\frac{x^{m+1}e^{\text{sech}^{-1}\left(ax^3\right)}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^3]*x^m,x]
```

```
[Out] (-3*x^(-2 + m))/(a*(2 + m - m^2)) + (E^ArcSech[a*x^3]*x^(1 + m))/(1 + m) -
(3*x^(-2 + m)*Sqrt[(1 + a*x^3)^(-1)]*Sqrt[1 + a*x^3]*Hypergeometric2F1[1/2,
 (-2 + m)/6, (4 + m)/6, a^2*x^6])/(a*(2 + m - m^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax^3)}x^m\,dx = \frac{e^{\operatorname{sech}^{-1}(ax^3)}x^{1+m}}{1+m} + \frac{3\int x^{-3+m}\,dx}{a(1+m)} + \frac{\left(3\sqrt{\frac{1}{1+ax^3}}\sqrt{1+ax^3}\right)\int \frac{x^{-3+m}}{\sqrt{1-ax^3}\sqrt{1+ax^3}}\,dx}{a(1+m)}$$

$$= -\frac{3x^{-2+m}}{a\left(2+m-m^2\right)} + \frac{e^{\operatorname{sech}^{-1}(ax^3)}x^{1+m}}{1+m} + \frac{\left(3\sqrt{\frac{1}{1+ax^3}}\sqrt{1+ax^3}\right)\int \frac{x^{-3+m}}{\sqrt{1-a^2x^6}}\,dx}{a(1+m)}$$

$$= -\frac{3x^{-2+m}}{a\left(2+m-m^2\right)} + \frac{e^{\operatorname{sech}^{-1}(ax^3)}x^{1+m}}{1+m} - \frac{3x^{-2+m}\sqrt{\frac{1}{1+ax^3}}\sqrt{1+ax^3}\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-2+m);\frac{4+m}{6};\right.}{a\left(2+m-m^2\right)}$$

Mathematica [A] time = 2.52, size = 159, normalized size = 1.46

$$\frac{2^{\frac{m+1}{3}}x^{m+1}\left(ax^3\right)^{\frac{1}{3}(-m-1)}e^{\operatorname{sech}^{-1}(ax^3)}\left(\frac{e^{\operatorname{sech}^{-1}(ax^3)}}{e^{2\operatorname{sech}^{-1}(ax^3)}+1}\right)^{\frac{m+1}{3}}\left((m+10)\,{}_2F_1\left(1,\frac{2-m}{6};\frac{m+10}{6};-e^{2\operatorname{sech}^{-1}(ax^3)}\right)-(m+4)e^{2\operatorname{sech}}\right.}{(m+4)(m+10)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^3]*x^m,x]
```

```
[Out] (2^((1 + m)/3)*E^ArcSech[a*x^3]*(E^ArcSech[a*x^3]/(1 + E^(2*ArcSech[a*x^3])
))^((1 + m)/3)*x^(1 + m)*(a*x^3)^((-1 - m)/3)*((10 + m)*Hypergeometric2F1[1
, (2 - m)/6, (10 + m)/6, -E^(2*ArcSech[a*x^3])] - E^(2*ArcSech[a*x^3])*(4 +
 m)*Hypergeometric2F1[1, (8 - m)/6, (16 + m)/6, -E^(2*ArcSech[a*x^3])]))/((
4 + m)*(10 + m))
```

fricas [F] time = 2.37, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{ax^3x^m\sqrt{\frac{ax^3+1}{ax^3}}\sqrt{-\frac{ax^3-1}{ax^3}}+x^m}{ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x, algorithm="f
ricas")
```

```
[Out] integral((a*x^3*x^m*sqrt((a*x^3 + 1)/(a*x^3))*sqrt(-(a*x^3 - 1)/(a*x^3)) +
x^m)/(a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\sqrt{\frac{1}{ax^3}+1}\sqrt{\frac{1}{ax^3}-1}+\frac{1}{ax^3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x, algorithm="g
iac")
```

```
[Out] integrate(x^m*(sqrt(1/(a*x^3) + 1)*sqrt(1/(a*x^3) - 1) + 1/(a*x^3)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(\frac{1}{x^3a}+\sqrt{\frac{1}{x^3a}-1}\,\sqrt{\frac{1}{x^3a}+1}\right)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^3/a+(1/x^3/a-1)^(1/2)*(1/x^3/a+1)^(1/2))*x^m,x)
```

```
[Out] int((1/x^3/a+(1/x^3/a-1)^(1/2)*(1/x^3/a+1)^(1/2))*x^m,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-3>0)', see `assume?` for more det
ails)Is m-3 equal to -1?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\left(\sqrt{\frac{1}{a\,x^3}-1}\,\sqrt{\frac{1}{a\,x^3}+1}+\frac{1}{a\,x^3}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((1/(a*x^3) - 1)^(1/2)*(1/(a*x^3) + 1)^(1/2) + 1/(a*x^3)),x)
```

```
[Out] int(x^m*((1/(a*x^3) - 1)^(1/2)*(1/(a*x^3) + 1)^(1/2) + 1/(a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{m}}{x^{3}}\, dx + \int a x^{m} \sqrt{-1 + \frac{1}{a x^{3}}} \sqrt{1 + \frac{1}{a x^{3}}}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**3+(1/a/x**3-1)**(1/2)*(1/a/x**3+1)**(1/2))*x**m,x)
```

```
[Out] (Integral(x**m/x**3, x) + Integral(a*x**m*sqrt(-1 + 1/(a*x**3))*sqrt(1 + 1/
(a*x**3)), x))/a
```

3.57 $\int e^{\text{sech}^{-1}(ax^2)} x^m\,dx$

Optimal. Leaf size=107

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,x^{m-1}\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{4};\frac{m+3}{4};a^2x^4\right)}{a\left(1-m^2\right)}-\frac{2x^{m-1}}{a\left(1-m^2\right)}+\frac{x^{m+1}e^{\text{sech}^{-1}\left(ax^2\right)}}{m+1}$$

```
[Out] -2*x^(-1+m)/a/(-m^2+1)+(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^(1+m
)/(1+m)-2*x^(-1+m)*hypergeom([1/2, -1/4+1/4*m],[3/4+1/4*m],a^2*x^4)*(1/(a*x
^2+1))^(1/2)*(a*x^2+1)^(1/2)/a/(-m^2+1)
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6335, 30, 259, 364}

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,x^{m-1}\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{4};\frac{m+3}{4};a^2x^4\right)}{a\left(1-m^2\right)}-\frac{2x^{m-1}}{a\left(1-m^2\right)}+\frac{x^{m+1}e^{\text{sech}^{-1}\left(ax^2\right)}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x^m,x]
```

```
[Out] (-2*x^(-1 + m))/(a*(1 - m^2)) + (E^ArcSech[a*x^2]*x^(1 + m))/(1 + m) - (2*x
^(-1 + m)*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Hypergeometric2F1[1/2, (-1
 + m)/4, (3 + m)/4, a^2*x^4])/(a*(1 - m^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\text{sech}^{-1}(ax^2)}x^m\,dx = \frac{e^{\text{sech}^{-1}(ax^2)}x^{1+m}}{1+m} + \frac{2\int x^{-2+m}\,dx}{a(1+m)} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{x^{-2+m}}{\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{a(1+m)}$$
$$= -\frac{2x^{-1+m}}{a\left(1-m^2\right)} + \frac{e^{\text{sech}^{-1}(ax^2)}x^{1+m}}{1+m} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{x^{-2+m}}{\sqrt{1-a^2x^4}}\,dx}{a(1+m)}$$
$$= -\frac{2x^{-1+m}}{a\left(1-m^2\right)} + \frac{e^{\text{sech}^{-1}(ax^2)}x^{1+m}}{1+m} - \frac{2x^{-1+m}\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+m);\frac{3+m}{4};a^2x^4\right)}{a\left(1-m^2\right)}$$

Mathematica [A] time = 2.39, size = 159, normalized size = 1.49

$$\frac{2^{\frac{m+1}{2}}x^{m+1}\left(ax^2\right)^{\frac{1}{2}(-m-1)}e^{\text{sech}^{-1}(ax^2)}\left(\frac{e^{\text{sech}^{-1}(ax^2)}}{e^{2\text{sech}^{-1}(ax^2)}+1}\right)^{\frac{m+1}{2}}\left((m+7)\,{}_2F_1\left(1,\frac{1-m}{4};\frac{m+7}{4};-e^{2\text{sech}^{-1}(ax^2)}\right)-(m+3)e^{2\text{sech}^{-1}(ax^2}\right.}{(m+3)(m+7)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^m,x]
```

```
[Out] (2^((1 + m)/2)*E^ArcSech[a*x^2]*(E^ArcSech[a*x^2]/(1 + E^(2*ArcSech[a*x^2])
))^((1 + m)/2)*x^(1 + m)*(a*x^2)^((-1 - m)/2)*((7 + m)*Hypergeometric2F1[1,
 (1 - m)/4, (7 + m)/4, -E^(2*ArcSech[a*x^2])] - E^(2*ArcSech[a*x^2])*(3 + m
)*Hypergeometric2F1[1, (5 - m)/4, (11 + m)/4, -E^(2*ArcSech[a*x^2])]))/((3
+ m)*(7 + m))
```

fricas [F] time = 1.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax^2x^m\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+x^m}{ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x, algorithm="f
ricas")
```

```
[Out] integral((a*x^2*x^m*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) +
x^m)/(a*x^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{a\,x^2}+\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}\right)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x)
```

```
[Out] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is m-2 equal to -1?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\left(\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] int(x^m*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{x^2}\,dx+\int ax^m\sqrt{-1+\frac{1}{ax^2}}\,\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**m,x)
```

```
[Out] (Integral(x**m/x**2, x) + Integral(a*x**m*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/
(a*x**2)), x))/a
```

3.58 $\int e^{\text{sech}^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=91

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\,x^m\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};a^2x^2\right)}{am(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax)}}{m+1}+\frac{x^m}{am(m+1)}$$

```
[Out] x^m/a/m/(1+m)+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^(1+m)/(1+m)+x^m*hyp
ergeom([1/2, 1/2*m],[1+1/2*m],a^2*x^2)*(1/(a*x+1))^(1/2)*(a*x+1)^(1/2)/a/m/
(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6335, 30, 125, 364}

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\,x^m\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};a^2x^2\right)}{am(m+1)}+\frac{x^m}{am(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax)}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]*x^m,x]
```

```
[Out] x^m/(a*m*(1 + m)) + (E^ArcSech[a*x]*x^(1 + m))/(1 + m) + (x^m*Sqrt[(1 + a*x
)^(-1)]*Sqrt[1 + a*x]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, a^2*x^2])/(a*m
*(1 + m))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 125

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0] && GtQ[a, 0] && GtQ[c, 0
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax)}x^m\,dx = \frac{e^{\operatorname{sech}^{-1}(ax)}x^{1+m}}{1+m} + \frac{\int x^{-1+m}\,dx}{a(1+m)} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{x^{-1+m}}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a(1+m)}$$
$$= \frac{x^m}{am(1+m)} + \frac{e^{\operatorname{sech}^{-1}(ax)}x^{1+m}}{1+m} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{x^{-1+m}}{\sqrt{1-a^2x^2}}\,dx}{a(1+m)}$$
$$= \frac{x^m}{am(1+m)} + \frac{e^{\operatorname{sech}^{-1}(ax)}x^{1+m}}{1+m} + \frac{x^m\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};a^2x^2\right)}{am(1+m)}$$

Mathematica [A] time = 0.31, size = 145, normalized size = 1.59

$$-\frac{2^{m+1}x^m(ax)^{-m}e^{2\operatorname{sech}^{-1}(ax)}\left(\frac{e^{\operatorname{sech}^{-1}(ax)}}{e^{2\operatorname{sech}^{-1}(ax)}+1}\right)^m\left(e^{2\operatorname{sech}^{-1}(ax)}+1\right)^m\left((m+2)e^{2\operatorname{sech}^{-1}(ax)}\,{}_2F_1\left(\frac{m}{2}+2,m+2;\frac{m}{2}+3;-e^{2\operatorname{se}}\right.\right.}{a(m+2)(m+4)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]*x^m,x]
```

```
[Out] -((2^(1 + m)*E^(2*ArcSech[a*x])*(E^ArcSech[a*x]/(1 + E^(2*ArcSech[a*x])))^m
*(1 + E^(2*ArcSech[a*x]))^m*x^m*(-((4 + m)*Hypergeometric2F1[1 + m/2, 2 + m
, 2 + m/2, -E^(2*ArcSech[a*x])]) + E^(2*ArcSech[a*x])*(2 + m)*Hypergeometri
c2F1[2 + m/2, 2 + m, 3 + m/2, -E^(2*ArcSech[a*x])]))/(a*(2 + m)*(4 + m)*(a*
x)^m))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{axx^m\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+x^m}{ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*x*x^m*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + x^m)/(a*x)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x)
```

```
[Out] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{ax+1}\,\sqrt{-ax+1}\,x^m}{x}\,dx}{a} + \frac{x^m}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^m/x, x)/a + x^m/(a*m)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}+\frac{1}{a\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] int(x^m*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{x}\,dx + \int a x^m \sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**m,x)
```

```
[Out] (Integral(x**m/x, x) + Integral(a*x**m*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))
, x))/a
```

3.59 $\int e^{\text{sech}^{-1}\left(\frac{a}{x}\right)} x^m \, dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{\frac{1}{\frac{a}{x}+1}}\sqrt{\frac{a}{x}+1}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-2);-\frac{m}{2};\frac{a^2}{x^2}\right)}{a\left(m^2+3m+2\right)}-\frac{x^{m+2}}{a\left(m^2+3m+2\right)}+\frac{x^{m+1}e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}}{m+1}$$

```
[Out] (x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^(1+m)/(1+m)-x^(2+m)/a/(m^2+3*m+2)-x^(2
+m)*hypergeom([1/2, -1-1/2*m],[-1/2*m],a^2/x^2)*(1/(1+a/x))^(1/2)*(1+a/x)^(
1/2)/a/(m^2+3*m+2)
```

Rubi [A] time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6335, 30, 259, 339, 364}

$$-\frac{\sqrt{\frac{1}{\frac{a}{x}+1}}\sqrt{\frac{a}{x}+1}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-2);-\frac{m}{2};\frac{a^2}{x^2}\right)}{a\left(m^2+3m+2\right)}-\frac{x^{m+2}}{a\left(m^2+3m+2\right)}+\frac{x^{m+1}e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a/x]*x^m,x]
```

```
[Out] (E^ArcSech[a/x]*x^(1 + m))/(1 + m) - x^(2 + m)/(a*(2 + 3*m + m^2)) - (Sqrt[
(1 + a/x)^(-1)]*Sqrt[1 + a/x]*x^(2 + m)*Hypergeometric2F1[1/2, (-2 - m)/2,
-m/2, a^2/x^2])/(a*(2 + 3*m + m^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
```

]

Rubi steps

$$\int e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}x^m\,dx=\frac{e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}x^{1+m}}{1+m}-\frac{\int x^{1+m}\,dx}{a(1+m)}-\frac{\left(\sqrt{\frac{1}{1+\frac{a}{x}}}\sqrt{1+\frac{a}{x}}\right)\int\frac{x^{1+m}}{\sqrt{1-\frac{a}{x}}\sqrt{1+\frac{a}{x}}}\,dx}{a(1+m)}$$

$$=\frac{e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}x^{1+m}}{1+m}-\frac{x^{2+m}}{a\left(2+3m+m^2\right)}-\frac{\left(\sqrt{\frac{1}{1+\frac{a}{x}}}\sqrt{1+\frac{a}{x}}\right)\int\frac{x^{1+m}}{\sqrt{1-\frac{a^2}{x^2}}}\,dx}{a(1+m)}$$

$$=\frac{e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}x^{1+m}}{1+m}-\frac{x^{2+m}}{a\left(2+3m+m^2\right)}+\frac{\left(\sqrt{\frac{1}{1+\frac{a}{x}}}\sqrt{1+\frac{a}{x}}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-3-m}}{\sqrt{1-a^2x^2}}\,dx,x,\frac{1}{x}\right)}{a(1+m)}$$

$$=\frac{e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}x^{1+m}}{1+m}-\frac{x^{2+m}}{a\left(2+3m+m^2\right)}-\frac{\sqrt{\frac{1}{1+\frac{a}{x}}}\sqrt{1+\frac{a}{x}}\,x^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2-m);-\frac{m}{2};\frac{a^2}{x^2}\right)}{a\left(2+3m+m^2\right)}$$

Mathematica [A] time = 0.84, size = 139, normalized size = 1.28

$$-\frac{a2^{-m-1}x^m\left(\frac{a}{x}\right)^m e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}\left(\frac{e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}}{e^{2\text{sech}^{-1}\left(\frac{a}{x}\right)}+1}\right)^{-m-1}\left(me^{2\text{sech}^{-1}\left(\frac{a}{x}\right)}{}_2F_1\left(1,\frac{m}{2}+2;2-\frac{m}{2};-e^{2\text{sech}^{-1}\left(\frac{a}{x}\right)}\right)-(m-2)\,{}_2F_1\left(1,\frac{m}{2}+\right.\right.}{(m-2)m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a/x]*x^m,x]
```

```
[Out] -((2^(-1 - m)*a*E^ArcSech[a/x]*(E^ArcSech[a/x]/(1 + E^(2*ArcSech[a/x])))^(-
1 - m)*(a/x)^m*x^m*(-((-2 + m)*Hypergeometric2F1[1, 1 + m/2, 1 - m/2, -E^(2
*ArcSech[a/x])]) + E^(2*ArcSech[a/x])*m*Hypergeometric2F1[1, 2 + m/2, 2 - m
/2, -E^(2*ArcSech[a/x])]))/((-2 + m)*m))
```

fricas [F] time = 2.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax^m\sqrt{\frac{a+x}{a}}\sqrt{-\frac{a-x}{a}}+xx^m}{a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x^m*sqrt((a + x)/a)*sqrt(-(a - x)/a) + x*x^m)/a, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\sqrt{\frac{x}{a}+1}\sqrt{\frac{x}{a}-1}+\frac{x}{a}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*(sqrt(x/a + 1)*sqrt(x/a - 1) + x/a), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \left(\frac{x}{a}+\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\right)x^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x)
[Out] int((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^{2}x^{m}}{a(m+2)}+\frac{\int\sqrt{a+x}\,\sqrt{-a+x}\,x^{m}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x, algorithm="maxima")
[Out] x^2*x^m/(a*(m + 2)) + integrate(sqrt(a + x)*sqrt(-a + x)*x^m, x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m}\left(\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}+\frac{x}{a}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((x/a - 1)^(1/2)*(x/a + 1)^(1/2) + x/a),x)
[Out] int(x^m*((x/a - 1)^(1/2)*(x/a + 1)^(1/2) + x/a), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int xx^{m}\,dx+\int ax^{m}\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/a+(-1+x/a)**(1/2)*(1+x/a)**(1/2))*x**m,x)
[Out] (Integral(x*x**m, x) + Integral(a*x**m*sqrt(-1 + x/a)*sqrt(1 + x/a), x))/a
```

3.60 $\int e^{\text{sech}^{-1}(ax^p)}x^m\,dx$

Optimal. Leaf size=133

$$\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,x^{m-p+1}\,{}_2F_1\left(\frac{1}{2},\frac{m-p+1}{2p};\frac{m+p+1}{2p};a^2x^{2p}\right)}{a(m+1)(m-p+1)}+\frac{px^{m-p+1}}{a(m+1)(m-p+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^p)}}{m+1}$$

```
[Out] (1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^(1+m)/(1+m)+p*x^(1+m-
p)/a/(1+m)/(1+m-p)+p*x^(1+m-p)*hypergeom([1/2, 1/2*(1+m-p)/p],[1/2*(1+m+p)/
p],a^2*x^(2*p))*(1/(1+a*x^p))^(1/2)*(1+a*x^p)^(1/2)/a/(1+m)/(1+m-p)
```

Rubi [A] time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6335, 30, 259, 364}

$$\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,x^{m-p+1}\,{}_2F_1\left(\frac{1}{2},\frac{m-p+1}{2p};\frac{m+p+1}{2p};a^2x^{2p}\right)}{a(m+1)(m-p+1)}+\frac{px^{m-p+1}}{a(m+1)(m-p+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^p)}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^p]*x^m,x]
```

```
[Out] (E^ArcSech[a*x^p]*x^(1 + m))/(1 + m) + (p*x^(1 + m - p))/(a*(1 + m)*(1 + m
- p)) + (p*x^(1 + m - p)*Sqrt[(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeomet
ric2F1[1/2, (1 + m - p)/(2*p), (1 + m + p)/(2*p), a^2*x^(2*p)])/(a*(1 + m)*
(1 + m - p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax^p)}x^m\,dx = \frac{e^{\operatorname{sech}^{-1}(ax^p)}x^{1+m}}{1+m} + \frac{p\int x^{m-p}\,dx}{a(1+m)} + \frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int \frac{x^{m-p}}{\sqrt{1-ax^p}\sqrt{1+ax^p}}\,dx}{a(1+m)}$$

$$= \frac{e^{\operatorname{sech}^{-1}(ax^p)}x^{1+m}}{1+m} + \frac{px^{1+m-p}}{a(1+m)(1+m-p)} + \frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int \frac{x^{m-p}}{\sqrt{1-a^2x^{2p}}}\,dx}{a(1+m)}$$

$$= \frac{e^{\operatorname{sech}^{-1}(ax^p)}x^{1+m}}{1+m} + \frac{px^{1+m-p}}{a(1+m)(1+m-p)} + \frac{px^{1+m-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\;{}_2F_1\left(\frac{1}{2},\frac{1+m-p}{2p};\frac{1+m+p}{2p}\right.}{a(1+m)(1+m-p)}$$

Mathematica [A] time = 5.17, size = 186, normalized size = 1.40

$$\frac{2^{\frac{m+1}{p}}x^{m+1}(ax^p)^{-\frac{m+1}{p}}e^{\operatorname{sech}^{-1}(ax^p)}\left(\frac{e^{\operatorname{sech}^{-1}(ax^p)}}{e^{2\operatorname{sech}^{-1}(ax^p)}+1}\right)^{\frac{m+1}{p}}\left((m+3p+1)\,{}_2F_1\left(1,1-\frac{m+p+1}{2p};\frac{m+3p+1}{2p};-e^{2\operatorname{sech}^{-1}(ax^p)}\right)-(m\right.}{(m+p+1)(m+3p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^p]*x^m,x]
```

```
[Out] (2^((1 + m)/p)*E^ArcSech[a*x^p]*(E^ArcSech[a*x^p]/(1 + E^(2*ArcSech[a*x^p])
))^((1 + m)/p)*x^(1 + m)*(-(E^(2*ArcSech[a*x^p])*(1 + m + p)*Hypergeometric
2F1[1, -1/2*(1 + m - 3*p)/p, (1 + m + 5*p)/(2*p), -E^(2*ArcSech[a*x^p])]) +
 (1 + m + 3*p)*Hypergeometric2F1[1, 1 - (1 + m + p)/(2*p), (1 + m + 3*p)/(2
*p), -E^(2*ArcSech[a*x^p])]))/((1 + m + p)*(1 + m + 3*p)*(a*x^p)^((1 + m)/p
))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x, algori
thm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\sqrt{\frac{1}{ax^p}+1}\sqrt{\frac{1}{ax^p}-1}+\frac{1}{ax^p}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x, algori
thm="giac")
```

```
[Out] integrate(x^m*(sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p)), x)
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int\left(\frac{x^{-p}}{a}+\sqrt{\frac{x^{-p}}{a}-1}\sqrt{\frac{x^{-p}}{a}+1}\right)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x)
```

```
[Out] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-p>0)', see `assume?` for more det
ails)Is m-p equal to -1?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(\sqrt{\frac{1}{a\,x^p}-1}\,\sqrt{\frac{1}{a\,x^p}+1}+\frac{1}{a\,x^p}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p)),x)
```

```
[Out] int(x^m*((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x^m x^{-p}\,dx + \int a x^m \sqrt{-1+\frac{x^{-p}}{a}}\,\sqrt{1+\frac{x^{-p}}{a}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))*x**m,x)
```

```
[Out] (Integral(x**m*x**(-p), x) + Integral(a*x**m*sqrt(-1 + x**(-p)/a)*sqrt(1 +
x**(-p)/a), x))/a
```

3.61 $\int e^{\text{sech}^{-1}(ax^p)} x \, dx$

Optimal. Leaf size=119

$$\frac{px^{2-p}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(\frac{2}{p}-1\right);\frac{1}{2}\left(1+\frac{2}{p}\right);a^2x^{2p}\right)}{2a(2-p)}+\frac{px^{2-p}}{2a(2-p)}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^p)}$$

```
[Out] 1/2*(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^2+1/2*p*x^(2-p)/a
/(2-p)+1/2*p*x^(2-p)*hypergeom([1/2, -1/2+1/p],[1/2+1/p],a^2*x^(2*p))*(1/(1
+a*x^p))^(1/2)*(1+a*x^p)^(1/2)/a/(2-p)
```

Rubi [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6335, 30, 259, 364}

$$\frac{px^{2-p}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(\frac{2}{p}-1\right);\frac{1}{2}\left(1+\frac{2}{p}\right);a^2x^{2p}\right)}{2a(2-p)}+\frac{px^{2-p}}{2a(2-p)}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^p]*x,x]
```

```
[Out] (E^ArcSech[a*x^p]*x^2)/2 + (p*x^(2 - p))/(2*a*(2 - p)) + (p*x^(2 - p)*Sqrt[
(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeometric2F1[1/2, (-1 + 2/p)/2, (1 +
 2/p)/2, a^2*x^(2*p)])/(2*a*(2 - p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax^p)}x\,dx = \frac{1}{2}e^{\operatorname{sech}^{-1}(ax^p)}x^2 + \frac{p\int x^{1-p}\,dx}{2a} + \frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int \frac{x^{1-p}}{\sqrt{1-ax^p}\sqrt{1+ax^p}}\,dx}{2a}$$
$$= \frac{1}{2}e^{\operatorname{sech}^{-1}(ax^p)}x^2 + \frac{px^{2-p}}{2a(2-p)} + \frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int \frac{x^{1-p}}{\sqrt{1-a^2x^{2p}}}\,dx}{2a}$$
$$= \frac{1}{2}e^{\operatorname{sech}^{-1}(ax^p)}x^2 + \frac{px^{2-p}}{2a(2-p)} + \frac{px^{2-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(-1+\frac{2}{p}\right);\frac{1}{2}\left(1+\frac{2}{p}\right);a^2x^{2p}\right)}{2a(2-p)}$$

Mathematica [**A**] time = 0.31, size = 159, normalized size = 1.34

$$\frac{x^{2-p}\left(\frac{a^2px^{2p}\sqrt{\frac{1-ax^p}{ax^p+1}}\sqrt{1-a^2x^{2p}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{p};\frac{3}{2}+\frac{1}{p};a^2x^{2p}\right)}{(p+2)(ax^p-1)} - ax^p\sqrt{\frac{1-ax^p}{ax^p+1}} - \sqrt{\frac{1-ax^p}{ax^p+1}} - 1\right)}{a(p-2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^p]*x,x]
```

```
[Out] (x^(2 - p)*(-1 - Sqrt[(1 - a*x^p)/(1 + a*x^p)] - a*x^p*Sqrt[(1 - a*x^p)/(1
+ a*x^p)] + (a^2*p*x^(2*p)*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*Sqrt[1 - a^2*x^(2*
p)]*Hypergeometric2F1[1/2, 1/2 + p^(-1), 3/2 + p^(-1), a^2*x^(2*p)])/((2 +
p)*(-1 + a*x^p))))/(a*(-2 + p))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x, algorith
m="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\sqrt{\frac{1}{ax^p}+1}\sqrt{\frac{1}{ax^p}-1}+\frac{1}{ax^p}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x, algorith
m="giac")
```

```
[Out] integrate(x*(sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p)), x)
```

maple [**F**] time = 1.06, size = 0, normalized size = 0.00

$$\int \left(\frac{x^{-p}}{a}+\sqrt{\frac{x^{-p}}{a}-1}\,\sqrt{\frac{x^{-p}}{a}+1}\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x)
```

```
[Out] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-p>0)', see `assume?` for more det
ails)Is 1-p equal to -1?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(\sqrt{\frac{1}{a\,x^p}-1}\,\sqrt{\frac{1}{a\,x^p}+1}+\frac{1}{a\,x^p}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p)),x)
```

```
[Out] int(x*((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int xx^{-p}\, dx + \int ax\sqrt{-1 + \frac{x^{-p}}{a}} \sqrt{1 + \frac{x^{-p}}{a}}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))*x,x)
```

```
[Out] (Integral(x*x**(-p), x) + Integral(a*x*sqrt(-1 + x**(-p)/a)*sqrt(1 + x**(-p
)/a), x))/a
```

3.62 $\int e^{\text{sech}^{-1}(ax^p)}\,dx$

Optimal. Leaf size=105

$$\frac{px^{1-p}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(\frac{1}{p}-1\right);\frac{p+1}{2p};a^2x^{2p}\right)}{a(1-p)}+\frac{px^{1-p}}{a(1-p)}+xe^{\text{sech}^{-1}(ax^p)}$$

```
[Out] (1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x+p*x^(1-p)/a/(1-p)+p*x
^(1-p)*hypergeom([1/2, -1/2+1/2/p],[1/2*(1+p)/p],a^2*x^(2*p))*(1/(1+a*x^p))
^(1/2)*(1+a*x^p)^(1/2)/a/(1-p)
```

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6330, 30, 259, 364}

$$\frac{px^{1-p}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(\frac{1}{p}-1\right);\frac{p+1}{2p};a^2x^{2p}\right)}{a(1-p)}+\frac{px^{1-p}}{a(1-p)}+xe^{\text{sech}^{-1}(ax^p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^p],x]
```

```
[Out] E^ArcSech[a*x^p]*x + (p*x^(1 - p))/(a*(1 - p)) + (p*x^(1 - p)*Sqrt[(1 + a*x
^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeometric2F1[1/2, (-1 + p^(-1))/2, (1 + p)/(
2*p), a^2*x^(2*p)])/(a*(1 - p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6330

```
Int[E^ArcSech[(a_.)*(x_)^(p_)], x_Symbol] :> Simp[x*E^ArcSech[a*x^p], x] +
(Dist[p/a, Int[1/x^p, x], x] + Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])
/a, Int[1/(x^p*Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, p}, x
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax^p)}\,dx = e^{\operatorname{sech}^{-1}(ax^p)}x + \frac{p\int x^{-p}\,dx}{a} + \frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int \frac{x^{-p}}{\sqrt{1-ax^p}\sqrt{1+ax^p}}\,dx}{a}$$

$$= e^{\operatorname{sech}^{-1}(ax^p)}x + \frac{px^{1-p}}{a(1-p)} + \frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int \frac{x^{-p}}{\sqrt{1-a^2x^{2p}}}\,dx}{a}$$

$$= e^{\operatorname{sech}^{-1}(ax^p)}x + \frac{px^{1-p}}{a(1-p)} + \frac{px^{1-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(-1+\frac{1}{p}\right);\frac{1+p}{2p};a^2x^{2p}\right)}{a(1-p)}$$

Mathematica [A] time = 0.41, size = 139, normalized size = 1.32

$$\frac{x\left(-\frac{a^2px^p\sqrt{\frac{1-ax^p}{ax^p+1}}\sqrt{1-a^2x^{2p}}\ {}_2F_1\left(\frac{1}{2},\frac{p+1}{2p};\frac{1}{2}\left(3+\frac{1}{p}\right);a^2x^{2p}\right)}{(p+1)(ax^p-1)}+(a+x^{-p})\sqrt{\frac{1-ax^p}{ax^p+1}}+x^{-p}\right)}{a-ap}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^p],x]
```

```
[Out] (x*(x^(-p) + (a + x^(-p))*Sqrt[(1 - a*x^p)/(1 + a*x^p)] - (a^2*p*x^p*Sqrt[(
1 - a*x^p)/(1 + a*x^p)]*Sqrt[1 - a^2*x^(2*p)]*Hypergeometric2F1[1/2, (1 + p
)/(2*p), (3 + p^(-1))/2, a^2*x^(2*p)])/((1 + p)*(-1 + a*x^p))))/(a - a*p)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1}{ax^p}+1}\sqrt{\frac{1}{ax^p}-1}+\frac{1}{ax^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p), x)
```

maple [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{x^{-p}}{a}+\sqrt{\frac{x^{-p}}{a}-1}\,\sqrt{\frac{x^{-p}}{a}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x)
```

```
[Out] int(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-p>0)', see `assume?` for more deta
ils)Is -p equal to -1?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{a\,x^p}-1}\,\sqrt{\frac{1}{a\,x^p}+1}+\frac{1}{a\,x^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p),x)
```

```
[Out] int((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x^{-p}\,dx+\int a\sqrt{-1+\frac{x^{-p}}{a}}\,\sqrt{1+\frac{x^{-p}}{a}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2),x)
```

```
[Out] (Integral(x**(-p), x) + Integral(a*sqrt(-1 + x**(-p)/a)*sqrt(1 + x**(-p)/a)
, x))/a
```

3.63 $\int \frac{e^{\text{sech}^{-1}(ax^p)}}{x}\,dx$

Optimal. Leaf size=87

$$-\frac{x^{-p}}{ap}-\frac{\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,\sin^{-1}(ax^p)}{p}-\frac{x^{-p}\sqrt{1-ax^p}}{ap\sqrt{\frac{1}{ax^p+1}}}$$

```
[Out] -1/a/p/(x^p)-(1-a*x^p)^(1/2)/a/p/(x^p)/(1/(1+a*x^p))^(1/2)-arcsin(a*x^p)*(1
/(1+a*x^p))^(1/2)*(1+a*x^p)^(1/2)/p
```

Rubi [A] time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6334, 259, 345, 242, 277, 216}

$$-\frac{x^{-p}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\sqrt{1-a^2x^{2p}}}{ap}-\frac{x^{-p}}{ap}-\frac{\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,\csc^{-1}\left(\frac{x^{-p}}{a}\right)}{p}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x^p]/x,x]
```

```
[Out] -(1/(a*p*x^p)) - (Sqrt[(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Sqrt[1 - a^2*x^(2*
p)])/(a*p*x^p) - (Sqrt[(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*ArcCsc[1/(a*x^p)])
/p
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 6334

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]/(x_), x_Symbol] :> -Simp[(a*p*x^p)^(-1), x]
 + Dist[(Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/a, Int[(Sqrt[1 + a*x^p]*Sqrt[
1 - a*x^p])/x^(p + 1), x], x] /; FreeQ[{a, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x}\,dx &= -\frac{x^{-p}}{ap}+\frac{\left(\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int x^{-1-p}\sqrt{1-ax^p}\sqrt{1+ax^p}\,dx}{a}\\
&= -\frac{x^{-p}}{ap}+\frac{\left(\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int x^{-1-p}\sqrt{1-a^2x^{2p}}\,dx}{a}\\
&= -\frac{x^{-p}}{ap}-\frac{\left(\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\operatorname{Subst}\left(\int\sqrt{1-\frac{a^2}{x^2}}\,dx,x,x^{-p}\right)}{ap}\\
&= -\frac{x^{-p}}{ap}+\frac{\left(\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1-a^2x^2}}{x^2}\,dx,x,x^p\right)}{ap}\\
&= -\frac{x^{-p}}{ap}-\frac{x^{-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\sqrt{1-a^2x^{2p}}}{ap}-\frac{\left(a\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-a^2x^2}}\,dx,x,x^p\right)}{p}\\
&= -\frac{x^{-p}}{ap}-\frac{x^{-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\sqrt{1-a^2x^{2p}}}{ap}-\frac{\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\sin^{-1}(ax^p)}{p}
\end{aligned}$$

Mathematica [C] time = 0.16, size = 96, normalized size = 1.10

$$-\frac{i\left(a\log\left(2\sqrt{\frac{1-ax^p}{ax^p+1}}(ax^p+1)-2iax^p\right)-i(a+x^{-p})\sqrt{\frac{1-ax^p}{ax^p+1}}-ix^{-p}\right)}{ap}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^p]/x,x]
```

```
[Out] ((-I)*((-I)/x^p - I*(a + x^(-p))*Sqrt[(1 - a*x^p)/(1 + a*x^p)] + a*Log[(-2*
I)*a*x^p + 2*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*(1 + a*x^p)]))/(a*p)
```

fricas [A] time = 0.70, size = 102, normalized size = 1.17

$$-\frac{ax^p\sqrt{\frac{ax^p+1}{ax^p}}\sqrt{-\frac{ax^p-1}{ax^p}}-ax^p\arctan\left(\sqrt{\frac{ax^p+1}{ax^p}}\sqrt{-\frac{ax^p-1}{ax^p}}\right)+1}{apx^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x, algorith
m="fricas")
```

```
[Out] -(a*x^p*sqrt((a*x^p + 1)/(a*x^p))*sqrt(-(a*x^p - 1)/(a*x^p)) - a*x^p*arctan
(sqrt((a*x^p + 1)/(a*x^p))*sqrt(-(a*x^p - 1)/(a*x^p))) + 1)/(a*p*x^p)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{1}{ax^p}+1}\sqrt{\frac{1}{ax^p}-1}+\frac{1}{ax^p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x, algorith
m="giac")
```

```
[Out] integrate((sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p))/x, x)
```

maple [C] time = 0.15, size = 145, normalized size = 1.67

$$-\frac{\sqrt{-\frac{(a\,x^{p}-1)x^{-p}}{a}}\,\sqrt{\frac{(1+a\,x^{p})x^{-p}}{a}}\,\operatorname{csgn}(a)\arctan\left(\frac{\operatorname{csgn}(a)a\,x^{p}}{\sqrt{1-a^{2}x^{2p}}}\right)x^{p}a}{p\sqrt{1-a^{2}x^{2p}}}-\frac{\sqrt{-\frac{(a\,x^{p}-1)x^{-p}}{a}}\,\sqrt{\frac{(1+a\,x^{p})x^{-p}}{a}}\,\operatorname{csgn}(a)^{2}}{p}-\frac{x^{-p}}{ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x)
```

```
[Out] -1/p*(-(a*x^p-1)/a/(x^p))^(1/2)*((1+a*x^p)/a/(x^p))^(1/2)*csgn(a)/(-(x^p)^2
*a^2+1)^(1/2)*arctan(csgn(a)*a*x^p/(-(x^p)^2*a^2+1)^(1/2))*x^p*a-1/p*(-(a*x
^p-1)/a/(x^p))^(1/2)*((1+a*x^p)/a/(x^p))^(1/2)*csgn(a)^2-1/a/p/(x^p)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{ax^{p}+1}\,\sqrt{-ax^{p}+1}}{xx^{p}}\,dx}{a}-\frac{1}{apx^{p}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(a*x^p + 1)*sqrt(-a*x^p + 1)/(x*x^p), x)/a - 1/(a*p*x^p)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{a\,x^{p}}-1}\,\sqrt{\frac{1}{a\,x^{p}}+1}+\frac{1}{a\,x^{p}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p))/x,x)
```

```
[Out] int(((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))/x,x)
```

```
[Out] Timed out
```

3.64 $\int \frac{e^{\text{sech}^{-1}(ax^p)}}{x^2} dx$

Optimal. Leaf size=107

$$\frac{px^{-p-1}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,{}_2F_1\left(\frac{1}{2},-\frac{p+1}{2p};-\frac{1-p}{2p};a^2x^{2p}\right)}{a(p+1)}+\frac{px^{-p-1}}{a(p+1)}-\frac{e^{\text{sech}^{-1}(ax^p)}}{x}$$

```
[Out] -(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x+p*x^(-1-p)/a/(1+p)+p
*x^(-1-p)*hypergeom([1/2, 1/2*(-1-p)/p],[1/2*(-1+p)/p],a^2*x^(2*p))*(1/(1+a
*x^p))^(1/2)*(1+a*x^p)^(1/2)/a/(1+p)
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6335, 30, 259, 364}

$$\frac{px^{-p-1}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,{}_2F_1\left(\frac{1}{2},-\frac{p+1}{2p};-\frac{1-p}{2p};a^2x^{2p}\right)}{a(p+1)}+\frac{px^{-p-1}}{a(p+1)}-\frac{e^{\text{sech}^{-1}(ax^p)}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^p]/x^2,x]
```

```
[Out] -(E^ArcSech[a*x^p]/x) + (p*x^(-1 - p))/(a*(1 + p)) + (p*x^(-1 - p)*Sqrt[(1
+ a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeometric2F1[1/2, -(1 + p)/(2*p), -(1 -
 p)/(2*p), a^2*x^(2*p)])/(a*(1 + p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6335

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*E^
ArcSech[a*x^p])/(m + 1), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[(p*Sqrt[1 + a*x^p]*Sqrt[1/(1 + a*x^p)])/(a*(m + 1)), Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x^2}\,dx = -\frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x} - \frac{p\int x^{-2-p}\,dx}{a} - \frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int \frac{x^{-2-p}}{\sqrt{1-ax^p}\sqrt{1+ax^p}}\,dx}{a}$$

$$= -\frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x} + \frac{px^{-1-p}}{a(1+p)} - \frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int \frac{x^{-2-p}}{\sqrt{1-a^2x^{2p}}}\,dx}{a}$$

$$= -\frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x} + \frac{px^{-1-p}}{a(1+p)} + \frac{px^{-1-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\,{}_2F_1\left(\frac{1}{2},-\frac{1+p}{2p};-\frac{1-p}{2p};a^2x^{2p}\right)}{a(1+p)}$$

Mathematica [A] time = 0.28, size = 156, normalized size = 1.46

$$x^{-p-1}\left(\frac{apx^{2p}\sqrt{\frac{1-ax^p}{ax^p+1}}\sqrt{1-a^2x^{2p}}\,{}_2F_1\left(\frac{1}{2},\frac{p-1}{2p};\frac{3}{2}-\frac{1}{2p};a^2x^{2p}\right)}{(p-1)(p+1)\left(ax^p-1\right)} - \frac{\sqrt{\frac{1-ax^p}{ax^p+1}}\left(ax^p+1\right)}{a(p+1)} - \frac{1}{ap+a}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^p]/x^2,x]
```

```
[Out] x^(-1 - p)*(-(a + a*p)^(-1) - (Sqrt[(1 - a*x^p)/(1 + a*x^p)]*(1 + a*x^p))/(
a*(1 + p)) + (a*p*x^(2*p)*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*Sqrt[1 - a^2*x^(2*p
)]*Hypergeometric2F1[1/2, (-1 + p)/(2*p), 3/2 - 1/(2*p), a^2*x^(2*p)])/((-1
 + p)*(1 + p)*(-1 + a*x^p)))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x, algori
thm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{ax^p}+1}\sqrt{\frac{1}{ax^p}-1}+\frac{1}{ax^p}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x, algori
thm="giac")
```

```
[Out] integrate((sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p))/x^2, x)
```

maple [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\frac{x^{-p}}{a}+\sqrt{\frac{x^{-p}}{a}-1}\sqrt{\frac{x^{-p}}{a}+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x)
```

```
[Out] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{ax^p+1}\sqrt{-ax^p+1}}{x^2x^p}\,dx}{a} - \frac{x^{-p-1}}{a\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(a*x^p + 1)*sqrt(-a*x^p + 1)/(x^2*x^p), x)/a - x^(-p - 1)/(a*
(p + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{a\,x^p}-1}\,\sqrt{\frac{1}{a\,x^p}+1}+\frac{1}{a\,x^p}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p))/x^2,x)
```

```
[Out] int(((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{-p}}{x^2}\,dx + \int \frac{a\sqrt{-1+\frac{x^{-p}}{a}}\sqrt{1+\frac{x^{-p}}{a}}}{x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))/x**2,x)
```

```
[Out] (Integral(x**(-p)/x**2, x) + Integral(a*sqrt(-1 + x**(-p)/a)*sqrt(1 + x**(-
p)/a)/x**2, x))/a
```

3.65 $\int e^{2\operatorname{sech}^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=203

$$\frac{(1-ax)(ax+1)^4}{5a^5}+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(5-6\sqrt{\frac{1-ax}{ax+1}}\right)(ax+1)^4}{10a^5}-\frac{\left(45\sqrt{\frac{1-ax}{ax+1}}+4\right)(ax+1)^3}{30a^5}+\frac{5\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2}{4a^5}+\frac{\left(4-\sqrt{\frac{1-ax}{ax+1}}\right.}{4a}$$

```
[Out] 1/5*(-a*x+1)*(a*x+1)^4/a^5-1/2*arctan(((-a*x+1)/(a*x+1))^(1/2))/a^5+1/4*(a*
x+1)*(4-((-a*x+1)/(a*x+1))^(1/2))/a^5+5/4*(a*x+1)^2*((-a*x+1)/(a*x+1))^(1/2
)/a^5+1/10*(a*x+1)^4*(5-6*((-a*x+1)/(a*x+1))^(1/2))*((-a*x+1)/(a*x+1))^(1/2
)/a^5-1/30*(a*x+1)^3*(4+45*((-a*x+1)/(a*x+1))^(1/2))/a^5
```

Rubi [A] time = 0.70, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6337, 1804, 1811, 1814, 639, 203}

$$\frac{(1-ax)(ax+1)^4}{5a^5}+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(5-6\sqrt{\frac{1-ax}{ax+1}}\right)(ax+1)^4}{10a^5}-\frac{\left(45\sqrt{\frac{1-ax}{ax+1}}+4\right)(ax+1)^3}{30a^5}+\frac{5\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2}{4a^5}+\frac{\left(4-\sqrt{\frac{1-ax}{ax+1}}\right.}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])*x^4,x]
```

```
[Out] (5*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(4*a^5) + ((1 - a*x)*(1 + a*x)^4)
/(5*a^5) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^4*(5 - 6*Sqrt[(1 - a*x)/(1
+ a*x)]))/(10*a^5) + ((1 + a*x)*(4 - Sqrt[(1 - a*x)/(1 + a*x)]))/(4*a^5) -
((1 + a*x)^3*(4 + 45*Sqrt[(1 - a*x)/(1 + a*x)]))/(30*a^5) - ArcTan[Sqrt[(1
- a*x)/(1 + a*x)]]/(2*a^5)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1811

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int e^{2\operatorname{sech}^{-1}(ax)}x^4\,dx = \int x^4\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2 dx$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{(-1+x)^2x(1+x)^6}{\left(1+x^2\right)^6}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^5}$$

$$= \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{2\,\text{Subst}\left(\int \frac{-42x-40x^2+130x^3+80x^4-30x^5-40x^6-10x^7}{\left(1+x^2\right)^5}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{5a^5}$$

$$= \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{2\,\text{Subst}\left(\int \frac{x\left(-42-40x+130x^2+80x^3-30x^4-40x^5-10x^6\right)}{\left(1+x^2\right)^5}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{5a^5}$$

$$= \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4\left(5-6\sqrt{\frac{1-ax}{1+ax}}\right)}{10a^5} - \frac{\text{Subst}\left(\int \frac{160-48x-960x^2+160x^3+320x^4+80x}{\left(1+x^2\right)^4}\right.}{20a^5}$$

$$= \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4\left(5-6\sqrt{\frac{1-ax}{1+ax}}\right)}{10a^5} - \frac{(1+ax)^3\left(4+45\sqrt{\frac{1-ax}{1+ax}}\right)}{30a^5} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{5\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2}{4a^5} + \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4\left(5-6\sqrt{\frac{1-ax}{1+ax}}\right)}{10a^5} - \frac{(1+ax)^3\left(4+45\right.}{30a^5}$$

$$= \frac{5\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2}{4a^5} + \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4\left(5-6\sqrt{\frac{1-ax}{1+ax}}\right)}{10a^5} + \frac{(1+ax)\left(4-\sqrt{\frac{1}{1}}\right.}{4a^5}$$

$$= \frac{5\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2}{4a^5} + \frac{(1-ax)(1+ax)^4}{5a^5} + \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4\left(5-6\sqrt{\frac{1-ax}{1+ax}}\right)}{10a^5} + \frac{(1+ax)\left(4-\sqrt{\frac{1}{1}}\right.}{4a^5}$$

Mathematica [C] time = 0.16, size = 105, normalized size = 0.52

$$\frac{-12a^5x^5+40a^3x^3-15a\sqrt{\frac{1-ax}{ax+1}}\left(-2a^3x^4-2a^2x^3+ax^2+x\right)+15i\log\left(2\sqrt{\frac{1-ax}{ax+1}}(ax+1)-2iax\right)}{60a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])*x^4,x]
```

```
[Out] (40*a^3*x^3 - 12*a^5*x^5 - 15*a*Sqrt[(1 - a*x)/(1 + a*x)]*(x + a*x^2 - 2*a^
2*x^3 - 2*a^3*x^4) + (15*I)*Log[(-2*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1
 + a*x)])/(60*a^5)
```

fricas [A] time = 0.89, size = 103, normalized size = 0.51

$$-\frac{12\,a^5x^5-40\,a^3x^3-15\left(2\,a^4x^4-a^2x^2\right)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}+15\,\arctan\left(\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\right)}{60\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x, algorithm="frica
s")
```

```
[Out] -1/60*(12*a^5*x^5 - 40*a^3*x^3 - 15*(2*a^4*x^4 - a^2*x^2)*sqrt((a*x + 1)/(a
*x))*sqrt(-(a*x - 1)/(a*x)) + 15*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x -
1)/(a*x))))/a^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,%%%{-4,[1,1]%
%%},0,%%%{4,[4,4]%%%}] at parameters values [86,-97]Warning, choosing root
of [1,0,%%%{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-82,7]Warn
ing, integration of abs or sign assumes constant sign by intervals (correct
 if the argument is real):Check [abs(t_nostep)]Warning, choosing root of [1
,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-27,26]Warning
, choosing root of [1,0,%%%{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters va
lues [-89,63]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%
%%}] at parameters values [-49,-86]Warning, choosing root of [1,0,%%%{4,[1,
1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-64,-30]Warning, choosing r
oot of [1,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [70,22
]Warning, choosing root of [1,0,%%%{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at param
eters values [42,56]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%},0,%%%{4
,[4,4]%%%}] at parameters values [-9,-13]Warning, choosing root of [1,0,%%%
{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [46,24]Warning, choosi
ng root of [1,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [4
9,-6]Unable to divide, perhaps due to rounding error%%%{-1,[4,2,0,0]%%%}+%%
%{2,[3,1,1,1]%%%}+%%%{2,[2,0,0,0]%%%} / %%%{1,[0,2,0,0]%%%} Error: Bad Argu
ment Value
```

maple [C] time = 0.06, size = 123, normalized size = 0.61

$$-\frac{x^5}{5}+\frac{2x^3}{3a^2}+\frac{\sqrt{-\frac{ax-1}{ax}}\;x\sqrt{\frac{ax+1}{ax}}\;\left(2\,\mathrm{csgn}(a)x^3a^3\sqrt{-a^2x^2+1}\;-x\sqrt{-a^2x^2+1}\;\mathrm{csgn}(a)a+\arctan\left(\frac{\mathrm{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)\right)\mathrm{csgn}}{4a^4\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x)
```

```
[Out] -1/5*x^5+2/3*x^3/a^2+1/4/a^4*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(2*
csgn(a)*x^3*a^3*(-a^2*x^2+1)^(1/2)-x*(-a^2*x^2+1)^(1/2)*csgn(a)*a+arctan(cs
gn(a)*a*x/(-a^2*x^2+1)^(1/2)))*csgn(a)/(-a^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,x^3}{3\,a^2}+\frac{2\left(-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{4\,a^2}+\frac{\sqrt{-a^2x^2+1}\,x}{8\,a^2}+\frac{\arcsin\left(ax\right)}{8\,a^3}\right)}{a^2}-\int x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x, algorithm="maxim
a")
```

```
[Out] 2/3*x^3/a^2 + 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^2, x)/a^2 - integr
ate(x^4, x)
```

mupad [B] time = 19.73, size = 808, normalized size = 3.98

$$-\frac{\frac{1\mathrm{i}}{512\,a^5}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 3\mathrm{i}}{64\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4 53\mathrm{i}}{256\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6 87\mathrm{i}}{128\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8 657\mathrm{i}}{512\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10} 121\mathrm{i}}{128\,a^5\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{6\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{12}}}-\frac{\frac{1\mathrm{i}}{16\,a^5}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] (log((a*(-(a - 1/x)/a)^(1/2)*2i - 2/x + 2*a*((a + 1/x)/a)^(1/2))/(2*a + 1/x
 - 2*a*((a + 1/x)/a)^(1/2)))*1i)/a^5 - (1i/(512*a^5) - (((1/(a*x) - 1)^(1/2
) - 1i)^2*3i)/(64*a^5*((1/(a*x) + 1)^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/2)
- 1i)^4*53i)/(256*a^5*((1/(a*x) + 1)^(1/2) - 1)^4) + (((1/(a*x) - 1)^(1/2)
- 1i)^6*87i)/(128*a^5*((1/(a*x) + 1)^(1/2) - 1)^6) + (((1/(a*x) - 1)^(1/2)
- 1i)^8*657i)/(512*a^5*((1/(a*x) + 1)^(1/2) - 1)^8) + (((1/(a*x) - 1)^(1/2)
 - 1i)^10*121i)/(128*a^5*((1/(a*x) + 1)^(1/2) - 1)^10))/(((1/(a*x) - 1)^(1/
2) - 1i)^4/((1/(a*x) + 1)^(1/2) - 1)^4 + (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/(
(1/(a*x) + 1)^(1/2) - 1)^6 + (6*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1
)^(1/2) - 1)^8 + (4*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1
)^10 + ((1/(a*x) - 1)^(1/2) - 1i)^12/((1/(a*x) + 1)^(1/2) - 1)^12) - (log((
(1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/(4*a^5) - (1i/(16*
a^5) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(8*a^5*((1/(a*x) + 1)^(1/2) - 1)^2
) - (((1/(a*x) - 1)^(1/2) - 1i)^4*15i)/(16*a^5*((1/(a*x) + 1)^(1/2) - 1)^4)
)/(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + (2*((1/(a*x)
- 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a*x) - 1)^(1/2) - 1i
)^6/((1/(a*x) + 1)^(1/2) - 1)^6) - (log((a*(1/(a*x) - 1)^(1/2)*1i + a*(1/(a
*x) + 1)^(1/2) - 1/x)/(2*a - 2*a*(1/(a*x) + 1)^(1/2) + 1/x))*3i)/(4*a^5) -
(((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(128*a^5*((1/(a*x) + 1)^(1/2) - 1)^2) - (
((1/(a*x) - 1)^(1/2) - 1i)^4*1i)/(512*a^5*((1/(a*x) + 1)^(1/2) - 1)^4) - (x
^5*(a^2/5 - 2/(3*x^2)))/a^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x**4,x)
```

```
[Out] Timed out
```

3.66 $\int e^{2\text{sech}^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=117

$$\frac{(1-ax)(ax+1)^3}{4a^4}+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(4-3\sqrt{\frac{1-ax}{ax+1}}\right)(ax+1)^3}{6a^4}+\frac{\left(3-8\sqrt{\frac{1-ax}{ax+1}}\right)(ax+1)^2}{6a^4}-\frac{x}{a^3}$$

```
[Out] -x/a^3+1/4*(-a*x+1)*(a*x+1)^3/a^4+1/6*(a*x+1)^2*(3-8*((-a*x+1)/(a*x+1))^(1/
2))/a^4+1/6*(a*x+1)^3*(4-3*((-a*x+1)/(a*x+1))^(1/2))*((-a*x+1)/(a*x+1))^(1/
2)/a^4
```

Rubi [A] time = 0.55, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6337, 1804, 1811, 1814, 12, 261}

$$\frac{(1-ax)(ax+1)^3}{4a^4}+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(4-3\sqrt{\frac{1-ax}{ax+1}}\right)(ax+1)^3}{6a^4}+\frac{\left(3-8\sqrt{\frac{1-ax}{ax+1}}\right)(ax+1)^2}{6a^4}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])*x^3,x]
```

```
[Out] -(x/a^3) + ((1 - a*x)*(1 + a*x)^3)/(4*a^4) + ((1 + a*x)^2*(3 - 8*Sqrt[(1 -
a*x)/(1 + a*x)]))/(6*a^4) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^3*(4 - 3*S
qrt[(1 - a*x)/(1 + a*x)]))/(6*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1811

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
```

```
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int e^{2\text{sech}^{-1}(ax)}x^3\,dx = \int x^3\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2 dx$$

$$= \frac{4\,\text{Subst}\left(\int \frac{(-1+x)x(1+x)^5}{\left(1+x^2\right)^5}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^4}$$

$$= \frac{(1-ax)(1+ax)^3}{4a^4} - \frac{\text{Subst}\left(\int \frac{24x+32x^2-32x^3-32x^4-8x^5}{\left(1+x^2\right)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{2a^4}$$

$$= \frac{(1-ax)(1+ax)^3}{4a^4} - \frac{\text{Subst}\left(\int \frac{x\left(24+32x-32x^2-32x^3-8x^4\right)}{\left(1+x^2\right)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{2a^4}$$

$$= \frac{(1-ax)(1+ax)^3}{4a^4} + \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3\left(4-3\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4} + \frac{\text{Subst}\left(\int \frac{-64-48x+192x^2+48x^3}{\left(1+x^2\right)^3}\,dx, x, \sqrt{\frac{1\cdots}{1\cdots}}\right.}{12a^4}$$

$$= \frac{(1-ax)(1+ax)^3}{4a^4} + \frac{(1+ax)^2\left(3-8\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4} + \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3\left(4-3\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4} - \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{(1-ax)(1+ax)^3}{4a^4} + \frac{(1+ax)^2\left(3-8\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4} + \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3\left(4-3\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4} + \frac{4\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{x}{a^3} + \frac{(1-ax)(1+ax)^3}{4a^4} + \frac{(1+ax)^2\left(3-8\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4} + \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3\left(4-3\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4}$$

Mathematica [A] time = 0.07, size = 52, normalized size = 0.44

$$\frac{2(ax-1)\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^2}{3a^4} + \frac{x^2}{a^2} - \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcSech[a*x])*x^3,x]
```

```
[Out] x^2/a^2 - x^4/4 + (2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(3*a
^4)
```

fricas [A] time = 1.54, size = 62, normalized size = 0.53

$$-\frac{3\,a^3x^4 - 12\,ax^2 - 8\left(a^2x^3 - x\right)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}}{12\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x, algorithm="frica
s")
```

```
[Out] -1/12*(3*a^3*x^4 - 12*a*x^2 - 8*(a^2*x^3 - x)*sqrt((a*x + 1)/(a*x))*sqrt(-(
a*x - 1)/(a*x)))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,%%%{-4,[1,1]%
%%},0,%%%{4,[4,4]%%%}] at parameters values [86,-97]Warning, choosing root
of [1,0,%%%{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-82,7]Warn
ing, integration of abs or sign assumes constant sign by intervals (correct
 if the argument is real):Check [abs(t_nostep)]Warning, choosing root of [1
,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-27,26]Warning
, choosing root of [1,0,%%%{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters va
lues [-89,63]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%
%%}] at parameters values [-49,-86]Warning, choosing root of [1,0,%%%{4,[1,
1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-64,-30]Warning, choosing r
oot of [1,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [70,22
]Warning, choosing root of [1,0,%%%{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at param
eters values [42,56]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%},0,%%%{4
,[4,4]%%%}] at parameters values [-9,-13]Warning, choosing root of [1,0,%%%
{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [46,24]Warning, choosi
ng root of [1,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [4
9,-6]Unable to divide, perhaps due to rounding error%%%{-1,[3,2,0,0]%%%}+%%
%{2,[2,1,1,1]%%%}+%%%{2,[1,0,0,0]%%%} / %%%{1,[0,2,0,0]%%%} Error: Bad Argu
ment Value
```

maple [A] time = 0.05, size = 72, normalized size = 0.62

$$\frac{-\frac{1}{4}a^{2}x^{4}+\frac{1}{2}x^{2}}{a^{2}}+\frac{2\sqrt{-\frac{ax-1}{ax}}\,x\sqrt{\frac{ax+1}{ax}}\,\left(a^{2}x^{2}-1\right)}{3a^{3}}+\frac{x^{2}}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x)
```

```
[Out] 1/a^2*(-1/4*a^2*x^4+1/2*x^2)+2/3/a^3*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(
1/2)*(a^2*x^2-1)+1/2*x^2/a^2
```

maxima [A] time = 0.41, size = 42, normalized size = 0.36

$$-\frac{1}{4}\,x^{4}+\frac{x^{2}}{a^{2}}+\frac{2\left(a^{2}x^{2}-1\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{3\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x, algorithm="maxim
a")
```

```
[Out] -1/4*x^4 + x^2/a^2 + 2/3*(a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)/a^4
```

mupad [B] time = 1.75, size = 63, normalized size = 0.54

$$\frac{x^2}{a^2} - \frac{x^4}{4} - \sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,x\,\sqrt{\frac{1}{a\,x}+1}}{3\,a^3} - \frac{2\,x^3\,\sqrt{\frac{1}{a\,x}+1}}{3\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] x^2/a^2 - x^4/4 - (1/(a*x) - 1)^(1/2)*((2*x*(1/(a*x) + 1)^(1/2))/(3*a^3) -
(2*x^3*(1/(a*x) + 1)^(1/2))/(3*a))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x**3,x)
```

```
[Out] Timed out
```

3.67 $\int e^{2\text{sech}^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=169

$$\frac{(ax+1)^3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}{12a^3}-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}{6a^3}+\frac{(ax+1)\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{2a^3}-\frac{2\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right.}{a^3}$$

```
[Out] -2*arctan(((-a*x+1)/(a*x+1))^(1/2))/a^3+1/2*(a*x+1)*(1-((-a*x+1)/(a*x+1))^(
1/2))*(1+((-a*x+1)/(a*x+1))^(1/2))/a^3-1/6*(a*x+1)^2*((-a*x+1)/(a*x+1))^(1/
2)*(1+((-a*x+1)/(a*x+1))^(1/2))^3/a^3+1/12*(a*x+1)^3*(1+((-a*x+1)/(a*x+1))^
(1/2))^4/a^3
```

Rubi [A] time = 0.47, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6337, 821, 12, 729, 723, 203}

$$\frac{(ax+1)^3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}{12a^3}-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}{6a^3}+\frac{(ax+1)\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{2a^3}-\frac{2\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right.}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])*x^2,x]
```

```
[Out] ((1 + a*x)*(1 - Sqrt[(1 - a*x)/(1 + a*x)])*(1 + Sqrt[(1 - a*x)/(1 + a*x)]))
/(2*a^3) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(1 + Sqrt[(1 - a*x)/(1 +
a*x)])^3)/(6*a^3) + ((1 + a*x)^3*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^4)/(12*a^3
) - (2*ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]])/a^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 729

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^m*(2*c*x)*(a + c*x^2)^(p + 1))/(4*a*c*(p + 1)), x] - Dist[(m*(2
*c*d))/(4*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /
; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3,
 0] && LtQ[p, -1]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
```

```
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int e^{2\text{sech}^{-1}(ax)}x^2\,dx = \int x^2\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2 dx$$

$$= -\frac{4\,\text{Subst}\left(\int\frac{x(1+x)^4}{\left(1+x^2\right)^4}dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^3}$$

$$= \frac{(1+ax)^3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4}{12a^3} - \frac{2\,\text{Subst}\left(\int\frac{4(1+x)^3}{\left(1+x^2\right)^3}dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{3a^3}$$

$$= \frac{(1+ax)^3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4}{12a^3} - \frac{8\,\text{Subst}\left(\int\frac{(1+x)^3}{\left(1+x^2\right)^3}dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{3a^3}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3}{6a^3} + \frac{(1+ax)^3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4}{12a^3} - \frac{2\,\text{Subst}\left(\int\frac{(1+x)^2}{\left(1+x^2\right)^2}dx, x, \sqrt{\frac{1-a}{1+a}}\right.}{a^3}$$

$$= \frac{(1+ax)\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^3} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3}{6a^3} + \frac{(1+ax)^3\left(1+\sqrt{\frac{1-a}{1+a}}\right.}{12a^3}$$

$$= \frac{(1+ax)\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^3} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3}{6a^3} + \frac{(1+ax)^3\left(1+\sqrt{\frac{1-a}{1+a}}\right.}{12a^3}$$

Mathematica [C] time = 0.08, size = 86, normalized size = 0.51

$$\frac{i\log\left(2\sqrt{\frac{1-ax}{ax+1}}(ax+1)-2iax\right)}{a^3}+\sqrt{\frac{1-ax}{ax+1}}\left(\frac{x}{a^2}+\frac{x^2}{a}\right)+\frac{2x}{a^2}-\frac{x^3}{3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])*x^2,x]
```

```
[Out] (2*x)/a^2 - x^3/3 + Sqrt[(1 - a*x)/(1 + a*x)]*(x/a^2 + x^2/a) + (I*Log[(-2*
I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/a^3
```

fricas [A] time = 1.53, size = 87, normalized size = 0.51

$$-\frac{a^3x^3-3\,a^2x^2\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-6\,ax+3\,\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x, algorithm="frica
s")
```

```
[Out] -1/3*(a^3*x^3 - 3*a^2*x^2*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 6*
a*x + 3*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,%%%{-4,[1,1]%
%%},0,%%%{4,[4,4]%%%}] at parameters values [86,-97]Warning, choosing root
of [1,0,%%%{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-82,7]Warn
ing, integration of abs or sign assumes constant sign by intervals (correct
 if the argument is real):Check [abs(t_nostep)]Warning, choosing root of [1
,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-27,26]Warning
, choosing root of [1,0,%%%{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters va
lues [-89,63]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%
%%}] at parameters values [-49,-86]Warning, choosing root of [1,0,%%%{4,[1,
1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [-64,-30]Warning, choosing r
oot of [1,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [70,22
]Warning, choosing root of [1,0,%%%{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at param
eters values [42,56]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%},0,%%%{4
,[4,4]%%%}] at parameters values [-9,-13]Warning, choosing root of [1,0,%%%
{4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [46,24]Warning, choosi
ng root of [1,0,%%%{-4,[1,1]%%%},0,%%%{4,[4,4]%%%}] at parameters values [4
9,-6]Unable to divide, perhaps due to rounding error%%%{-1,[2,2,0,0]%%%}+%%
%{2,[1,1,1,1]%%%}+%%%{2,[0,0,0,0]%%%} / %%%{1,[0,2,0,0]%%%} Error: Bad Argu
ment Value
```

maple [C] time = 0.06, size = 97, normalized size = 0.57

$$-\frac{x^{3}}{3}+\frac{2x}{a^{2}}+\frac{\sqrt{-\frac{ax-1}{ax}}\, x\sqrt{\frac{ax+1}{ax}}\, \left(x\sqrt{-a^{2}x^{2}+1}\, \mathrm{csgn}(a)a+\arctan\left(\frac{\mathrm{csgn}(a)ax}{\sqrt{-a^{2}x^{2}+1}}\right)\right)\mathrm{csgn}(a)}{a^{2}\sqrt{-a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x)
```

```
[Out] -1/3*x^3+2*x/a^2+1/a^2*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(x*(-a^2*
x^2+1)^(1/2)*csgn(a)*a+arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1/2)))/(-a^2*x^2+1)
^(1/2)*csgn(a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,x}{a^{2}}+\frac{2\left(\frac{1}{2}\,\sqrt{-a^{2}x^{2}+1}\,x+\frac{\arcsin(ax)}{2\,a}\right)}{a^{2}}-\int x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x, algorithm="maxim
a")
```

```
[Out] 2*x/a^2 + 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1), x)/a^2 - integrate(x^2,
 x)
```

mupad [B] time = 8.96, size = 420, normalized size = 2.49

$$\frac{\frac{1i}{16\,a^3}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-i\right)^2\,1i}{8\,a^3\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-i\right)^4\,15i}{16\,a^3\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-i\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{2\,\left(\sqrt{\frac{1}{a\,x}-1}-i\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-i\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}}-\frac{x^3\,\left(\frac{a^2}{3}-\frac{2}{x^2}\right)}{a^2}+\frac{\left(\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-i\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-i}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\right)\,2i}{a^3}+\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-i}{\sqrt{\frac{1}{a\,x}+1}-1}\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] ((log(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1) - log((
(1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))*2i)/a^3 + (log(((1/(a
*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/a^3 + (1i/(16*a^3) + ((
(1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(8*a^3*((1/(a*x) + 1)^(1/2) - 1)^2) - (((1/
(a*x) - 1)^(1/2) - 1i)^4*15i)/(16*a^3*((1/(a*x) + 1)^(1/2) - 1)^4))/(((1/(a
*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + (2*((1/(a*x) - 1)^(1/2
) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a*x) - 1)^(1/2) - 1i)^6/((1/(
a*x) + 1)^(1/2) - 1)^6) - (log((a*(-(a - 1/x)/a)^(1/2)*2i - 2/x + 2*a*((a +
 1/x)/a)^(1/2))/(2*a + 1/x - 2*a*((a + 1/x)/a)^(1/2)))*1i)/a^3 + (((1/(a*x)
 - 1)^(1/2) - 1i)^2*1i)/(16*a^3*((1/(a*x) + 1)^(1/2) - 1)^2) - (x^3*(a^2/3
- 2/x^2))/a^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x**2,x)
```

```
[Out] Timed out
```

3.68 $\int e^{2\text{sech}^{-1}(ax)}x\,dx$

Optimal. Leaf size=85

$$-\frac{(ax+1)^2}{2a^2}+\frac{\left(2\sqrt{\frac{1-ax}{ax+1}}+1\right)(ax+1)}{a^2}+\frac{2\log(ax+1)}{a^2}+\frac{4\log\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}{a^2}$$

```
[Out] -1/2*(a*x+1)^2/a^2+2*ln(a*x+1)/a^2+4*ln(1-((-a*x+1)/(a*x+1))^(1/2))/a^2+(a*
x+1)*(1+2*((-a*x+1)/(a*x+1))^(1/2))/a^2
```

Rubi [A] time = 0.43, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6337, 1647, 1593, 801, 260}

$$-\frac{(ax+1)^2}{2a^2}+\frac{\left(2\sqrt{\frac{1-ax}{ax+1}}+1\right)(ax+1)}{a^2}+\frac{2\log(ax+1)}{a^2}+\frac{4\log\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])*x,x]
```

```
[Out] -(1 + a*x)^2/(2*a^2) + ((1 + a*x)*(1 + 2*Sqrt[(1 - a*x)/(1 + a*x)]))/a^2 +
(2*Log[1 + a*x])/a^2 + (4*Log[1 - Sqrt[(1 - a*x)/(1 + a*x)]])/a^2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int e^{2\text{sech}^{-1}(ax)}x\,dx=\int x\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2dx$$
$$=\frac{4\,\text{Subst}\left(\int\frac{x(1+x)^3}{(-1+x)\left(1+x^2\right)^3}dx,x,\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$
$$=-\frac{(1+ax)^2}{2a^2}-\frac{\text{Subst}\left(\int\frac{-12x-4x^2}{(-1+x)\left(1+x^2\right)^2}dx,x,\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$
$$=-\frac{(1+ax)^2}{2a^2}-\frac{\text{Subst}\left(\int\frac{(-12-4x)x}{(-1+x)\left(1+x^2\right)^2}dx,x,\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$
$$=-\frac{(1+ax)^2}{2a^2}+\frac{(1+ax)\left(1+2\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}+\frac{\text{Subst}\left(\int\frac{8+8x}{(-1+x)\left(1+x^2\right)}dx,x,\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2}$$
$$=-\frac{(1+ax)^2}{2a^2}+\frac{(1+ax)\left(1+2\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}+\frac{\text{Subst}\left(\int\left(\frac{8}{-1+x}-\frac{8x}{1+x^2}\right)dx,x,\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2}$$
$$=-\frac{(1+ax)^2}{2a^2}+\frac{(1+ax)\left(1+2\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}+\frac{4\log\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}-\frac{4\,\text{Subst}\left(\int\frac{x}{1+x^2}dx,x,\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$
$$=-\frac{(1+ax)^2}{2a^2}+\frac{(1+ax)\left(1+2\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}+\frac{2\log(1+ax)}{a^2}+\frac{4\log\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$

Mathematica [A] time = 0.06, size = 89, normalized size = 1.05

$$\frac{-a^2x^2+4\sqrt{\frac{1-ax}{ax+1}}(ax+1)-4\log\left(ax\sqrt{\frac{1-ax}{ax+1}}+\sqrt{\frac{1-ax}{ax+1}}+1\right)+8\log(x)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])*x,x]
```

```
[Out] (-(a^2*x^2) + 4*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + 8*Log[x] - 4*Log[1 +
Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(2*a^2)
```

fricas [A] time = 0.50, size = 124, normalized size = 1.46

$$-\frac{a^2x^2-4\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+2\,\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-2\,\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)-4\,\log(x)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x, algorithm="fricas"
)
```

```
[Out] -1/2*(a^2*x^2 - 4*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 2*log(
a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - 2*log(a*x*sqrt((a*x
 + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 4*log(x))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x, algorithm="giac")
```

```
[Out] integrate(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

maple [A] time = 0.06, size = 89, normalized size = 1.05

$$-\frac{x^2}{2}+\frac{2\ln(x)}{a^2}-\frac{2\sqrt{-\frac{ax-1}{ax}}\,x\sqrt{\frac{ax+1}{ax}}\left(-\sqrt{-a^2x^2+1}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)}{a\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x)
```

```
[Out] -1/2*x^2+2*ln(x)/a^2-2/a*(-(a*x-1)/a/x)^(1/2)*x*((a*x+1)/a/x)^(1/2)*(-(-a^2
*x^2+1)^(1/2)+arctanh(1/(-a^2*x^2+1)^(1/2)))/(-a^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x, algorithm="maxima"
)
```

```
[Out] integrate(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

mupad [B] time = 3.69, size = 56, normalized size = 0.66

$$\frac{2\,x\,\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}}{a}-\frac{2\,\mathrm{acosh}\left(\frac{1}{a\,x}\right)}{a^2}-\frac{x^2}{2}-\frac{2\,\ln\left(\frac{1}{x}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] (2*x*(1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2))/a - (2*acosh(1/(a*x)))/a^2 -
x^2/2 - (2*log(1/x))/a^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x,x)
```

```
[Out] Timed out
```

3.69 $\int e^{2\text{sech}^{-1}(ax)}\,dx$

Optimal. Leaf size=57

$$-\frac{4}{a\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}+\frac{4\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a}-x$$

[Out] -x+4*arctan(((-a*x+1)/(a*x+1))^(1/2))/a-4/a/(1-((-a*x+1)/(a*x+1))^(1/2))

Rubi [A] time = 0.17, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6332, 1647, 12, 801, 203}

$$-\frac{4}{a\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}+\frac{4\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a}-x$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcSech[a*x]),x]

[Out] -x - 4/(a*(1 - Sqrt[(1 - a*x)/(1 + a*x)])) + (4*ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]])/a

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6332

```
Int[E^(ArcSech[u_]*(n_.)), x_Symbol] :> Int[(1/u + Sqrt[(1 - u)/(1 + u)] +
(1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; IntegerQ[n]
```

Rubi steps

$$\int e^{2\text{sech}^{-1}(ax)}\,dx = \int \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2 dx$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{x(1+x)^2}{(-1+x)^2(1+x^2)^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

$$= -x + \frac{2\,\text{Subst}\left(\int -\frac{4x}{(-1+x)^2(1+x^2)}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

$$= -x - \frac{8\,\text{Subst}\left(\int \frac{x}{(-1+x)^2(1+x^2)}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

$$= -x - \frac{8\,\text{Subst}\left(\int \left(\frac{1}{2(-1+x)^2} - \frac{1}{2(1+x^2)}\right)dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

$$= -x - \frac{4}{a\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)} + \frac{4\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

$$= -x - \frac{4}{a\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)} + \frac{4\tan^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

Mathematica [A] time = 0.10, size = 75, normalized size = 1.32

$$-\frac{a^2x^2 + 2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1) + 2ax\tan^{-1}\left(\frac{ax}{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}\right) + 2}{a^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x]),x]
```

```
[Out] -((2 + a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + 2*a*x*ArcTan[(a*x)
/(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))])/(a^2*x))
```

fricas [A] time = 1.02, size = 85, normalized size = 1.49

$$-\frac{a^2x^2 + 2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 2\,ax\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right) + 2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*x^2 + 2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2*a*x*arct
an(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))) + 2)/(a^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1} + \frac{1}{ax}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2,x, algorithm="giac")
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

maple [C] time = 0.06, size = 98, normalized size = 1.72

$$-x-\frac{2}{a^{2}x}-\frac{2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\left(\arctan\left(\frac{\mathrm{csgn}(a)ax}{\sqrt{-a^{2}x^{2}+1}}\right)xa+\sqrt{-a^{2}x^{2}+1}\,\mathrm{csgn}(a)\right)\mathrm{csgn}(a)}{a\sqrt{-a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2,x)
[Out] -x-2/a^2/x-2/a*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*(arctan(csgn(a)*a*x
/(-a^2*x^2+1)^(1/2))*x*a+(-a^2*x^2+1)^(1/2)*csgn(a))*csgn(a)/(-a^2*x^2+1)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-x+\frac{2\left(-a\arcsin\left(ax\right)-\frac{\sqrt{-a^{2}x^{2}+1}}{x}\right)}{a^{2}}+\frac{-\frac{1}{x}}{a^{2}}-\frac{1}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2,x, algorithm="maxima")
[Out] -x + 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^2, x)/a^2 + integrate(x^(-2
), x)/a^2 - 1/(a^2*x)
```

mupad [B] time = 4.64, size = 162, normalized size = 2.84

$$-x-\frac{\left(\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{2}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{2}}+1\right)-\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\right)2\mathrm{i}}{a}-\frac{2}{a^{2}\,x}+\frac{\left(1+\sqrt{-\frac{a-\frac{1}{x}}{a}}\,1\mathrm{i}\right)^{2}\left(\sqrt{\frac{a+\frac{1}{x}}{a}}-1\right)^{2}4\mathrm{i}}{a\left(\sqrt{\frac{a+\frac{1}{x}}{a}}\,1\mathrm{i}+\sqrt{-\frac{a-\frac{1}{x}}{a}}-2\mathrm{i}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
[Out] (((-(a - 1/x)/a)^(1/2)*1i + 1)^2*(((a + 1/x)/a)^(1/2) - 1)^2*4i)/(a*(((a +
1/x)/a)^(1/2)*1i + (-(a - 1/x)/a)^(1/2) - 2i)^2) - ((log((((1/(a*x) - 1)^(1/
2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1) - log(((1/(a*x) - 1)^(1/2) - 1i
)/((1/(a*x) + 1)^(1/2) - 1)))*2i)/a - 2/(a^2*x) - x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\left(-a^{2}\right)dx+\int\frac{2}{x^{2}}\,dx+\int\frac{2a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2,x)
[Out] (Integral(-a**2, x) + Integral(2/x**2, x) + Integral(2*a*sqrt(-1 + 1/(a*x))
*sqrt(1 + 1/(a*x))/x, x))/a**2
```

3.70 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=86

$$\frac{2}{1-\sqrt{\frac{1-ax}{ax+1}}}-\frac{2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\log(ax+1)-2\log\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)$$

```
[Out] -ln(a*x+1)-2*ln(1-((-a*x+1)/(a*x+1))^(1/2))-2/(1-((-a*x+1)/(a*x+1))^(1/2))^
2+2/(1-((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A] time = 0.45, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6337, 1629, 260}

$$\frac{2}{1-\sqrt{\frac{1-ax}{ax+1}}}-\frac{2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\log(ax+1)-2\log\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])/x,x]
```

```
[Out] -2/(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2 + 2/(1 - Sqrt[(1 - a*x)/(1 + a*x)]) -
Log[1 + a*x] - 2*Log[1 - Sqrt[(1 - a*x)/(1 + a*x)]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x}\,dx$$

$$= 4\,\text{Subst}\left(\int \frac{x(1+x)}{(-1+x)^3\left(1+x^2\right)}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$

$$= 4\,\text{Subst}\left(\int \left(\frac{1}{(-1+x)^3}+\frac{1}{2(-1+x)^2}-\frac{1}{2(-1+x)}+\frac{x}{2\left(1+x^2\right)}\right)dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$

$$= -\frac{2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{2}{1-\sqrt{\frac{1-ax}{1+ax}}}-2\log\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)+2\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$

$$= -\frac{2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{2}{1-\sqrt{\frac{1-ax}{1+ax}}}-\log(1+ax)-2\log\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)$$

Mathematica [A] time = 0.06, size = 86, normalized size = 1.00

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{a^2x^2}-\frac{1}{a^2x^2}+\log\left(ax\sqrt{\frac{1-ax}{ax+1}}+\sqrt{\frac{1-ax}{ax+1}}+1\right)-2\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])/x,x]
```

```
[Out] -(1/(a^2*x^2)) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a^2*x^2) - 2*Log[x]
 + Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]]
```

fricas [A] time = 0.85, size = 138, normalized size = 1.60

$$\frac{a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)-2\,a^2x^2\log(x)-2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-2}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x,x, algorithm="fricas"
)
```

```
[Out] 1/2*(a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a^
2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 2*a^2*x^2
*log(x) - 2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2)/(a^2*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x, x)
```

maple [A] time = 0.06, size = 97, normalized size = 1.13

$$-\ln(x)-\frac{1}{x^2a^2}-\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\left(-a^2x^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\sqrt{-a^2x^2+1}\right)}{ax\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x,x)
```

```
[Out] -ln(x)-1/x^2/a^2-1/a*(-(a*x-1)/a/x)^(1/2)/x*((a*x+1)/a/x)^(1/2)*(-a^2*x^2*a
rctanh(1/(-a^2*x^2+1)^(1/2))+(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(\frac{1}{2}\,a^2\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{1}{2}\,\sqrt{-a^2x^2+1}\,a^2-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{2\,x^2}\right)}{a^2}-\frac{1}{a^2x^2}-\int\frac{1}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x,x, algorithm="maxima"
)
```

```
[Out] 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^3, x)/a^2 - 1/(a^2*x^2) - integr
ate(1/x, x)
```

mupad [B] time = 11.21, size = 323, normalized size = 3.76

$$\ln\left(\frac{1}{x}\right)-4\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)+2\,\mathrm{acosh}\left(\frac{1}{a\,x}\right)+\frac{\frac{28\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{28\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x}+1}-1}}{1+\frac{6\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x,x)
```

```
[Out] log(1/x) - 4*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)) +
2*acosh(1/(a*x)) + ((28*((1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2)
- 1)^3 + (28*((1/(a*x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 + (4
*((1/(a*x) - 1)^(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (4*((1/(a*x) -
 1)^(1/2) - 1i))/((1/(a*x) + 1)^(1/2) - 1))/((6*((1/(a*x) - 1)^(1/2) - 1i)^
4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (4*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x)
 + 1)^(1/2) - 1)^2 - (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2)
- 1)^6 + ((1/(a*x) - 1)^(1/2) - 1i)^8/((1/(a*x) + 1)^(1/2) - 1)^8 + 1) - 1/
(a^2*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{2}{x^3}\,dx+\int\left(-\frac{a^2}{x}\right)dx+\int\frac{2a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^2}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x,x)
```

```
[Out] (Integral(2/x**3, x) + Integral(-a**2/x, x) + Integral(2*a*sqrt(-1 + 1/(a*x
))*sqrt(1 + 1/(a*x))/x**2, x))/a**2
```

3.71 $\int \frac{e^{2\mathbf{sech}^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=57

$$\frac{2a}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{4a}{3\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}$$

```
[Out] -4/3*a/(1-((-a*x+1)/(a*x+1))^(1/2))^3+2*a/(1-((-a*x+1)/(a*x+1))^(1/2))^2
```

Rubi [A] time = 0.39, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6337, 43}

$$\frac{2a}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{4a}{3\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])/x^2,x]
```

```
[Out] (-4*a)/(3*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^3) + (2*a)/(1 - Sqrt[(1 - a*x)/(1
 + a*x)])^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^2}\,dx &= \int \frac{\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^2}\,dx\\
&= -\left((4a)\operatorname{Subst}\left(\int \frac{x}{(-1+x)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)\\
&= -\left((4a)\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^4}+\frac{1}{(-1+x)^3}\right)dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)\\
&= -\frac{4a}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{2a}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 52, normalized size = 0.91

$$\frac{3a^2x^2+2(ax-1)\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2-2}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcSech[a*x])/x^2,x]
```

```
[Out] (-2 + 3*a^2*x^2 + 2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(3*a^
2*x^3)
```

fricas [A] time = 1.06, size = 61, normalized size = 1.07

$$\frac{3\,a^2x^2 + 2\left(a^3x^3 - ax\right)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}} - 2}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x, algorithm="frica
s")
```

```
[Out] 1/3*(3*a^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a
*x)) - 2)/(a^2*x^3)
```

giac [B] time = 0.21, size = 122, normalized size = 2.14

$$\frac{3\left(a^2 + \frac{a}{x}\right)a^2 - \left(9\,a^2 + \left(a^2 + \frac{a}{x}\right)\left(\frac{2\left(a^2+\frac{a}{x}\right)}{a^2} - 7\right)\right)\sqrt{a^2 + \frac{a}{x}}\,\sqrt{-a^2 + \frac{a}{x}} + 3\left(2\,a^2 - \frac{a}{x}\right)\sqrt{a^2 + \frac{a}{x}}\,\sqrt{-a^2 + \frac{a}{x}} - \frac{2\,a}{x^3}}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x, algorithm="giac"
)
```

```
[Out] 1/3*(3*(a^2 + a/x)*a^2 - (9*a^2 + (a^2 + a/x)*(2*(a^2 + a/x)/a^2 - 7))*sqrt
(a^2 + a/x)*sqrt(-a^2 + a/x) + 3*(2*a^2 - a/x)*sqrt(a^2 + a/x)*sqrt(-a^2 +
a/x) - 2*a/x^3)/a^3
```

maple [A] time = 0.06, size = 73, normalized size = 1.28

$$\frac{\frac{a^2}{x} - \frac{1}{3x^3}}{a^2} + \frac{2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\left(a^2x^2 - 1\right)}{3a\,x^2} - \frac{1}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x)
```

```
[Out] 1/a^2*(a^2/x-1/3/x^3)+2/3/a*(-(a*x-1)/a/x)^(1/2)/x^2*((a*x+1)/a/x)^(1/2)*(a
^2*x^2-1)-1/3/a^2/x^3
```

maxima [A] time = 0.35, size = 46, normalized size = 0.81

$$\frac{1}{x} + \frac{2\left(a^2x^3 - x\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{3\,a^2x^4} - \frac{2}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x, algorithm="maxim
a")
```

```
[Out] 1/x + 2/3*(a^2*x^3 - x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^2*x^4) - 2/3/(a^2*x
^3)
```

mupad [B] time = 1.80, size = 67, normalized size = 1.18

$$\frac{a^2\,x^2 - \frac{2}{3}}{a^2\,x^3} - \frac{\sqrt{\frac{1}{a\,x} - 1}\,\left(\frac{2\,\sqrt{\frac{1}{a\,x}+1}}{3\,a} - \frac{2\,a\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{3}\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^2,x)
```

```
[Out] (a^2*x^2 - 2/3)/(a^2*x^3) - ((1/(a*x) - 1)^(1/2)*((2*(1/(a*x) + 1)^(1/2))/(
3*a) - (2*a*x^2*(1/(a*x) + 1)^(1/2))/3))/x^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2}{x^4}\,dx + \int \left(-\frac{a^2}{x^2}\right)dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^3}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**2,x)
```

```
[Out] (Integral(2/x**4, x) + Integral(-a**2/x**2, x) + Integral(2*a*sqrt(-1 + 1/(
a*x))*sqrt(1 + 1/(a*x))/x**3, x))/a**2
```

3.72 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=147

$$\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{3a^2}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{2a^2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{a^2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)$$

```
[Out] 1/2*a^2*arctanh(((-a*x+1)/(a*x+1))^(1/2))-a^2/(1-((-a*x+1)/(a*x+1))^(1/2))^
4+2*a^2/(1-((-a*x+1)/(a*x+1))^(1/2))^3-3/2*a^2/(1-((-a*x+1)/(a*x+1))^(1/2))
^2+1/2*a^2/(1-((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A] time = 0.45, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6337, 1612, 207}

$$\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{3a^2}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{2a^2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{a^2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])/x^3,x]
```

```
[Out] -(a^2/(1 - Sqrt[(1 - a*x)/(1 + a*x)])^4) + (2*a^2)/(1 - Sqrt[(1 - a*x)/(1 +
 a*x)])^3 - (3*a^2)/(2*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) + a^2/(2*(1 - Sqr
t[(1 - a*x)/(1 + a*x)])) + (a^2*ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]])/2
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^3}\,dx$$
$$= \left(4a^2\right)\text{Subst}\left(\int \frac{x\left(1+x^2\right)}{(-1+x)^5(1+x)}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$
$$= \left(4a^2\right)\text{Subst}\left(\int \left(\frac{1}{(-1+x)^5}+\frac{3}{2(-1+x)^4}+\frac{3}{4(-1+x)^3}+\frac{1}{8(-1+x)^2}-\frac{1}{8\left(-1+x^2\right)}\right)dx, x, \sqrt{\right.$$
$$= -\frac{a^2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{2a^2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{3a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{-1+x\right.$$
$$= -\frac{a^2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{2a^2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{3a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right.$$

Mathematica [A] time = 0.13, size = 121, normalized size = 0.82

$$\frac{a^4(-\log(x))+a^4\log\left(ax\sqrt{\frac{1-ax}{ax+1}}+\sqrt{\frac{1-ax}{ax+1}}+1\right)+\frac{(ax+1)\left(a^2x^2\sqrt{\frac{1-ax}{ax+1}}+2ax-2\sqrt{\frac{1-ax}{ax+1}}-2\right)}{x^4}}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])/x^3,x]
```

```
[Out] (((1 + a*x)*(-2 + 2*a*x - 2*Sqrt[(1 - a*x)/(1 + a*x)] + a^2*x^2*Sqrt[(1 - a
*x)/(1 + a*x)]))/x^4 - a^4*Log[x] + a^4*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] +
 a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(4*a^2)
```

fricas [A] time = 1.02, size = 146, normalized size = 0.99

$$\frac{a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+4\,a^2x^2+2\left(a^3x^3-2\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-4}{8\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^3,x, algorithm="frica
s")
```

```
[Out] 1/8*(a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a^
4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 4*a^2*x^2
 + 2*(a^3*x^3 - 2*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 4)/(a
^2*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^3, x)
```

maple [A] time = 0.06, size = 131, normalized size = 0.89

$$\frac{\frac{a^2}{2x^2}-\frac{1}{4x^4}}{a^2}+\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)x^4a^4+a^2x^2\sqrt{-a^2x^2+1}-2\sqrt{-a^2x^2+1}\right)}{4a\,x^3\sqrt{-a^2x^2+1}}-\frac{1}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^3,x)
```

```
[Out] 1/a^2*(1/2*a^2/x^2-1/4/x^4)+1/4/a*(-(a*x-1)/a/x)^(1/2)/x^3*((a*x+1)/a/x)^(1
/2)*(arctanh(1/(-a^2*x^2+1)^(1/2))*x^4*a^4+a^2*x^2*(-a^2*x^2+1)^(1/2)-2*(-a
^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/4/a^2/x^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(\frac{1}{8}\,a^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{1}{8}\sqrt{-a^2x^2+1}\,a^4-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2}{8\,x^2}-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{4\,x^4}\right)}{a^2}-\frac{1}{2\,a^2x^4}-\int\frac{1}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^3,x, algorithm="maxim
a")
```

```
[Out] 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^5, x)/a^2 - 1/2/(a^2*x^4) - inte
grate(x^(-3), x)
```

mupad [B] time = 46.99, size = 885, normalized size = 6.02

$$a^2\operatorname{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)-\frac{\frac{28\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{28\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{4\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{4\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x}+1}-1}}{1+\frac{6\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}}-\frac{\frac{23\,a^2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}\cdots}{1+\frac{28(\cdots}{(\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^3,x)
```

```
[Out] a^2*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)) - ((28*a^2*
((1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) - 1)^3 + (28*a^2*((1/(a*
x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 + (4*a^2*((1/(a*x) - 1)^
(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (4*a^2*((1/(a*x) - 1)^(1/2) -
1i))/((1/(a*x) + 1)^(1/2) - 1))/((6*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x)
 + 1)^(1/2) - 1)^4 - (4*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2)
- 1)^2 - (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + ((1
/(a*x) - 1)^(1/2) - 1i)^8/((1/(a*x) + 1)^(1/2) - 1)^8 + 1) - ((23*a^2*((1/(
a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) - 1)^3 + (333*a^2*((1/(a*x) -
 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 + (671*a^2*((1/(a*x) - 1)^(1
/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (671*a^2*((1/(a*x) - 1)^(1/2) -
1i)^9)/((1/(a*x) + 1)^(1/2) - 1)^9 + (333*a^2*((1/(a*x) - 1)^(1/2) - 1i)^11
)/((1/(a*x) + 1)^(1/2) - 1)^11 + (23*a^2*((1/(a*x) - 1)^(1/2) - 1i)^13)/((1
/(a*x) + 1)^(1/2) - 1)^13 - (3*a^2*((1/(a*x) - 1)^(1/2) - 1i)^15)/((1/(a*x)
 + 1)^(1/2) - 1)^15 - (3*a^2*((1/(a*x) - 1)^(1/2) - 1i))/((1/(a*x) + 1)^(1/
2) - 1))/((28*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (
8*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (56*((1/(a*x)
```

```
 - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (70*((1/(a*x) - 1)^(1/2)
 - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (56*((1/(a*x) - 1)^(1/2) - 1i)^10)/
((1/(a*x) + 1)^(1/2) - 1)^10 + (28*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x)
 + 1)^(1/2) - 1)^12 - (8*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2
) - 1)^14 + ((1/(a*x) - 1)^(1/2) - 1i)^16/((1/(a*x) + 1)^(1/2) - 1)^16 + 1)
 + 1/(2*x^2) - 1/(2*a^2*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2}{x^5}\,dx + \int \left(-\frac{a^2}{x^3}\right)dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^4}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**3,x)
```

```
[Out] (Integral(2/x**5, x) + Integral(-a**2/x**3, x) + Integral(2*a*sqrt(-1 + 1/(
a*x))*sqrt(1 + 1/(a*x))/x**4, x))/a**2
```

3.73 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=183

$$-\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{a^3}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{3a^3}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{7a^3}{3\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}+\frac{2a^3}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}-\frac{4a^3}{5\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^5}$$

```
[Out] -4/5*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))^5+2*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))^
4-7/3*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))^3+3/2*a^3/(1-((-a*x+1)/(a*x+1))^(1/2
))^2-1/4*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))-1/4*a^3/(1+((-a*x+1)/(a*x+1))^(1/
2))
```

Rubi [A] time = 0.50, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6337, 1612}

$$-\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{a^3}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{3a^3}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{7a^3}{3\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}+\frac{2a^3}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}-\frac{4a^3}{5\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])/x^4,x]
```

```
[Out] (-4*a^3)/(5*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^5) + (2*a^3)/(1 - Sqrt[(1 - a*x
)/(1 + a*x)])^4 - (7*a^3)/(3*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^3) + (3*a^3)/(
2*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) - a^3/(4*(1 - Sqrt[(1 - a*x)/(1 + a*x)
])) - a^3/(4*(1 + Sqrt[(1 - a*x)/(1 + a*x)]))
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^4}\,dx &= \int \frac{\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^4}\,dx\\
&= -\left((4a)\,\text{Subst}\left(\int \frac{x\left(a+ax^2\right)^2}{(-1+x)^6(1+x)^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)\\
&= -\left((4a)\,\text{Subst}\left(\int \left(\frac{a^2}{(-1+x)^6}+\frac{2a^2}{(-1+x)^5}+\frac{7a^2}{4(-1+x)^4}+\frac{3a^2}{4(-1+x)^3}+\frac{a^2}{16(-1+x)^2}-\frac{}{16(}\right.\right.\right.\\
&= -\frac{4a^3}{5\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{2a^3}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}-\frac{7a^3}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{3a^3}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}-\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 69, normalized size = 0.38

$$\frac{5a^2x^2 + 2\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2\left(2a^3x^3 - 2a^2x^2 + 3ax - 3\right) - 6}{15a^2x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])/x^4,x]
```

```
[Out] (-6 + 5*a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-3 + 3*a*x - 2*a
^2*x^2 + 2*a^3*x^3))/(15*a^2*x^5)
```

fricas [A] time = 0.74, size = 69, normalized size = 0.38

$$\frac{5\,a^2x^2 + 2\left(2\,a^5x^5 + a^3x^3 - 3\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 6}{15\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^4,x, algorithm="frica
s")
```

```
[Out] 1/15*(5*a^2*x^2 + 2*(2*a^5*x^5 + a^3*x^3 - 3*a*x)*sqrt((a*x + 1)/(a*x))*sqr
t(-(a*x - 1)/(a*x)) - 6)/(a^2*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^4,x, algorithm="giac"
)
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^4, x)
```

maple [A] time = 0.06, size = 84, normalized size = 0.46

$$\frac{\frac{a^2}{3x^3} - \frac{1}{5x^5}}{a^2} + \frac{2\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\left(a^2x^2-1\right)\left(2a^2x^2+3\right)}{15a\,x^4} - \frac{1}{5a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^4,x)
```

```
[Out] 1/a^2*(1/3*a^2/x^3-1/5/x^5)+2/15/a*(-(a*x-1)/a/x)^(1/2)/x^4*((a*x+1)/a/x)^(
1/2)*(a^2*x^2-1)*(2*a^2*x^2+3)-1/5/a^2/x^5
```

maxima [A] time = 0.40, size = 56, normalized size = 0.31

$$\frac{1}{3\,x^3} + \frac{2\left(2\,a^4x^5 + a^2x^3 - 3\,x\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{15\,a^2x^6} - \frac{2}{5\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^4,x, algorithm="maxim
a")
```

```
[Out] 1/3/x^3 + 2/15*(2*a^4*x^5 + a^2*x^3 - 3*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^
2*x^6) - 2/5/(a^2*x^5)
```

mupad [B] time = 1.94, size = 86, normalized size = 0.47

$$\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,a\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{15}-\frac{2\,\sqrt{\frac{1}{a\,x}+1}}{5\,a}+\frac{4\,a^3\,x^4\,\sqrt{\frac{1}{a\,x}+1}}{15}\right)}{x^4}+\frac{\frac{a^2\,x^2}{3}-\frac{2}{5}}{a^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^4,x)
```

```
[Out] ((1/(a*x) - 1)^(1/2)*((2*a*x^2*(1/(a*x) + 1)^(1/2))/15 - (2*(1/(a*x) + 1)^(
1/2))/(5*a) + (4*a^3*x^4*(1/(a*x) + 1)^(1/2))/15))/x^4 + ((a^2*x^2)/3 - 2/5
)/(a^2*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2}{x^6}\,dx+\int \left(-\frac{a^2}{x^4}\right)dx+\int \frac{2a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^5}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**4,x)
```

```
[Out] (Integral(2/x**6, x) + Integral(-a**2/x**4, x) + Integral(2*a*sqrt(-1 + 1/(
a*x))*sqrt(1 + 1/(a*x))/x**5, x))/a**2
```

3.74 $\int \frac{e^{2\mathbf{sech}^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=267

$$\frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}+\frac{a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{11a^4}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{8a^4}{3\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{3a^4}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}+\frac{2a^4}{\left(1-\sqrt{\frac{1-a}{ax+}}\right.}$$

```
[Out] 1/4*a^4*arctanh(((-a*x+1)/(a*x+1))^(1/2))-2/3*a^4/(1-((-a*x+1)/(a*x+1))^(1/
2))^6+2*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))^5-3*a^4/(1-((-a*x+1)/(a*x+1))^(1/2
))^4+8/3*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))^3-11/8*a^4/(1-((-a*x+1)/(a*x+1))^
(1/2))^2+3/8*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))-1/8*a^4/(1+((-a*x+1)/(a*x+1))
^(1/2))^2+1/8*a^4/(1+((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A] time = 0.54, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6337, 1612, 207}

$$\frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}+\frac{a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{11a^4}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{8a^4}{3\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{3a^4}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}+\frac{2a^4}{\left(1-\sqrt{\frac{1-a}{ax+}}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])/x^5,x]
```

```
[Out] (-2*a^4)/(3*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^6) + (2*a^4)/(1 - Sqrt[(1 - a*x
)/(1 + a*x)])^5 - (3*a^4)/(1 - Sqrt[(1 - a*x)/(1 + a*x)])^4 + (8*a^4)/(3*(1
 - Sqrt[(1 - a*x)/(1 + a*x)])^3) - (11*a^4)/(8*(1 - Sqrt[(1 - a*x)/(1 + a*x
)])^2) + (3*a^4)/(8*(1 - Sqrt[(1 - a*x)/(1 + a*x)])) - a^4/(8*(1 + Sqrt[(1
- a*x)/(1 + a*x)])^2) + a^4/(8*(1 + Sqrt[(1 - a*x)/(1 + a*x)])) + (a^4*ArcT
anh[Sqrt[(1 - a*x)/(1 + a*x)]])/4
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^5}\,dx$$

$$= (4a)\operatorname{Subst}\left(\int \frac{x\left(a+ax^2\right)^3}{(-1+x)^7(1+x)^3}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$

$$= (4a)\operatorname{Subst}\left(\int\left(\frac{a^3}{(-1+x)^7}+\frac{5a^3}{2(-1+x)^6}+\frac{3a^3}{(-1+x)^5}+\frac{2a^3}{(-1+x)^4}+\frac{11a^3}{16(-1+x)^3}+\frac{3a^3}{32(-1+\right.\right.$$

$$= -\frac{2a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^6}+\frac{2a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}-\frac{3a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{8a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{11a^4}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}$$

$$= -\frac{2a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^6}+\frac{2a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}-\frac{3a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{8a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{11a^4}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}$$

Mathematica [A] time = 0.15, size = 137, normalized size = 0.51

$$\frac{-3a^6x^6\log(x)+3a^6x^6\log\left(ax\sqrt{\frac{1-ax}{ax+1}}+\sqrt{\frac{1-ax}{ax+1}}+1\right)+6a^2x^2+\sqrt{\frac{1-ax}{ax+1}}\left(3a^5x^5+3a^4x^4+2a^3x^3+2a^2x^2-8ax\right.}{24a^2x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])/x^5,x]
```

```
[Out] (-8 + 6*a^2*x^2 + Sqrt[(1 - a*x)/(1 + a*x)]*(-8 - 8*a*x + 2*a^2*x^2 + 2*a^3
*x^3 + 3*a^4*x^4 + 3*a^5*x^5) - 3*a^6*x^6*Log[x] + 3*a^6*x^6*Log[1 + Sqrt[(
1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(24*a^2*x^6)
```

fricas [A] time = 0.55, size = 156, normalized size = 0.58

$$\frac{3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+12\,a^2x^2+2\left(3\,a^5x^5+2\,a^3x^3-8\,ax\right)}{48\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x, algorithm="frica
s")
```

```
[Out] 1/48*(3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) -
 3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 12*a
^2*x^2 + 2*(3*a^5*x^5 + 2*a^3*x^3 - 8*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x
 - 1)/(a*x)) - 16)/(a^2*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x, algorithm="giac"
)
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^5, x)
```

maple [A] time = 0.09, size = 153, normalized size = 0.57

$$\frac{-\frac{1}{6x^6}+\frac{a^2}{4x^4}}{a^2}+\frac{\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\left(3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)x^6a^6+3\sqrt{-a^2x^2+1}\,x^4a^4+2a^2x^2\sqrt{-a^2x^2+1}-8\sqrt{-a^2x^2+1}\right.}{24a\,x^5\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x)
```

```
[Out] 1/a^2*(-1/6/x^6+1/4*a^2/x^4)+1/24/a*(-(a*x-1)/a/x)^(1/2)/x^5*((a*x+1)/a/x)^
(1/2)*(3*arctanh(1/(-a^2*x^2+1)^(1/2))*x^6*a^6+3*(-a^2*x^2+1)^(1/2)*x^4*a^4
+2*a^2*x^2*(-a^2*x^2+1)^(1/2)-8*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/6/
a^2/x^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(\frac{1}{16}\,a^6\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{1}{16}\sqrt{-a^2x^2+1}\,a^6-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}a^4}{16\,x^2}-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2}{8\,x^4}-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{6\,x^6}\right)}{a^2}-\frac{1}{3\,a^2x^6}-\int\frac{1}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x, algorithm="maxim
a")
```

```
[Out] 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^7, x)/a^2 - 1/3/(a^2*x^6) - inte
grate(x^(-5), x)
```

mupad [B] time = 65.19, size = 2480, normalized size = 9.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^5,x)
```

```
[Out] ((311*a^4*((1/(a*x) - 1)^(1/2) - 1i)^5)/(2*((1/(a*x) + 1)^(1/2) - 1)^5) - (
175*a^4*((1/(a*x) - 1)^(1/2) - 1i)^3)/(6*((1/(a*x) + 1)^(1/2) - 1)^3) + (83
61*a^4*((1/(a*x) - 1)^(1/2) - 1i)^7)/(2*((1/(a*x) + 1)^(1/2) - 1)^7) + (422
59*a^4*((1/(a*x) - 1)^(1/2) - 1i)^9)/(3*((1/(a*x) + 1)^(1/2) - 1)^9) + (252
95*a^4*((1/(a*x) - 1)^(1/2) - 1i)^11)/((1/(a*x) + 1)^(1/2) - 1)^11 + (25295
*a^4*((1/(a*x) - 1)^(1/2) - 1i)^13)/((1/(a*x) + 1)^(1/2) - 1)^13 + (42259*a
^4*((1/(a*x) - 1)^(1/2) - 1i)^15)/(3*((1/(a*x) + 1)^(1/2) - 1)^15) + (8361*
a^4*((1/(a*x) - 1)^(1/2) - 1i)^17)/(2*((1/(a*x) + 1)^(1/2) - 1)^17) + (311*
a^4*((1/(a*x) - 1)^(1/2) - 1i)^19)/(2*((1/(a*x) + 1)^(1/2) - 1)^19) - (175*
a^4*((1/(a*x) - 1)^(1/2) - 1i)^21)/(6*((1/(a*x) + 1)^(1/2) - 1)^21) + (5*a^
4*((1/(a*x) - 1)^(1/2) - 1i)^23)/(2*((1/(a*x) + 1)^(1/2) - 1)^23) + (5*a^4*
((1/(a*x) - 1)^(1/2) - 1i))/(2*((1/(a*x) + 1)^(1/2) - 1)))/((66*((1/(a*x) -
 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (12*((1/(a*x) - 1)^(1/2) -
 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (220*((1/(a*x) - 1)^(1/2) - 1i)^6)/((
1/(a*x) + 1)^(1/2) - 1)^6 + (495*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) +
1)^(1/2) - 1)^8 - (792*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2)
- 1)^10 + (924*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12
- (792*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + (495*(
(1/(a*x) - 1)^(1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) - 1)^16 - (220*((1/(a*x)
 - 1)^(1/2) - 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 + (66*((1/(a*x) - 1)^(1/
2) - 1i)^20)/((1/(a*x) + 1)^(1/2) - 1)^20 - (12*((1/(a*x) - 1)^(1/2) - 1i)^
22)/((1/(a*x) + 1)^(1/2) - 1)^22 + ((1/(a*x) - 1)^(1/2) - 1i)^24/((1/(a*x)
```

```
+ 1)^(1/2) - 1)^24 + 1) + (a^4*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) +
 1)^(1/2) - 1)))/2 - ((a^4*((1/(a*x) - 1)^(1/2) - 1i)^6*4096i)/(3*((1/(a*x)
 + 1)^(1/2) - 1)^6) + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^8*8192i)/(3*((1/(a*x)
 + 1)^(1/2) - 1)^8) + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^10*24576i)/(5*((1/(a*
x) + 1)^(1/2) - 1)^10) + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^12*8192i)/(3*((1/(
a*x) + 1)^(1/2) - 1)^12) + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^14*4096i)/(3*((1
/(a*x) + 1)^(1/2) - 1)^14))/((45*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) +
1)^(1/2) - 1)^4 - (10*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) -
1)^2 - (120*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (21
0*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (252*((1/(a*x
) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + (210*((1/(a*x) - 1)^(
1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (120*((1/(a*x) - 1)^(1/2) - 1
i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + (45*((1/(a*x) - 1)^(1/2) - 1i)^16)/((
1/(a*x) + 1)^(1/2) - 1)^16 - (10*((1/(a*x) - 1)^(1/2) - 1i)^18)/((1/(a*x) +
 1)^(1/2) - 1)^18 + ((1/(a*x) - 1)^(1/2) - 1i)^20/((1/(a*x) + 1)^(1/2) - 1)
^20 + 1) + ((a^4*((1/(a*x) - 1)^(1/2) - 1i)^6*20480i)/((1/(a*x) + 1)^(1/2)
- 1)^6 + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^8*40960i)/((1/(a*x) + 1)^(1/2) - 1
)^8 + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^10*73728i)/((1/(a*x) + 1)^(1/2) - 1)^
10 + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^12*40960i)/((1/(a*x) + 1)^(1/2) - 1)^1
2 + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^14*20480i)/((1/(a*x) + 1)^(1/2) - 1)^14
)/(15*((45*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (10*
((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (120*((1/(a*x)
- 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (210*((1/(a*x) - 1)^(1/2)
 - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (252*((1/(a*x) - 1)^(1/2) - 1i)^10)
/((1/(a*x) + 1)^(1/2) - 1)^10 + (210*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*
x) + 1)^(1/2) - 1)^12 - (120*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^
(1/2) - 1)^14 + (45*((1/(a*x) - 1)^(1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) - 1
)^16 - (10*((1/(a*x) - 1)^(1/2) - 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 + ((
1/(a*x) - 1)^(1/2) - 1i)^20/((1/(a*x) + 1)^(1/2) - 1)^20 + 1)) + ((23*a^4*(
(1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) - 1)^3 + (333*a^4*((1/(a*
x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 + (671*a^4*((1/(a*x) - 1
)^(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (671*a^4*((1/(a*x) - 1)^(1/2
) - 1i)^9)/((1/(a*x) + 1)^(1/2) - 1)^9 + (333*a^4*((1/(a*x) - 1)^(1/2) - 1i
)^11)/((1/(a*x) + 1)^(1/2) - 1)^11 + (23*a^4*((1/(a*x) - 1)^(1/2) - 1i)^13)
/((1/(a*x) + 1)^(1/2) - 1)^13 - (3*a^4*((1/(a*x) - 1)^(1/2) - 1i)^15)/((1/(
a*x) + 1)^(1/2) - 1)^15 - (3*a^4*((1/(a*x) - 1)^(1/2) - 1i))/((1/(a*x) + 1)
^(1/2) - 1))/((28*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4
 - (8*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (56*((1/(
a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (70*((1/(a*x) - 1)^(
1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (56*((1/(a*x) - 1)^(1/2) - 1i)^
10)/((1/(a*x) + 1)^(1/2) - 1)^10 + (28*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(
a*x) + 1)^(1/2) - 1)^12 - (8*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^
(1/2) - 1)^14 + ((1/(a*x) - 1)^(1/2) - 1i)^16/((1/(a*x) + 1)^(1/2) - 1)^16
+ 1) + 1/(4*x^4) - 1/(3*a^2*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2}{x^7}\,dx + \int \left(-\frac{a^2}{x^5}\right)dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^6}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**5,x)
```

```
[Out] (Integral(2/x**7, x) + Integral(-a**2/x**5, x) + Integral(2*a*sqrt(-1 + 1/(
a*x))*sqrt(1 + 1/(a*x))/x**6, x))/a**2
```

3.75 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^6}\,dx$

Optimal. Leaf size=301

$$-\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{a^5}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{11a^5}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{a^5}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{35a^5}{12\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{a^5}{12\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}+\frac{}{\left(1-\right.}$$

```
[Out] -4/7*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^7+2*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^
6-18/5*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^5+4*a^5/(1-((-a*x+1)/(a*x+1))^(1/2)
)^4-35/12*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^3+11/8*a^5/(1-((-a*x+1)/(a*x+1))
^(1/2))^2-1/4*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))-1/12*a^5/(1+((-a*x+1)/(a*x+1
))^(1/2))^3+1/8*a^5/(1+((-a*x+1)/(a*x+1))^(1/2))^2-1/4*a^5/(1+((-a*x+1)/(a*
x+1))^(1/2))
```

Rubi [A] time = 0.58, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6337, 1612}

$$-\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{a^5}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{11a^5}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{a^5}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{35a^5}{12\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{a^5}{12\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}+\frac{}{\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])/x^6,x]
```

```
[Out] (-4*a^5)/(7*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^7) + (2*a^5)/(1 - Sqrt[(1 - a*x
)/(1 + a*x)])^6 - (18*a^5)/(5*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^5) + (4*a^5)/
(1 - Sqrt[(1 - a*x)/(1 + a*x)])^4 - (35*a^5)/(12*(1 - Sqrt[(1 - a*x)/(1 + a
*x)])^3) + (11*a^5)/(8*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) - a^5/(4*(1 - Sqr
t[(1 - a*x)/(1 + a*x)])) - a^5/(12*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^3) + a^5
/(8*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^2) - a^5/(4*(1 + Sqrt[(1 - a*x)/(1 + a*
x)]))
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^6}\,dx = \int \frac{\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^6}\,dx$$
$$= -\left((4a)\operatorname{Subst}\left(\int \frac{x\left(a+ax^2\right)^4}{(-1+x)^8(1+x)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$
$$= -\left((4a)\operatorname{Subst}\left(\int \left(\frac{a^4}{(-1+x)^8}+\frac{3a^4}{(-1+x)^7}+\frac{9a^4}{2(-1+x)^6}+\frac{4a^4}{(-1+x)^5}+\frac{35a^4}{16(-1+x)^4}+\frac{1}{16(-\right.\right.\right.$$
$$= -\frac{4a^5}{7\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^7}+\frac{2a^5}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^6}-\frac{18a^5}{5\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{4a^5}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}-\frac{35a^5}{12\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}$$

Mathematica [A] time = 0.10, size = 85, normalized size = 0.28

$$\frac{21a^2x^2+2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^2\left(8a^5x^5-8a^4x^4+12a^3x^3-12a^2x^2+15ax-15\right)-30}{105a^2x^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])/x^6,x]
```

```
[Out] (-30 + 21*a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-15 + 15*a*x -
 12*a^2*x^2 + 12*a^3*x^3 - 8*a^4*x^4 + 8*a^5*x^5))/(105*a^2*x^7)
```

fricas [A] time = 0.76, size = 78, normalized size = 0.26

$$\frac{21\,a^2x^2+2\left(8\,a^7x^7+4\,a^5x^5+3\,a^3x^3-15\,ax\right)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-30}{105\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x, algorithm="frica
s")
```

```
[Out] 1/105*(21*a^2*x^2 + 2*(8*a^7*x^7 + 4*a^5*x^5 + 3*a^3*x^3 - 15*a*x)*sqrt((a*
x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 30)/(a^2*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{\frac{1}{ax}+1}\,\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x, algorithm="giac"
)
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^6, x)
```

maple [A] time = 0.07, size = 92, normalized size = 0.31

$$\frac{-\frac{1}{7x^7}+\frac{a^2}{5x^5}}{a^2}+\frac{2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\left(a^2x^2-1\right)\left(8x^4a^4+12a^2x^2+15\right)}{105a\,x^6}-\frac{1}{7a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x)
```

```
[Out] 1/a^2*(-1/7/x^7+1/5*a^2/x^5)+2/105/a*(-(a*x-1)/a/x)^(1/2)/x^6*((a*x+1)/a/x)
^(1/2)*(a^2*x^2-1)*(8*a^4*x^4+12*a^2*x^2+15)-1/7/a^2/x^7
```

maxima [A] time = 0.40, size = 65, normalized size = 0.22

$$\frac{1}{5\,x^5}+\frac{2\left(8\,a^6x^7+4\,a^4x^5+3\,a^2x^3-15\,x\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{105\,a^2x^8}-\frac{2}{7\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x, algorithm="maxim
a")
```

```
[Out] 1/5/x^5 + 2/105*(8*a^6*x^7 + 4*a^4*x^5 + 3*a^2*x^3 - 15*x)*sqrt(a*x + 1)*sq
rt(-a*x + 1)/(a^2*x^8) - 2/7/(a^2*x^7)
```

mupad [B] time = 2.12, size = 105, normalized size = 0.35

$$\frac{\frac{a^2\,x^2}{5}-\frac{2}{7}}{a^2\,x^7}+\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,a\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{35}-\frac{2\,\sqrt{\frac{1}{a\,x}+1}}{7\,a}+\frac{8\,a^3\,x^4\,\sqrt{\frac{1}{a\,x}+1}}{105}+\frac{16\,a^5\,x^6\,\sqrt{\frac{1}{a\,x}+1}}{105}\right)}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^6,x)
```

```
[Out] ((a^2*x^2)/5 - 2/7)/(a^2*x^7) + ((1/(a*x) - 1)^(1/2)*((2*a*x^2*(1/(a*x) + 1
)^(1/2))/35 - (2*(1/(a*x) + 1)^(1/2))/(7*a) + (8*a^3*x^4*(1/(a*x) + 1)^(1/2
))/105 + (16*a^5*x^6*(1/(a*x) + 1)^(1/2))/105))/x^6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2}{x^8}\,dx+\int\left(-\frac{a^2}{x^6}\right)dx+\int\frac{2a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^7}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**6,x)
```

```
[Out] (Integral(2/x**8, x) + Integral(-a**2/x**6, x) + Integral(2*a*sqrt(-1 + 1/(
a*x))*sqrt(1 + 1/(a*x))/x**7, x))/a**2
```

3.76 $\int e^{-\text{sech}^{-1}(ax)} x^4 \, dx$

Optimal. Leaf size=147

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)^5}{5a^5}+\frac{\left(16\sqrt{\frac{1-ax}{ax+1}}+5\right)(ax+1)^4}{20a^5}-\frac{\left(17\sqrt{\frac{1-ax}{ax+1}}+15\right)(ax+1)^3}{15a^5}+\frac{\left(4\sqrt{\frac{1-ax}{ax+1}}+9\right)(ax+1)^2}{6a^5}-\frac{x}{a^4}$$

```
[Out] -x/a^4-1/5*(a*x+1)^5*((-a*x+1)/(a*x+1))^(1/2)/a^5+1/6*(a*x+1)^2*(9+4*((-a*x
+1)/(a*x+1))^(1/2))/a^5+1/20*(a*x+1)^4*(5+16*((-a*x+1)/(a*x+1))^(1/2))/a^5-
1/15*(a*x+1)^3*(15+17*((-a*x+1)/(a*x+1))^(1/2))/a^5
```

Rubi [A] time = 0.62, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6337, 1804, 1814, 12, 261}

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)^5}{5a^5}+\frac{\left(16\sqrt{\frac{1-ax}{ax+1}}+5\right)(ax+1)^4}{20a^5}-\frac{\left(17\sqrt{\frac{1-ax}{ax+1}}+15\right)(ax+1)^3}{15a^5}+\frac{\left(4\sqrt{\frac{1-ax}{ax+1}}+9\right)(ax+1)^2}{6a^5}-\frac{x}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^ArcSech[a*x],x]
```

```
[Out] -(x/a^4) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^5)/(5*a^5) + ((1 + a*x)^2*(
9 + 4*Sqrt[(1 - a*x)/(1 + a*x)]))/(6*a^5) + ((1 + a*x)^4*(5 + 16*Sqrt[(1 -
a*x)/(1 + a*x)]))/(20*a^5) - ((1 + a*x)^3*(15 + 17*Sqrt[(1 - a*x)/(1 + a*x)
]))/(15*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^4\,dx = \int \frac{x^4}{\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}}\,dx$$

$$= \frac{4\,\text{Subst}\left(\int \frac{(-1+x)^5x(1+x)^3}{\left(1+x^2\right)^6}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^5}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^5}{5a^5} - \frac{2\,\text{Subst}\left(\int \frac{-16+10x+140x^2-30x^3-80x^4+30x^5+20x^6-10x^7}{\left(1+x^2\right)^5}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{5a^5}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^5}{5a^5} + \frac{(1+ax)^4\left(5+16\sqrt{\frac{1-ax}{1+ax}}\right)}{20a^5} + \frac{\text{Subst}\left(\int \frac{-128+560x+800x^2-320x^3-160x^4+80x^5}{\left(1+x^2\right)^4}\,d\right.}{20a^5}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^5}{5a^5} + \frac{(1+ax)^4\left(5+16\sqrt{\frac{1-ax}{1+ax}}\right)}{20a^5} - \frac{(1+ax)^3\left(15+17\sqrt{\frac{1-ax}{1+ax}}\right)}{15a^5} - \frac{\text{Subst}\left(\int \frac{-32}{}\right.}{}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^5}{5a^5} + \frac{(1+ax)^2\left(9+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^5} + \frac{(1+ax)^4\left(5+16\sqrt{\frac{1-ax}{1+ax}}\right)}{20a^5} - \frac{(1+ax)^3\left(15+\right.}{15a^5}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^5}{5a^5} + \frac{(1+ax)^2\left(9+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^5} + \frac{(1+ax)^4\left(5+16\sqrt{\frac{1-ax}{1+ax}}\right)}{20a^5} - \frac{(1+ax)^3\left(15+\right.}{15a^5}$$

$$= -\frac{x}{a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^5}{5a^5} + \frac{(1+ax)^2\left(9+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^5} + \frac{(1+ax)^4\left(5+16\sqrt{\frac{1-ax}{1+ax}}\right)}{20a^5} - \frac{(1+ax)^3\left(\right.}{}$$

Mathematica [A] time = 0.10, size = 65, normalized size = 0.44

$$\frac{15a^4x^4-4\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^2\left(3a^3x^3-3a^2x^2+2ax-2\right)}{60a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/E^ArcSech[a*x],x]
```

```
[Out] (15*a^4*x^4 - 4*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-2 + 2*a*x - 3*a^2*x
^2 + 3*a^3*x^3))/(60*a^5)
```

fricas [A] time = 1.97, size = 65, normalized size = 0.44

$$\frac{15\,a^3x^4-4\left(3\,a^4x^5-a^2x^3-2\,x\right)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}}{60\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas"
)
```

```
[Out] 1/60*(15*a^3*x^4 - 4*(3*a^4*x^5 - a^2*x^3 - 2*x)*sqrt((a*x + 1)/(a*x))*sqrt
(-(a*x - 1)/(a*x)))/a^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

maple [C] time = 0.36, size = 531, normalized size = 3.61

$$-\frac{(ax-1)\left(15a^{10}x^{10}\left(\frac{ax+1}{ax}\right)^{\frac{7}{2}}\left(-\frac{ax-1}{ax}\right)^{\frac{5}{2}}+30a^8x^8\left(\frac{ax+1}{ax}\right)^{\frac{7}{2}}\left(-\frac{ax-1}{ax}\right)^{\frac{5}{2}}+30\left(-\frac{ax-1}{ax}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax}\right)^{\frac{7}{2}}x^8a^8+30x^6\ln\left(a^2x^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)
```

```
[Out] -1/60*(a*x-1)/x^7*(15*a^10*x^10*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(5/2)+30
*a^8*x^8*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(5/2)+30*(-(a*x-1)/a/x)^(3/2)*(
(a*x+1)/a/x)^(7/2)*x^8*a^8+30*x^6*ln(a^2*x^2)*((a*x+1)/a/x)^(7/2)*(-(a*x-1)
/a/x)^(5/2)*a^6-30*a^7*x^7*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(3/2)+60*(-(a
*x-1)/a/x)^(3/2)*((a*x+1)/a/x)^(7/2)*ln(a^2*x^2)*x^6*a^6+12*x^11*a^11-60*x^
5*ln(a^2*x^2)*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(3/2)*a^5+30*(-(a*x-1)/a/x
)^(1/2)*((a*x+1)/a/x)^(7/2)*ln(a^2*x^2)*x^6*a^6+12*x^10*a^10-60*(-(a*x-1)/a
/x)^(1/2)*((a*x+1)/a/x)^(7/2)*ln(a^2*x^2)*x^5*a^5-40*x^9*a^9+30*x^4*ln(a^2*
x^2)*((a*x+1)/a/x)^(7/2)*(-(a*x-1)/a/x)^(1/2)*a^4-40*x^8*a^8+40*x^7*a^7+40*
x^6*a^6-20*x^3*a^3-20*a^2*x^2+8*a*x+8)/a^12/((a*x+1)/a/x)^(7/2)/(-(a*x-1)/a
/x)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima"
)
```

```
[Out] integrate(x^4/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

mupad [B] time = 2.14, size = 73, normalized size = 0.50

$$\frac{x^4}{4\,a}+\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,x}{15\,a^4}+\frac{2}{15\,a^5}-\frac{x^5}{5}-\frac{x^4}{5\,a}+\frac{x^3}{15\,a^2}+\frac{x^2}{15\,a^3}\right)}{\sqrt{\frac{1}{a\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] x^4/(4*a) + ((1/(a*x) - 1)^(1/2)*((2*x)/(15*a^4) + 2/(15*a^5) - x^5/5 - x^4
/(5*a) + x^3/(15*a^2) + x^2/(15*a^3)))/(1/(a*x) + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{x^5}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
```

```
[Out] a*Integral(x**5/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

3.77 $\int e^{-\mathbf{sech}^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=163

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^4}{4a^4}+\frac{\left(9\sqrt{\frac{1-ax}{ax+1}}+4\right)(ax+1)^3}{12a^4}-\frac{\left(5\sqrt{\frac{1-ax}{ax+1}}+8\right)(ax+1)^2}{8a^4}+\frac{\left(\sqrt{\frac{1-ax}{ax+1}}+8\right)(ax+1)}{8a^4}+\frac{\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{4a^4}$$

```
[Out] 1/4*arctan(((-a*x+1)/(a*x+1))^(1/2))/a^4-1/4*(a*x+1)^4*((-a*x+1)/(a*x+1))^(
1/2)/a^4+1/8*(a*x+1)*(8+((-a*x+1)/(a*x+1))^(1/2))/a^4-1/8*(a*x+1)^2*(8+5*((
-a*x+1)/(a*x+1))^(1/2))/a^4+1/12*(a*x+1)^3*(4+9*((-a*x+1)/(a*x+1))^(1/2))/a
^4
```

Rubi [A] time = 0.57, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6337, 1804, 1814, 639, 203}

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^4}{4a^4}+\frac{\left(9\sqrt{\frac{1-ax}{ax+1}}+4\right)(ax+1)^3}{12a^4}-\frac{\left(5\sqrt{\frac{1-ax}{ax+1}}+8\right)(ax+1)^2}{8a^4}+\frac{\left(\sqrt{\frac{1-ax}{ax+1}}+8\right)(ax+1)}{8a^4}+\frac{\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^ArcSech[a*x],x]
```

```
[Out] -(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^4)/(4*a^4) + ((1 + a*x)*(8 + Sqrt[(1
- a*x)/(1 + a*x)]))/(8*a^4) - ((1 + a*x)^2*(8 + 5*Sqrt[(1 - a*x)/(1 + a*x)]
))/(8*a^4) + ((1 + a*x)^3*(4 + 9*Sqrt[(1 - a*x)/(1 + a*x)]))/(12*a^4) + Arc
Tan[Sqrt[(1 - a*x)/(1 + a*x)]]/(4*a^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int e^{-\text{sech}^{-1}(ax)}x^3\,dx = \int \frac{x^3}{\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}}\,dx$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{(-1+x)^4x(1+x)^2}{(1+x^2)^5}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^4}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4}{4a^4}+\frac{\text{Subst}\left(\int \frac{8-8x-48x^2+16x^3+16x^4-8x^5}{(1+x^2)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{2a^4}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4}{4a^4}+\frac{(1+ax)^3\left(4+9\sqrt{\frac{1-ax}{1+ax}}\right)}{12a^4}-\frac{\text{Subst}\left(\int \frac{24-144x-96x^2+48x^3}{(1+x^2)^3}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{12a^4}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4}{4a^4}-\frac{(1+ax)^2\left(8+5\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4}+\frac{(1+ax)^3\left(4+9\sqrt{\frac{1-ax}{1+ax}}\right)}{12a^4}+\frac{\text{Subst}\left(\int \frac{24-192x}{(1+x^2)^2}\right.}{48a}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4}{4a^4}+\frac{(1+ax)\left(8+\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4}-\frac{(1+ax)^2\left(8+5\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4}+\frac{(1+ax)^3\left(4+9\sqrt{\right.}}{12a^4}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4}{4a^4}+\frac{(1+ax)\left(8+\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4}-\frac{(1+ax)^2\left(8+5\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4}+\frac{(1+ax)^3\left(4+9\sqrt{\right.}}{12a^4}$$

Mathematica [C] time = 0.13, size = 97, normalized size = 0.60

$$\frac{8a^3x^3+3a\sqrt{\frac{1-ax}{ax+1}}\left(-2a^3x^4-2a^2x^3+ax^2+x\right)-3i\log\left(2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)-2iax\right)}{24a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^ArcSech[a*x],x]
```

```
[Out] (8*a^3*x^3 + 3*a*Sqrt[(1 - a*x)/(1 + a*x)]*(x + a*x^2 - 2*a^2*x^3 - 2*a^3*x
^4) - (3*I)*Log[(-2*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/(24*a^
4)
```

fricas [A] time = 0.78, size = 95, normalized size = 0.58

$$\frac{8\,a^3x^3-3\left(2\,a^4x^4-a^2x^2\right)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}+3\,\arctan\left(\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas"
)
```

```
[Out] 1/24*(8*a^3*x^3 - 3*(2*a^4*x^4 - a^2*x^2)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x
- 1)/(a*x)) + 3*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\frac{1}{ax}+1}\,\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
[Out] integrate(x^3/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)
[Out] int(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\frac{1}{ax}+1}\,\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima"
)
[Out] integrate(x^3/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

mupad [B] time = 19.83, size = 795, normalized size = 4.88

$$\frac{\ln\left(\frac{a\sqrt{\frac{1}{a\,x}+1}-\frac{1}{x}+a\sqrt{\frac{1}{a\,x}-1}\,1\mathrm{i}}{2\,a-2\,a\sqrt{\frac{1}{a\,x}+1}+\frac{1}{x}}\right)3\mathrm{i}}{8\,a^4}+\frac{\frac{1\mathrm{i}}{1024\,a^4}-\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2\,3\mathrm{i}}{128\,a^4\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}-\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4\,53\mathrm{i}}{512\,a^4\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^6\,87\mathrm{i}}{256\,a^4\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^6}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^8\,6\ldots}{1024\,a^4\,\left(\sqrt{\frac{1}{a\,x}+1\ldots}\right.}}{\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}+\frac{4\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^6}+\frac{6\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^8}+\frac{4\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{10}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{10}}+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
[Out] (log((a*(1/(a*x) - 1)^(1/2)*1i + a*(1/(a*x) + 1)^(1/2) - 1/x)/(2*a - 2*a*(1
/(a*x) + 1)^(1/2) + 1/x))*3i)/(8*a^4) + (1i/(1024*a^4) - (((1/(a*x) - 1)^(1
/2) - 1i)^2*3i)/(128*a^4*((1/(a*x) + 1)^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/
2) - 1i)^4*53i)/(512*a^4*((1/(a*x) + 1)^(1/2) - 1)^4) + (((1/(a*x) - 1)^(1/
2) - 1i)^6*87i)/(256*a^4*((1/(a*x) + 1)^(1/2) - 1)^6) + (((1/(a*x) - 1)^(1/
2) - 1i)^8*657i)/(1024*a^4*((1/(a*x) + 1)^(1/2) - 1)^8) + (((1/(a*x) - 1)^(
1/2) - 1i)^10*121i)/(256*a^4*((1/(a*x) + 1)^(1/2) - 1)^10))/(((1/(a*x) - 1)
^(1/2) - 1i)^4/((1/(a*x) + 1)^(1/2) - 1)^4 + (4*((1/(a*x) - 1)^(1/2) - 1i)^
6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (6*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x)
 + 1)^(1/2) - 1)^8 + (4*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2)
 - 1)^10 + ((1/(a*x) - 1)^(1/2) - 1i)^12/((1/(a*x) + 1)^(1/2) - 1)^12) + (1
```

```
og(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/(8*a^4) + (1i/
(32*a^4) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(16*a^4*((1/(a*x) + 1)^(1/2) -
 1)^2) - (((1/(a*x) - 1)^(1/2) - 1i)^4*15i)/(32*a^4*((1/(a*x) + 1)^(1/2) -
1)^4))/(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + (2*((1/(
a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a*x) - 1)^(1/2)
 - 1i)^6/((1/(a*x) + 1)^(1/2) - 1)^6) - (log((a*(-(a - 1/x)/a)^(1/2)*2i - 2
/x + 2*a*((a + 1/x)/a)^(1/2))/(2*a + 1/x - 2*a*((a + 1/x)/a)^(1/2)))*1i)/(2
*a^4) + x^3/(3*a) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(256*a^4*((1/(a*x) +
1)^(1/2) - 1)^2) + (((1/(a*x) - 1)^(1/2) - 1i)^4*1i)/(1024*a^4*((1/(a*x) +
1)^(1/2) - 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{x^4}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
```

```
[Out] a*Integral(x**4/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

3.78 $\int e^{-\operatorname{sech}^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^3}{3a^3}+\frac{\left(4\sqrt{\frac{1-ax}{ax+1}}+3\right)(ax+1)^2}{6a^3}-\frac{x}{a^2}$$

```
[Out] -x/a^2-1/3*(a*x+1)^3*((-a*x+1)/(a*x+1))^(1/2)/a^3+1/6*(a*x+1)^2*(3+4*((-a*x
+1)/(a*x+1))^(1/2))/a^3
```

Rubi [A] time = 0.51, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6337, 1804, 1814, 12, 261}

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^3}{3a^3}+\frac{\left(4\sqrt{\frac{1-ax}{ax+1}}+3\right)(ax+1)^2}{6a^3}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^ArcSech[a*x],x]
```

```
[Out] -(x/a^2) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^3)/(3*a^3) + ((1 + a*x)^2*(
3 + 4*Sqrt[(1 - a*x)/(1 + a*x)]))/(6*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
```

```
Q[n]
```

Rubi steps

$$\int e^{-\text{sech}^{-1}(ax)}x^2\,dx = \int \frac{x^2}{\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}}\,dx$$

$$= \frac{4\,\text{Subst}\left(\int \frac{(-1+x)^3x(1+x)}{\left(1+x^2\right)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^3}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3}{3a^3} - \frac{2\,\text{Subst}\left(\int \frac{-4+6x+12x^2-6x^3}{\left(1+x^2\right)^3}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{3a^3}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3}{3a^3} + \frac{(1+ax)^2\left(3+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^3} + \frac{\text{Subst}\left(\int \frac{24x}{\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{6a^3}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3}{3a^3} + \frac{(1+ax)^2\left(3+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^3} + \frac{4\,\text{Subst}\left(\int \frac{x}{\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^3}$$

$$= -\frac{x}{a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3}{3a^3} + \frac{(1+ax)^2\left(3+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^3}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 0.64

$$\frac{3a^2x^2 - 2(ax-1)\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^2}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^ArcSech[a*x],x]
```

```
[Out] (3*a^2*x^2 - 2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(6*a^3)
```

fricas [A] time = 0.54, size = 54, normalized size = 0.72

$$\frac{3\,ax^2 - 2\left(a^2x^3 - x\right)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas"
)
```

```
[Out] 1/6*(3*a*x^2 - 2*(a^2*x^3 - x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))
)/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

maple [C] time = 0.32, size = 269, normalized size = 3.59

$$-\frac{(ax-1)\left(3a^6x^6\left(\frac{ax+1}{ax}\right)^{\frac{5}{2}}\left(-\frac{ax-1}{ax}\right)^{\frac{3}{2}}+3x^4\ln\left(a^2x^2\right)\left(\frac{ax+1}{ax}\right)^{\frac{5}{2}}\left(-\frac{ax-1}{ax}\right)^{\frac{3}{2}}a^4+3\ln\left(a^2x^2\right)\sqrt{-\frac{ax-1}{ax}}\left(\frac{ax+1}{ax}\right)^{\frac{5}{2}}x^4a^4-\cdots\right.}{6x^5a^8\left(\frac{ax+1}{ax}\right)^{\frac{5}{2}}\left(-\frac{ax}{\cdots}\right.\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)
```

```
[Out] -1/6*(a*x-1)/x^5*(3*a^6*x^6*((a*x+1)/a/x)^(5/2)*(-(a*x-1)/a/x)^(3/2)+3*x^4*
ln(a^2*x^2)*((a*x+1)/a/x)^(5/2)*(-(a*x-1)/a/x)^(3/2)*a^4+3*ln(a^2*x^2)*(-(a
*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(5/2)*x^4*a^4-2*x^7*a^7-3*x^3*ln(a^2*x^2)*((
a*x+1)/a/x)^(5/2)*(-(a*x-1)/a/x)^(1/2)*a^3-2*x^6*a^6+6*x^5*a^5+6*x^4*a^4-6*
x^3*a^3-6*a^2*x^2+2*a*x+2)/a^8/((a*x+1)/a/x)^(5/2)/(-(a*x-1)/a/x)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima"
)
```

```
[Out] integrate(x^2/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

mupad [B] time = 2.06, size = 57, normalized size = 0.76

$$\frac{x^2}{2\,a}+\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{x}{3\,a^2}+\frac{1}{3\,a^3}-\frac{x^3}{3}-\frac{x^2}{3\,a}\right)}{\sqrt{\frac{1}{a\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] x^2/(2*a) + ((1/(a*x) - 1)^(1/2)*(x/(3*a^2) + 1/(3*a^3) - x^3/3 - x^2/(3*a)
))/(1/(a*x) + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{x^3}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
```

```
[Out] a*Integral(x**3/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

3.79 $\int e^{-\operatorname{sech}^{-1}(ax)} x \, dx$

Optimal. Leaf size=94

$$\frac{(ax+1)^2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}{4a^2}+\frac{(ax+1)\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{2a^2}+\frac{\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a^2}$$

```
[Out] arctan(((-a*x+1)/(a*x+1))^(1/2))/a^2+1/4*(a*x+1)^2*(1-((-a*x+1)/(a*x+1))^(1
/2))^2/a^2+1/2*(a*x+1)*(1+((-a*x+1)/(a*x+1))^(1/2))/a^2
```

Rubi [A] time = 0.31, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6337, 819, 639, 203}

$$\frac{(ax+1)^2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}{4a^2}+\frac{(ax+1)\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{2a^2}+\frac{\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^ArcSech[a*x],x]
```

```
[Out] ((1 + a*x)^2*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2)/(4*a^2) + ((1 + a*x)*(1 + S
qrt[(1 - a*x)/(1 + a*x)]))/(2*a^2) + ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]]/a^2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int e^{-\operatorname{sech}^{-1}(ax)}x\,dx=\int \frac{x}{\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}}\,dx$$
$$=-\frac{4\,\mathrm{Subst}\left(\int \frac{(-1+x)^2x}{\left(1+x^2\right)^3}\,dx,x,\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$
$$=\frac{(1+ax)^2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}{4a^2}-\frac{\mathrm{Subst}\left(\int \frac{-2+2x}{\left(1+x^2\right)^2}\,dx,x,\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$
$$=\frac{(1+ax)^2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}{4a^2}+\frac{(1+ax)\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2}+\frac{\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$
$$=\frac{(1+ax)^2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}{4a^2}+\frac{(1+ax)\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2}+\frac{\tan^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$

Mathematica [**C**] time = 0.07, size = 75, normalized size = 0.80

$$-\frac{-2ax+ax\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)+i\log\left(2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)-2iax\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^ArcSech[a*x],x]
```

```
[Out] -1/2*(-2*a*x + a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + I*Log[(-2*I)*a*x + 
 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/a^2
```

fricas [**A**] time = 0.85, size = 79, normalized size = 0.84

$$-\frac{a^2x^2\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-2\,ax-\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*x^2*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2*a*x - arctan
(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^2
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)
```

```
[Out] int(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

mupad [B] time = 9.22, size = 407, normalized size = 4.33

$$\frac{x}{a}-\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)1\mathrm{i}}{2\,a^2}-\frac{\frac{1\mathrm{i}}{32\,a^2}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{16\,a^2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4 15\mathrm{i}}{32\,a^2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}}-\frac{\left(\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\right)1\mathrm{i}}{a^2}+\frac{\ln\left(\frac{2\,a\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] x/a - (log(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/(2*a^2
) - (1i/(32*a^2) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(16*a^2*((1/(a*x) + 1)
^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/2) - 1i)^4*15i)/(32*a^2*((1/(a*x) + 1)^
(1/2) - 1)^4))/(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 +
(2*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a*x) -
1)^(1/2) - 1i)^6/((1/(a*x) + 1)^(1/2) - 1)^6) - ((log(((1/(a*x) - 1)^(1/2)
- 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1) - log(((1/(a*x) - 1)^(1/2) - 1i)/(
(1/(a*x) + 1)^(1/2) - 1)))*1i)/a^2 + (log((a*(-(a - 1/x)/a)^(1/2)*2i - 2/x
+ 2*a*((a + 1/x)/a)^(1/2))/(2*a + 1/x - 2*a*((a + 1/x)/a)^(1/2)))*1i)/(2*a^
2) - (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(32*a^2*((1/(a*x) + 1)^(1/2) - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{x^2}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
```

```
[Out] a*Integral(x**2/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

3.80 $\int e^{-\text{sech}^{-1}(ax)}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{a}+\frac{\log(ax+1)}{a}+\frac{2\log\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{a}$$

```
[Out] ln(a*x+1)/a+2*ln(1+((-a*x+1)/(a*x+1))^(1/2))/a-(a*x+1)*((-a*x+1)/(a*x+1))^(
1/2)/a
```

Rubi [A] time = 0.17, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6332, 1647, 801, 260}

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{a}+\frac{\log(ax+1)}{a}+\frac{2\log\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-ArcSech[a*x]),x]
```

```
[Out] -((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/a) + Log[1 + a*x]/a + (2*Log[1 + Sq
rt[(1 - a*x)/(1 + a*x)]])/a
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6332

```
Int[E^(ArcSech[u_]*(n_.)), x_Symbol] :> Int[(1/u + Sqrt[(1 - u)/(1 + u)] +
(1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; IntegerQ[n]
```

Rubi steps

$$\int e^{-\text{sech}^{-1}(ax)}\,dx = \int \frac{1}{\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}}\,dx$$
$$= \frac{4\,\text{Subst}\left(\int \frac{(-1+x)x}{(1+x)(1+x^2)^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$
$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{a} - \frac{2\,\text{Subst}\left(\int \frac{-1+x}{(1+x)(1+x^2)}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$
$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{a} - \frac{2\,\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{x}{1+x^2}\right)dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$
$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{a} + \frac{2\log\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{a} - \frac{2\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$
$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{a} + \frac{\log(1+ax)}{a} + \frac{2\log\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.11

$$\frac{\log\left(ax\sqrt{\frac{1-ax}{ax+1}}+\sqrt{\frac{1-ax}{ax+1}}+1\right)-\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-ArcSech[a*x]),x]
```

```
[Out] (-(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)) + Log[1 + Sqrt[(1 - a*x)/(1 + a*x)]
 + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/a
```

fricas [A] time = 0.53, size = 115, normalized size = 1.77

$$-\frac{2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)+\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)-2\,\log(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - log(a*x*sqrt((a*
x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) + log(a*x*sqrt((a*x + 1)/(a*x))*s
qrt(-(a*x - 1)/(a*x)) - 1) - 2*log(x))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

maple [B] time = 0.31, size = 2612, normalized size = 40.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)
```

```
[Out] -1/2*(a*x-1)/x^3*(x^2*ln(a^2*x^2)*((a*x+1)/a/x)^(3/2)*(-(a*x-1)/a/x)^(1/2)*
(-a^2*x^2+1)^(1/2)*a^2+2*(-a^2*x^2+1)^(1/2)*x^3*a^3-ln(2*(((-((a*x+1)/a/x)^
(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*(((a
*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^
(1/2))*a^2/(a*x+1)/(a*x-1)/x^2)^(1/2)*x+(-a^2*x^2+1)^(1/2)+1)*a^2/(a^2*x+((
-((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)
*x)^(1/2))*(((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2
)*(-(a*x-1)*x)^(1/2))*a^2/(a*x+1)/(a*x-1)/x^2)^(1/2)))*x^3*a^3-ln(-2*(-((-(
(a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x
)^(1/2))*(((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*
(-(a*x-1)*x)^(1/2))*a^2/(a*x+1)/(a*x-1)/x^2)^(1/2)*x+(-a^2*x^2+1)^(1/2)+1)*
a^2/(-a^2*x+((-((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(
1/2)*(-(a*x-1)*x)^(1/2))*(((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((
a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*a^2/(a*x+1)/(a*x-1)/x^2)^(1/2)))*x^3*a^
3+2*a^2*x^2*(-a^2*x^2+1)^(1/2)-x^2*ln(2*(((-((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-
1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*(((a*x+1)/a/x)^(1/2)*
x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*a^2/(a*x+1
)/(a*x-1)/x^2)^(1/2)*x+(-a^2*x^2+1)^(1/2)+1)*a^2/(a^2*x+((-((a*x+1)/a/x)^(1
/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*(((a*x
+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1
/2))*a^2/(a*x+1)/(a*x-1)/x^2)^(1/2)))*a^2-x^2*ln(-2*(-((-((a*x+1)/a/x)^(1/2
)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*(((a*x+1
)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2
))*a^2/(a*x+1)/(a*x-1)/x^2)^(1/2)*x+(-a^2*x^2+1)^(1/2)+1)*a^2/(-a^2*x+((-((
a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)
^(1/2))*(((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(
-(a*x-1)*x)^(1/2))*a^2/(a*x+1)/(a*x-1)/x^2)^(1/2)))*a^2-2*a*x*(-a^2*x^2+1)^
(1/2)+x*ln(2*(((-((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)
^(1/2)*(-(a*x-1)*x)^(1/2))*(((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+
((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*a^2/(a*x+1)/(a*x-1)/x^2)^(1/2)*x+(-a^
2*x^2+1)^(1/2)+1)*a^2/(a^2*x+((-((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2
)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*(((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1
)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*a^2/(a*x+1)/(a*x-1)/x^
2)^(1/2)))*a+x*ln(-2*(-((-((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((
a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*(((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)
^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*a^2/(a*x+1)/(a*x-1)/x^2)^(1/
2)*x+(-a^2*x^2+1)^(1/2)+1)*a^2/(-a^2*x+((-((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)
/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*(((a*x+1)/a/x)^(1/2)*x^
2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*a^2/(a*x+1)/
(a*x-1)/x^2)^(1/2)))*a-2*(-a^2*x^2+1)^(1/2)+ln(2*(((-((a*x+1)/a/x)^(1/2)*x^
2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*(((a*x+1)/a/
x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*a
^2/(a*x+1)/(a*x-1)/x^2)^(1/2)*x+(-a^2*x^2+1)^(1/2)+1)*a^2/(a^2*x+((-((a*x+1
)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2
))*(((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x
-1)*x)^(1/2))*a^2/(a*x+1)/(a*x-1)/x^2)^(1/2)))+ln(-2*(-((-((a*x+1)/a/x)^(1/
2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/2))*(((a*x+
1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x)^(1/
2))*a^2/(a*x+1)/(a*x-1)/x^2)^(1/2)*x+(-a^2*x^2+1)^(1/2)+1)*a^2/(-a^2*x+((-(
(a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*(-(a*x-1)*x
)^(1/2))*(((a*x+1)/a/x)^(1/2)*x^2*(-(a*x-1)/a/x)^(1/2)*a+((a*x+1)*x)^(1/2)*
(-(a*x-1)*x)^(1/2))*a^2/(a*x+1)/(a*x-1)/x^2)^(1/2))))/a^4/((a*x+1)/a/x)^(3/
2)/(-(a*x-1)/a/x)^(3/2)/(-a^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

mupad [B] time = 4.17, size = 47, normalized size = 0.72

$$\frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)}{a}-\frac{\ln\left(\frac{1}{x}\right)}{a}-x\,\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
[Out] acosh(1/(a*x))/a - log(1/x)/a - x*(1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{x}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
[Out] a*Integral(x/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

3.81 $\int \frac{e^{-\text{sech}^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=46

$$-\frac{2}{\sqrt{\frac{1-ax}{ax+1}}+1}-2\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)$$

```
[Out] -2*arctan(((-a*x+1)/(a*x+1))^(1/2))-2/(1+((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A] time = 0.40, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6337, 801, 203}

$$-\frac{2}{\sqrt{\frac{1-ax}{ax+1}}+1}-2\tan^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x),x]
```

```
[Out] -2/(1 + Sqrt[(1 - a*x)/(1 + a*x)]) - 2*ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)}\,dx$$
$$= -\left(4\,\mathrm{Subst}\left(\int \frac{x}{(1+x)^2\left(1+x^2\right)}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$
$$= -\left(4\,\mathrm{Subst}\left(\int \left(-\frac{1}{2(1+x)^2}+\frac{1}{2\left(1+x^2\right)}\right)dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$
$$= -\frac{2}{1+\sqrt{\frac{1-ax}{1+ax}}} - 2\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$
$$= -\frac{2}{1+\sqrt{\frac{1-ax}{1+ax}}} - 2\tan^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

Mathematica [C] time = 0.04, size = 74, normalized size = 1.61

$$\sqrt{\frac{1-ax}{ax+1}}\left(\frac{1}{ax}+1\right)-\frac{1}{ax}+i\log\left(2\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)-2iax\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x),x]
```

```
[Out] -(1/(a*x)) + (1 + 1/(a*x))*Sqrt[(1 - a*x)/(1 + a*x)] + I*Log[(-2*I)*a*x + 2
*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)]
```

fricas [A] time = 1.70, size = 76, normalized size = 1.65

$$\frac{ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - ax\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)-1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="fricas"
)
```

```
[Out] (a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - a*x*arctan(sqrt((a*x +
1)/(a*x))*sqrt(-(a*x - 1)/(a*x))) - 1)/(a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(1/(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x)
```

```
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="maxima"
)
```

```
[Out] integrate(1/(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

mupad [B] time = 3.85, size = 184, normalized size = 4.00

$$\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)1\mathrm{i}-\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)1\mathrm{i}-\frac{1}{a\,x}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 8\mathrm{i}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2\left(1+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] log(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)*1i - log(
((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i - 1/(a*x) - (((1/(
a*x) - 1)^(1/2) - 1i)^2*8i)/(((1/(a*x) + 1)^(1/2) - 1)^2*(((1/(a*x) - 1)^(1
/2) - 1i)^4/((1/(a*x) + 1)^(1/2) - 1)^4 - (2*((1/(a*x) - 1)^(1/2) - 1i)^2)/
((1/(a*x) + 1)^(1/2) - 1)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{1}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x,x)
```

```
[Out] a*Integral(1/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

3.82 $\int \frac{e^{-\mathrm{sech}^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=72

$$\frac{a}{\sqrt{\frac{1-ax}{ax+1}}+1}-\frac{a}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+a\left(-\tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)\right)$$

```
[Out] -a*arctanh(((-a*x+1)/(a*x+1))^(1/2))-a/(1+((-a*x+1)/(a*x+1))^(1/2))^2+a/(1+
((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A] time = 0.38, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6337, 77, 207}

$$\frac{a}{\sqrt{\frac{1-ax}{ax+1}}+1}-\frac{a}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+a\left(-\tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x^2),x]
```

```
[Out] -(a/(1 + Sqrt[(1 - a*x)/(1 + a*x)])^2) + a/(1 + Sqrt[(1 - a*x)/(1 + a*x)])
- a*ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)}\,dx$$
$$= (4a)\,\text{Subst}\left(\int \frac{x}{(-1+x)(1+x)^3}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$
$$= (4a)\,\text{Subst}\left(\int \left(\frac{1}{2(1+x)^3}-\frac{1}{4(1+x)^2}+\frac{1}{4\left(-1+x^2\right)}\right)dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$
$$= -\frac{a}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a}{1+\sqrt{\frac{1-ax}{1+ax}}}+a\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$
$$= -\frac{a}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a}{1+\sqrt{\frac{1-ax}{1+ax}}}-a\tanh^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

Mathematica [A] time = 0.06, size = 92, normalized size = 1.28

$$\frac{1}{2}\left(\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{ax^2}-\frac{1}{ax^2}+a\log(x)-a\log\left(ax\sqrt{\frac{1-ax}{ax+1}}+\sqrt{\frac{1-ax}{ax+1}}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x^2),x]
```

```
[Out] (-(1/(a*x^2)) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*x^2) + a*Log[x] -
a*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/2
```

fricas [A] time = 0.51, size = 128, normalized size = 1.78

$$-\frac{a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)-2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+2}{4\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="frica
s")
```

```
[Out] -1/4*(a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a
^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 2*a*x*sq
rt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 2)/(a*x^2)
```

giac [A] time = 0.17, size = 110, normalized size = 1.53

$$-\frac{1}{2}\left(\sqrt{a^2+\frac{a}{x}}\sqrt{-a^2+\frac{a}{x}}\left(\frac{1}{a^2}-\frac{a^2+\frac{a}{x}}{a^4}\right)-\frac{2\left(a^2+\frac{a}{x}\right)a^2-\left(a^2+\frac{a}{x}\right)^2}{a^4}-2\log\left(\sqrt{a^2+\frac{a}{x}}-\sqrt{-a^2+\frac{a}{x}}\right)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="giac"
)
```

```
[Out] -1/2*(sqrt(a^2 + a/x)*sqrt(-a^2 + a/x)*(1/a^2 - (a^2 + a/x)/a^4) - (2*(a^2
+ a/x)*a^2 - (a^2 + a/x)^2)/a^4 - 2*log(sqrt(a^2 + a/x) - sqrt(-a^2 + a/x))
)*a
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1}\,\sqrt{1 + \frac{1}{ax}}\right) x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x)
```

```
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{\frac{1}{ax} + 1}\,\sqrt{\frac{1}{ax} - 1} + \frac{1}{ax}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x^2*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

mupad [B] time = 12.05, size = 323, normalized size = 4.49

$$2\,a\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)-a\,\mathrm{acosh}\left(\frac{1}{a\,x}\right)-\frac{1}{2\,a\,x^2}-\frac{a\left(\frac{14\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{14\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{2\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x}+1}-1}\right)}{1+\frac{6\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] 2*a*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)) - a*acosh(1
/(a*x)) - 1/(2*a*x^2) - (a*((14*((1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1
)^(1/2) - 1)^3 + (14*((1/(a*x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1
)^5 + (2*((1/(a*x) - 1)^(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (2*((1
/(a*x) - 1)^(1/2) - 1i))/((1/(a*x) + 1)^(1/2) - 1)))/((6*((1/(a*x) - 1)^(1/
2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (4*((1/(a*x) - 1)^(1/2) - 1i)^2)/
((1/(a*x) + 1)^(1/2) - 1)^2 - (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) +
1)^(1/2) - 1)^6 + ((1/(a*x) - 1)^(1/2) - 1i)^8/((1/(a*x) + 1)^(1/2) - 1)^8
+ 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \int \frac{1}{ax^2\sqrt{-1 + \frac{1}{ax}}\,\sqrt{1 + \frac{1}{ax}} + x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**2,x)
```

```
[Out] a*Integral(1/(a*x**2*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x), x)
```

3.83 $\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=116

$$-\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{a^2}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{a^2}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{2a^2}{3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}$$

```
[Out] -1/2*a^2/(1-((-a*x+1)/(a*x+1))^(1/2))-2/3*a^2/(1+((-a*x+1)/(a*x+1))^(1/2))^
3+a^2/(1+((-a*x+1)/(a*x+1))^(1/2))^2-1/2*a^2/(1+((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A] time = 0.42, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6337, 1612}

$$-\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{a^2}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{a^2}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{2a^2}{3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x^3),x]
```

```
[Out] -a^2/(2*(1 - Sqrt[(1 - a*x)/(1 + a*x)])) - (2*a^2)/(3*(1 + Sqrt[(1 - a*x)/(
1 + a*x)])^3) + a^2/(1 + Sqrt[(1 - a*x)/(1 + a*x)])^2 - a^2/(2*(1 + Sqrt[(1
 - a*x)/(1 + a*x)]))
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^3}\,dx &= \int \frac{1}{x^3\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)}\,dx\\
&= -\left(\left(4a^2\right)\text{Subst}\left(\int \frac{x\left(1+x^2\right)}{(-1+x)^2(1+x)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)\\
&= -\left(\left(4a^2\right)\text{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}-\frac{1}{2(1+x)^4}+\frac{1}{2(1+x)^3}-\frac{1}{8(1+x)^2}\right)dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)\\
&= -\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{2a^2}{3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{a^2}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2}-\frac{a^2}{2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 43, normalized size = 0.37

$$-\frac{(ax-1)\sqrt{\frac{1-ax}{ax+1}}(ax+1)^2+1}{3ax^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x^3),x]
[Out] -1/3*(1 + (-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(a*x^3)
```

fricas [A] time = 0.79, size = 52, normalized size = 0.45

$$-\frac{\left(a^3x^3-ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="frica
s")
[Out] -1/3*((a^3*x^3 - a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1)/(a*
x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="giac"
)
[Out] integrate(1/(x^3*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\sqrt{1+\frac{1}{ax}}\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x)
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="maxim
a")
[Out] integrate(1/(x^3*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

mupad [B] time = 2.14, size = 58, normalized size = 0.50

$$\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{x}{3}-\frac{a\,x^2}{3}+\frac{1}{3\,a}-\frac{a^2\,x^3}{3}\right)}{x^3\,\sqrt{\frac{1}{a\,x}+1}}-\frac{1}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] ((1/(a*x) - 1)^(1/2)*(x/3 - (a*x^2)/3 + 1/(3*a) - (a^2*x^3)/3))/(x^3*(1/(a*
x) + 1)^(1/2)) - 1/(3*a*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int\frac{1}{ax^3\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**3,x)
```

```
[Out] a*Integral(1/(a*x**3*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**2), x)
```

3.84 $\int \frac{e^{-\mathrm{sech}^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=200

$$\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}+\frac{a^3}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{a^3}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{a^3}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}-\frac{a^3}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}-\frac{1}{4}a^3\tanh^{-1}$$

```
[Out] -1/4*a^3*arctanh(((-a*x+1)/(a*x+1))^(1/2))-1/4*a^3/(1-((-a*x+1)/(a*x+1))^(1
/2))^2+1/4*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))-1/2*a^3/(1+((-a*x+1)/(a*x+1))^(
1/2))^4+a^3/(1+((-a*x+1)/(a*x+1))^(1/2))^3-a^3/(1+((-a*x+1)/(a*x+1))^(1/2))
^2+1/2*a^3/(1+((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A] time = 0.50, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6337, 1612, 207}

$$\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}+\frac{a^3}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{a^3}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{a^3}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}-\frac{a^3}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}-\frac{1}{4}a^3\tanh^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x^4),x]
```

```
[Out] -a^3/(4*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) + a^3/(4*(1 - Sqrt[(1 - a*x)/(1
+ a*x)])) - a^3/(2*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^4) + a^3/(1 + Sqrt[(1 -
a*x)/(1 + a*x)])^3 - a^3/(1 + Sqrt[(1 - a*x)/(1 + a*x)])^2 + a^3/(2*(1 + Sq
rt[(1 - a*x)/(1 + a*x)])) - (a^3*ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]])/4
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^4} dx = \int \frac{1}{x^4 \left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)} dx$$

$$= (4a) \text{Subst}\left(\int \frac{x\left(a+ax^2\right)^2}{(-1+x)^3(1+x)^5} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$

$$= (4a) \text{Subst}\left(\int \left(\frac{a^2}{8(-1+x)^3} + \frac{a^2}{16(-1+x)^2} + \frac{a^2}{2(1+x)^5} - \frac{3a^2}{4(1+x)^4} + \frac{a^2}{2(1+x)^3} - \frac{a^2}{8(1+x)^2}\right.\right.$$

$$= -\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{a^3}{2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4} + \frac{a^3}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3} - \frac{a^3}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2} +$$

$$= -\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2} + \frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{a^3}{2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4} + \frac{a^3}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3} - \frac{a^3}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2} +$$

Mathematica [A] time = 0.11, size = 110, normalized size = 0.55

$$-\frac{-a^4x^4\log(x) + a^4x^4\log\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1\right) + \sqrt{\frac{1-ax}{ax+1}}\left(a^3x^3 + a^2x^2 - 2ax - 2\right) + 2}{8ax^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x^4),x]
```

```
[Out] -1/8*(2 + Sqrt[(1 - a*x)/(1 + a*x)]*(-2 - 2*a*x + a^2*x^2 + a^3*x^3) - a^4*
x^4*Log[x] + a^4*x^4*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)
/(1 + a*x)]])/(a*x^4)
```

fricas [A] time = 0.63, size = 138, normalized size = 0.69

$$-\frac{a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 1\right) - a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - 1\right) + 2\left(a^3x^3 - 2\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 4}{16\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="frica
s")
```

```
[Out] -1/16*(a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) -
a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*(a^3*
x^3 - 2*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 4)/(a*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1} + \frac{1}{ax}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="giac"
)
```

```
[Out] integrate(1/(x^4*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x)
```

```
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\sqrt{\frac{1}{ax}+1}\,\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x^4*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

mupad [B] time = 43.71, size = 1511, normalized size = 7.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] ((a^3*((1/(a*x) - 1)^(1/2) - 1i)^4*192i)/((1/(a*x) + 1)^(1/2) - 1)^4 + (a^3
*((1/(a*x) - 1)^(1/2) - 1i)^6*128i)/((1/(a*x) + 1)^(1/2) - 1)^6 + (a^3*((1/
(a*x) - 1)^(1/2) - 1i)^8*192i)/((1/(a*x) + 1)^(1/2) - 1)^8)/(3*((15*((1/(a*
x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (6*((1/(a*x) - 1)^(1/2
) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (20*((1/(a*x) - 1)^(1/2) - 1i)^6)/
((1/(a*x) + 1)^(1/2) - 1)^6 + (15*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) +
 1)^(1/2) - 1)^8 - (6*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) -
 1)^10 + ((1/(a*x) - 1)^(1/2) - 1i)^12/((1/(a*x) + 1)^(1/2) - 1)^12 + 1)) -
 ((a^3*((1/(a*x) - 1)^(1/2) - 1i)^4*64i)/((1/(a*x) + 1)^(1/2) - 1)^4 + (a^3
*((1/(a*x) - 1)^(1/2) - 1i)^6*128i)/(3*((1/(a*x) + 1)^(1/2) - 1)^6) + (a^3*
((1/(a*x) - 1)^(1/2) - 1i)^8*64i)/((1/(a*x) + 1)^(1/2) - 1)^8)/((15*((1/(a*
x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (6*((1/(a*x) - 1)^(1/2
) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (20*((1/(a*x) - 1)^(1/2) - 1i)^6)/
((1/(a*x) + 1)^(1/2) - 1)^6 + (15*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) +
 1)^(1/2) - 1)^8 - (6*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) -
 1)^10 + ((1/(a*x) - 1)^(1/2) - 1i)^12/((1/(a*x) + 1)^(1/2) - 1)^12 + 1) -
(a^3*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))/2 + ((14*
a^3*((1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) - 1)^3 + (14*a^3*((1
/(a*x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 + (2*a^3*((1/(a*x) -
 1)^(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (2*a^3*((1/(a*x) - 1)^(1/2
) - 1i))/((1/(a*x) + 1)^(1/2) - 1))/((6*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(
a*x) + 1)^(1/2) - 1)^4 - (4*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1
/2) - 1)^2 - (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 +
 ((1/(a*x) - 1)^(1/2) - 1i)^8/((1/(a*x) + 1)^(1/2) - 1)^8 + 1) + ((23*a^3*(
(1/(a*x) - 1)^(1/2) - 1i)^3)/(2*((1/(a*x) + 1)^(1/2) - 1)^3) + (333*a^3*((1
/(a*x) - 1)^(1/2) - 1i)^5)/(2*((1/(a*x) + 1)^(1/2) - 1)^5) + (671*a^3*((1/(
a*x) - 1)^(1/2) - 1i)^7)/(2*((1/(a*x) + 1)^(1/2) - 1)^7) + (671*a^3*((1/(a*
x) - 1)^(1/2) - 1i)^9)/(2*((1/(a*x) + 1)^(1/2) - 1)^9) + (333*a^3*((1/(a*x)
 - 1)^(1/2) - 1i)^11)/(2*((1/(a*x) + 1)^(1/2) - 1)^11) + (23*a^3*((1/(a*x)
```

```
- 1)^(1/2) - 1i)^13)/(2*((1/(a*x) + 1)^(1/2) - 1)^13) - (3*a^3*((1/(a*x) -
1)^(1/2) - 1i)^15)/(2*((1/(a*x) + 1)^(1/2) - 1)^15) - (3*a^3*((1/(a*x) - 1)
^(1/2) - 1i))/(2*((1/(a*x) + 1)^(1/2) - 1)))/((28*((1/(a*x) - 1)^(1/2) - 1i
)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (8*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*
x) + 1)^(1/2) - 1)^2 - (56*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/
2) - 1)^6 + (70*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 -
 (56*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + (28*((1/
(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (8*((1/(a*x) - 1)
^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + ((1/(a*x) - 1)^(1/2) - 1i)^
16/((1/(a*x) + 1)^(1/2) - 1)^16 + 1) - 1/(4*a*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{1}{ax^4\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**4,x)
```

```
[Out] a*Integral(1/(a*x**4*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**3), x)
```

3.85 $\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5} dx$

Optimal. Leaf size=233

$$-\frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{3a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{a^4}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{a^4}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{a^4}{6\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{4a^4}{3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}+\frac{a^4}{\left(\sqrt{\frac{1-ax}{ax+1}}\right.}$$

```
[Out] -1/6*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))^3+1/4*a^4/(1-((-a*x+1)/(a*x+1))^(1/2)
)^2-3/8*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))-2/5*a^4/(1+((-a*x+1)/(a*x+1))^(1/2
))^5+a^4/(1+((-a*x+1)/(a*x+1))^(1/2))^4-4/3*a^4/(1+((-a*x+1)/(a*x+1))^(1/2)
)^3+a^4/(1+((-a*x+1)/(a*x+1))^(1/2))^2-3/8*a^4/(1+((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A] time = 0.50, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6337, 1612}

$$-\frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{3a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{a^4}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{a^4}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{a^4}{6\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{4a^4}{3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}+\frac{a^4}{\left(\sqrt{\frac{1-ax}{ax+1}}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x^5),x]
```

```
[Out] -a^4/(6*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^3) + a^4/(4*(1 - Sqrt[(1 - a*x)/(1
+ a*x)])^2) - (3*a^4)/(8*(1 - Sqrt[(1 - a*x)/(1 + a*x)])) - (2*a^4)/(5*(1 +
 Sqrt[(1 - a*x)/(1 + a*x)])^5) + a^4/(1 + Sqrt[(1 - a*x)/(1 + a*x)])^4 - (4
*a^4)/(3*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^3) + a^4/(1 + Sqrt[(1 - a*x)/(1 +
a*x)])^2 - (3*a^4)/(8*(1 + Sqrt[(1 - a*x)/(1 + a*x)]))
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{-\mathrm{sech}^{-1}(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)}\,dx$$
$$= -\left((4a)\,\mathrm{Subst}\left(\int \frac{x\left(a+ax^2\right)^3}{(-1+x)^4(1+x)^6}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$
$$= -\left((4a)\,\mathrm{Subst}\left(\int \left(\frac{a^3}{8(-1+x)^4}+\frac{a^3}{8(-1+x)^3}+\frac{3a^3}{32(-1+x)^2}-\frac{a^3}{2(1+x)^6}+\frac{a^3}{(1+x)^5}-\frac{a^3}{(1+\right.\right.\right.$$
$$= -\frac{a^4}{6\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{a^4}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}-\frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{2a^4}{5\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{a^4}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.26

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}\left(2a^3x^3-2a^2x^2+3ax-3\right)(ax+1)^2+3}{15ax^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x^5),x]
```

```
[Out] -1/15*(3 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-3 + 3*a*x - 2*a^2*x^2 +
2*a^3*x^3))/(a*x^5)
```

fricas [A] time = 0.74, size = 60, normalized size = 0.26

$$-\frac{\left(2\,a^5x^5+a^3x^3-3\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+3}{15\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="frica
s")
```

```
[Out] -1/15*((2*a^5*x^5 + a^3*x^3 - 3*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/
(a*x)) + 3)/(a*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="giac"
)
```

```
[Out] integrate(1/(x^5*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x)
```

```
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x^5*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

mupad [B] time = 2.37, size = 75, normalized size = 0.32

$$-\frac{1}{5\,a\,x^5}-\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{a\,x^2}{15}-\frac{x}{5}-\frac{1}{5\,a}+\frac{a^2\,x^3}{15}+\frac{2\,a^3\,x^4}{15}+\frac{2\,a^4\,x^5}{15}\right)}{x^5\,\sqrt{\frac{1}{a\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] - 1/(5*a*x^5) - ((1/(a*x) - 1)^(1/2)*((a*x^2)/15 - x/5 - 1/(5*a) + (a^2*x^3
)/15 + (2*a^3*x^4)/15 + (2*a^4*x^5)/15))/(x^5*(1/(a*x) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{1}{ax^5\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**5,x)
```

```
[Out] a*Integral(1/(a*x**5*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**4), x)
```

3.86 $\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^6}\,dx$

Optimal. Leaf size=320

$$\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}+\frac{3a^5}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{3a^5}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{a^5}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}+\frac{19a^5}{12\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}-\frac{}{8\left(1-\right.}$$

```
[Out] -1/8*a^5*arctanh(((-a*x+1)/(a*x+1))^(1/2))-1/8*a^5/(1-((-a*x+1)/(a*x+1))^(1
/2))^4+1/4*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^3-3/8*a^5/(1-((-a*x+1)/(a*x+1))
^(1/2))^2+1/4*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))-1/3*a^5/(1+((-a*x+1)/(a*x+1)
)^(1/2))^6+a^5/(1+((-a*x+1)/(a*x+1))^(1/2))^5-13/8*a^5/(1+((-a*x+1)/(a*x+1)
)^(1/2))^4+19/12*a^5/(1+((-a*x+1)/(a*x+1))^(1/2))^3-a^5/(1+((-a*x+1)/(a*x+1
))^(1/2))^2+3/8*a^5/(1+((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A] time = 0.57, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6337, 1612, 207}

$$\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}+\frac{3a^5}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{3a^5}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{a^5}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}+\frac{19a^5}{12\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}-\frac{}{8\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x^6),x]
```

```
[Out] -a^5/(8*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^4) + a^5/(4*(1 - Sqrt[(1 - a*x)/(1
+ a*x)])^3) - (3*a^5)/(8*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) + a^5/(4*(1 - S
qrt[(1 - a*x)/(1 + a*x)])) - a^5/(3*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^6) + a^
5/(1 + Sqrt[(1 - a*x)/(1 + a*x)])^5 - (13*a^5)/(8*(1 + Sqrt[(1 - a*x)/(1 +
a*x)])^4) + (19*a^5)/(12*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^3) - a^5/(1 + Sqrt
[(1 - a*x)/(1 + a*x)])^2 + (3*a^5)/(8*(1 + Sqrt[(1 - a*x)/(1 + a*x)])) - (a
^5*ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]])/8
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^6}\,dx = \int \frac{1}{x^6\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)}\,dx$$

$$= (4a)\,\text{Subst}\left(\int \frac{x\left(a+ax^2\right)^4}{(-1+x)^5(1+x)^7}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$

$$= (4a)\,\text{Subst}\left(\int\left(\frac{a^4}{8(-1+x)^5}+\frac{3a^4}{16(-1+x)^4}+\frac{3a^4}{16(-1+x)^3}+\frac{a^4}{16(-1+x)^2}+\frac{a^4}{2(1+x)^7}-\frac{5a^4}{4(1+\ldots}\right.\right.$$

$$= -\frac{a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{3a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{a^5}{3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^6}+\ldots$$

$$= -\frac{a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{3a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{a^5}{3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^6}+\ldots$$

Mathematica [A] time = 0.14, size = 129, normalized size = 0.40

$$-\frac{-3a^6x^6\log(x)+3a^6x^6\log\left(ax\sqrt{\frac{1-ax}{ax+1}}+\sqrt{\frac{1-ax}{ax+1}}+1\right)+\sqrt{\frac{1-ax}{ax+1}}\left(3a^5x^5+3a^4x^4+2a^3x^3+2a^2x^2-8ax-8\right)+8}{48ax^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x^6),x]
```

```
[Out] -1/48*(8 + Sqrt[(1 - a*x)/(1 + a*x)]*(-8 - 8*a*x + 2*a^2*x^2 + 2*a^3*x^3 +
3*a^4*x^4 + 3*a^5*x^5) - 3*a^6*x^6*Log[x] + 3*a^6*x^6*Log[1 + Sqrt[(1 - a*x
)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(a*x^6)
```

fricas [A] time = 0.83, size = 148, normalized size = 0.46

$$-\frac{3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+2\left(3\,a^5x^5+2\,a^3x^3-8\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\ldots}}{96\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="frica
s")
```

```
[Out] -1/96*(3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1)
- 3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*(
3*a^5*x^5 + 2*a^3*x^3 - 8*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))
 + 16)/(a*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="giac"
)
```

```
[Out] integrate(1/(x^6*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x)
```

```
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6}\left(\sqrt{\frac{1}{ax}+1}\,\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x^6*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

mupad [B] time = 65.40, size = 2479, normalized size = 7.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] ((a^5*((1/(a*x) - 1)^(1/2) - 1i)^6*10240i)/((1/(a*x) + 1)^(1/2) - 1)^6 + (a
^5*((1/(a*x) - 1)^(1/2) - 1i)^8*20480i)/((1/(a*x) + 1)^(1/2) - 1)^8 + (a^5*
((1/(a*x) - 1)^(1/2) - 1i)^10*36864i)/((1/(a*x) + 1)^(1/2) - 1)^10 + (a^5*(
(1/(a*x) - 1)^(1/2) - 1i)^12*20480i)/((1/(a*x) + 1)^(1/2) - 1)^12 + (a^5*((
1/(a*x) - 1)^(1/2) - 1i)^14*10240i)/((1/(a*x) + 1)^(1/2) - 1)^14)/(15*((45*
((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (10*((1/(a*x) -
 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (120*((1/(a*x) - 1)^(1/2)
- 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (210*((1/(a*x) - 1)^(1/2) - 1i)^8)/(
(1/(a*x) + 1)^(1/2) - 1)^8 - (252*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x)
+ 1)^(1/2) - 1)^10 + (210*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/
2) - 1)^12 - (120*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^
14 + (45*((1/(a*x) - 1)^(1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) - 1)^16 - (10*
((1/(a*x) - 1)^(1/2) - 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 + ((1/(a*x) - 1
)^(1/2) - 1i)^20/((1/(a*x) + 1)^(1/2) - 1)^20 + 1)) - (a^5*atanh(((1/(a*x)
- 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))/4 - ((a^5*((1/(a*x) - 1)^(1/2)
 - 1i)^6*2048i)/(3*((1/(a*x) + 1)^(1/2) - 1)^6) + (a^5*((1/(a*x) - 1)^(1/2)
 - 1i)^8*4096i)/(3*((1/(a*x) + 1)^(1/2) - 1)^8) + (a^5*((1/(a*x) - 1)^(1/2)
 - 1i)^10*12288i)/(5*((1/(a*x) + 1)^(1/2) - 1)^10) + (a^5*((1/(a*x) - 1)^(1
/2) - 1i)^12*4096i)/(3*((1/(a*x) + 1)^(1/2) - 1)^12) + (a^5*((1/(a*x) - 1)^
(1/2) - 1i)^14*2048i)/(3*((1/(a*x) + 1)^(1/2) - 1)^14))/((45*((1/(a*x) - 1)
^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (10*((1/(a*x) - 1)^(1/2) - 1i
)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (120*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(
a*x) + 1)^(1/2) - 1)^6 + (210*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^
(1/2) - 1)^8 - (252*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1
)^10 + (210*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (
120*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + (45*((1/(
a*x) - 1)^(1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) - 1)^16 - (10*((1/(a*x) - 1)
```

```
^(1/2) - 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 + ((1/(a*x) - 1)^(1/2) - 1i)^
20/((1/(a*x) + 1)^(1/2) - 1)^20 + 1) - ((311*a^5*((1/(a*x) - 1)^(1/2) - 1i)
^5)/(4*((1/(a*x) + 1)^(1/2) - 1)^5) - (175*a^5*((1/(a*x) - 1)^(1/2) - 1i)^3
)/(12*((1/(a*x) + 1)^(1/2) - 1)^3) + (8361*a^5*((1/(a*x) - 1)^(1/2) - 1i)^7
)/(4*((1/(a*x) + 1)^(1/2) - 1)^7) + (42259*a^5*((1/(a*x) - 1)^(1/2) - 1i)^9
)/(6*((1/(a*x) + 1)^(1/2) - 1)^9) + (25295*a^5*((1/(a*x) - 1)^(1/2) - 1i)^1
1)/(2*((1/(a*x) + 1)^(1/2) - 1)^11) + (25295*a^5*((1/(a*x) - 1)^(1/2) - 1i)
^13)/(2*((1/(a*x) + 1)^(1/2) - 1)^13) + (42259*a^5*((1/(a*x) - 1)^(1/2) - 1
i)^15)/(6*((1/(a*x) + 1)^(1/2) - 1)^15) + (8361*a^5*((1/(a*x) - 1)^(1/2) -
1i)^17)/(4*((1/(a*x) + 1)^(1/2) - 1)^17) + (311*a^5*((1/(a*x) - 1)^(1/2) -
1i)^19)/(4*((1/(a*x) + 1)^(1/2) - 1)^19) - (175*a^5*((1/(a*x) - 1)^(1/2) -
1i)^21)/(12*((1/(a*x) + 1)^(1/2) - 1)^21) + (5*a^5*((1/(a*x) - 1)^(1/2) - 1
i)^23)/(4*((1/(a*x) + 1)^(1/2) - 1)^23) + (5*a^5*((1/(a*x) - 1)^(1/2) - 1i)
)/(4*((1/(a*x) + 1)^(1/2) - 1)))/((66*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*
x) + 1)^(1/2) - 1)^4 - (12*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/
2) - 1)^2 - (220*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6
+ (495*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (792*((1
/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + (924*((1/(a*x) -
 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (792*((1/(a*x) - 1)^(1/2
) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + (495*((1/(a*x) - 1)^(1/2) - 1i)^
16)/((1/(a*x) + 1)^(1/2) - 1)^16 - (220*((1/(a*x) - 1)^(1/2) - 1i)^18)/((1/
(a*x) + 1)^(1/2) - 1)^18 + (66*((1/(a*x) - 1)^(1/2) - 1i)^20)/((1/(a*x) + 1
)^(1/2) - 1)^20 - (12*((1/(a*x) - 1)^(1/2) - 1i)^22)/((1/(a*x) + 1)^(1/2) -
 1)^22 + ((1/(a*x) - 1)^(1/2) - 1i)^24/((1/(a*x) + 1)^(1/2) - 1)^24 + 1) -
((23*a^5*((1/(a*x) - 1)^(1/2) - 1i)^3)/(2*((1/(a*x) + 1)^(1/2) - 1)^3) + (3
33*a^5*((1/(a*x) - 1)^(1/2) - 1i)^5)/(2*((1/(a*x) + 1)^(1/2) - 1)^5) + (671
*a^5*((1/(a*x) - 1)^(1/2) - 1i)^7)/(2*((1/(a*x) + 1)^(1/2) - 1)^7) + (671*a
^5*((1/(a*x) - 1)^(1/2) - 1i)^9)/(2*((1/(a*x) + 1)^(1/2) - 1)^9) + (333*a^5
*((1/(a*x) - 1)^(1/2) - 1i)^11)/(2*((1/(a*x) + 1)^(1/2) - 1)^11) + (23*a^5*
((1/(a*x) - 1)^(1/2) - 1i)^13)/(2*((1/(a*x) + 1)^(1/2) - 1)^13) - (3*a^5*((
1/(a*x) - 1)^(1/2) - 1i)^15)/(2*((1/(a*x) + 1)^(1/2) - 1)^15) - (3*a^5*((1/
(a*x) - 1)^(1/2) - 1i))/(2*((1/(a*x) + 1)^(1/2) - 1)))/((28*((1/(a*x) - 1)^
(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (8*((1/(a*x) - 1)^(1/2) - 1i)^
2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (56*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x
) + 1)^(1/2) - 1)^6 + (70*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2
) - 1)^8 - (56*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10
+ (28*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (8*((1/
(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + ((1/(a*x) - 1)^(1
/2) - 1i)^16/((1/(a*x) + 1)^(1/2) - 1)^16 + 1) - 1/(6*a*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{1}{ax^6\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**6,x)
```

```
[Out] a*Integral(1/(a*x**6*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**5), x)
```

3.87 $\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^7}\,dx$

Optimal. Leaf size=353

$$-\frac{5a^6}{16\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{5a^6}{16\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{3a^6}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{a^6}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{5a^6}{12\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{11a^6}{6\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}+\cdots$$

```
[Out] -1/10*a^6/(1-((-a*x+1)/(a*x+1))^(1/2))^5+1/4*a^6/(1-((-a*x+1)/(a*x+1))^(1/2
))^4-5/12*a^6/(1-((-a*x+1)/(a*x+1))^(1/2))^3+3/8*a^6/(1-((-a*x+1)/(a*x+1))^
(1/2))^2-5/16*a^6/(1-((-a*x+1)/(a*x+1))^(1/2))-2/7*a^6/(1+((-a*x+1)/(a*x+1)
)^(1/2))^7+a^6/(1+((-a*x+1)/(a*x+1))^(1/2))^6-19/10*a^6/(1+((-a*x+1)/(a*x+1
))^(1/2))^5+9/4*a^6/(1+((-a*x+1)/(a*x+1))^(1/2))^4-11/6*a^6/(1+((-a*x+1)/(a
*x+1))^(1/2))^3+a^6/(1+((-a*x+1)/(a*x+1))^(1/2))^2-5/16*a^6/(1+((-a*x+1)/(a
*x+1))^(1/2))
```

Rubi [A] time = 0.60, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6337, 1612}

$$-\frac{5a^6}{16\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{5a^6}{16\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{3a^6}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{a^6}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{5a^6}{12\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{11a^6}{6\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x^7),x]
```

```
[Out] -a^6/(10*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^5) + a^6/(4*(1 - Sqrt[(1 - a*x)/(1
 + a*x)])^4) - (5*a^6)/(12*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^3) + (3*a^6)/(8*
(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) - (5*a^6)/(16*(1 - Sqrt[(1 - a*x)/(1 + a
*x)])) - (2*a^6)/(7*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^7) + a^6/(1 + Sqrt[(1 -
 a*x)/(1 + a*x)])^6 - (19*a^6)/(10*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^5) + (9*
a^6)/(4*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^4) - (11*a^6)/(6*(1 + Sqrt[(1 - a*x
)/(1 + a*x)])^3) + a^6/(1 + Sqrt[(1 - a*x)/(1 + a*x)])^2 - (5*a^6)/(16*(1 +
 Sqrt[(1 - a*x)/(1 + a*x)]))
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6337

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1*Sqrt[(1 - u)/(1 + u)])/u)^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^7} dx = \int \frac{1}{x^7\left(\frac{1}{ax} + \sqrt{\frac{1-ax}{1+ax}} + \frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)} dx$$

$$= -\left((4a)\,\text{Subst}\left(\int \frac{x\left(a+ax^2\right)^5}{(-1+x)^6(1+x)^8} dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$

$$= -\left((4a)\,\text{Subst}\left(\int \left(\frac{a^5}{8(-1+x)^6} + \frac{a^5}{4(-1+x)^5} + \frac{5a^5}{16(-1+x)^4} + \frac{3a^5}{16(-1+x)^3} + \frac{5a^5}{64(-1+x)^2} - \frac{}{2}\right.\right.\right.$$

$$= -\frac{a^6}{10\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5} + \frac{a^6}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4} - \frac{5a^6}{12\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3} + \frac{3a^6}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2} - \frac{5a^6}{16\left(1-\sqrt{\frac{1-ax}{1+ax}}\right.}$$

Mathematica [A] time = 0.10, size = 76, normalized size = 0.22

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}\left(8a^5x^5 - 8a^4x^4 + 12a^3x^3 - 12a^2x^2 + 15ax - 15\right)(ax+1)^2 + 15}{105ax^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x^7),x]
```

```
[Out] -1/105*(15 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-15 + 15*a*x - 12*a^2*x
^2 + 12*a^3*x^3 - 8*a^4*x^4 + 8*a^5*x^5))/(a*x^7)
```

fricas [A] time = 0.81, size = 69, normalized size = 0.20

$$-\frac{\left(8\,a^7x^7 + 4\,a^5x^5 + 3\,a^3x^3 - 15\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 15}{105\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="frica
s")
```

```
[Out] -1/105*((8*a^7*x^7 + 4*a^5*x^5 + 3*a^3*x^3 - 15*a*x)*sqrt((a*x + 1)/(a*x))*
sqrt(-(a*x - 1)/(a*x)) + 15)/(a*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1} + \frac{1}{ax}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="giac"
)
```

```
[Out] integrate(1/(x^7*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1}{ax} + \sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x)
```

```
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x^7*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

mupad [B] time = 2.59, size = 91, normalized size = 0.26

$$-\frac{1}{7\,a\,x^7}-\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{a\,x^2}{35}-\frac{x}{7}-\frac{1}{7\,a}+\frac{a^2\,x^3}{35}+\frac{4\,a^3\,x^4}{105}+\frac{4\,a^4\,x^5}{105}+\frac{8\,a^5\,x^6}{105}+\frac{8\,a^6\,x^7}{105}\right)}{x^7\,\sqrt{\frac{1}{a\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] - 1/(7*a*x^7) - ((1/(a*x) - 1)^(1/2)*((a*x^2)/35 - x/7 - 1/(7*a) + (a^2*x^3
)/35 + (4*a^3*x^4)/105 + (4*a^4*x^5)/105 + (8*a^5*x^6)/105 + (8*a^6*x^7)/10
5))/(x^7*(1/(a*x) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{1}{ax^7\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**7,x)
```

```
[Out] a*Integral(1/(a*x**7*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**6), x)
```

3.88 $\int \frac{e^{\text{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,(dx)^m\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};c^2x^2\right)}{cm}+\frac{(dx)^m\,{}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};c^2x^2\right)}{cm}$$

```
[Out] (d*x)^m*hypergeom([1, 1/2*m],[1+1/2*m],c^2*x^2)/c/m+(d*x)^m*hypergeom([1/2,
 1/2*m],[1+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c/m
```

Rubi [A] time = 0.27, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6341, 6677, 125, 364}

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,(dx)^m\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};c^2x^2\right)}{cm}+\frac{(dx)^m\,{}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};c^2x^2\right)}{cm}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcSech[c*x]*(d*x)^m)/(1 - c^2*x^2),x]
```

```
[Out] ((d*x)^m*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Hypergeometric2F1[1/2, m/2, (2
+ m)/2, c^2*x^2])/(c*m) + ((d*x)^m*Hypergeometric2F1[1, m/2, (2 + m)/2, c^2
*x^2])/(c*m)
```

Rule 125

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0] && GtQ[a, 0] && GtQ[c, 0
]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6341

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[((d*x)^(m - 1)*Sqrt[1/(1 + c*x)])/Sqrt[1 - c*x],
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rule 6677

```
Int[(u_)*((c_.)*((a_.) + (b_.)*(x_))^(n_))^(p_), x_Symbol] :> Dist[(c^IntPa
rt[p]*(c*(a + b*x)^n)^FracPart[p])/(a + b*x)^(n*FracPart[p]), Int[u*(a + b*
x)^(n*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && !IntegerQ[p] && !MatchQ[
u, x^(n1_.)*(v_.) /; EqQ[n, n1 + 1]]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2}\,dx = \frac{d\int \frac{(dx)^{-1+m}\sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}}\,dx}{c} + \frac{d\int \frac{(dx)^{-1+m}}{1-c^2x^2}\,dx}{c}$$
$$= \frac{(dx)^m\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};c^2x^2\right)}{cm} + \frac{\left(d\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{(dx)^{-1+m}}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}$$
$$= \frac{(dx)^m\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};c^2x^2\right)}{cm} + \frac{\left(d\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{(dx)^{-1+m}}{\sqrt{1-c^2x^2}}\,dx}{c}$$
$$= \frac{(dx)^m\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};c^2x^2\right)}{cm} + \frac{(dx)^m\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};c^2x^2\right)}{cm}$$

Mathematica [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^ArcSech[c*x]*(d*x)^m)/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(E^ArcSech[c*x]*(d*x)^m)/(1 - c^2*x^2), x]
```

fricas [F] time = 1.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(dx)^m\,cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}} + (dx)^m}{c^3x^3-cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x,
algorithm="fricas")
```

```
[Out] integral(-((d*x)^m*c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + (d*x)
^m)/(c^3*x^3 - c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(dx)^m\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x,
algorithm="giac")
```

```
[Out] integrate(-(d*x)^m*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2
 - 1), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)(dx)^m}{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x)
```

```
[Out] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-d^m \int \frac{\sqrt{cx+1}\,\sqrt{-cx+1}\,x^m}{c^3x^3-cx}\,dx - d^m \int \frac{x^m}{2\,(cx+1)}\,dx - d^m \int \frac{x^m}{2\,(cx-1)}\,dx + \frac{d^m x^m}{cm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x,
algorithm="maxima")
```

```
[Out] -d^m*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m/(c^3*x^3 - c*x), x) - d^m*i
ntegrate(1/2*x^m/(c*x + 1), x) - d^m*integrate(1/2*x^m/(c*x - 1), x) + d^m*
x^m/(c*m)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}+\frac{1}{c\,x}\right)\,{\left(d\,x\right)}^m}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))*(d*x)^m)/(c^2*x^2
 - 1),x)
```

```
[Out] -int((((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))*(d*x)^m)/(c^2*x^2
 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(dx)^m}{c^2x^3-x}\,dx + \int \frac{cx(dx)^m\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{c^2x^3-x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*(d*x)**m/(-c**2*x**2+1
),x)
```

```
[Out] -(Integral((d*x)**m/(c**2*x**3 - x), x) + Integral(c*x*(d*x)**m*sqrt(-1 + 1
/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**3 - x), x))/c
```

3.89 $\int \frac{e^{\text{sech}^{-1}(cx)} x^4}{1-c^2x^2} dx$

Optimal. Leaf size=88

$$-\frac{2\sqrt{1-cx}}{3c^5\sqrt{\frac{1}{cx+1}}}-\frac{x^2\sqrt{1-cx}}{3c^3\sqrt{\frac{1}{cx+1}}}-\frac{x^2}{2c^3}-\frac{\log\left(1-c^2x^2\right)}{2c^5}$$

```
[Out] -1/2*x^2/c^3-1/2*ln(-c^2*x^2+1)/c^5-2/3*(-c*x+1)^(1/2)/c^5/(1/(c*x+1))^(1/2
)-1/3*x^2*(-c*x+1)^(1/2)/c^3/(1/(c*x+1))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6341, 1956, 100, 12, 74, 266, 43}

$$-\frac{x^2\sqrt{1-cx}}{3c^3\sqrt{\frac{1}{cx+1}}}-\frac{x^2}{2c^3}-\frac{\log\left(1-c^2x^2\right)}{2c^5}-\frac{2\sqrt{1-cx}}{3c^5\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcSech[c*x]*x^4)/(1 - c^2*x^2),x]
```

```
[Out] -x^2/(2*c^3) - (2*Sqrt[1 - c*x])/(3*c^5*Sqrt[(1 + c*x)^(-1)]) - (x^2*Sqrt[1
 - c*x])/(3*c^3*Sqrt[(1 + c*x)^(-1)]) - Log[1 - c^2*x^2]/(2*c^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1956

```
Int[(x_)^(m_.)*((e_.)*((a_) + (b_.)*(x_)^(n_.))^(r_.))^(p_)*((f_.)*((c_) +
(d_.)*(x_)^(n_.))^(s_))^(q_), x_Symbol] :> Dist[((e*(a + b*x^n)^r)^p*(f*(c
+ d*x^n)^s)^q)/((a + b*x^n)^(p*r)*(c + d*x^n)^(q*s)), Int[x^m*(a + b*x^n)^(
p*r)*(c + d*x^n)^(q*s), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s
}, x]
```

Rule 6341

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[((d*x)^(m - 1)*Sqrt[1/(1 + c*x)])/Sqrt[1 - c*x],
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^4}{1-c^2x^2}\,dx &= \frac{\int \frac{x^3\sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}}\,dx}{c}+\frac{\int \frac{x^3}{1-c^2x^2}\,dx}{c}\\
&= \frac{\operatorname{Subst}\left(\int \frac{x}{1-c^2x}\,dx,x,x^2\right)}{2c}+\frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x^3}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}\\
&= -\frac{x^2\sqrt{1-cx}}{3c^3\sqrt{\frac{1}{1+cx}}}+\frac{\operatorname{Subst}\left(\int \left(-\frac{1}{c^2}-\frac{1}{c^2(-1+c^2x)}\right)dx,x,x^2\right)}{2c}-\frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{2x}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{3c^3}\\
&= -\frac{x^2}{2c^3}-\frac{x^2\sqrt{1-cx}}{3c^3\sqrt{\frac{1}{1+cx}}}-\frac{\log\left(1-c^2x^2\right)}{2c^5}+\frac{\left(2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{3c^3}\\
&= -\frac{x^2}{2c^3}-\frac{2\sqrt{1-cx}}{3c^5\sqrt{\frac{1}{1+cx}}}-\frac{x^2\sqrt{1-cx}}{3c^3\sqrt{\frac{1}{1+cx}}}-\frac{\log\left(1-c^2x^2\right)}{2c^5}
\end{aligned}
$$

Mathematica [A] time = 0.20, size = 69, normalized size = 0.78

$$
-\frac{3c^2x^2+3\log\left(1-c^2x^2\right)+2\sqrt{\frac{1-cx}{cx+1}}\left(c^3x^3+c^2x^2+2cx+2\right)}{6c^5}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcSech[c*x]*x^4)/(1 - c^2*x^2),x]
```

```
[Out] -1/6*(3*c^2*x^2 + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + c^2*x^2 + c^3*x^
3) + 3*Log[1 - c^2*x^2])/c^5
```

fricas [A] time = 1.26, size = 69, normalized size = 0.78

$$
-\frac{3\,c^2x^2+2\left(c^3x^3+2\,cx\right)\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}+3\,\log\left(c^2x^2-1\right)}{6\,c^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] -1/6*(3*c^2*x^2 + 2*(c^3*x^3 + 2*c*x)*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)
/(c*x)) + 3*log(c^2*x^2 - 1))/c^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^4\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-x^4*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1
), x)
```

maple [A] time = 0.10, size = 69, normalized size = 0.78

$$-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,\left(c^2x^2+2\right)}{3c^4}-\frac{x^2}{2c^3}-\frac{\ln\left(c^2x^2-1\right)}{2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x)
```

```
[Out] -1/3*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(c^2*x^2+2)/c^4-1/2*x^2/c^3
-1/2/c^5*ln(c^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\frac{1}{2}x^2}{c^3}-\frac{\log\left(cx+1\right)}{2\,c^5}-\frac{\log\left(cx-1\right)}{2\,c^5}-\int\frac{\sqrt{cx+1}\,\sqrt{-cx+1}\,x^3}{c^3x^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] -integrate(x, x)/c^3 - 1/2*log(c*x + 1)/c^5 - 1/2*log(c*x - 1)/c^5 - integr
ate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^3/(c^3*x^2 - c), x)
```

mupad [B] time = 2.60, size = 76, normalized size = 0.86

$$-\frac{\ln\left(c^2\,x^2-1\right)+c^2\,x^2}{2\,c^5}-x^3\,\sqrt{\frac{1}{c\,x}-1}\,\left(\frac{\sqrt{\frac{1}{c\,x}+1}}{3\,c^2}+\frac{2\,\sqrt{\frac{1}{c\,x}+1}}{3\,c^4\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 - 1
),x)
```

```
[Out] - (log(c^2*x^2 - 1) + c^2*x^2)/(2*c^5) - x^3*(1/(c*x) - 1)^(1/2)*((1/(c*x)
+ 1)^(1/2)/(3*c^2) + (2*(1/(c*x) + 1)^(1/2))/(3*c^4*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x**4/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

3.90 $\int \frac{e^{\text{sech}^{-1}(cx)}x^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{2c^4}+\frac{\tanh^{-1}(cx)}{c^4}-\frac{x\sqrt{1-cx}}{2c^3\sqrt{\frac{1}{cx+1}}}-\frac{x}{c^3}$$

```
[Out] -x/c^3+arctanh(c*x)/c^4-1/2*x*(-c*x+1)^(1/2)/c^3/(1/(c*x+1))^(1/2)+1/2*arcs
in(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^4
```

Rubi [A] time = 0.17, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6341, 1956, 90, 41, 216, 321, 206}

$$-\frac{x\sqrt{1-cx}}{2c^3\sqrt{\frac{1}{cx+1}}}-\frac{x}{c^3}+\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{2c^4}+\frac{\tanh^{-1}(cx)}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcSech[c*x]*x^3)/(1 - c^2*x^2),x]
```

```
[Out] -(x/c^3) - (x*Sqrt[1 - c*x])/(2*c^3*Sqrt[(1 + c*x)^(-1)]) + (Sqrt[(1 + c*x)
^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(2*c^4) + ArcTanh[c*x]/c^4
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1956

```
Int[(x_)^(m_.)*((e_.)*((a_) + (b_.)*(x_)^(n_.))^(r_.))^(p_)*((f_.)*((c_) +
(d_.)*(x_)^(n_.))^(s_))^(q_), x_Symbol] :> Dist[((e*(a + b*x^n)^r)^p*(f*(c
+ d*x^n)^s)^q)/((a + b*x^n)^(p*r)*(c + d*x^n)^(q*s)), Int[x^m*(a + b*x^n)^(
p*r)*(c + d*x^n)^(q*s), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s
}, x]
```

Rule 6341

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[((d*x)^(m - 1)*Sqrt[1/(1 + c*x)])/Sqrt[1 - c*x],
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^3}{1-c^2x^2}\,dx &= \frac{\int \frac{x^2\sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}}\,dx}{c}+\frac{\int \frac{x^2}{1-c^2x^2}\,dx}{c}\\
&= -\frac{x}{c^3}+\frac{\int \frac{1}{1-c^2x^2}\,dx}{c^3}+\frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x^2}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}\\
&= -\frac{x}{c^3}-\frac{x\sqrt{1-cx}}{2c^3\sqrt{\frac{1}{1+cx}}}+\frac{\tanh^{-1}(cx)}{c^4}+\frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{2c^3}\\
&= -\frac{x}{c^3}-\frac{x\sqrt{1-cx}}{2c^3\sqrt{\frac{1}{1+cx}}}+\frac{\tanh^{-1}(cx)}{c^4}+\frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}}\,dx}{2c^3}\\
&= -\frac{x}{c^3}-\frac{x\sqrt{1-cx}}{2c^3\sqrt{\frac{1}{1+cx}}}+\frac{\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\sin^{-1}(cx)}{2c^4}+\frac{\tanh^{-1}(cx)}{c^4}
\end{aligned}
$$

Mathematica [C] time = 0.16, size = 110, normalized size = 1.47

$$
-\frac{c^2x^2\sqrt{\frac{1-cx}{cx+1}}+2cx+cx\sqrt{\frac{1-cx}{cx+1}}+\log(1-cx)-\log(cx+1)-i\log\left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1)-2icx\right)}{2c^4}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcSech[c*x]*x^3)/(1 - c^2*x^2),x]
```

```
[Out] -1/2*(2*c*x + c*x*Sqrt[(1 - c*x)/(1 + c*x)] + c^2*x^2*Sqrt[(1 - c*x)/(1 + c
*x)] + Log[1 - c*x] - Log[1 + c*x] - I*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1
 + c*x)]*(1 + c*x)])/c^4
```

fricas [B] time = 1.57, size = 91, normalized size = 1.21

$$
-\frac{c^2x^2\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}+2\,cx+\arctan\left(\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}\right)-\log\left(cx+1\right)+\log\left(cx-1\right)}{2\,c^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] -1/2*(c^2*x^2*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + 2*c*x + arctan
(sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x))) - log(c*x + 1) + log(c*x - 1
))/c^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^3\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-x^3*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1
), x)
```

maple [C] time = 0.08, size = 117, normalized size = 1.56

$$-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(x\sqrt{-c^2x^2+1}\,\mathrm{csgn}(c)c-\arctan\left(\frac{\mathrm{csgn}(c)cx}{\sqrt{-c^2x^2+1}}\right)\right)\mathrm{csgn}(c)}{2c^3\sqrt{-c^2x^2+1}}-\frac{x}{c^3}+\frac{\ln(cx+1)}{2c^4}-\frac{\ln(cx-1)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x)
```

```
[Out] -1/2*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/c^3*(x*(-c^2*x^2+1)^(1/2)*c
sgn(c)*c-arctan(csgn(c)*c*x/(-c^2*x^2+1)^(1/2)))/(-c^2*x^2+1)^(1/2)*csgn(c)
-x/c^3+1/2/c^4*ln(c*x+1)-1/2/c^4*ln(c*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x}{c^3}+\frac{\log(cx+1)}{2\,c^4}-\frac{\log(cx-1)}{2\,c^4}-\int\frac{\sqrt{cx+1}\sqrt{-cx+1}\,x^2}{c^3x^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] -x/c^3 + 1/2*log(c*x + 1)/c^4 - 1/2*log(c*x - 1)/c^4 - integrate(sqrt(c*x +
 1)*sqrt(-c*x + 1)*x^2/(c^3*x^2 - c), x)
```

mupad [B] time = 8.08, size = 340, normalized size = 4.53

$$\frac{\mathrm{atanh}(c\,x)-c\,x}{c^4}-\frac{\ln\left(\frac{\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{c\,x}+1}-1}\right)1\mathrm{i}}{2\,c^4}-\frac{\frac{1\mathrm{i}}{32\,c^4}+\frac{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{16\,c^4\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^4 15\mathrm{i}}{32\,c^4\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^2}+\frac{2\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^6}}+\frac{\ln\left(\frac{2\,c\sqrt{\frac{c+\frac{1}{x}}{c}}-\frac{2}{x}+c\sqrt{-\frac{c-\frac{1}{x}}{c}}\,2\mathrm{i}}{2\,c+\frac{1}{x}-2\,c\sqrt{\frac{c+\frac{1}{x}}{c}}}\right)1\mathrm{i}}{2\,c^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 - 1
),x)
```

```
[Out] (atanh(c*x) - c*x)/c^4 - (log(((1/(c*x) - 1)^(1/2) - 1i)/((1/(c*x) + 1)^(1/
2) - 1))*1i)/(2*c^4) - (1i/(32*c^4) + (((1/(c*x) - 1)^(1/2) - 1i)^2*1i)/(16
*c^4*((1/(c*x) + 1)^(1/2) - 1)^2) - (((1/(c*x) - 1)^(1/2) - 1i)^4*15i)/(32*
c^4*((1/(c*x) + 1)^(1/2) - 1)^4))/(((1/(c*x) - 1)^(1/2) - 1i)^2/((1/(c*x) +
 1)^(1/2) - 1)^2 + (2*((1/(c*x) - 1)^(1/2) - 1i)^4)/((1/(c*x) + 1)^(1/2) -
1)^4 + ((1/(c*x) - 1)^(1/2) - 1i)^6/((1/(c*x) + 1)^(1/2) - 1)^6) + (log((c*
(-(c - 1/x)/c)^(1/2)*2i - 2/x + 2*c*((c + 1/x)/c)^(1/2))/(2*c + 1/x - 2*c*(
(c + 1/x)/c)^(1/2)))*1i)/(2*c^4) - (((1/(c*x) - 1)^(1/2) - 1i)^2*1i)/(32*c^
4*((1/(c*x) + 1)^(1/2) - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^2}{c^2x^2-1}\,dx + \int \frac{cx^3\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{c^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x**3/(-c**2*x**2+1),x)
```

```
[Out] -(Integral(x**2/(c**2*x**2 - 1), x) + Integral(c*x**3*sqrt(-1 + 1/(c*x))*sq
rt(1 + 1/(c*x))/(c**2*x**2 - 1), x))/c
```

3.91 $\int \frac{e^{\text{sech}^{-1}(cx)}x^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=45

$$-\frac{\sqrt{1-cx}}{c^3\sqrt{\frac{1}{cx+1}}}-\frac{\log\left(1-c^2x^2\right)}{2c^3}$$

```
[Out] -1/2*ln(-c^2*x^2+1)/c^3-(-c*x+1)^(1/2)/c^3/(1/(c*x+1))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6341, 1956, 74, 260}

$$-\frac{\log\left(1-c^2x^2\right)}{2c^3}-\frac{\sqrt{1-cx}}{c^3\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcSech[c*x]*x^2)/(1 - c^2*x^2),x]
```

```
[Out] -(Sqrt[1 - c*x]/(c^3*Sqrt[(1 + c*x)^(-1)])) - Log[1 - c^2*x^2]/(2*c^3)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1956

```
Int[(x_)^(m_.)*((e_.)*((a_) + (b_.)*(x_)^(n_.))^(r_.))^(p_)*((f_.)*((c_) +
(d_.)*(x_)^(n_.))^(s_))^(q_), x_Symbol] :> Dist[((e*(a + b*x^n)^r)^p*(f*(c
+ d*x^n)^s)^q)/((a + b*x^n)^(p*r)*(c + d*x^n)^(q*s)), Int[x^m*(a + b*x^n)^(
p*r)*(c + d*x^n)^(q*s), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s
}, x]
```

Rule 6341

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[((d*x)^(m - 1)*Sqrt[1/(1 + c*x)])/Sqrt[1 - c*x],
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^2}{1-c^2x^2}\,dx = \frac{\int \frac{x\sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}}\,dx}{c} + \frac{\int \frac{x}{1-c^2x^2}\,dx}{c}$$
$$= -\frac{\log\left(1-c^2x^2\right)}{2c^3} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}$$
$$= -\frac{\sqrt{1-cx}}{c^3\sqrt{\frac{1}{1+cx}}} - \frac{\log\left(1-c^2x^2\right)}{2c^3}$$

Mathematica [A] time = 0.09, size = 44, normalized size = 0.98

$$-\frac{\log\left(1-c^2x^2\right)+2\sqrt{\frac{1-cx}{cx+1}}\,(cx+1)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcSech[c*x]*x^2)/(1 - c^2*x^2),x]
```

```
[Out] -1/2*(2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + Log[1 - c^2*x^2])/c^3
```

fricas [A] time = 0.50, size = 49, normalized size = 1.09

$$-\frac{2\,cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}+\log\left(c^2x^2-1\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] -1/2*(2*c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + log(c^2*x^2 - 1)
)/c^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-x^2*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1
), x)
```

maple [A] time = 0.07, size = 52, normalized size = 1.16

$$-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}}{c^2}-\frac{\ln\left(c^2x^2-1\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x)
```

```
[Out] -(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/c^2-1/2/c^3*ln(c^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(cx+1\right)}{2\,c^3}-\frac{\log\left(cx-1\right)}{2\,c^3}-\int\frac{\sqrt{cx+1}\,\sqrt{-cx+1}\,x}{c^3x^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] -1/2*log(c*x + 1)/c^3 - 1/2*log(c*x - 1)/c^3 - integrate(sqrt(c*x + 1)*sqrt
(-c*x + 1)*x/(c^3*x^2 - c), x)
```

mupad [B] time = 2.70, size = 44, normalized size = 0.98

$$-\frac{\ln\left(c^2\,x^2-1\right)}{2\,c^3}-\frac{x\,\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 - 1
),x)
```

```
[Out] - log(c^2*x^2 - 1)/(2*c^3) - (x*(1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2))/c^
2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{x}{c^2x^2-1}\,dx+\int\frac{cx^2\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{c^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x**2/(-c**2*x**2+1),x)
```

```
[Out] -(Integral(x/(c**2*x**2 - 1), x) + Integral(c*x**2*sqrt(-1 + 1/(c*x))*sqrt(
1 + 1/(c*x))/(c**2*x**2 - 1), x))/c
```

3.92 $\int \frac{e^{\text{sech}^{-1}(cx)}x}{1-c^2x^2}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\sin^{-1}(cx)}{c^2}+\frac{\tanh^{-1}(cx)}{c^2}$$

```
[Out] arctanh(c*x)/c^2+arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A] time = 0.11, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6341, 6677, 41, 216, 206}

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\sin^{-1}(cx)}{c^2}+\frac{\tanh^{-1}(cx)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcSech[c*x]*x)/(1 - c^2*x^2),x]
```

```
[Out] (Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/c^2 + ArcTanh[c*x]/c^2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6341

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[((d*x)^(m - 1)*Sqrt[1/(1 + c*x)])/Sqrt[1 - c*x],
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rule 6677

```
Int[(u_)*((c_.)*((a_.) + (b_.)*(x_))^(n_))^(p_), x_Symbol] :> Dist[(c^IntPa
rt[p]*(c*(a + b*x)^n)^FracPart[p])/(a + b*x)^(n*FracPart[p]), Int[u*(a + b*
x)^(n*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && !IntegerQ[p] && !MatchQ[
u, x^(n1_.)*(v_.) /; EqQ[n, n1 + 1]]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x}{1-c^2x^2}\,dx = \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}}\,dx}{c} + \frac{\int \frac{1}{1-c^2x^2}\,dx}{c}$$
$$= \frac{\tanh^{-1}(cx)}{c^2} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}$$
$$= \frac{\tanh^{-1}(cx)}{c^2} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}}\,dx}{c}$$
$$= \frac{\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\sin^{-1}(cx)}{c^2} + \frac{\tanh^{-1}(cx)}{c^2}$$

Mathematica [C] time = 0.05, size = 68, normalized size = 1.84

$$-\frac{\log(1-cx)}{2c^2} + \frac{\log(cx+1)}{2c^2} + \frac{i\log\left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1)-2icx\right)}{c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcSech[c*x]*x)/(1 - c^2*x^2),x]
```

```
[Out] -1/2*Log[1 - c*x]/c^2 + Log[1 + c*x]/(2*c^2) + (I*Log[(-2*I)*c*x + 2*Sqrt[(
1 - c*x)/(1 + c*x)]*(1 + c*x)])/c^2
```

fricas [B] time = 0.82, size = 53, normalized size = 1.43

$$-\frac{2\arctan\left(\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}\right) - \log(cx+1) + \log(cx-1)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x, algori
thm="fricas")
```

```
[Out] -1/2*(2*arctan(sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x))) - log(c*x + 1)
 + log(c*x - 1))/c^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x, algori
thm="giac")
```

```
[Out] integrate(-x*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1),
 x)
```

maple [C] time = 0.08, size = 92, normalized size = 2.49

$$\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,\arctan\left(\frac{\operatorname{csgn}(c)cx}{\sqrt{-(cx-1)(cx+1)}}\right)\operatorname{csgn}(c)}{\sqrt{-c^2x^2+1}\,c} + \frac{\ln(cx+1)}{2c^2} - \frac{\ln(cx-1)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x)
```

```
[Out] (-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*arctan(csgn(c)*c*x/(-(c*x-1)*(c*
x+1))^(1/2))/(-c^2*x^2+1)^(1/2)*csgn(c)/c+1/2/c^2*ln(c*x+1)-1/2/c^2*ln(c*x-
1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log\left(cx+1\right)}{2\,c^2}-\frac{\log\left(cx-1\right)}{2\,c^2}-\int\frac{\sqrt{cx+1}\,\sqrt{-cx+1}}{c^3x^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x, algori
thm="maxima")
```

```
[Out] 1/2*log(c*x + 1)/c^2 - 1/2*log(c*x - 1)/c^2 - integrate(sqrt(c*x + 1)*sqrt(
-c*x + 1)/(c^3*x^2 - c), x)
```

mupad [B] time = 3.57, size = 84, normalized size = 2.27

$$\frac{\mathrm{atanh}\left(c\,x\right)}{c^2}+\frac{\left(\ln\left(\frac{{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)}^2}+1\right)-\ln\left(\frac{\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{c\,x}+1}-1}\right)\right)\,1{}\mathrm{i}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 - 1),
x)
```

```
[Out] ((log(((1/(c*x) - 1)^(1/2) - 1i)^2/((1/(c*x) + 1)^(1/2) - 1)^2 + 1) - log((
(1/(c*x) - 1)^(1/2) - 1i)/((1/(c*x) + 1)^(1/2) - 1)))*1i)/c^2 + atanh(c*x)/
c^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{cx\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{c^2x^2-1}\,dx+\int\frac{1}{c^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x/(-c**2*x**2+1),x)
```

```
[Out] -(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**2 - 1), x) + I
ntegral(1/(c**2*x**2 - 1), x))/c
```

3.93 $\int \frac{e^{\text{sech}^{-1}(cx)}}{1-c^2x^2} dx$

Optimal. Leaf size=71

$$-\frac{\log\left(1-c^2x^2\right)}{2c}+\frac{\log(x)}{c}-\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right)}{c}$$

[Out] ln(x)/c-1/2*ln(-c^2*x^2+1)/c-arctanh((-c*x+1)^(1/2)*(c*x+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c

Rubi [A] time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6339, 1956, 92, 208, 266, 36, 29, 31}

$$-\frac{\log\left(1-c^2x^2\right)}{2c}+\frac{\log(x)}{c}-\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[E^ArcSech[c*x]/(1 - c^2*x^2),x]

[Out] -((Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/c) + Log[x]/c - Log[1 - c^2*x^2]/(2*c)

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1956

```
Int[(x_)^(m_.)*((e_.)*((a_) + (b_.)*(x_)^(n_.))^(r_.))^(p_)*((f_.)*((c_) +
(d_.)*(x_)^(n_.))^(s_))^(q_), x_Symbol] :> Dist[((e*(a + b*x^n)^r)^p*(f*(c
+ d*x^n)^s)^q)/((a + b*x^n)^(p*r)*(c + d*x^n)^(q*s)), Int[x^m*(a + b*x^n)^(
p*r)*(c + d*x^n)^(q*s), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s
}, x]
```

Rule 6339

```
Int[E^ArcSech[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Dist[1/(a*c),
 Int[Sqrt[1/(1 + c*x)]/(x*Sqrt[1 - c*x]), x], x] + Dist[1/c, Int[1/(x*(a +
b*x^2)), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2}\,dx &= \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{x\sqrt{1-cx}}\,dx}{c}+\frac{\int \frac{1}{x(1-c^2x^2)}\,dx}{c}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x(1-c^2x)}\,dx,x,x^2\right)}{2c}+\frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2c}+\frac{1}{2}c\operatorname{Subst}\left(\int \frac{1}{1-c^2x}\,dx,x,x^2\right)-\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{c-cx}\right.\\ &= -\frac{\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{1+cx}\right)}{c}+\frac{\log(x)}{c}-\frac{\log\left(1-c^2x^2\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 1.03

$$-\frac{\log\left(1-c^2x^2\right)}{2c}+\frac{2\log(x)}{c}-\frac{\log\left(cx\sqrt{\frac{1-cx}{cx+1}}+\sqrt{\frac{1-cx}{cx+1}}+1\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[c*x]/(1 - c^2*x^2),x]
```

```
[Out] (2*Log[x])/c - Log[1 - c^2*x^2]/(2*c) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] +
 c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/c
```

fricas [B] time = 1.91, size = 92, normalized size = 1.30

$$-\frac{\log\left(c^2x^2-1\right)+\log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}+1\right)-\log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}-1\right)-2\log(x)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x, algorith
m="fricas")
```

```
[Out] -1/2*(log(c^2*x^2 - 1) + log(c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x
)) + 1) - log(c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) - 1) - 2*log
(x))/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x, algorith
m="giac")
```

```
[Out] integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1), x
)
```

maple [A] time = 0.08, size = 87, normalized size = 1.23

$$-\frac{\sqrt{-\frac{cx-1}{cx}}\, x\sqrt{\frac{cx+1}{cx}}\, \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{\sqrt{-c^2x^2+1}}+\frac{\ln(x)}{c}-\frac{\ln(cx+1)}{2c}-\frac{\ln(cx-1)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x)
```

```
[Out] -(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))/(
-c^2*x^2+1)^(1/2)+ln(x)/c-1/2/c*ln(c*x+1)-1/2/c*ln(c*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log(x)}{c}-\frac{\log(cx+1)}{2c}-\frac{\log(cx-1)}{2c}-\int\frac{\sqrt{cx+1}\sqrt{-cx+1}}{c^3x^3-cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x, algorith
m="maxima")
```

```
[Out] integrate(1/x, x)/c - 1/2*log(c*x + 1)/c - 1/2*log(c*x - 1)/c - integrate(s
qrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^3 - c*x), x)
```

mupad [B] time = 2.92, size = 59, normalized size = 0.83

$$\frac{\ln(x)}{c}-\frac{4\,\operatorname{atanh}\left(\frac{\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{c\,x}+1}-1}\right)}{c}-\frac{\ln\left(3\,c^2\,x^2-3\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))/(c^2*x^2 - 1),x)
```

```
[Out] log(x)/c - (4*atanh(((1/(c*x) - 1)^(1/2) - 1i)/((1/(c*x) + 1)^(1/2) - 1)))/
c - log(3*c^2*x^2 - 3)/(2*c)
```

sympy [A] time = 16.43, size = 48, normalized size = 0.68

$$-\frac{\log\left(-1+\frac{1}{cx}\right)}{2c}-\frac{\log\left(\sqrt{1+\frac{1}{cx}}\right)}{c}-\frac{2\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{1+\frac{1}{cx}}}{2}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/(-c**2*x**2+1),x)
```

```
[Out] -log(-1 + 1/(c*x))/(2*c) - log(sqrt(1 + 1/(c*x)))/c - 2*acosh(sqrt(2)*sqrt(
1 + 1/(c*x))/2)/c
```

3.94 $\int \frac{e^{\text{sech}^{-1}(cx)}}{x(1-c^2x^2)}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{1-cx}}{cx\sqrt{\frac{1}{cx+1}}}-\frac{1}{cx}+\tanh^{-1}(cx)$$

```
[Out] -1/c/x+arctanh(c*x)-(-c*x+1)^(1/2)/c/x/(1/(c*x+1))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6341, 1956, 95, 325, 206}

$$-\frac{\sqrt{1-cx}}{cx\sqrt{\frac{1}{cx+1}}}-\frac{1}{cx}+\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[c*x]/(x*(1 - c^2*x^2)),x]
```

```
[Out] -(1/(c*x)) - Sqrt[1 - c*x]/(c*x*Sqrt[(1 + c*x)^(-1)]) + ArcTanh[c*x]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1956

```
Int[(x_)^(m_.)*((e_.)*((a_) + (b_.)*(x_)^(n_.))^(r_.))^(p_)*((f_.)*((c_) +
(d_.)*(x_)^(n_.))^(s_))^(q_), x_Symbol] :> Dist[((e*(a + b*x^n)^r)^p*(f*(c
+ d*x^n)^s)^q)/((a + b*x^n)^(p*r)*(c + d*x^n)^(q*s)), Int[x^m*(a + b*x^n)^(
p*r)*(c + d*x^n)^(q*s), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s
}, x]
```

Rule 6341

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[((d*x)^(m - 1)*Sqrt[1/(1 + c*x)])/Sqrt[1 - c*x],
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x\left(1-c^2x^2\right)}\,dx = \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{x^2\sqrt{1-cx}}\,dx}{c} + \frac{\int \frac{1}{x^2\left(1-c^2x^2\right)}\,dx}{c}$$

$$= -\frac{1}{cx} + c\int \frac{1}{1-c^2x^2}\,dx + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}$$

$$= -\frac{1}{cx} - \frac{\sqrt{1-cx}}{cx\sqrt{\frac{1}{1+cx}}} + \tanh^{-1}(cx)$$

Mathematica [A] time = 0.17, size = 59, normalized size = 1.40

$$-\sqrt{\frac{1-cx}{cx+1}}\left(\frac{1}{cx}+1\right) - \frac{1}{cx} - \frac{1}{2}\log(1-cx) + \frac{1}{2}\log(cx+1)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[c*x]/(x*(1 - c^2*x^2)),x]
```

```
[Out] -(1/(c*x)) - (1 + 1/(c*x))*Sqrt[(1 - c*x)/(1 + c*x)] - Log[1 - c*x]/2 + Log
[1 + c*x]/2
```

fricas [A] time = 0.92, size = 62, normalized size = 1.48

$$-\frac{2\,cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}} - cx\log\left(cx+1\right) + cx\log\left(cx-1\right) + 2}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x, algori
thm="fricas")
```

```
[Out] -1/2*(2*c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) - c*x*log(c*x + 1)
 + c*x*log(c*x - 1) + 2)/(c*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1} + \frac{1}{cx}}{\left(c^2x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x, algori
thm="giac")
```

```
[Out] integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((c^2*x^2 - 1)*x
), x)
```

maple [C] time = 0.08, size = 61, normalized size = 1.45

$$-\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,\mathrm{csgn}(c)^2 - \frac{1}{cx} + \frac{\ln\left(cx+1\right)}{2} - \frac{\ln\left(cx-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x)
```

```
[Out] -(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*csgn(c)^2-1/c/x+1/2*ln(c*x+1)-1/2
*ln(c*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\frac{1}{x}}{c} - \int \frac{\sqrt{cx+1}\sqrt{-cx+1}}{c^3x^4 - cx^2}\,dx + \frac{1}{2}\log(cx+1) - \frac{1}{2}\log(cx-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x, algori
thm="maxima")
```

```
[Out] integrate(x^(-2), x)/c - integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^4 -
c*x^2), x) + 1/2*log(c*x + 1) - 1/2*log(c*x - 1)
```

mupad [B] time = 2.53, size = 37, normalized size = 0.88

$$\operatorname{atanh}(c\,x) - \sqrt{\frac{1}{c\,x} - 1}\,\sqrt{\frac{1}{c\,x} + 1} - \frac{1}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))/(x*(c^2*x^2 - 1)),
x)
```

```
[Out] atanh(c*x) - (1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) - 1/(c*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^4-x^2}\,dx + \int \frac{1}{c^2x^4-x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/x/(-c**2*x**2+1),x)
```

```
[Out] -(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**4 - x**2), x)
+ Integral(1/(c**2*x**4 - x**2), x))/c
```

3.95 $\int \frac{e^{\text{sech}^{-1}(cx)}}{x^2(1-c^2x^2)}\,dx$

Optimal. Leaf size=108

$$-\frac{1}{2}c\log\left(1-c^2x^2\right)-\frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{cx+1}}}-\frac{1}{2cx^2}+c\log(x)-\frac{1}{2}c\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right)$$

```
[Out] -1/2/c/x^2+c*ln(x)-1/2*c*ln(-c^2*x^2+1)-1/2*(-c*x+1)^(1/2)/c/x^2/(1/(c*x+1)
)^(1/2)-1/2*c*arctanh((-c*x+1)^(1/2)*(c*x+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6341, 1956, 103, 12, 92, 208, 266, 44}

$$-\frac{1}{2}c\log\left(1-c^2x^2\right)-\frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{cx+1}}}-\frac{1}{2cx^2}+c\log(x)-\frac{1}{2}c\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[c*x]/(x^2*(1 - c^2*x^2)),x]
```

```
[Out] -1/(2*c*x^2) - Sqrt[1 - c*x]/(2*c*x^2*Sqrt[(1 + c*x)^(-1)]) - (c*Sqrt[(1 +
c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/2 + c*Log[x]
 - (c*Log[1 - c^2*x^2])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1956

```
Int[(x_)^(m_.)*((e_.)*((a_) + (b_.)*(x_)^(n_.))^(r_.))^(p_)*((f_.)*((c_) +
(d_.)*(x_)^(n_.))^(s_))^(q_), x_Symbol] :> Dist[((e*(a + b*x^n)^r)^p*(f*(c
+ d*x^n)^s)^q)/((a + b*x^n)^(p*r)*(c + d*x^n)^(q*s)), Int[x^m*(a + b*x^n)^(
p*r)*(c + d*x^n)^(q*s), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s
}, x]
```

Rule 6341

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[((d*x)^(m - 1)*Sqrt[1/(1 + c*x)])/Sqrt[1 - c*x],
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^2\left(1-c^2x^2\right)}\,dx = \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{x^3\sqrt{1-cx}}\,dx}{c} + \frac{\int \frac{1}{x^3\left(1-c^2x^2\right)}\,dx}{c}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-c^2x\right)}\,dx, x, x^2\right)}{2c} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^3\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}$$

$$= -\frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}} + \frac{\operatorname{Subst}\left(\int \left(\frac{1}{x^2}+\frac{c^2}{x}-\frac{c^4}{-1+c^2x}\right)dx, x, x^2\right)}{2c} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{c^2}{x\sqrt{1-cx}\sqrt{1+cx}}\,d}{2c}$$

$$= -\frac{1}{2cx^2} - \frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}} + c\log(x) - \frac{1}{2}c\log\left(1-c^2x^2\right) + \frac{1}{2}\left(c\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}}$$

$$= -\frac{1}{2cx^2} - \frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}} + c\log(x) - \frac{1}{2}c\log\left(1-c^2x^2\right) - \frac{1}{2}\left(c^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{}{c}\right.$$

$$= -\frac{1}{2cx^2} - \frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}} - \frac{1}{2}c\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{1+cx}\right) + c\log(x) - \frac{1}{2}c\log$$

Mathematica [A] time = 0.16, size = 108, normalized size = 1.00

$$\frac{1}{2}\left(-c\log\left(1-c^2x^2\right) - \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{cx^2} - \frac{1}{cx^2} + 3c\log(x) - c\log\left(cx\sqrt{\frac{1-cx}{cx+1}} + \sqrt{\frac{1-cx}{cx+1}} + 1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[c*x]/(x^2*(1 - c^2*x^2)),x]
```

[Out] (-(1/(c*x^2)) - (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(c*x^2) + 3*c*Log[x] - c*Log[1 - c^2*x^2] - c*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/2

fricas [B] time = 0.85, size = 156, normalized size = 1.44

$$-\frac{2\,c^2x^2\log\left(c^2x^2-1\right)+c^2x^2\log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}+1\right)-c^2x^2\log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}-1\right)-4\,c^2x^2\log(x)+2}{4\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x, algorithm="fricas")

[Out] -1/4*(2*c^2*x^2*log(c^2*x^2 - 1) + c^2*x^2*log(c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + 1) - c^2*x^2*log(c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) - 1) - 4*c^2*x^2*log(x) + 2*c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + 2)/(c*x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}}{\left(c^2x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x, algorithm="giac")

[Out] integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((c^2*x^2 - 1)*x^2), x)

maple [A] time = 0.09, size = 111, normalized size = 1.03

$$-\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2x^2+\sqrt{-c^2x^2+1}\right)}{2x\sqrt{-c^2x^2+1}}-\frac{1}{2c\,x^2}+c\ln(x)-\frac{c\ln(cx+1)}{2}-\frac{c\ln(cx-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x)

[Out] -1/2*(-(c*x-1)/c/x)^(1/2)/x*((c*x+1)/c/x)^(1/2)*(arctanh(1/(-c^2*x^2+1)^(1/2))*c^2*x^2+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-1/2/c/x^2+c*ln(x)-1/2*c*ln(c*x+1)-1/2*c*ln(c*x-1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c\int\frac{1}{x}\,dx-\frac{1}{2}\,c\log\left(cx+1\right)-\frac{1}{2}\,c\log\left(cx-1\right)+\frac{-\frac{1}{2\,x^2}}{c}-\int\frac{\sqrt{cx+1}\sqrt{-cx+1}}{c^3x^5-cx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x, algorithm="maxima")

[Out] c*integrate(1/x, x) - 1/2*c*log(c*x + 1) - 1/2*c*log(c*x - 1) + integrate(x^(-3), x)/c - integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^5 - c*x^3), x)

mupad [B] time = 14.53, size = 331, normalized size = 3.06

$$c\ln(x)+\frac{\frac{2c\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{cx}+1}-1}+\frac{14c\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^3}+\frac{14c\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^5}+\frac{2c\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^7}}{1+\frac{6\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^4}-\frac{4\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^8}-\frac{4\left(\sqrt{\frac{1}{cx}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{cx}+1}-1\right)^2}}-\frac{c\ln\left(c^2x^2-1\right)}{2}-2c\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{cx}-1}-\mathrm{i}}{\sqrt{\frac{1}{cx}+1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))/(x^2*(c^2*x^2 - 1)
),x)
```

```
[Out] ((2*c*((1/(c*x) - 1)^(1/2) - 1i))/((1/(c*x) + 1)^(1/2) - 1) + (14*c*((1/(c*
x) - 1)^(1/2) - 1i)^3)/((1/(c*x) + 1)^(1/2) - 1)^3 + (14*c*((1/(c*x) - 1)^(
1/2) - 1i)^5)/((1/(c*x) + 1)^(1/2) - 1)^5 + (2*c*((1/(c*x) - 1)^(1/2) - 1i)
^7)/((1/(c*x) + 1)^(1/2) - 1)^7)/((6*((1/(c*x) - 1)^(1/2) - 1i)^4)/((1/(c*x
) + 1)^(1/2) - 1)^4 - (4*((1/(c*x) - 1)^(1/2) - 1i)^2)/((1/(c*x) + 1)^(1/2)
 - 1)^2 - (4*((1/(c*x) - 1)^(1/2) - 1i)^6)/((1/(c*x) + 1)^(1/2) - 1)^6 + ((
1/(c*x) - 1)^(1/2) - 1i)^8/((1/(c*x) + 1)^(1/2) - 1)^8 + 1) - 2*c*atanh(((1
/(c*x) - 1)^(1/2) - 1i)/((1/(c*x) + 1)^(1/2) - 1)) - (c*log(c^2*x^2 - 1))/2
 + c*log(x) - 1/(2*c*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^5-x^3}\,dx+\int \frac{1}{c^2x^5-x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/x**2/(-c**2*x**2+1),x)
```

```
[Out] -(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**5 - x**3), x)
+ Integral(1/(c**2*x**5 - x**3), x))/c
```

3.96 $\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^3(1-c^2x^2)}\,dx$

Optimal. Leaf size=85

$$c^2\tanh^{-1}(cx)-\frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{cx+1}}}-\frac{1}{3cx^3}-\frac{2c\sqrt{1-cx}}{3x\sqrt{\frac{1}{cx+1}}}-\frac{c}{x}$$

```
[Out] -1/3/c/x^3-c/x+c^2*arctanh(c*x)-1/3*(-c*x+1)^(1/2)/c/x^3/(1/(c*x+1))^(1/2)-
2/3*c*(-c*x+1)^(1/2)/x/(1/(c*x+1))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6341, 1956, 103, 12, 95, 325, 206}

$$c^2\tanh^{-1}(cx)-\frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{cx+1}}}-\frac{1}{3cx^3}-\frac{2c\sqrt{1-cx}}{3x\sqrt{\frac{1}{cx+1}}}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[c*x]/(x^3*(1 - c^2*x^2)),x]
```

```
[Out] -1/(3*c*x^3) - c/x - Sqrt[1 - c*x]/(3*c*x^3*Sqrt[(1 + c*x)^(-1)]) - (2*c*Sq
rt[1 - c*x])/(3*x*Sqrt[(1 + c*x)^(-1)]) + c^2*ArcTanh[c*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1956

```
Int[(x_)^(m_.)*((e_.)*((a_) + (b_.)*(x_)^(n_.))^(r_.))^(p_)*((f_.)*((c_) +
(d_.)*(x_)^(n_.))^(s_))^(q_), x_Symbol] :> Dist[((e*(a + b*x^n)^r)^p*(f*(c
+ d*x^n)^s)^q)/((a + b*x^n)^(p*r)*(c + d*x^n)^(q*s)), Int[x^m*(a + b*x^n)^(
p*r)*(c + d*x^n)^(q*s), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s
}, x]
```

Rule 6341

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[((d*x)^(m - 1)*Sqrt[1/(1 + c*x)])/Sqrt[1 - c*x],
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^3\left(1-c^2x^2\right)}\,dx = \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{x^4\sqrt{1-cx}}\,dx}{c}+\frac{\int \frac{1}{x^4\left(1-c^2x^2\right)}\,dx}{c}$$

$$= -\frac{1}{3cx^3}+c\int \frac{1}{x^2\left(1-c^2x^2\right)}\,dx+\frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^4\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}$$

$$= -\frac{1}{3cx^3}-\frac{c}{x}-\frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{1+cx}}}+c^3\int \frac{1}{1-c^2x^2}\,dx-\frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{2c^2}{x^2\sqrt{1-cx}\sqrt{1+cx}}\,dx}{3c}$$

$$= -\frac{1}{3cx^3}-\frac{c}{x}-\frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{1+cx}}}+c^2\tanh^{-1}(cx)+\frac{1}{3}\left(2c\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{1-cx}\sqrt{1+cx}}$$

$$= -\frac{1}{3cx^3}-\frac{c}{x}-\frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{1+cx}}}-\frac{2c\sqrt{1-cx}}{3x\sqrt{\frac{1}{1+cx}}}+c^2\tanh^{-1}(cx)$$

Mathematica [A] time = 0.26, size = 90, normalized size = 1.06

$$-\frac{3c^3x^3\log(1-cx)-3c^3x^3\log(cx+1)+6c^2x^2+2\sqrt{\frac{1-cx}{cx+1}}\left(2c^3x^3+2c^2x^2+cx+1\right)+2}{6cx^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[c*x]/(x^3*(1 - c^2*x^2)),x]
```

```
[Out] -1/6*(2 + 6*c^2*x^2 + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2*c^2*x^2 + 2*
c^3*x^3) + 3*c^3*x^3*Log[1 - c*x] - 3*c^3*x^3*Log[1 + c*x])/(c*x^3)
```

fricas [A] time = 0.86, size = 89, normalized size = 1.05

$$\frac{3\,c^3x^3\log\left(cx+1\right)-3\,c^3x^3\log\left(cx-1\right)-6\,c^2x^2-2\left(2\,c^3x^3+cx\right)\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}-2}{6\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] 1/6*(3*c^3*x^3*log(c*x + 1) - 3*c^3*x^3*log(c*x - 1) - 6*c^2*x^2 - 2*(2*c^3
*x^3 + c*x)*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) - 2)/(c*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}}{\left(c^2x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((c^2*x^2 - 1)*x
^3), x)
```

maple [C] time = 0.10, size = 86, normalized size = 1.01

$$-\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\mathrm{csgn}(c)^2\left(2c^2x^2+1\right)}{3x^2}-\frac{1}{3c\,x^3}-\frac{c}{x}+\frac{c^2\ln\left(cx+1\right)}{2}-\frac{c^2\ln\left(cx-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x)
```

```
[Out] -1/3*(-(c*x-1)/c/x)^(1/2)/x^2*((c*x+1)/c/x)^(1/2)*csgn(c)^2*(2*c^2*x^2+1)-1
/3/c/x^3-c/x+1/2*c^2*ln(c*x+1)-1/2*c^2*ln(c*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,c^2\log\left(cx+1\right)-\frac{1}{2}\,c^2\log\left(cx-1\right)+c\int\frac{1}{x^2}\,dx+\frac{-\frac{1}{3\,x^3}}{c}-\int\frac{\sqrt{cx+1}\,\sqrt{-cx+1}}{c^3x^6-cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*c^2*log(c*x + 1) - 1/2*c^2*log(c*x - 1) + c*integrate(x^(-2), x) + inte
grate(x^(-4), x)/c - integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^6 - c*x^
4), x)
```

mupad [B] time = 2.51, size = 75, normalized size = 0.88

$$c^2\,\mathrm{atanh}\left(c\,x\right)-\frac{\left(\frac{\sqrt{\frac{1}{c\,x}+1}}{3}+\frac{2\,c^2\,x^2\,\sqrt{\frac{1}{c\,x}+1}}{3}\right)\sqrt{\frac{1}{c\,x}-1}}{x^2}-\frac{c^2\,x^2+\frac{1}{3}}{c\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))/(x^3*(c^2*x^2 - 1)
),x)
```

```
[Out] c^2*atanh(c*x) - (((1/(c*x) + 1)^(1/2)/3 + (2*c^2*x^2*(1/(c*x) + 1)^(1/2))/
3)*(1/(c*x) - 1)^(1/2))/x^2 - (c^2*x^2 + 1/3)/(c*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{c^2x^6-x^4}\,dx+\int \frac{1}{c^2x^6-x^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/x**3/(-c**2*x**2+1),x)
```

```
[Out] -(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**6 - x**4), x)
+ Integral(1/(c**2*x**6 - x**4), x))/c
```

3.97 $\int \frac{x\left(-1+ae^{\operatorname{sech}^{-1}(ax)}x\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=12

$$-\frac{xe^{\operatorname{sech}^{-1}(ax)}}{a}$$

[Out] -(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x/a

Rubi [B] time = 1.05, antiderivative size = 26, normalized size of antiderivative = 2.17, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6725, 260, 6341, 1956, 74}

$$-\frac{\sqrt{1-ax}}{a^2\sqrt{\frac{1}{ax+1}}}$$

Warning: Unable to verify antiderivative.

[In] Int[(x*(-1 + a*E^ArcSech[a*x]*x))/(1 - a^2*x^2),x]

[Out] -(Sqrt[1 - a*x]/(a^2*Sqrt[(1 + a*x)^(-1)]))

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1956

```
Int[(x_)^(m_.)*((e_.)*((a_) + (b_.)*(x_)^(n_.))^(r_.))^(p_)*((f_.)*((c_) +
(d_.)*(x_)^(n_.))^(s_))^(q_), x_Symbol] :> Dist[((e*(a + b*x^n)^r)^p*(f*(c
+ d*x^n)^s)^q)/((a + b*x^n)^(p*r)*(c + d*x^n)^(q*s)), Int[x^m*(a + b*x^n)^(
p*r)*(c + d*x^n)^(q*s), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s
}, x]
```

Rule 6341

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[((d*x)^(m - 1)*Sqrt[1/(1 + c*x)])/Sqrt[1 - c*x],
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x\left(-1+ae^{\text{sech}^{-1}(ax)}x\right)}{1-a^2x^2}\,dx = \int \left(\frac{x}{-1+a^2x^2} - \frac{ae^{\text{sech}^{-1}(ax)}x^2}{-1+a^2x^2}\right)dx$$
$$= -\left(a\int \frac{e^{\text{sech}^{-1}(ax)}x^2}{-1+a^2x^2}\,dx\right) + \int \frac{x}{-1+a^2x^2}\,dx$$
$$= \frac{\log\left(1-a^2x^2\right)}{2a^2} + \int \frac{x\sqrt{\frac{1}{1+ax}}}{\sqrt{1-ax}}\,dx - \int \frac{x}{-1+a^2x^2}\,dx$$
$$= \left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{x}{\sqrt{1-ax}\sqrt{1+ax}}\,dx$$
$$= -\frac{\sqrt{1-ax}}{a^2\sqrt{\frac{1}{1+ax}}}$$

Mathematica [B] time = 0.28, size = 28, normalized size = 2.33

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(-1 + a*E^ArcSech[a*x]*x))/(1 - a^2*x^2),x]
```

```
[Out] -((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/a^2)
```

fricas [A] time = 0.63, size = 35, normalized size = 2.92

$$-\frac{x\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),x
, algorithm="fricas")
```

```
[Out] -x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(ax\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)-1\right)x}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),x
, algorithm="giac")
```

```
[Out] integrate(-(a*x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)) - 1)*x/(a^2
*x^2 - 1), x)
```

maple [A] time = 0.05, size = 36, normalized size = 3.00

$$-\frac{x\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),x)
```

```
[Out] -1/a*x*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(ax\left(\sqrt{\frac{1}{ax}+1}\sqrt{\frac{1}{ax}-1}+\frac{1}{ax}\right)-1\right)x}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),x
, algorithm="maxima")
```

```
[Out] -integrate((a*x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)) - 1)*x/(a^2
*x^2 - 1), x)
```

mupad [B] time = 2.96, size = 76, normalized size = 6.33

$$\frac{\ln\left(\frac{1}{x}\right)}{a^2}-\frac{\ln\left(a+\frac{1}{x}\right)}{2\,a^2}-\frac{\ln\left(\frac{1}{x}-a\right)}{2\,a^2}+\frac{\ln\left(a^2\,x^2-1\right)}{2\,a^2}-\frac{x\,\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*x*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)) - 1))/(a^2
*x^2 - 1),x)
```

```
[Out] log(1/x)/a^2 - log(a + 1/x)/(2*a^2) - log(1/x - a)/(2*a^2) + log(a^2*x^2 -
1)/(2*a^2) - (x*(1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\int \frac{x^2\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-1+a*(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x)/(-a**2*x**2+
1),x)
```

```
[Out] -a*Integral(x**2*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/(a**2*x**2 - 1), x)
```

3.98 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=61

$$-\frac{\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(a+bx)}\right)}{2d}+\frac{\operatorname{sech}^{-1}(a+bx)^2}{2d}-\frac{\operatorname{sech}^{-1}(a+bx)\log\left(e^{2\operatorname{sech}^{-1}(a+bx)}+1\right)}{d}$$

```
[Out] 1/2*arcsech(b*x+a)^2/d-arcsech(b*x+a)*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(
1/(b*x+a)+1)^(1/2))^2)/d-1/2*polylog(2,-(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(
b*x+a)+1)^(1/2))^2)/d
```

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6319, 12, 6281, 5660, 3718, 2190, 2279, 2391}

$$-\frac{\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(a+bx)}\right)}{2d}+\frac{\operatorname{sech}^{-1}(a+bx)^2}{2d}-\frac{\operatorname{sech}^{-1}(a+bx)\log\left(e^{2\operatorname{sech}^{-1}(a+bx)}+1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ArcSech[a + b*x]^2/(2*d) - (ArcSech[a + b*x]*Log[1 + E^(2*ArcSech[a + b*x])
])/d - PolyLog[2, -E^(2*ArcSech[a + b*x])]/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6281

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rule 6319

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcSech[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx &= \frac{\text{Subst}\left(\int \frac{b\operatorname{sech}^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\operatorname{sech}^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{x}\,dx, x, \frac{1}{a+bx}\right)}{d}\\
&= -\frac{\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{1}{a+bx}\right)\right)}{d}\\
&= \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\frac{1}{a+bx}\right)\right)}{d}\\
&= \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} + \frac{\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx,\right.}{d}\\
&= \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\,\text{co}}\right.}{2d}\\
&= \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} - \frac{\text{Li}_2\left(-e^{2\cosh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 52, normalized size = 0.85

$$
\frac{\text{Li}_2\left(-e^{-2\operatorname{sech}^{-1}(a+bx)}\right) - \operatorname{sech}^{-1}(a+bx)\left(\operatorname{sech}^{-1}(a+bx) + 2\log\left(e^{-2\operatorname{sech}^{-1}(a+bx)}+1\right)\right)}{2d}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] (-(ArcSech[a + b*x]*(ArcSech[a + b*x] + 2*Log[1 + E^(-2*ArcSech[a + b*x])])
) + PolyLog[2, -E^(-2*ArcSech[a + b*x])])/(2*d)
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arsech}\,(bx+a)}{bdx+ad},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arcsech(b*x + a)/(b*d*x + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsech}\,(bx+a)}{dx+\frac{ad}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)/(d*x + a*d/b), x)
```

maple [A] time = 0.34, size = 104, normalized size = 1.70

$$\frac{\text{arcsech}\,(bx+a)^2}{2d}-\frac{\text{arcsech}\,(bx+a)\ln\left(1+\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)^2\right)}{d}-\frac{\text{polylog}\left(2,-\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\right.\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] 1/2*arcsech(b*x+a)^2/d-arcsech(b*x+a)*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(
1/(b*x+a)+1)^(1/2))^2)/d-1/2*polylog(2,-(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(
b*x+a)+1)^(1/2))^2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,\log\left(\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,bx+\sqrt{bx+a+1}\,\sqrt{-bx-a+1}\,a+bx+a\right)\log\left(bx+a\right)-3\,\log\left(bx+a\right)^2}{2\,d}-\frac{\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqr
t(-b*x - a + 1)*a + b*x + a)*log(b*x + a) - 3*log(b*x + a)^2)/d - 1/2*(log(
b*x + a + 1)*log(b*x + a) + dilog(-b*x - a))/d - 1/2*(log(b*x + a)*log(-b*x
 - a + 1) + dilog(b*x + a))/d + integrate((b^2*x + a*b)*log(b*x + a)/(b^2*d
*x^2 + 2*a*b*d*x + a^2*d + (b^2*d*x^2 + 2*a*b*d*x + a^2*d - d)*sqrt(b*x + a
 + 1)*sqrt(-b*x - a + 1) - d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\text{acosh}\left(\frac{1}{a+b\,x}\right)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))/(d*x + (a*d)/b),x)
```

```
[Out] int(acosh(1/(a + b*x))/(d*x + (a*d)/b), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{\operatorname{asech}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)/(a*d/b+d*x),x)
[Out] b*Integral(asech(a + b*x)/(a + b*x), x)/d
```

3.99 $\int x^3 \text{sech}^{-1}\left(a+bx^4\right)\,dx$

Optimal. Leaf size=57

$$\frac{\left(a+bx^4\right)\text{sech}^{-1}\left(a+bx^4\right)}{4b}-\frac{\tan^{-1}\left(\sqrt{\frac{-a-bx^4+1}{a+bx^4+1}}\right)}{2b}$$

```
[Out] 1/4*(b*x^4+a)*arcsech(b*x^4+a)/b-1/2*arctan(((-b*x^4-a+1)/(b*x^4+a+1))^(1/2
))/b
```

Rubi [A] time = 0.12, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6715, 6313, 1961, 12, 203}

$$\frac{\left(a+bx^4\right)\text{sech}^{-1}\left(a+bx^4\right)}{4b}-\frac{\tan^{-1}\left(\sqrt{\frac{-a-bx^4+1}{a+bx^4+1}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSech[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcSech[a + b*x^4])/(4*b) - ArcTan[Sqrt[(1 - a - b*x^4)/(1 + a
 + b*x^4)]]/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 6313

```
Int[ArcSech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcSech[c + d*
x])/d, x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; Free
Q[{c, d}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x^3\text{sech}^{-1}\left(a+bx^4\right)\,dx=\frac{1}{4}\text{Subst}\left(\int\text{sech}^{-1}(a+bx)\,dx,x,x^4\right)$$
$$=\frac{\left(a+bx^4\right)\text{sech}^{-1}\left(a+bx^4\right)}{4b}+\frac{1}{4}\text{Subst}\left(\int\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}}{1-a-bx}\,dx,x,x^4\right)$$
$$=\frac{\left(a+bx^4\right)\text{sech}^{-1}\left(a+bx^4\right)}{4b}-b\,\text{Subst}\left(\int\frac{1}{2b^2\left(1+x^2\right)}\,dx,x,\sqrt{\frac{1-a-bx^4}{1+a+bx^4}}\right)$$
$$=\frac{\left(a+bx^4\right)\text{sech}^{-1}\left(a+bx^4\right)}{4b}-\frac{\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\frac{1-a-bx^4}{1+a+bx^4}}\right)}{2b}$$
$$=\frac{\left(a+bx^4\right)\text{sech}^{-1}\left(a+bx^4\right)}{4b}-\frac{\tan^{-1}\left(\sqrt{\frac{1-a-bx^4}{1+a+bx^4}}\right)}{2b}$$

Mathematica [A] time = 0.21, size = 84, normalized size = 1.47

$$\frac{\frac{\sqrt{1-\left(a+bx^4\right)^2}\sin^{-1}\left(a+bx^4\right)}{\sqrt{-\frac{a+bx^4-1}{a+bx^4+1}}\left(a+bx^4+1\right)}+\left(a+bx^4\right)\text{sech}^{-1}\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSech[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcSech[a + b*x^4] + (Sqrt[1 - (a + b*x^4)^2]*ArcSin[a + b*x^4
])/(Sqrt[-((-1 + a + b*x^4)/(1 + a + b*x^4))]*(1 + a + b*x^4)))/(4*b)
```

fricas [B] time = 1.75, size = 283, normalized size = 4.96

$$\frac{2\,bx^4\log\left(\frac{\left(bx^4+a\right)\sqrt{-\frac{b^2x^8+2\,abx^4+a^2-1}{b^2x^8+2\,abx^4+a^2}}+1}{bx^4+a}\right)+a\log\left(\frac{\left(bx^4+a\right)\sqrt{-\frac{b^2x^8+2\,abx^4+a^2-1}{b^2x^8+2\,abx^4+a^2}}+1}{x^4}\right)-a\log\left(\frac{\left(bx^4+a\right)\sqrt{-\frac{b^2x^8+2\,abx^4+a^2-1}{b^2x^8+2\,abx^4+a^2}}-1}{x^4}\right)-2}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b*x^4*log(((b*x^4 + a)*sqrt(-(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/(b^2*x^
8 + 2*a*b*x^4 + a^2)) + 1)/(b*x^4 + a)) + a*log(((b*x^4 + a)*sqrt(-(b^2*x^8
 + 2*a*b*x^4 + a^2 - 1)/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 1)/x^4) - a*log(((b*
x^4 + a)*sqrt(-(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/(b^2*x^8 + 2*a*b*x^4 + a^2))
 - 1)/x^4) - 2*arctan((b^2*x^8 + 2*a*b*x^4 + a^2)*sqrt(-(b^2*x^8 + 2*a*b*x^
4 + a^2 - 1)/(b^2*x^8 + 2*a*b*x^4 + a^2))/(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)))
/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,\text{arsech}\left(bx^4+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsech(b*x^4 + a), x)
```

maple [A] time = 0.10, size = 62, normalized size = 1.09

$$\frac{\operatorname{arcsech}\left(b\,x^{4}+a\right)x^{4}}{4}+\frac{\operatorname{arcsech}\left(b\,x^{4}+a\right)a}{4b}-\frac{\arctan\left(\sqrt{\frac{1}{b\,x^{4}+a}-1}\,\sqrt{\frac{1}{b\,x^{4}+a}+1}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsech(b*x^4+a),x)
```

```
[Out] 1/4*arcsech(b*x^4+a)*x^4+1/4/b*arcsech(b*x^4+a)*a-1/4/b*arctan((1/(b*x^4+a)
-1)^(1/2)*(1/(b*x^4+a)+1)^(1/2))
```

maxima [A] time = 0.36, size = 38, normalized size = 0.67

$$\frac{\left(bx^{4}+a\right)\operatorname{arsech}\left(bx^{4}+a\right)-\arctan\left(\sqrt{\frac{1}{\left(bx^{4}+a\right)^{2}}-1}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*((b*x^4 + a)*arcsech(b*x^4 + a) - arctan(sqrt(1/(b*x^4 + a)^2 - 1)))/b
```

mupad [B] time = 2.99, size = 56, normalized size = 0.98

$$\frac{\operatorname{atan}\left(\frac{1}{\sqrt{\frac{1}{b\,x^{4}+a}-1}\,\sqrt{\frac{1}{b\,x^{4}+a}+1}}\right)}{4\,b}+\frac{\operatorname{acosh}\left(\frac{1}{b\,x^{4}+a}\right)\left(b\,x^{4}+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(1/(a + b*x^4)),x)
```

```
[Out] atan(1/((1/(a + b*x^4) - 1)^(1/2)*(1/(a + b*x^4) + 1)^(1/2)))/(4*b) + (acos
h(1/(a + b*x^4))*(a + b*x^4))/(4*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\operatorname{asech}\left(a+bx^{4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asech(b*x**4+a),x)
```

```
[Out] Integral(x**3*asech(a + b*x**4), x)
```

3.100 $\int x^{-1+n}\mathbf{sech}^{-1}(a+bx^n)\,dx$

Optimal. Leaf size=58

$$\frac{(a+bx^n)\,\mathrm{sech}^{-1}(a+bx^n)}{bn}-\frac{2\tan^{-1}\left(\sqrt{\frac{-a-bx^n+1}{a+bx^n+1}}\right)}{bn}$$

```
[Out] (a+b*x^n)*arcsech(a+b*x^n)/b/n-2*arctan(((1-a-b*x^n)/(1+a+b*x^n))^(1/2))/b/
n
```

Rubi [A] time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6715, 6313, 1961, 12, 203}

$$\frac{(a+bx^n)\,\mathrm{sech}^{-1}(a+bx^n)}{bn}-\frac{2\tan^{-1}\left(\sqrt{\frac{-a-bx^n+1}{a+bx^n+1}}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcSech[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcSech[a + b*x^n])/(b*n) - (2*ArcTan[Sqrt[(1 - a - b*x^n)/(1
+ a + b*x^n)]])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 6313

```
Int[ArcSech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcSech[c + d*
x])/d, x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; Free
Q[{c, d}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x^{-1+n}\text{sech}^{-1}(a+bx^n)\,dx=\frac{\text{Subst}\left(\int \text{sech}^{-1}(a+bx)\,dx,x,x^n\right)}{n}$$
$$=\frac{(a+bx^n)\,\text{sech}^{-1}(a+bx^n)}{bn}+\frac{\text{Subst}\left(\int \frac{\sqrt{\frac{1-a-bx}{1+a+bx}}}{1-a-bx}\,dx,x,x^n\right)}{n}$$
$$=\frac{(a+bx^n)\,\text{sech}^{-1}(a+bx^n)}{bn}-\frac{(4b)\,\text{Subst}\left(\int \frac{1}{2b^2(1+x^2)}\,dx,x,\sqrt{\frac{1-a-bx^n}{1+a+bx^n}}\right)}{n}$$
$$=\frac{(a+bx^n)\,\text{sech}^{-1}(a+bx^n)}{bn}-\frac{2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{\frac{1-a-bx^n}{1+a+bx^n}}\right)}{bn}$$
$$=\frac{(a+bx^n)\,\text{sech}^{-1}(a+bx^n)}{bn}-\frac{2\tan^{-1}\left(\sqrt{\frac{1-a-bx^n}{1+a+bx^n}}\right)}{bn}$$

Mathematica [**A**] time = 0.26, size = 84, normalized size = 1.45

$$\frac{\frac{\sqrt{1-(a+bx^n)^2}\,\sin^{-1}(a+bx^n)}{\sqrt{-\frac{a+bx^n-1}{a+bx^n+1}}\,(a+bx^n+1)}+(a+bx^n)\,\text{sech}^{-1}(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcSech[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcSech[a + b*x^n] + (Sqrt[1 - (a + b*x^n)^2]*ArcSin[a + b*x^n
])/(Sqrt[-((-1 + a + b*x^n)/(1 + a + b*x^n))]*(1 + a + b*x^n)))/(b*n)
```

fricas [**B**] time = 0.92, size = 385, normalized size = 6.64

$$2\left(b\cosh\left(n\log(x)\right)+b\sinh\left(n\log(x)\right)\right)\log\left(\frac{\sqrt{-\frac{2\,ab+\left(a^2+b^2-1\right)\cosh\left(n\log(x)\right)-\left(a^2-b^2-1\right)\sinh\left(n\log(x)\right)}{\cosh\left(n\log(x)\right)-\sinh\left(n\log(x)\right)}}+1}{b\cosh\left(n\log(x)\right)+b\sinh\left(n\log(x)\right)+a}\right)+a\log\left(\frac{\sqrt{-\frac{2\,ab+\left(a^2+b^2-1\right)}{\text{co}}}}{\text{cosh}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsech(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*cosh(n*log(x)) + b*sinh(n*log(x)))*log((sqrt(-(2*a*b + (a^2 + b^2
 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*sinh(n*log(x)))/(cosh(n*log(x)) - si
nh(n*log(x)))) + 1)/(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)) + a*log((sqr
t(-(2*a*b + (a^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*sinh(n*log(x))
)/(cosh(n*log(x)) - sinh(n*log(x)))) + 1)/(cosh(n*log(x)) + sinh(n*log(x)))
) - a*log((sqrt(-(2*a*b + (a^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*
sinh(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x)))) - 1)/(cosh(n*log(x)) + s
inh(n*log(x)))) - 2*arctan((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)*sqrt(-
(2*a*b + (a^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*sinh(n*log(x)))/(
cosh(n*log(x)) - sinh(n*log(x))))/(b^2*cosh(n*log(x))^2 + b^2*sinh(n*log(x)
)^2 + 2*a*b*cosh(n*log(x)) + a^2 + 2*(b^2*cosh(n*log(x)) + a*b)*sinh(n*log(
x)) - 1)))/(b*n)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{n-1}\,\text{arsech}\left(bx^n+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsech(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 1)*arcsech(b*x^n + a), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int x^{-1+n}\mathrm{arcsech}\left(a+b\,x^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*arcsech(a+b*x^n),x)
```

```
[Out] int(x^(-1+n)*arcsech(a+b*x^n),x)
```

maxima [A] time = 0.38, size = 40, normalized size = 0.69

$$\frac{\left(bx^{n}+a\right)\mathrm{arsech}\left(bx^{n}+a\right)-\arctan\left(\sqrt{\frac{1}{\left(bx^{n}+a\right)^{2}}-1}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsech(a+b*x^n),x, algorithm="maxima")
```

```
[Out] ((b*x^n + a)*arcsech(b*x^n + a) - arctan(sqrt(1/(b*x^n + a)^2 - 1)))/(b*n)
```

mupad [B] time = 2.35, size = 54, normalized size = 0.93

$$\frac{\mathrm{atan}\left(\frac{1}{\sqrt{\frac{1}{a+b\,x^{n}}-1}\,\sqrt{\frac{1}{a+b\,x^{n}}+1}}\right)+\mathrm{acosh}\left(\frac{1}{a+b\,x^{n}}\right)\,\left(a+b\,x^{n}\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*acosh(1/(a + b*x^n)),x)
```

```
[Out] (atan(1/((1/(a + b*x^n) - 1)^(1/2)*(1/(a + b*x^n) + 1)^(1/2))) + acosh(1/(a
 + b*x^n))*(a + b*x^n))/(b*n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*asech(a+b*x**n),x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```